

LIGHTNING PHENOMENOLOGY NOTES

Note 1

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THE RESPONSE OF A TRANSMISSION LINE ILLUMINATED
BY LIGHTNING-INDUCED ELECTROMAGNETIC FIELDS

Harold J. Price and Ashok K. Agrawal

Mission Research Corporation
1400 San Mateo S.E., Suite A
Albuquerque, NM 87108

ABSTRACT

This paper presents the theory and procedures used to estimate the voltage and currents induced on long transmission lines by cloud-to-cloud lightning. A model for the cloud-to-cloud lightning phenomena is presented, and the theory necessary to calculate the electromagnetic fields created by the lightning stroke is derived. The time-domain transmission-line equations are derived from Maxwell's equations in the presence of external electromagnetic fields. A time-domain formulation is more convenient, if in the future nonlinear effects are to be included. The results of sample calculations, using finite-difference techniques for the solution of the transmission-line equations, are presented.

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SECTION 1

INTRODUCTION

It is not unusual for electronic and electrical subsystems to be connected by transmission lines. Electromagnetic energy can couple to these transmission lines from lightning strokes, and this electromagnetic energy, after being transmitted, can cause damage. Thus, the determination of the response of a transmission line illuminated by lightning-induced electromagnetic fields is very important.

Electromagnetic fields due to a vertical lightning stroke have been calculated by Uman, *et al*, [2]. In this paper, we consider horizontal lightning strokes (cloud-to-cloud lightning), and unlike Uman, we formulate the problem in terms of a cartesian coordinate system, for ease in integration of the vector fields. We also consider the effects of static fields which occur prior to the discharge.

In Section 2, we model the cloud-to-cloud lightning phenomena, and derive the theory necessary to calculate the electromagnetic fields created by the lightning stroke. The static fields which occur prior to the lightning discharge are included, since they impose a non-zero initial condition on the transmission line.

The time-domain, transmission line equations are derived in Section 3, primarily to point out the little known fact that two formulations are possible, depending on how the line voltage is defined. One of these formulations is considerably more convenient to apply than the other. The relation of the static electric field, which is present prior to the

lightning discharge, to the transmission-line initial conditions is also presented.

Finally, in Section 4, we present the results of sample calculations, using finite-difference techniques to represent the transmission line equations.

The model used herein contains approximations; the most significant being that the ground is assumed perfectly conducting, and the transmission line is assumed to be above ground and lossless. The assumption of a perfectly conducting ground will tend to underestimate the induced effects, since the reflected electric field at the ground interferes with the electric field of the incident wave. Thus, the transmission-line drive is lower than that for a more realistic ground. The above-ground, lossless line tends to overestimate the induced effects, particularly as compared with a buried line. These two assumptions, then, tend to cancel.

SECTION 2

ELECTROMAGNETIC FIELDS FROM CLOUD-TO-CLOUD LIGHTNING

Figure 1 is an illustration of the physical situation being considered. We assume that cloud #1 is charged with respect to cloud #2, creating a static field in the surrounding vicinity. At some time ($t=0$), a traveling wave of current moves from cloud #1 to cloud #2 discharging the clouds.

The discharge current, and the associated diminishing charge sources on the clouds, create electromagnetic fields. To model the discharge, we assume that point charge sources exist in the clouds, and that the current travels along a horizontal, straight line.

The geometry of the problem is illustrated in Figure 2. A situation similar to this has previously been considered by Uman, *et al*, [2], for vertical lightning strokes. Here we consider horizontal lightning strokes, and unlike Uman, we formulate the problem in terms of a cartesian coordinate system, for ease in integration of the vector fields. In addition, to increase the speed of numerical computation, the derivative of electric field is used to compute the horizontal electric field close to the ground.

The total transient electromagnetic fields are obtained by integration of the fields from an elemental dipole over the length of the current discharge. The fields of the elemental dipole are obtained from the corresponding vector potential. Initially, for ease in formulation, the dipole fields are derived in the frequency domain. Conversion to the time domain is easily accomplished by simple Fourier transform relationships.

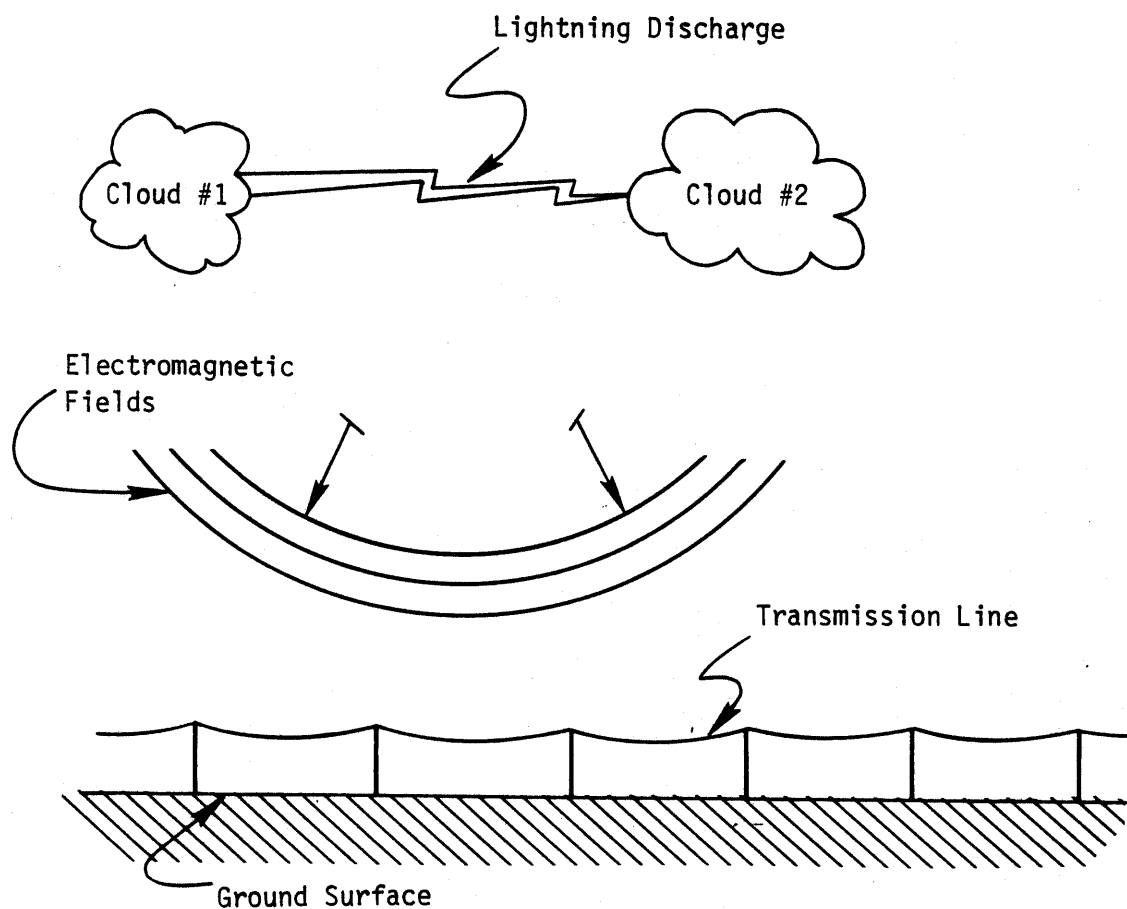


Figure 1. Configuration of Lightning and Transmission Line.

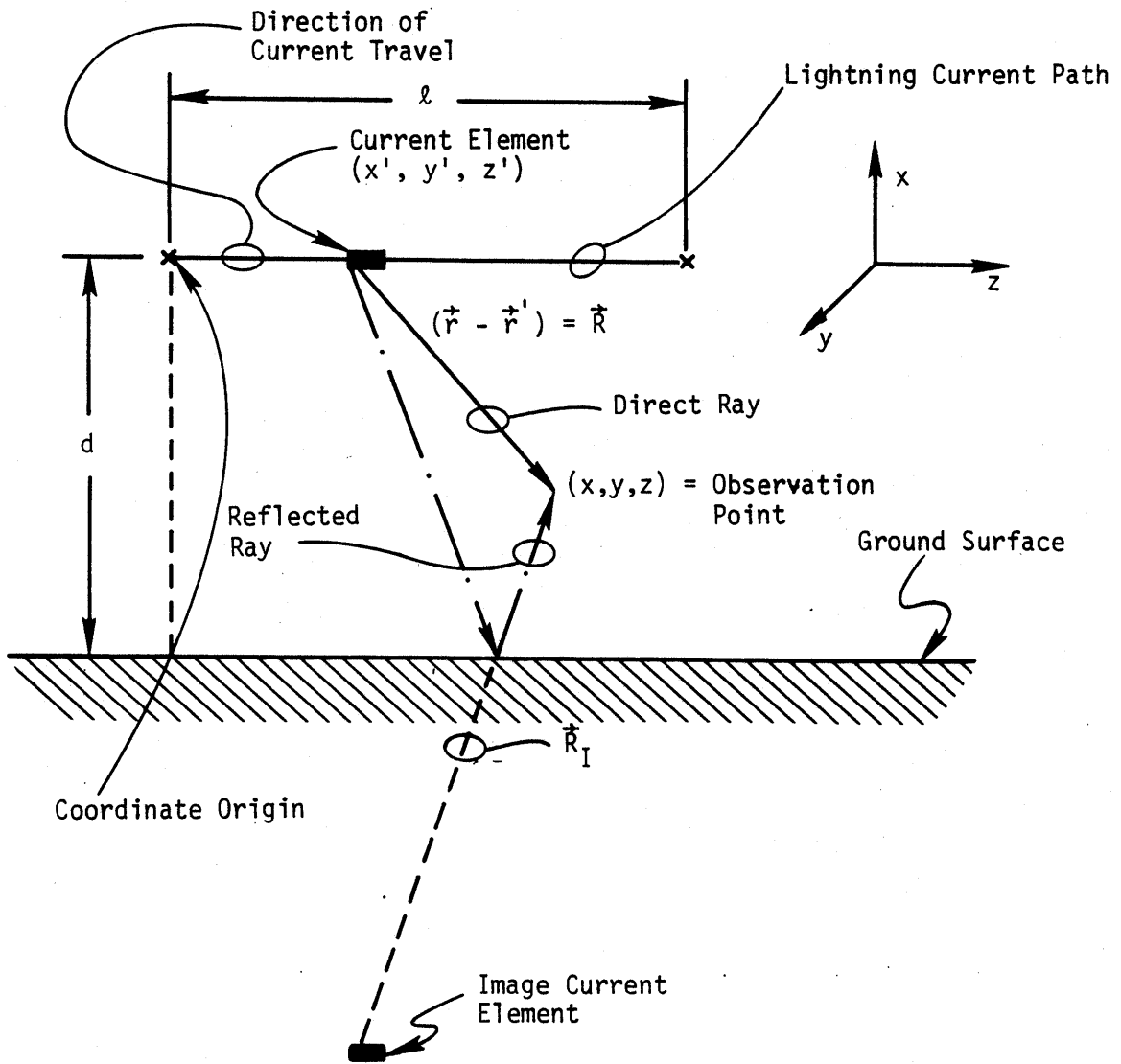


Figure 2. Geometry of Lightning Discharge.

The vector potential at point (x, y, z) generated by a current element at point (x', y', z') , and oriented in the positive z direction is [1]

$$\vec{A} = \vec{a}_z A_z = \vec{a}_z \frac{I dz'}{4\pi |\vec{r} - \vec{r}'|} e^{ik|\vec{r} - \vec{r}'|} ; \quad (1)$$

where \vec{a}_z is a unit vector in the z direction,
 A_z is the z -component of potential \vec{A} ,
 I is the value of the current element,
 dz' is the length of the current element,
 $k = \frac{\omega}{c}$ is the propagation constant,
 c is the speed of light,

ω = radian frequency

$|\vec{r} - \vec{r}'| = \left[(x-x')^2 + (y-y')^2 + (z-z')^2 \right]^{1/2}$ is the radial distance

from the observation point to the source point.

In Equation 1, we have assumed time variations of the form $\exp(-i\omega t)$.

In Section 3, we show that knowledge of the electric field is sufficient to determine the response of a transmission line. Thus, we derive only the electric field components of the lightning discharge.

The electric field is obtained from the vector potential by means of the relationship [1]

$$\vec{E} = \frac{1}{i\omega\epsilon} \nabla(\nabla \cdot \vec{A}) - i\omega\mu\vec{A} ; \quad (2)$$

where ϵ and μ are the dielectric constant and permeability of free space, respectively.

Since the vector potential of Equation 1 has only a single component, it follows that

$$\begin{aligned}
 dE_x &= \frac{1}{i\omega\epsilon} \frac{\partial^2 A_z}{\partial x \partial z} \\
 dE_y &= \frac{1}{i\omega\epsilon} \frac{\partial^2 A_z}{\partial y \partial z} \\
 dE_z &= \frac{1}{i\omega\epsilon} \frac{\partial^2 A_z}{\partial z^2} - i\omega\mu A_z,
 \end{aligned} \tag{3}$$

Insertion of the vector potential (Equation 1) into the electric field expressions (Equation 3) results in

$$\begin{aligned}
 dE_x &= \frac{1}{i\omega\epsilon} \frac{Idz'}{4\pi} e^{ikR} \left(-\frac{k^2 \bar{z}\bar{x}}{R^3} - \frac{3ik\bar{z}\bar{x}}{R^4} + \frac{3\bar{z}\bar{x}}{R^5} \right) \\
 dE_y &= \frac{1}{i\omega\epsilon} \frac{Idz'}{4\pi} e^{ikR} \left(-\frac{k^2 \bar{y}\bar{z}}{R^3} - \frac{3iky\bar{z}}{R^4} + \frac{3\bar{y}\bar{z}}{R^5} \right), \\
 dE_z &= \frac{1}{i\omega\epsilon} \frac{Idz'}{4\pi} e^{ikR} \left(\frac{ik}{R^2} - \frac{(k^2 \bar{z}^2 + 1)}{R^3} - \frac{3ik\bar{z}^2}{R^4} + \frac{3\bar{z}^2}{R^5} \right) \\
 &\quad - i\omega\mu \frac{Id\ell}{4\pi} \frac{e^{ikR}}{R}.
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 \text{where } R &= |\vec{r} - \vec{r}'| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}, \\
 \bar{x} &= (x-x'), \\
 \bar{y} &= (y-y'), \\
 \bar{z} &= (z-z').
 \end{aligned}$$

Equation 4 represents the fields from a current element located at (x', y', z') . We assume that the current travels from $z'=0$ to $z'=\ell$ at uniform speed v ; that is, in the time domain $i \equiv i(t-z'/v)$. Therefore, it follows that in the frequency domain

$$I \equiv I(z') = I(\omega) e^{i\gamma z'}; \quad (5)$$

where $I(\omega)$ is the Fourier transform of the current time waveform,
 $\gamma = \omega/v$.

In Equation 5 we have made use of the Fourier transform relationship

$$F\left[g\left(t-\frac{z'}{v}\right)\right] = G(\omega) e^{i\gamma z'}; \quad (6)$$

where $F[\cdot] \Rightarrow$ Fourier transform,

$G(\omega)$ is the Fourier transform of $g(t)$.

To convert Equation 5 to the time domain, we use Equation 6, together with the additional Fourier transform relationship

$$F\left[\frac{d}{dt} g(t)\right] = -i\omega G(\omega). \quad (7)$$

Thus,

$$\begin{aligned} dE_x(R, z', t) &= \frac{1}{4\pi\epsilon} \left\{ -\frac{\bar{x}\bar{z}}{C^2 R^3} \frac{\partial}{\partial t} (i(\tau)) - \frac{3\bar{z}\bar{x}}{CR^4} i(\tau) + \frac{3\bar{z}\bar{x}}{R^5} \int_0^t i(\tau) dt \right\} dz', \\ dE_y(R, z', t) &= \frac{1}{4\pi\epsilon} \left\{ -\frac{\bar{y}\bar{z}}{C^2 R^3} \frac{\partial}{\partial t} (i(\tau)) - \frac{3\bar{y}\bar{z}}{CR^4} i(\tau) + \frac{3\bar{y}\bar{z}}{R^5} \int_0^t i(\tau) dt \right\} dz', \\ dE_z(R, z', t) &= \frac{1}{4\pi\epsilon} \left\{ -\left(\frac{1}{CR^2} + \frac{3\bar{z}^2}{R^4} + \frac{1}{C^2 R}\right) i(\tau) - \frac{\bar{z}^2}{C^2 R^3} \frac{\partial}{\partial t} (i(\tau)) \right. \\ &\quad \left. + \left(\frac{1}{R^3} + \frac{3\bar{z}^2}{R^5}\right) \int_0^t i(\tau) dt \right\} dz'; \end{aligned} \quad (8)$$

$$\text{where } \tau = t - \frac{R}{C} - \frac{z'}{v}$$

If the current waveform in the discharge is known, either Equation 4 (in the frequency domain) or Equation 8 (in the time domain) can be integrated along the discharge (with respect to z') to obtain the fields. These fields, however, do not account for the ground reflection, or the static charge which is present prior to the discharge.

In order to account for the ground reflection, the ground is assumed to be a perfect conductor. Therefore, the ground reflection is entirely accounted for from the image current shown in Figure 2. The total field is obtained by adding the fields from the two currents.

The field which drives the transmission line is the horizontal electric field. Since the transmission line is close to the ground surface, we must determine the horizontal electric field in the vicinity of the ground. The horizontal electric fields from the direct and image currents oppose each other; indeed, they cancel at the surface.

The horizontal electric fields from the direct and image currents are nearly equal, so that the sum of the direct and image fields amounts to calculating the difference between two nearly equal numbers. This process can lead to large errors, unless the numerical integrations along the direct and image discharge are calculated to a large number of significant digits. In turn, this precise integration is quite time consuming.

Rather than sum the current element fields directly, a much better numerical procedure is to make use of the approximate relation

$$dE_{zT} \approx 2\Delta x \frac{\partial}{\partial x}(dE_z) ; \quad (9)$$

where dE_{zT} is the total field close to the ground surface at height Δx .

For small Δx (compared to the height of the lightning stroke) which is the case for the transmission line, Equation 9 has negligible error. Therefore, integration of Equation 9 along the discharge can be done with much less precision, resulting in a greatly reduced computation time; that is, it is not necessary to obtain the difference between two large, nearly equal numbers. This relation is, of course, valid in either the time or frequency domain.

In order to use Equation 9, it is necessary to determine $\frac{\partial}{\partial x}(dE_z)$. This derivative can be obtained in the frequency domain from Equation 4 and converted to the time domain by means of Equation 7. The result in the frequency domain is

$$\frac{\partial}{\partial x}(dE_z) = \frac{I(\omega)\epsilon^{\gamma z'}}{i\omega 4\pi\epsilon} \frac{e^{ikR_x}}{R^2} \left\{ ik^3 - \frac{2k^2}{R} - \frac{[ik^3(z-z')^2 + 3ik]}{R^2} + \frac{[6k^2(z-z')^2 + 3]}{R^3} + \frac{15ik(z-z')^2}{R^4} - \frac{15(z-z')^2}{R^5} \right\} \quad (10)$$

Here we have included the factor $e^{\gamma z'}$ to account for the time delay of the traveling current wave to reach point z' .

When Equation 10 is converted to the time domain, the result is

$$\begin{aligned} \frac{\partial}{\partial x}(dE_z) &= \frac{x}{4\pi\epsilon R^2} \left[-\frac{1}{C^3} \left\{ 1 - \frac{(z-z')^2}{R^2} \right\} \frac{\partial^2}{\partial t^2} i(\tau) - \frac{1}{C^2} \left\{ \frac{2}{R} - \frac{6(z-z')^2}{R^3} \right\} \right. \\ &\quad \left. \frac{\partial}{\partial t} i(\tau) + \frac{1}{C} \left\{ \frac{3}{R^2} + \frac{15(z-z')^2}{R^4} \right\} i(\tau) - \left\{ \frac{3}{R^3} - \frac{15(z-z')^2}{R^5} \right\} \right. \\ &\quad \left. \int_{-\infty}^t i(\tau) dt \right] dz' \quad (11) \end{aligned}$$

Equations 4, 8, 10, and 11 do not contain the proper static charge. That is, if one computes the electric field from these equations, it starts at zero and ends up with a constant value when the discharge is over. This results from the fact that the current flows from one point into another. Since Maxwell's equations conserve charge, a negative charge ends up on cloud #1 and a positive charge on cloud #2. The magnitude of these charges is equal to the time integral of the current.

To correct this situation, it is necessary to insert positive charge on cloud #1 and negative charge on cloud #2 prior to the discharge, and corresponding image charges at the source and terminations of the image current. These charges create a static field prior to the discharge. If the magnitude of these charges is equal to the integral of the current, the electric field will be zero following the discharge.

Figure 3 illustrates the static charges and their images. The horizontal and vertical static fields in the plane of the charges are

$$\begin{aligned}
 E_z &= \frac{q}{4\pi\epsilon} \left(\frac{\cos\theta_1}{R_1^2} - \frac{\cos\theta_3}{R_3^2} + \frac{\cos\theta_2}{R_2^2} - \frac{\cos\theta_4}{R_4^2} \right) \\
 &= \frac{q}{4\pi\epsilon} \left(\frac{z}{R_1^3} - \frac{z}{R_3^3} + \frac{\ell-z}{R_2^3} - \frac{\ell-z}{R_4^3} \right) \\
 E_x &= \frac{q}{4\pi\epsilon} \left(-\frac{\sin\theta_1}{R_1^2} - \frac{\sin\theta_3}{R_3^2} + \frac{\sin\theta_2}{R_2^2} + \frac{\sin\theta_4}{R_4^2} \right) \\
 &= \frac{q}{4\pi\epsilon} \left(-\frac{d-x}{R_1^3} - \frac{d+x}{R_3^3} + \frac{d-x}{R_2^3} + \frac{d+x}{R_4^3} \right);
 \end{aligned} \tag{12}$$

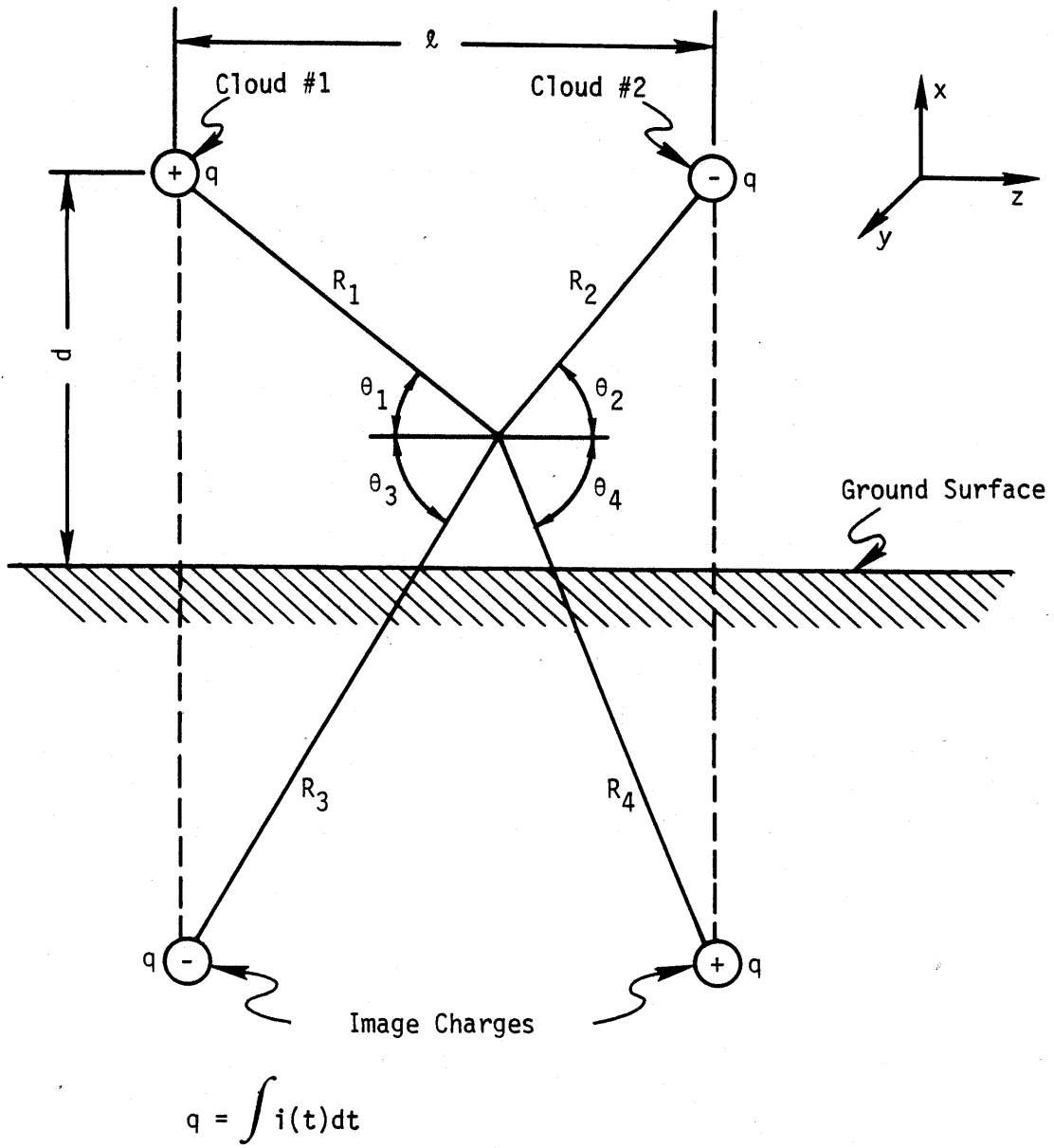


Figure 3. Configuration of Charges and Their Images.

$$\begin{aligned}\text{where } R_1^2 &= (d-x)^2 + z^2, \\ R_2^2 &= (d-x)^2 + (l-z)^2, \\ R_3^2 &= (d+x)^2 + z^2, \\ R_4^2 &= (d+x)^2 + (l-z)^2.\end{aligned}$$

We have limited the static fields to the plane of the charges, since in the next section we assume that the transmission line is in the plane of the charges. This situation will provide maximum coupling to the line.

The static fields of Equation 12 must be added to the corresponding dynamic fields obtained by integrating the differential field expressions, previously derived, over the length of the discharge. This completes the derivation of the driving electromagnetic fields on the transmission line.

SECTION 3

DERIVATION OF THE TRANSMISSION LINE EQUATIONS FROM MAXWELL'S EQUATIONS

The relationship of the transmission line equations to Maxwell's equations in the presence of external electromagnetic fields is contained in a note by Taylor, *et al* [3]. This formulation is in the frequency domain.

It is not pointed out that (in terms of sources) there are two equivalent formulations of the line equations. The source terms appear quite differently in the two formulations.

Since Reference 3 does not contain a convenient formulation, we derive the line equations in this section. Furthermore, a time domain formulation is more convenient, if at some future time nonlinear effects are included. Therefore, we derive the equations in the time domain.

The starting point in the derivation is the Maxwell equation

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (13)$$

Figure 4 shows a small section of transmission line. We integrate Equation 13 over the area included by the dotted line; that is,

$$\oint \nabla \times \vec{E} \cdot d\vec{A} = \oint \vec{E} \cdot d\vec{l} = - \frac{\partial}{\partial t} \oint \vec{B} \cdot d\vec{A} \quad (14)$$

Therefore,

$$\int_0^h E_x(z+\Delta z) dx + R\Delta z i(z) - \int_0^h E_x(z) dx \approx -\Delta z \frac{\partial}{\partial t} \int_0^h B_y(z) dx \quad (15)$$

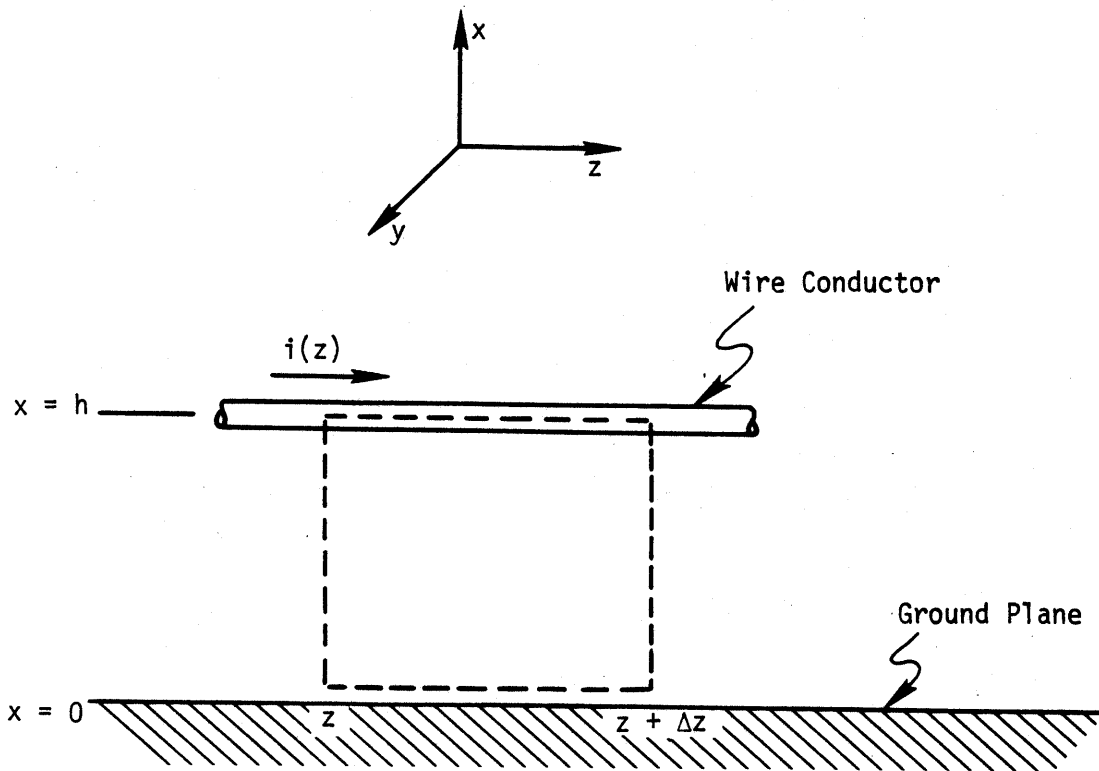


Figure 4. Small Section of Transmission Line.

Here, we have suppressed all but the z dependence of E_x and B_y . We have also assumed that the net series resistance of the transmission line and ground is R ohms per unit length.

Now divide Equation 15 by Δz and take the limit as $\Delta z \rightarrow 0$; the result is

$$\frac{\partial}{\partial z} \int_0^h E_x(z) dx + Ri(z) = - \frac{\partial}{\partial t} \int_0^h B_y(z) dx \quad (16)$$

The fields in Equation 16 are total fields; that is, they consist of the sum of an incident or impressed field and a scattered field. The scattered field results from the interaction of the incident field with the conductors.

It is convenient to separate the incident and scattered fields in Equation 16. If this is done, then

$$\frac{\partial}{\partial z} \int_0^h E_x^i dx + \frac{\partial}{\partial z} \int_0^h E_x^s dx + Ri(z) = - \frac{\partial}{\partial t} \int_0^h B_y^i dx - \frac{\partial}{\partial t} \int_0^h B_y^s dx \quad (17)$$

For convenience the explicit dependence of E_x and B_y on the coordinates is suppressed. The superscripts i and s refer to incident and scattered, respectively.

Now, it is assumed that the following expressions are valid:*

$$\int_0^h E_x^s dx = -v^s = - \frac{Q}{C} \quad (18a)$$

* These are approximately true for spacings small compared to the frequencies of interest.

and

$$\int_0^h B_y^s dx = - Li ; \quad (18b)$$

where v^s is the voltage associated with the scattered wave.

C is the line capacity per unit length.

L is the line inductance per unit length.

Therefore equation 17 can be written

$$\frac{\partial v^s}{\partial z} + Ri + L \frac{\partial i}{\partial t} = \frac{\partial}{\partial z} \int_0^h E_x^i dx + \frac{\partial}{\partial t} \int_0^h B_y^i dx \quad (19)$$

Equation 19 is one of the transmission line equations.

The second transmission line equation

$$\frac{\partial i}{\partial z} + \frac{\partial Q}{\partial t} = 0 , \quad (20)$$

is obtained from the divergence of the Maxwell magnetic field equation

$$\nabla_x \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} , \quad (21)$$

by integrating over a closed surface surrounding the conductors. In terms of the scattered voltage v_s , Equation 20 is

$$\frac{\partial i}{\partial z} + C \frac{\partial v^s}{\partial t} = 0 \quad (22)$$

We have assumed in Equations 20 and 22 that any conductance associated with the dielectric is negligible.

Equations 19 and 22 are the transmission line equations in terms of scattered voltage. The source term in Equation 19 can be considerably simplified. Again consider Equation 14 integrated over the rectangle shown in Figure 4. Now, however, we consider only the impressed fields \vec{E}^i and \vec{B}^i . The result is

$$\frac{\partial}{\partial z} \int_0^h E_x^i dx + \frac{\partial}{\partial t} \int_0^h B_y^i dx = E_z^i(h) - E_z^i(0) .$$

Therefore, Equation 19 can be written as

$$\frac{\partial v^s}{\partial z} + Ri + L \frac{\partial i}{\partial t} = E_z^i(h) - E_z^i(0) . \quad (23)$$

Obviously, if the net horizontal electric field along the transmission line is zero, there is no line response.

The line equations can also be written in terms of total voltage v^T . The total voltage is the sum of the voltages in the incident and scattered waves; that is,

$$v^T = v^i + v^s = - \int_0^h E_x^i dx + v^s \quad (24)$$

Writing Equations 19 and 22 in terms of Equation 24 yields the line equations in terms of the total voltage; i.e.,

$$\begin{aligned} \frac{\partial v^T}{\partial z} + Ri + L \frac{\partial i}{\partial t} &= \frac{\partial}{\partial t} \int_0^h B_y^i dx , \\ \frac{\partial i}{\partial z} + C \frac{\partial v^T}{\partial t} &= -C \frac{\partial}{\partial t} \int_0^h E_x^i dx . \end{aligned} \quad (25)$$

In the calculations which follow, we use the scattered voltage formulation (Equations 22 and 23), because of the simplified driving source. Due to the static field which is present prior to the lightning discharge, the scattered voltage prior to the discharge is not zero. We derive the initial condition on scattered voltage here.

It is assumed that the static field has been applied for a long time, so that the transmission line has reached a steady-state condition. Thus, the time derivatives of the line variables are zero, so that Equation 23 for the scattered voltage becomes

$$\frac{\partial v^s}{\partial z} = E_z^i ; \quad (26)$$

where $E_z^i = E_z^i(h) - E_z^i(0)$ = net horizontal electric field per unit length driving the transmission line.

Equation 26 can be integrated to yield

$$v^s = \int_{z_0}^z E_z^i dz + K . \quad (27)$$

Here, K is a constant of integration and z_0 is the coordinate of the end of the line. The constant K is evaluated from the transmission-line boundary condition at $z = z_0$. Since there is no current flow in the static condition, the line boundary condition, for all but a perfect open circuit, is

$$v^s(z_0) - \int_0^h E_x^i(z=z_0) dx = 0 ; \quad (28)$$

where $E_x^i(z=z_0)$ is the vertical component of the incident electric field at $z=z_0$.

From Equations 27 and 28, the constant K is

$$K = \int_0^h E_x^i(z=z_0) dx .$$

Therefore, the initial condition on scattered voltage is (from Equation 27)

$$v^s(z) = \int_{z_0}^z E_z^i dz + \int_0^h E_x^i(z=z_0) dx \quad (29)$$

Equation 29 can be simplified by noting that the line integral of static electric field around any closed path is zero; i.e.,

$$\oint \vec{E} \cdot d\vec{\ell} = 0. \quad (30)$$

If we take as the path the rectangular loop formed by the transmission line from $z=z_0$ to z , Equation 30 yields

$$\int_0^h E_x^i(z=z_0) dx + \int_{z_0}^z E_z^i dz + \int_h^0 E_x^i(z) dx = 0 .$$

Thus, the initial condition on scattered voltage is

$$v^s(z) = \int_0^h E_x^i(z) dx . \quad (31)$$

SECTION 4
NUMERICAL RESULTS

In this section, we present the results of numerical calculations for the induced currents and voltages on a long, lossless line. The lossless-line assumption will cause real line responses to be overestimated somewhat; however, the overestimates are compensated by the assumption of a perfectly conducting ground. That is, the assumption of a perfectly conducting ground under estimates, to some extent, the horizontal, electric, lightning field which drives the line.

The current pulse used in the calculation of the electric fields is [4]

$$i(t) = 2.0 \times 10^4 \times 1.935 \left[\exp(-5 \times 10^4 t) - \exp(-2 \times 10^5 t) \right. \\ \left. + \left\{ \exp(-5 \times 10^3 t) - \exp(-2 \times 10^4 t) \right\} \right] / 3 \quad . \quad (32)$$

This pulse has a peak amplitude of 20 kiloamperes. The transmission line Equations 22 and 23 are solved using finite-difference techniques; a computer code, WIRES [5], is used for this purpose. The voltages on the line are calculated from the scattered and incident voltages using Equation 24.

The transmission line, assumed for calculational purposes, consists of an insulated conductor of 14 miles length spaced 3 meters above a perfectly conducting ground plane (see Figure 5). The line is terminated in 0.1 ohm and 10^4 ohms at the near and far ends. These impedances of 0.1 ohm and 10^4 ohms represent nearly short-and-open circuits. This transmission line is intended to be typical of a long, above-ground line.

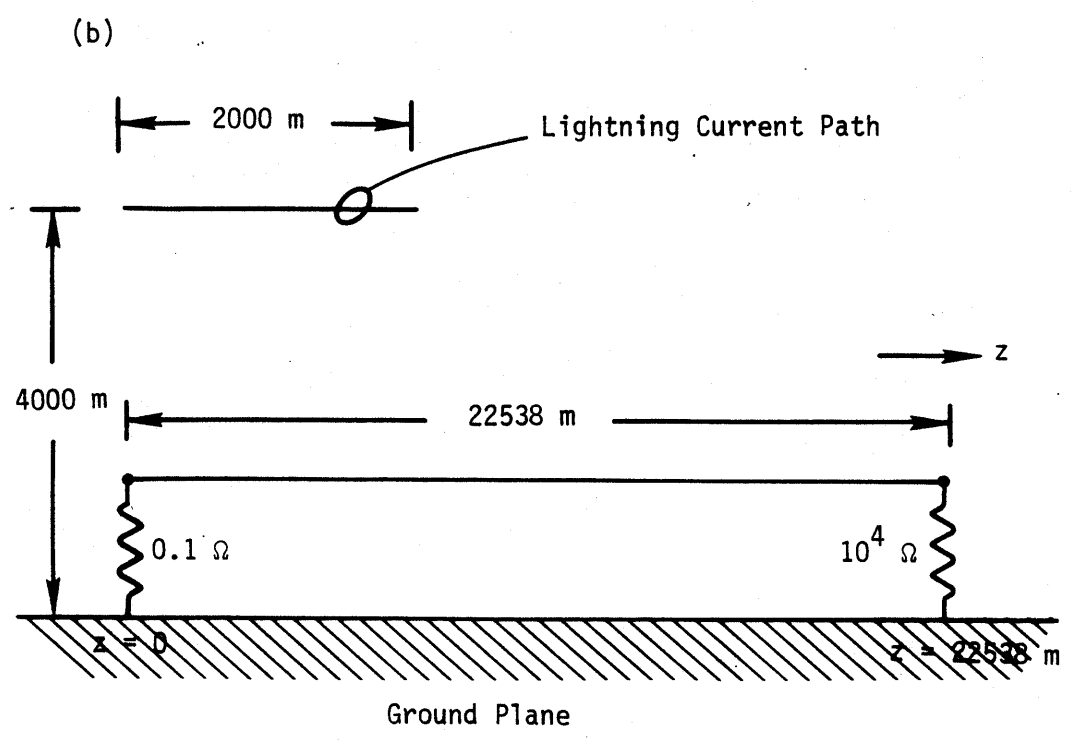
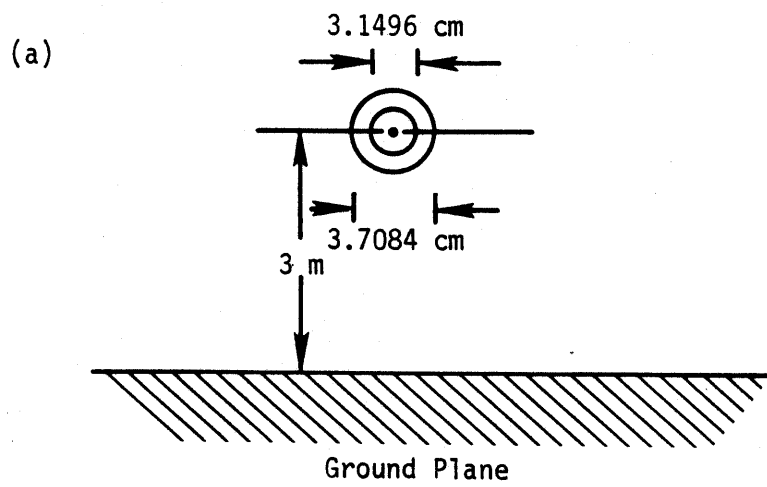


Figure 5. Transmission Line Configuration (a) Cross-Sectional View (b) Side View.

The lightning channel is assumed to be directly above the cable as shown in Figure 5. The length and height of the lightning channel are 2000 meters and 4000 meters* respectively. The speed of the current discharge is assumed to be 2×10^7 meters/sec [4, 6].

Figures 6, 7, and 8 show the horizontal component of the electric field created by the lightning discharge at $z=0$, 7 miles and 14 miles. These fields are shown at line height. Similarly, Figures 9, 10 and 11 show the vertical component of the electric field at these same points.

It is noted that discontinuities appear in these calculated electric fields. These can be attributed to the discontinuous time derivative of the assumed current waveform (Equation 32). If a "smoother" waveform is used, these discontinuities would not be present. The line acts as a filter to these discontinuities, however, so their presence will not appreciably affect the line behavior.

The total and scattered voltages across the 0.1 ohm resistor at $z=0$ are shown in Figures 12 and 13. Figure 14 shows the current through the 0.1 ohm resistor at $z=0$. Figure 15 shows the total voltage across the 10^4 ohm resistor at $z=14$ miles. The "ringing" in the calculated voltages and currents is the noise generated by the iterative techniques used**. The current through the 10^4 ohm resistor is essentially zero and is not shown.

* The discharge height of 4000 meters seems to be a lower limit observed for cloud-to-cloud or intracloud lightning.

** At the expense of additional computer cost, a finer grid size will reduce the numerical noise. Such increased costs are not deemed appropriate at this time. When specific line situations are completely known, a re-run of this program may be appropriate.

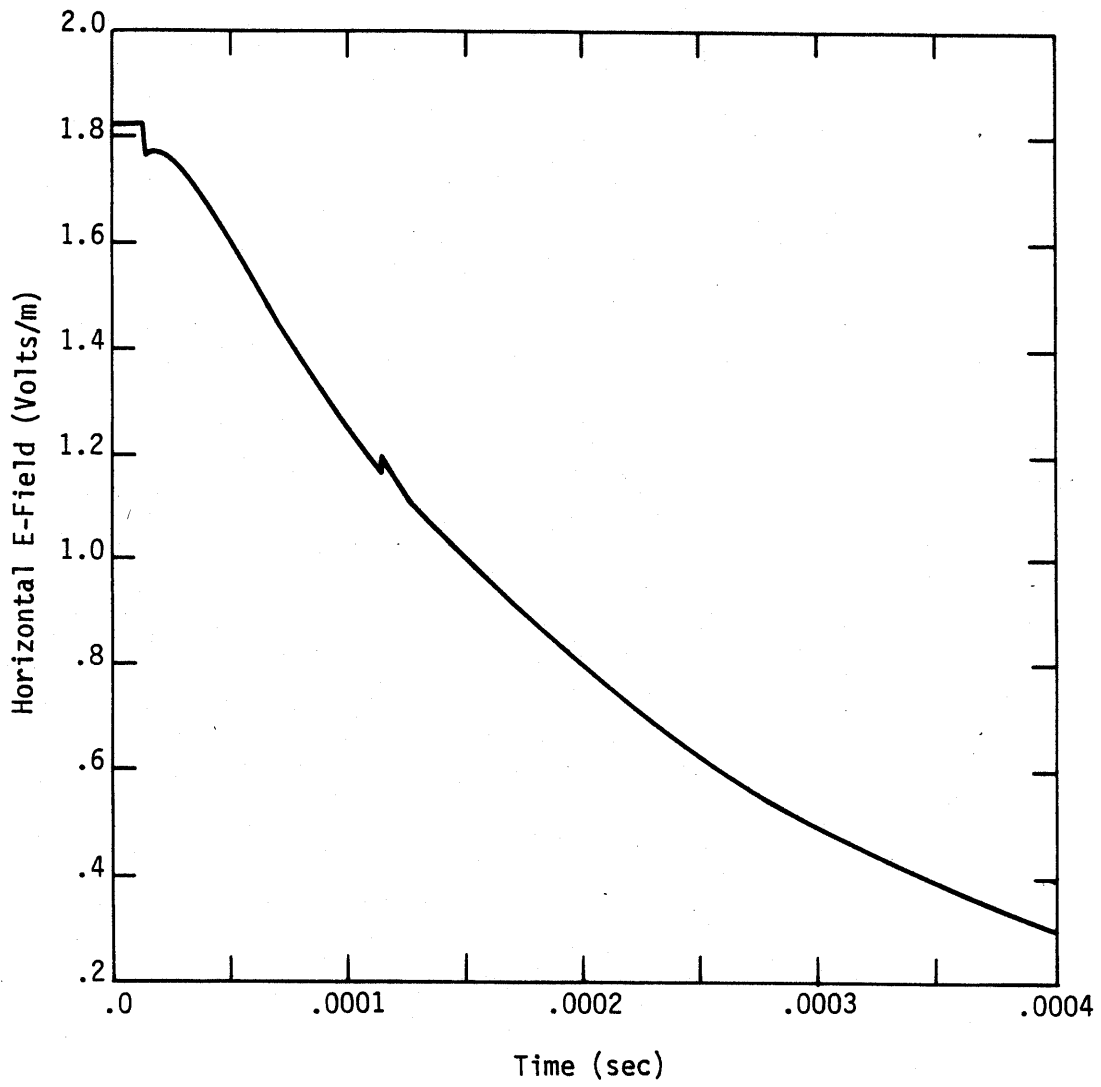


Figure 6. Horizontal Electric Field at $z = 0$.

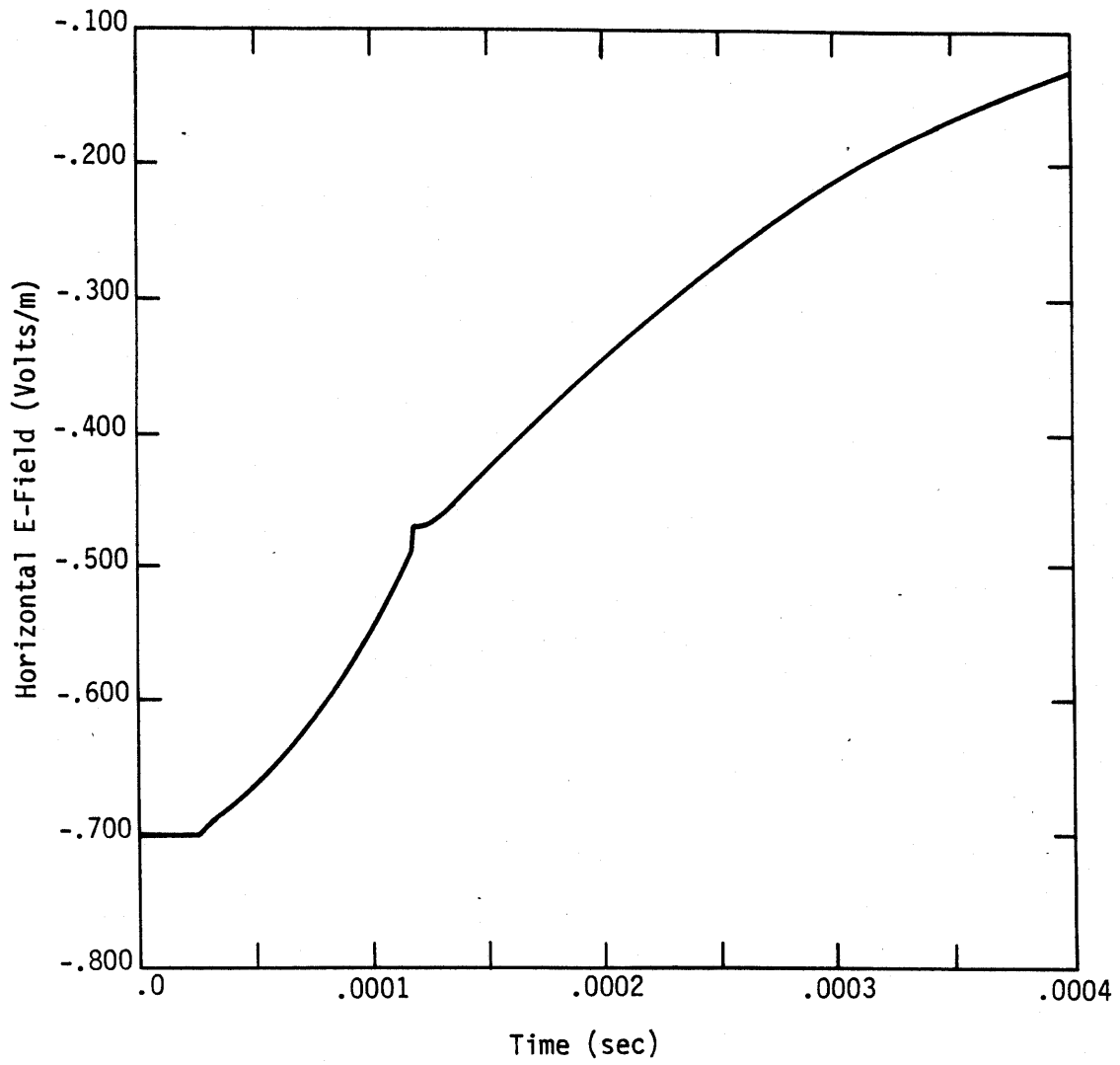


Figure 7. Horizontal Electric Field at $z = 7$ Miles.

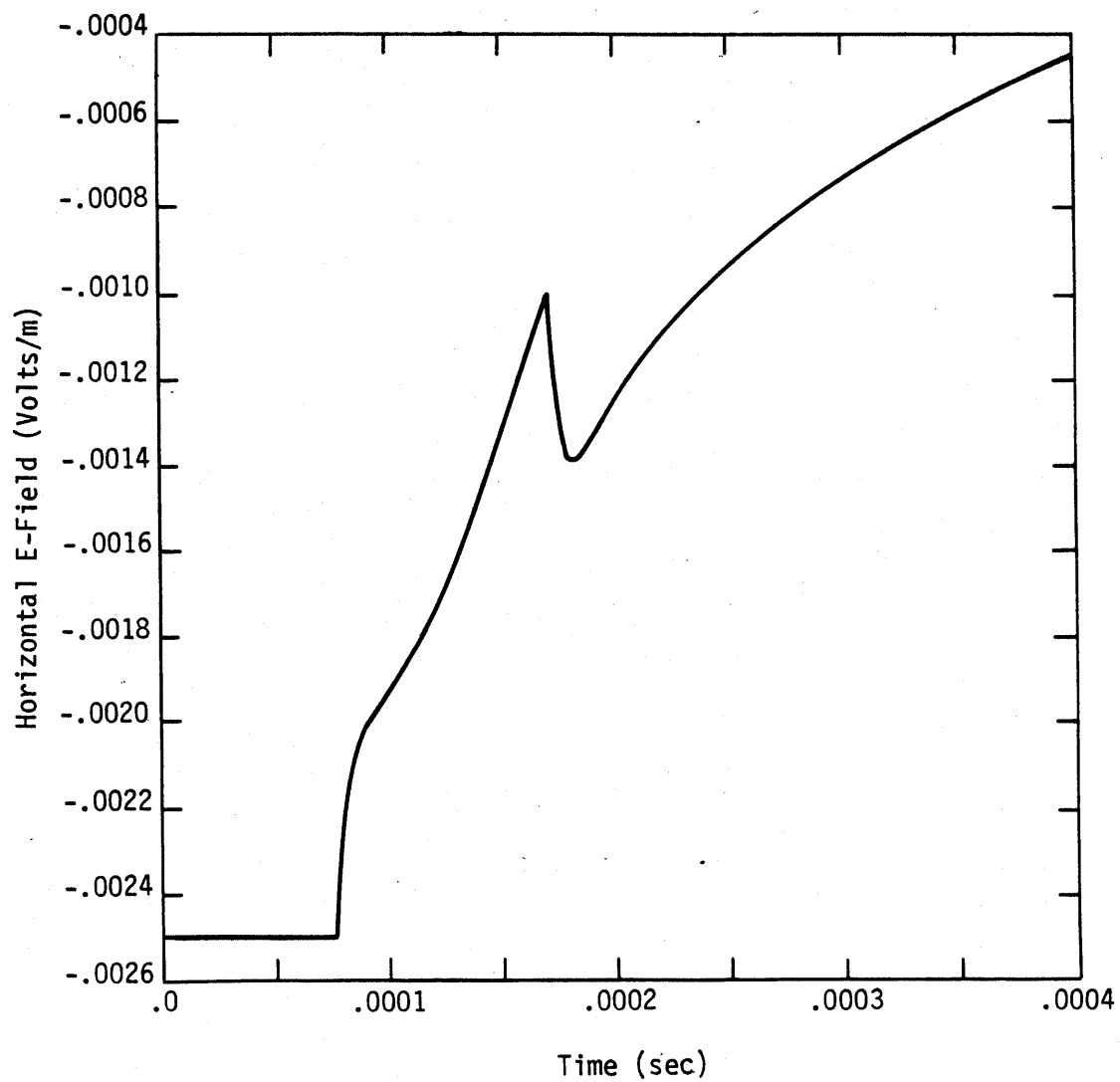


Figure 8. Horizontal Electric Field at $z = 14$ Miles.

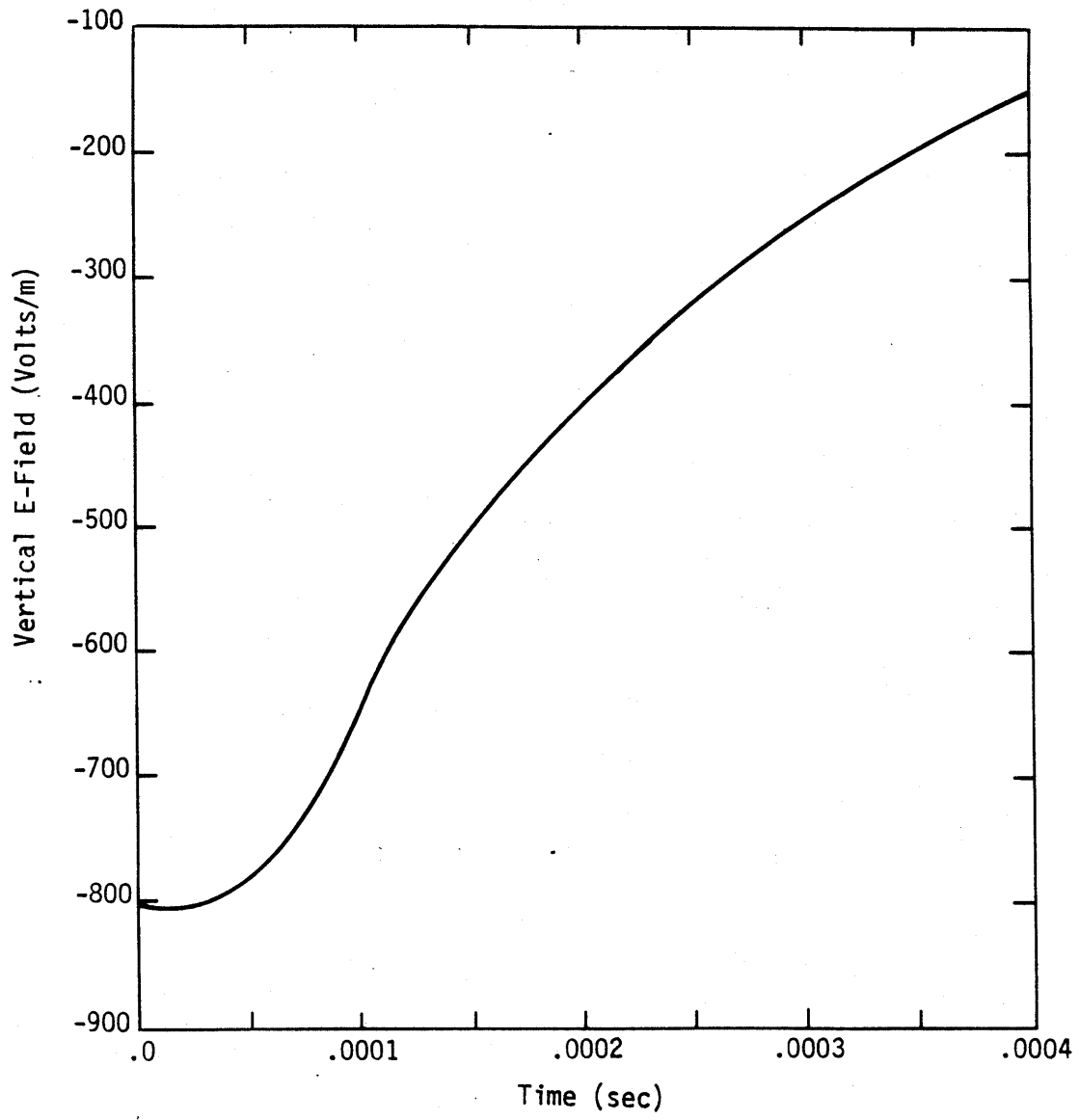


Figure 9. Vertical Electric Field at $z = 0$.

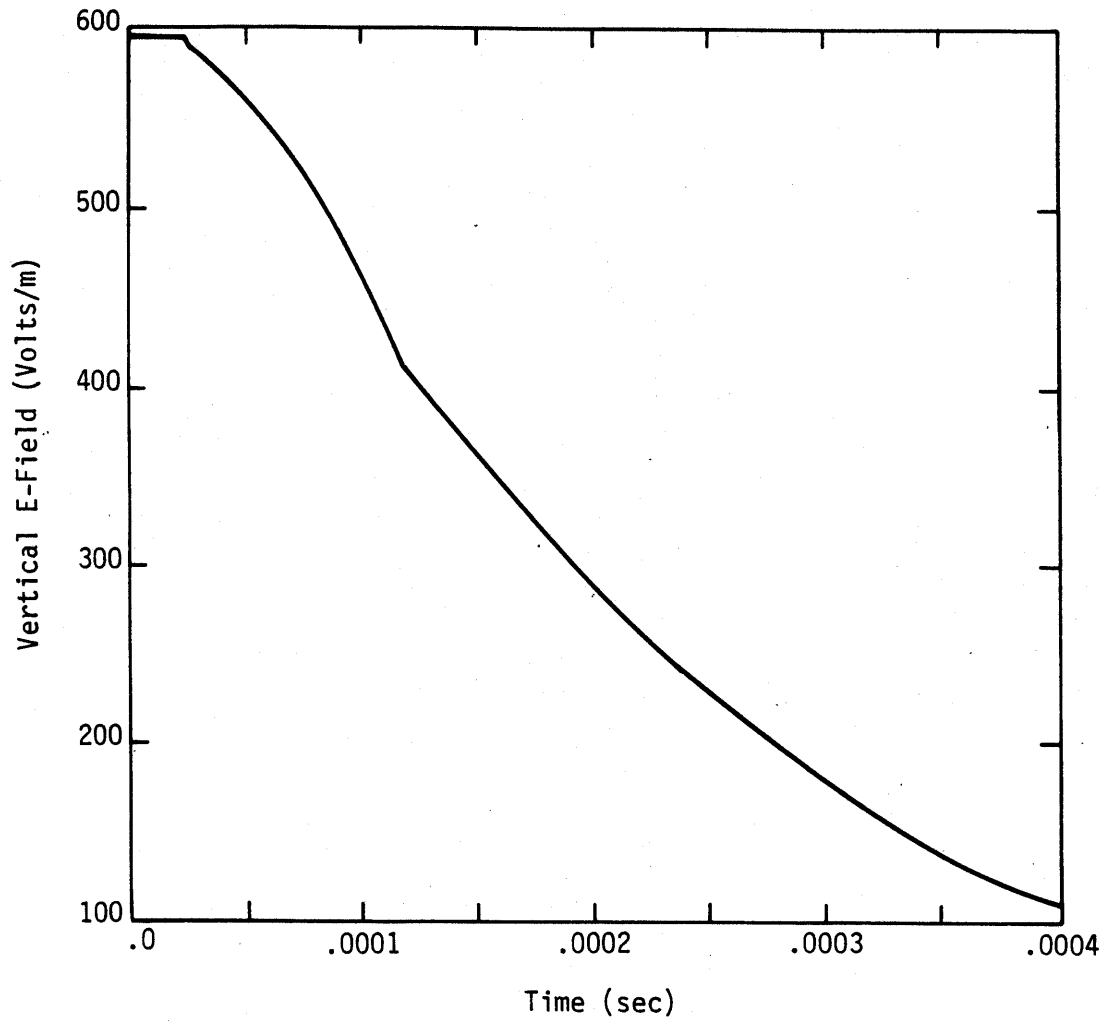


Figure 10. Vertical Electric Field at $z = 7$ Miles.

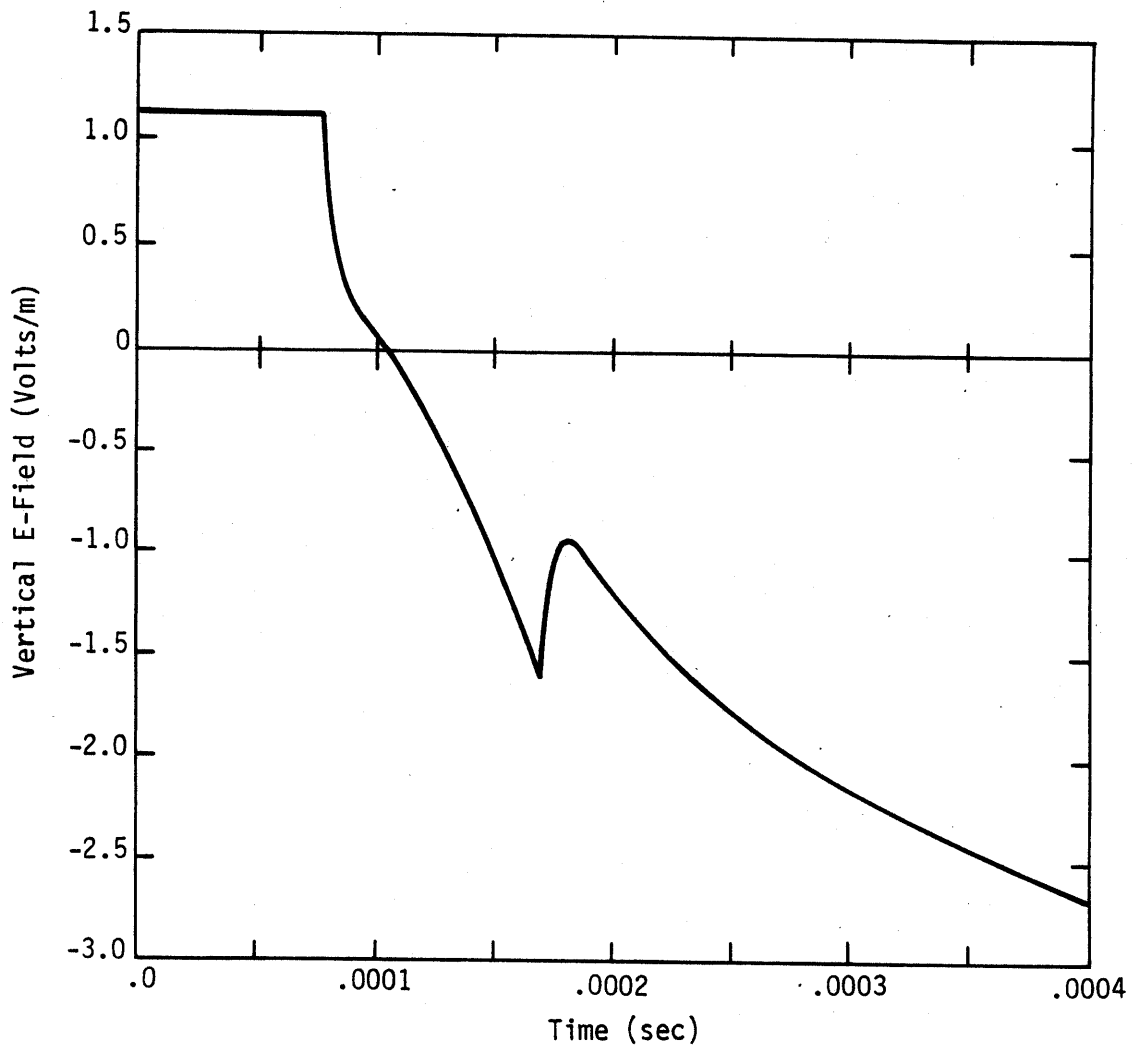


Figure 11. Vertical Electric Field at $z = 14$ Miles.

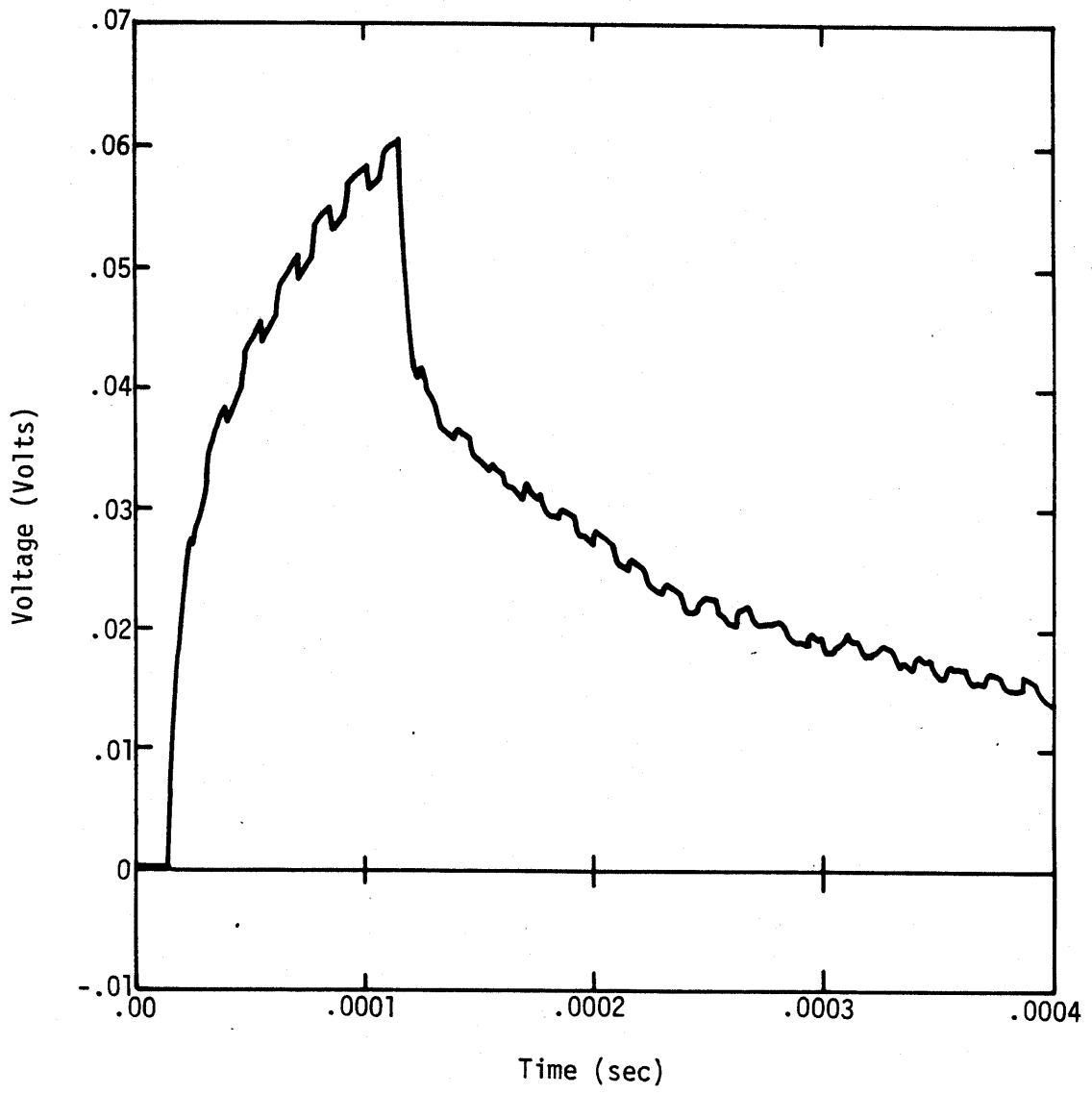


Figure 12. The Total Voltage $z = 0$.

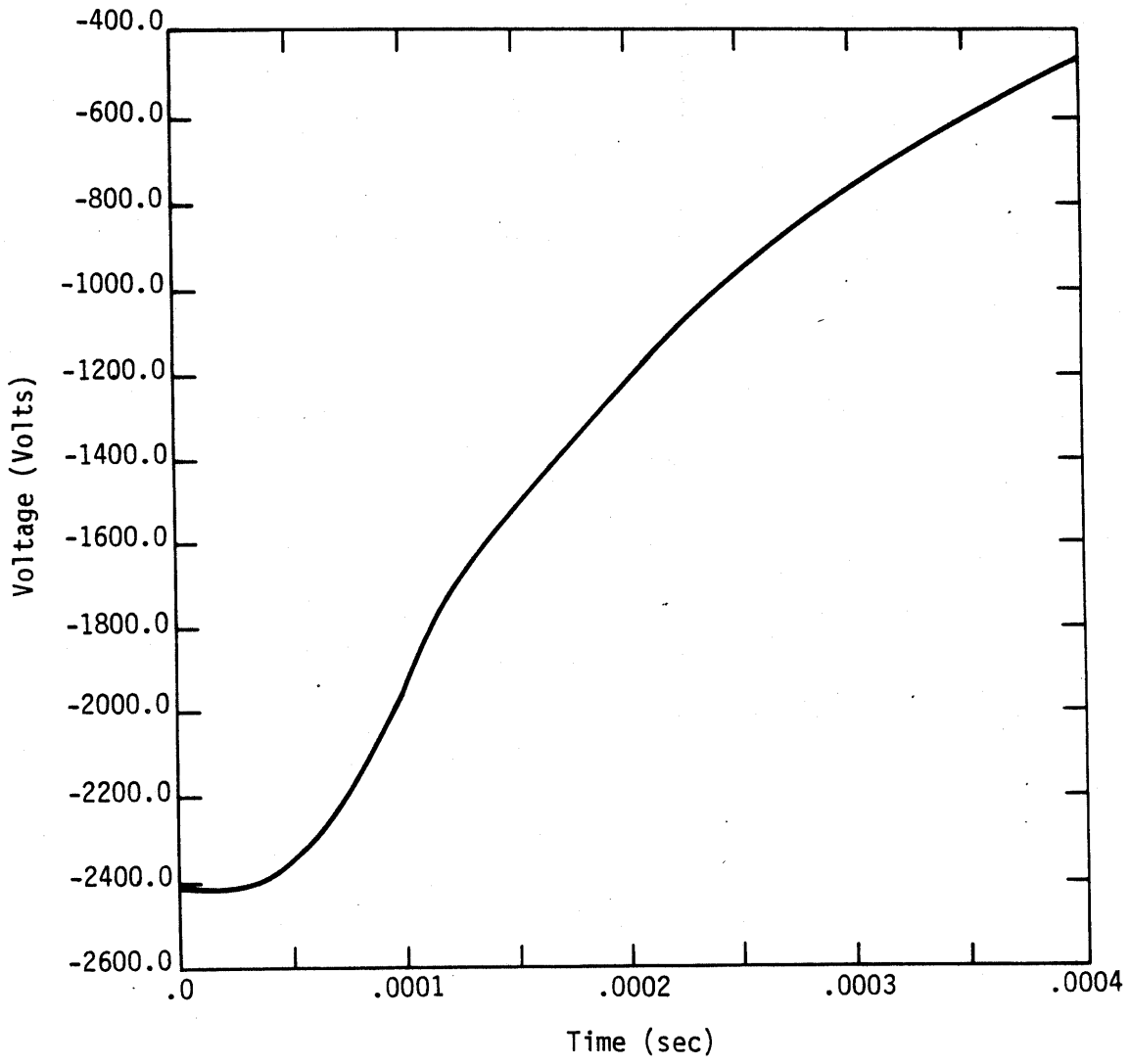


Figure 13. The Scattered Voltage at $z = 0$.

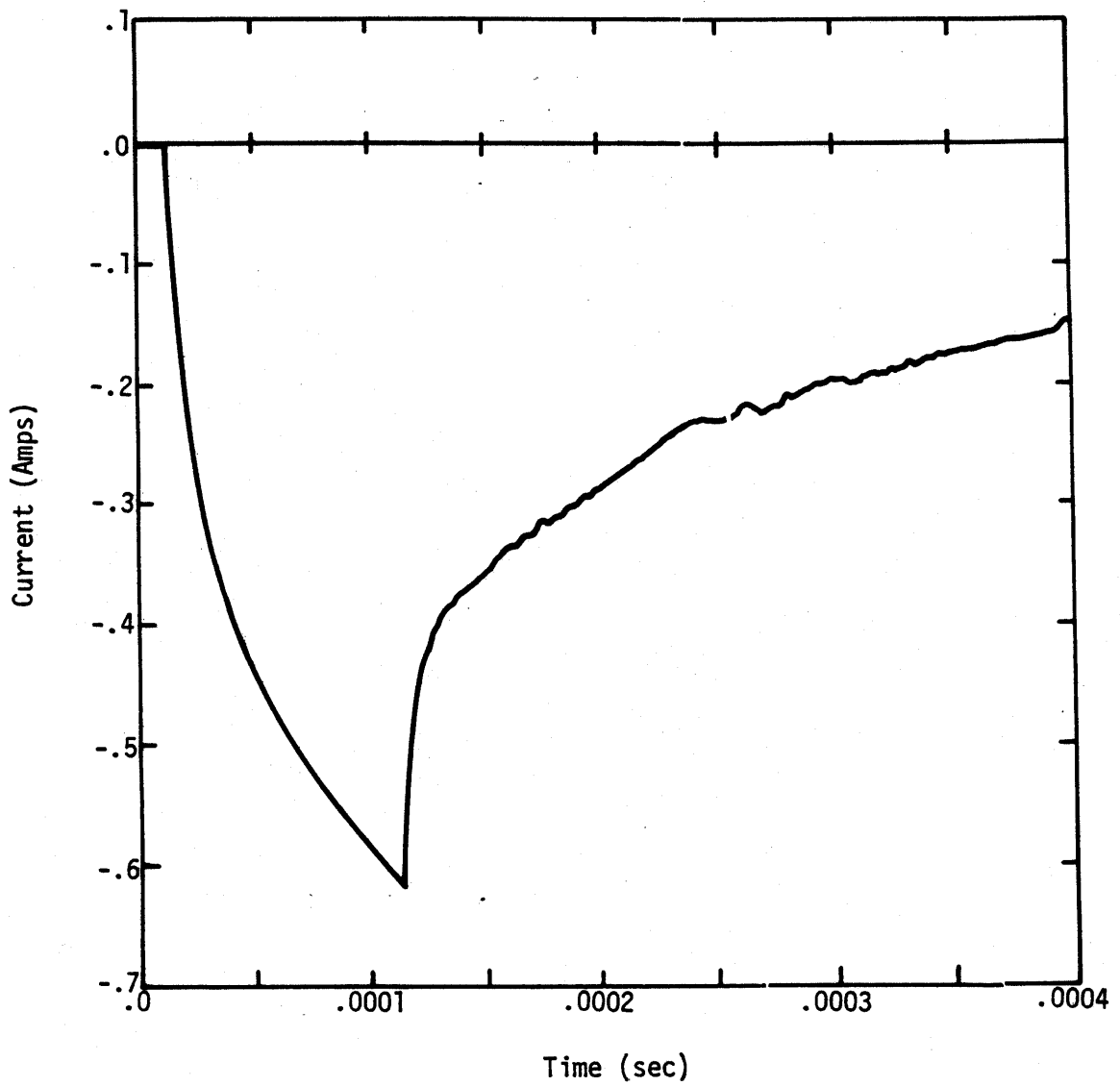


Figure 14. Current Through 0.1 Ohm Resistor at $z = 0$.

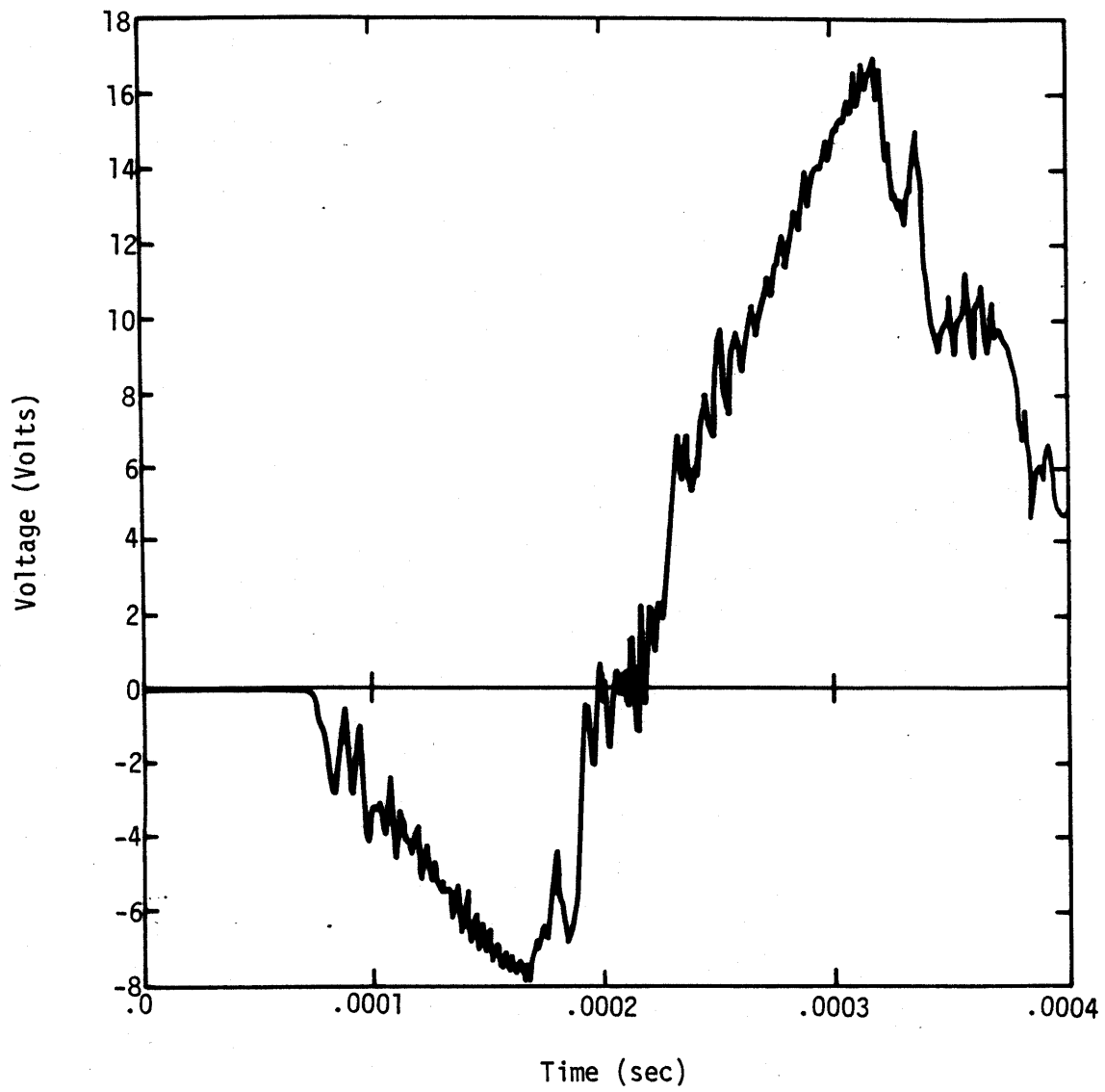


Figure 15. The Total Voltage at $z = 14$ Miles.

To verify the accuracy of the calculations, they were performed for several situations in which the solution of the transmission line equations can be obtained analytically. The theoretical and numerical results were found to be in good agreement.

It appears that the calculated voltages and currents induced on the line by a cloud-to-cloud lightning discharge are probably not large enough to cause damage to components.

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