Interaction Notes

Note 620

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Dynamics of Recovery of Coupled Infrastructures Following a Natural Disaster or Malicious Insult

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ABSTRACT

The interdependency between infrastructures comprising our country is reasonably well understood on a qualitative level under steady state conditions. Modeling the response of the infrastructure to predict its temporal behavior following a massive perturbation to the system, such as for example HEMP—the electromagnetic pulse produced a high altitude nuclear detonation, is challenging. Studies reveal that large systems can be susceptible to nuclear-induced high altitude electromagnetic pulses (HEMP) and that when large systems such as infrastructures are coupled in such a way that the control of each one depends on the state of the other, new control paradigms may be necessary to insure survivability.

INTRODUCTION

We have identified HEMP-related scenarios where there is strong dynamic coupling between the Electric Power Grid (EPG) and Public Data Network (PDN). This occurs for the following condition: (1) the EPG depends heavily on the real-time data supplied by the PDN, and (2) the backup power for telecommunications runs out, and it must rely completely on commercial power. Scenarios that fall in this category may lead to a condition we call *negative reinforcement.* This is the case where the EPG and supporting telecommunication networks both collapse due to their mutual interdependence, whereas they would not collapse if they were not dynamically coupled. The same theoretical formalism that predicts negative reinforcement also identifies the condition for the opposite case, *mutual recovery*. We regard negative reinforcement as being fundamentally and physically different from "cascading failure". The latter applies to significant collapse of segments of either the EPG or telephone networks when there is no dynamic coupling between them. Experience-based analytical relationships that describe the dependence of the PDN on electric power are used with a generic model of the EPG in a non-linear state variable theory approach. Notwithstanding the approximations inherent in our exploratory model we show how total collapse of coupled power and telecommunications systems could occur.

BACKGROUND AND SUMMARY

In the near future the EPG will depend on the PDN for more than 50% of its real-time data needs. This increased dependence, shown in **Figure 1,** is intended to provide more accurate and reliable real-time data transport at lower cost to the power utilities.

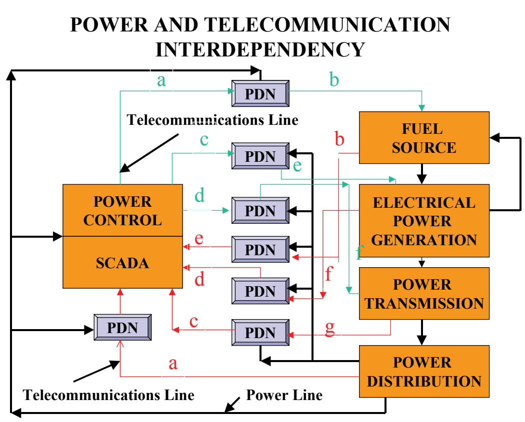

Figure 1 Anticipated Dependence of Electric Power Grid onPublic Data Networks [Source: D. Ambrose, Ref. 1]

The black lines show the flow of power from the power supply chain: fuel sources, electric power generation, power transmission, and power distribution to elements of the PDN and the power control and Supervisory Control and Data Acquisition (SCADA) system units. Data from the power supply chain to the power control and SCADA units is shown in red. Green lines depict data transport from the power control and SCADA back to the power supply chain. As shown in **Figure 1,** data transport through the PDN affects all aspects of the EPG. Failure of the PDN to provide real-time reliable data transport can lead to all kinds of problems for the EPG, including network wide power system instability and in extreme cases power system collapse.

Even in the absence of a natural disaster or malicious insult there's a reason for concern. Current power system control strategies are based on no more than one extreme contingency, and there are rules for dealing with these **[2]**. We can only hypothesize as to what would have happened in the recent August 14, 2003 power grid failure if telephones were not available. This is not quite the same as losing data, but the idea is similar—we may not know what to do. If the EPG depends on the PDN, but the PDN does not concurrently depend on the EPG, these infrastructures are not interdependent. The converse is also true. *Interdependence occurs when both infrastructures depend on each other at the same time—the systems are dynamically coupled to one another.*

We have examined scenarios where there is strong dynamic coupling between the EPG and PDN. This occurs for the following condition: (1) the EPG depends heavily on the real-time data supplied by the PDN, and (2) the backup power for telecommunications runs out, and it must then rely completely on commercial power. Scenarios that fall in this category may lead to a condition we call *negative reinforcement.* This is the case where the EPG and supporting telecommunication networks both collapse due to their mutual interdependence, whereas they would not collapse if they were not dynamically coupled. The same theoretical formalism that predicts negative reinforcement also identifies the condition for the opposite case, *mutual recovery*.

Predicting negative reinforcement and mutual recovery following a massive insult requires that we model the dynamics of both the EPG and that part of the PDN that supplies data to the EPG. The general features of such a model were described by Graham and Kohlberg **[3]**, and a systems level computer based model was developed by Morrison, Ambrose, and Kohlberg **[4]**. The telephone companies have in place good analytical system models that describe the recovery process with or without commercial power from the EPG. Unfortunately, we don't have a systems level model for the EPG that takes into account the breakdown of data communications. We're forced to use a general model of the EPG based on state variable theory **[2]**. We've been able to modify the standard state variable approach to include data flow as a variable. Theoretically, this is all we need to approximate dynamic coupling between power and telecommunications.

Notwithstanding the approximations inherent in our illustrative model we show how total collapse of coupled power and telecommunications systems could occur. When an explicit model of the EPG's dependence on data flow becomes available, we'll be able to predict recovery or collapse for coupled power-telecom systems in a specific HEMP scenario.

TELECOMMUNICATIONS AND POWER INTERDEPENDENCE

Under normal conditions telecommunication networks draw their power from the EPG. When there is a power failure these networks rely initially on battery back-up power, which lasts typically between 2-4 hours. If commercial power is still not available when the battery power is depleted, local power generation is used. Local power generation is programmed to last up to a maximum of three days, but on the average is less than this.

Battery back-up power is assumed to be immune to electromagnetic attacks such as HEMP, and is essentially 100% reliable. In this study we assume that the average combined battery-plus-local generation back-up power is one day. Should we discover that local power generation is robust, we can increase the duration to more than one day. The assumption of combined one-day back-up power does not limit the analysis. Timelines for various phases of the recovery processes for power and telecommunications are illustrated in **Figure 2.** Before the insult the power and telecommunications infrastructures operate at their nominal ratings. As noted in **Figure 2** the timeline for telecommunications recovery has four segments:

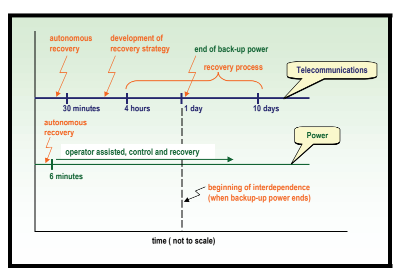

Figure 2. Timelines for Interdependence Between Power and Telecommunications

-The first segment, lasting for approximately 30 minutes is the autonomous recovery epoch. In this time regime the recovery process is determined solely by the physical response of the equipment and network control algorithms. There is no human intervention.
-The second segment, lasting from 30 minutes to 4 hours is a planning stage for manual recovery. During this period of time the state of the telecommunications network remains unchanged from its state at 30 minutes.
-The third segment, lasting from 4 hours to 1 day, is that phase of the manual recovery process that relies on back-up power. For this segment, as well as the first two, there is no dependence on commercial power.
-The fourth segment is that phase of the recovery process where telecommunications depends on the power grid. This is the regime of interdependence between power and telecommunications.

The electric power response after the onset of the insult is shown in **Figure 2**. Up to about 300 seconds, the response is autonomous, depending only on the equipment and specified control strategies. When the operator notices abnormal behavior, human intervention begins. From this point on, the power grid behavior is determined by a combination of automatic controls combined with discrete time dependent actions taken by the operators of the grid.

The distinction between dependence and interdependence of power and telecommunications is further illustrated in **Figure 3.** This sketch shows the temporal response of a telecommunications network based on hypothetical transients of the power grid. The metric that describes the capability of telecommunications is the Probability-of-Call-Blocking, P_b. The Probability-of-Call Completion is given by:

$$P_c = 1 - P_b \tag{1}$$

The metric that describes the capability of the power grid is the total power generated, Q. [The authors are aware that Q is traditionally used to represent reactive power in power grid analyses. We use Q to represent real power to avoid confusion with the symbols P_b or P_c.].

The left hand vertical axis of **Figure 3** is the Probability-of-Call Blocking, P_b. The arrow pointing in the downward direction signifies recovery; 100% call blocking means that $P_b = 1.0$ and no calls get through. This is the worst case, and as noted occurs immediately after the HEMP event (the upper left hand corner of **Figure 3**). When $P_b = 1.0$, the Probability-of-Call Completion is identically zero.

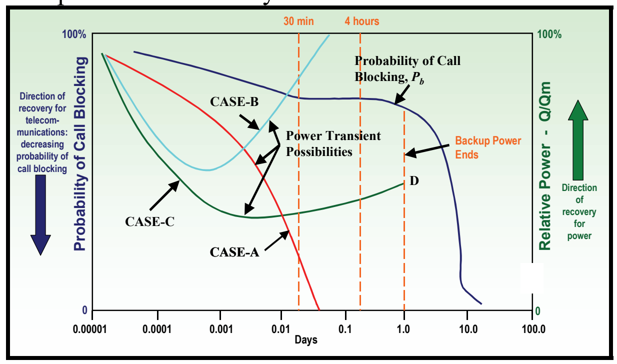

Figure 3 Probability-of-Call-Blocking and Possible Power Transients

The amount of power required to keep all the equipment used in the telecom network working is Q_m. The vertical axis on the right hand side of **Figure 3** is the relative power, $(Q(t)/Q_m)$. Having 100% of the electrical power available, $Q(t) = Q_m$, does not guarantee that all calls will be completed since telecom equipment could be rendered inoperative directly from the HEMP event. Up until the backup power runs out, recovery of the telecom network assumes 100% of the power it needs is available. The recovery curve is the one that is labeled "Probability-of-Call-Blocking, P_b" in **Figure 3**. Recovery of telecom is always improves with time as long as 100% of the power is available. Three hypothetical cases of power response are shown in this figure.

-In Case A the power transient never recovers from the HEMP event. When this happens and the backup power quits, the probability of call blocking becomes 1.0 and there is no longer any telecommunications capability.

-Case B depicts a power transient that recovers completely from the HEMP event before the 1-day mark. For this case the recovery of telecommunications takes place for 100% of power availability. Telecommunication response is unaffected by this type of power transient.

-Case C is the case of dynamic interdependency. The telecommunication network reverts back to depending on commercial power while both are in the recovery phase. The power infrastructure continues to depend in part on the probability of call blocking, while the recovery for telecommunications depends on the available power. The latter is the case being modeled in this study.

TELECOMMUNICATION RESPONSE WITH LOSS OF POWER

The top curve in **Figure 3** is the function

$$P_b = G(t) \tag{2}$$

This function describes the recovery of the telephone system under the assumption that 100% of the power necessary to operate the telephone network was available. For this part of the problem it's more convenient to use the Probability-of-Call Completion, P_c, instead of P_b.

$$P_c(t) = 1 - G(t) \tag{3}$$

Relating P_c to power is a two-step process. The first step is to relate P_c to the fraction of equipment available, *F*. The second step is to relate *F* to the power available for the network. At all times *F* is defined as

$$F \equiv \frac{N(t)}{N_{\max}} \tag{4}$$

In the foregoing expression $N(t)$ is the total amount of equipment that is available to support network operations at time *t*, and $N_{\max}$ is the maximum amount of equipment available to support network operations. The maximum value of F is unity.

The network is conceptualized as calls primarily consisting of originating equipment and terminating equipment. Intermediate equipment is considered less crucial to overall call completion, since there are often multiple paths available in the interior of the telecom network. Calls can be completed only if the originating equipment is available <u>and</u> the terminating equipment is available. This chain of reasoning leads us to the approximation

$$P_c \cong (F)^2 = \left(\frac{N(t)}{N_{\max}}\right)^2 \tag{5}$$

To a good approximation $F(t)$ is a linear function of time in the *Primary Reconstitution* period.

$$F = \frac{N(t)}{N_{\max}} = F_0 + Kt \tag{6}$$

The initial value at $t = 0$ is F_0, and the recovery rate is *K*. When power drops at a set of locations, call completion drops at those locations because the fraction of equipment available to support telecommunication network operations drops. If we assume that Q_m is the electrical power available to support all the equipment, $N_{\max}$, and $Q(t)$ is the actual power ($Q(t) \leq Q_m$), then the *maximum* amount of equipment that can support network telecommunications operations at time, *t*, is $N_{DM}(t)$. It is given by

$$N_{DM}(t) = N_{\max}\left(\frac{Q(t)}{Q_m}\right) \tag{8}$$

During the recovery process we require that $N(t)$ will always be less than or equal to $N_{DM}(t)$ because $Q(t) \leq Q_m$. Now suppose that during the recovery process $Q_1(t_0)$ is the power to the telecommunications network and $N_1(t_0)$ is the amount of equipment operating at some arbitrary time t_0. Suddenly, there is a step function decrease in available power to the telecommunications system so that at $t = t_0^+$ the available power drops to $Q_2(t_0^+)$. A discontinuous drop in power, ΔQ, occurs.

$$\Delta Q = Q_1 - Q_2 \tag{9}$$

We assume that the available power is spread homogeneously over all the equipment in the network, including that equipment that has been impaired by the HEMP insult and not yet fixed. The amount of operating equipment is then proportional to the available power. Thus, the discontinuous change in available equipment is

$$N_2 = N_1\left(\frac{Q_2}{Q_1}\right) = N_1\left(1 - \frac{\Delta Q}{Q_1}\right) \tag{10}$$

Discontinuous changes network capability and equipment will occur in a telecommunications network when it reverts back to commercial power from backup power.

Equation (10) is also used to relate continuous changes in *N* and *Q*—only when *Q* is decreasing. *The converse does not apply!* We don't get an increase in the amount of available equipment because more power is available to the network. Although power must be available for recovery of equipment, the recovery rate is controlled by the rate at which it can be repaired: this is given by the derivative of equation. (6). We get

$$\frac{dN}{dt} = N_{\max} K \qquad (11)$$

Actually, equation (11) applies when two conditions are met. As mentioned in the previous paragraph, the first condition is that $(dQ/dt) \geq 0$. The second condition is a restriction on *N* imposed by equation (8): we can't have the number of recovered equipment items exceed the maximum number allowed by the total available power: this means $N(t) \leq N_{DM}(t)$.

We ask the question "What happens if $N_{\max} K$ is very large, so that it causes $N(t)$ to catch up to $N_{DM}(t)$? Since $N(t) \leq N_{DM}(t)$, catch up in this case means that $N(t) = N_{DM}(t)$ from then on. When this happens the recovery rate drops. Instead of the fractional recovery rate being *K,* it's now less than *K*. The work force must slow down.

Let's begin at time t_0: $N(t_0)$ is the amount of equipment at t_0, $Q(t_0)$ is the power, and

$$N_{DM}(t_0) = N_{\max} \left(\frac{Q(t_0)}{Q_m} \right) \qquad (12)$$

Suppose $N(t_0) \leq N_{DM}(t_0)$. Integrating equation (11) gives

$$N(t) = N(t_0) + N_{\max} K t \qquad (13)$$

If $Q(t)$ and hence $N_{DM}(t)$ grow at a slower rate than $N(t)$, $N_{DM}(t)$ and $N(t)$ will intersect at time $\tilde{t}$ determined from the equation

$$N(t_0) + N_{\max} K \tilde{t} = N_{DM}(\tilde{t}) = \frac{N_{\max} Q(\tilde{t})}{Q_m} \qquad (14)$$

After time, $\tilde{t}$, $N(t)$ equals $N_{DM}(t)$ for reasons previously discussed. In the domain $t \geq \tilde{t}$ we have

$$N(t) = N_{DM} = \frac{N_{\max} Q(\tilde{t})}{Q_{\max}} \qquad (15)$$

The recovery rate must be reduced to satisfy equation (15). Instead of equation (11), the increase in available equipment is now given by

$$\frac{dN}{dt} = N_{\max} K_r(t) \qquad (16)$$

Differentiating equation (15) and inserting the result into equation (16) gives

$$K_r(t) = \frac{1}{Q_m} \frac{dQ}{dt} \qquad (17)$$

If $Q(t)$ and hence $N_{DM}(t)$ grow more rapidly than $N(t)$, $N_{DM}(t)$ and $N(t)$ will never intersect; $N_{DM}(t)$ will reach its terminal value, $N_{\max}$, while $N(t)$ is still growing. The time for total time for recovery is then given by

$$t_r = \frac{N_{\max} - N(t_0)}{K N_{\max}} \qquad (18)$$

Now let's turn our attention to recovery when $(dQ/dt) < 0$. Applying equation (10) in the limit of infinitesimal changes we write

$$N_2 = N_1 + \Delta N \qquad (19)$$

$$Q_2 = Q_1 - \Delta Q \qquad (20)$$

Substituting equations (19) and (20) into equation (10) gives

$$\frac{\Delta N}{\Delta t} = -\frac{N_1}{Q_1} \frac{\Delta Q}{\Delta t} \qquad (21)$$

In the limit of $\Delta \to 0$ we get (see equation (9) for definition of ΔQ)

$$\frac{\Delta N}{\Delta t} \to \frac{dN}{dt} \qquad (22)$$

$$\frac{\Delta Q}{\Delta t} = \frac{Q_1 - Q_2}{\Delta t} = -\frac{(Q_2 - Q_1)}{\Delta t} = -\frac{dQ}{dt} \qquad (23)$$

Inserting equatios (22) and (23) into equation (21) gives

$$\frac{dN}{dt} = \frac{N}{Q} \frac{dQ}{dt} \qquad (24)$$

Equation (24) applies only when $\left(dQ/dt\right) < 0$. The total recovery equation when power is decreasing is the combination of equations (11) and (24).

$$\frac{dN}{dt} = N_{\max} K + \frac{N}{Q}\frac{dQ}{dt} \tag{25}$$

We see from equation (25) that there are two competing effects for recovery when power is decreasing. The first term on the right hand side of equation (25) increases $N(t)$ from recovery procedures. The second term on the right hand side decreases $N(t)$ because power is decreasing.

DYNAMIC INTERDEPENDENCE BETWEEN POWER AND TELECOMMUNICATIONS

Predicting transient behavior and stability of a large segment of the electric power grid using computer models is challenging because hundreds to thousands of components need to be modeled. State variable theory is used to model the dynamics of the electric power grid. The mathematical foundations of state variable theory are rooted in the stability of ordinary differential equations, nonlinear differential and integral equations, and modern control theory. Kundar has provided an excellent discussion of the application of state variable theory to power system transient response as things are today (essentially no dynamic dependence on data communications) **[2]**.

In this section we link the equations that connect power and telecommunications. We explore conditions where the linkage (interdependency) leads to either recovery or negative reinforcement and possibly collapse. The power grid is just beginning to make more use of the public data network. Thus, we don't have an experience base that connects changes in grid power to changes in P_c. Despite this drawback, we can still gain insight into the dynamics of interdependency between power and telecommunications based on general properties of the power equation.

By combining the terms in equation (42) we can write it in the deceivingly simple form

$$\frac{dQ}{dt} = \Psi(\Theta, P_c) \tag{26}$$

All the power variables are grouped in a set denoted as $\in$.

We do not have the capability to solve equation (26) for a HEMP event combined with dependence on the Probability-of-Call Completion. What we have succeeded in doing is isolating the dependence of the total power on the instantaneous value of P_c. This is a key step in developing linked power and telecom equations, and is sufficient for illustrating conditions where interdependency could lead to negative reinforcement and collapse. This is demonstrated in the next section.

Figure 4 shows the state of affairs when telecom backup power runs out. This figure is an amplification of **Figure 3**, tailored to the conditions when backup power runs out. The telecommunications recovery curve has already been discussed, and applies for 100% of power availability. For the work of this section it is applicable only until the backup power runs out. Our job is to find out what could happen when the telecom network is suddenly connected back to the power grid when the latter is strongly dependent on P_c. For simplicity we assume that backup power runs at the same time all over the telecom network. This of course is not true, but it provides insight into the linkage issues. A refined calculation will handle a distribution of power run out times.

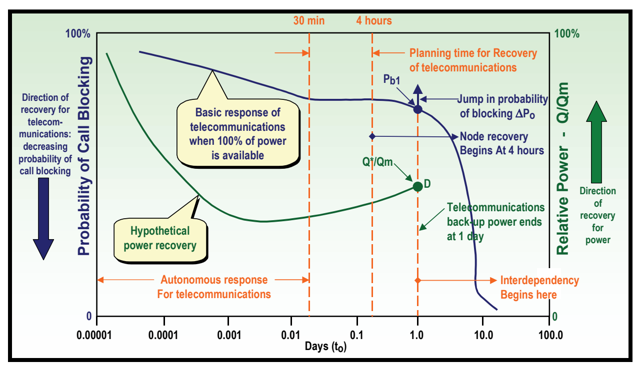

Figure 4 Transition of Telecommunication Backup Power to EPG

As observed in **Figure 4**, power recovery is assumed to occur, so that at the transition time $(dQ/dt) > 0$. During the backup interval the telecom network was operating with 100% backup power. This changes abruptly when the backup power runs out. The available power to the telecom network drops suddenly to $Q(t_0)$ at the transition time, t_0. This produces a discontinuous change in available equipment is calculated from Eq. (10).

Just before the transition, at $t = t_{0-}$, the amount of operating equipment was N_1. The amount of power available was Q_m. The new level of equipment is N_2. From Eq. (10) we get

$$N_2 = N_1(\frac{Q(t_0)}{Q_m}) \tag{27}$$

Before the transition the Probability-of-Call Blocking was

$$P_b(t = t_{0-}) = 1 - P_c(t = t_{0-}) = 1 - \left(\frac{N_1}{N_{\max}}\right)^2 \tag{28}$$

Now the Probability-of-Call Blocking is

$$\begin{aligned} P_b(t = t_{0+}) &= 1 - P_c(t = t_{0+}) = \\ 1 - \left(\frac{N_2}{N_{\max}}\right)^2 &= 1 - \left(\frac{N_1}{N_{\max}}\right)^2 \left(\frac{Q(t_0)}{Q_m}\right)^2 \end{aligned} \tag{29}$$

Comparing equations (28) and (29), we see that the Probability-of-Call Blocking has increased instantaneously. The jump at $t = t_0$ is

$$\Delta P_b = P_b(t = t_{0+}) - P_b(t = t_{0-}) = \left(\frac{N_1}{N_{\max}}\right)^2 \left(1 - \left(\frac{Q(t_0)}{Q_m}\right)^2\right) \tag{30}$$

This abrupt jump is shown in **Figure 4** as a upward pointing arrow.

Up until time t_0 the power recovery was assumed to be operating with 100% telecommunications capability: $P_c = 1.0$ The power recovery equation was

$$\frac{dQ}{dt} = \Psi(\Theta, P_c = 1.0) \tag{31}$$

Physical reasoning tells us that the rate of power recovery should be an increasing monotonic function of P_c. It's inconceivable that power recovery will improve with less telecommunication capability. The rate of change of power will slow down from that given by equation (31) because P_c will be less than unity until full recovery of telecom occurs. In fact, the initial drop in P_c may be large enough to cause $\Psi(\in, P_c)$ to be negative.

For lack of better information, we make the assumption that in the time interval shortly following t_0; changes in $\Psi(\in, P_c)$ will be mostly governed by P_c. One may make a case for this approximation based on the fact that the state variables for power depend on dynamic changes of large pieces of machinery—there's a lot of inertial in the power grid. We thus use the approximation

$$\frac{dQ}{dt} \cong \Psi(\Theta(t_0), P_c) \tag{32}$$

Since $\in(t_0)$ is assumed to be a constant we ignore it in time dependent equations and write

$$\frac{dQ}{dt} = \Psi(P_c) \tag{33}$$

We allow for the fact that $\Psi(P_c)$ could be positive or negative so that, depending on the value of P_c, the rate of change of power could be positive or negative. This allows us to assume that there is some value of P_c above which the rate of change of power will recover, and below which the rate of change of power will decrease. Let this value be $P_c = \hat{P}_c$.

Without loss in generality we can then write $\Psi(P_c)$ in the form

$$\Psi(P_c) \equiv -\overline{\Psi}(\hat{P}_c - P_c) \tag{34}$$

The following conditions apply:

$$\overline{\Psi}(\hat{P}_c - P_c) \geq 0 \; if \; \hat{P}_c - P_c \geq 0 \tag{35a}$$

$$\overline{\Psi}(\hat{P}_c - P_c) < 0 \; if \; \hat{P}_c - P_c < 0 \tag{35b}$$

Equation (33) then becomes

$$\frac{dQ}{dt} = -\overline{\Psi}(\hat{P}_c - P_c) \tag{36}$$

We solve the dynamical interdependency equations in this section using dimensionless variables. In this way a few calculations are applicable for a wide range of scenarios. The mathematics is made easier if we express P_c and $\hat{P}_c$

in terms of normalized dimensionless variables, n^2 and $\hat{n}^2$, defined by the equations

$$P_c = \left(\frac{N}{N_{\max}}\right)^2 = n^2 \quad (37a)$$

$$\hat{P}_c = \left(\frac{\hat{N}}{N_{\max}}\right)^2 = \hat{n}^2 \quad (37b)$$

Using equation (37) in equation (36) we can generate a more useful form of relating the change of power to the available telecommunications equipment. We write

$$\frac{dQ}{dt} = -\Phi(\hat{n} - n) \quad (38)$$

$$\Phi(\hat{n} - n) \equiv \overline{\Psi}(\hat{P}_c = \hat{n}^2, P_c = n^2) \quad (39)$$

The transformation of equation (37) results in two independent variables: Φ is a function of $\hat{n} - n$ and $\hat{n}$. It's not necessary to display the dependence of Φ on $\hat{n}$ since by itself $\hat{n}$ does enter into the dynamics.

Equation (38) is also a very general statement of the conditions for power recovery or decay, and analogous to equation (35) we have

$$\Phi(\hat{n} - n) \geq 0 \text{ if } \hat{n} - n \geq 0 \quad (40a)$$

$$\Phi(\hat{n} - n) < 0 \text{ if } \hat{n} - n < 0 \quad (40b)$$

There remains now the problem of connecting equations (38) to (40) to equations (11) and (25). We repeat the latter two:

$$\frac{dN}{dt} = N_{\max} K, \text{ if } \left(\frac{dQ}{dt}\right) > 0 \quad (41)$$

$$\frac{dN}{dt} = N_{\max} K + \frac{N}{Q}\frac{dQ}{dt}, \text{ if } \left(\frac{dQ}{dt}\right) \leq 0 \quad (42)$$

We next simplify equations (41) and (42) by introducing the dimensionless power, and the dimensionless time, respectively, through the equations

$$q \equiv \frac{Q}{Q_m} \quad (43)$$

$$\tau \equiv Kt \quad (44)$$

In place of equations (41) and (42) we get

$$\frac{dn}{d\tau} = 1, \text{ if } \left(\frac{dq}{dt}\right) > 0 \quad (45)$$

$$\frac{dn}{d\tau} = 1 + \frac{n}{q}\frac{dq}{d\tau}, \text{ if } \left(\frac{dq}{dt}\right) \leq 0 \quad (46)$$

The power equation now becomes

$$\frac{dq}{d\tau} = -\frac{\Phi(\hat{n} - n)}{KQ_m} \quad (47)$$

The function, $\Phi(\hat{n} - n)$, has the dimensions of power per unit time, as can be seen from inspection of equation (38). We can express $\Phi(\hat{n} - n)$ in the power series

$$\Phi(\hat{n} - n) = C\sum_{m=1}^{m=\infty} b_m (\hat{n} - n)^m = C\Gamma(\hat{n} - n) \quad (48)$$

$$\Gamma(\hat{n} - n) = \sum_{m=1}^{m=\infty} b_m (\hat{n} - n)^m \quad (49)$$

The dimensionality of $\Phi(\hat{n} - n)$ is expressed entirely in the positive constant, *C*; it has the dimensions of power per unit time. The coefficients in the expansion, b_m, are dimensionless constants. Since the product, KQ_m, has the dimensions of power per unit time, we define the dimensionless constant

$$\mathrm{A} \equiv \frac{C}{KQ_m} \quad (50)$$

From equation (40), we deduce the function Γ, satisfies the equation

$$\Gamma(\hat{n} - n) \geq 0 \text{ if } \hat{n} - n \geq 0 \quad (51a)$$

$$\Gamma(\hat{n} - n) < 0 \text{ if } \hat{n} - n < 0 \quad (51b)$$

Equation (47) becomes

$$\frac{dq}{d\tau} = -\mathrm{A}\Gamma(\hat{n} - n) \quad (52)$$

Equations (45) and (46) now become, respectively

$$\frac{dn}{d\tau} = 1, \;\; if \;\; n > \hat{n} \tag{53}$$

$$\frac{dn}{d\tau} = 1 + \frac{n}{q}\frac{dq}{d\tau}, if \;\; n \leq \hat{n} \tag{54}$$

Substituting equation (52) into equation (54) we get

$$\frac{dn}{d\tau} = 1 + \frac{n}{q}\mathrm{A}\Gamma(\hat{n} - n), if \;\; n \leq \hat{n} \tag{55}$$

To complete the analysis we need to include the constraint of equation (8); it is restated here: the <u>maximum</u> amount of equipment that can support network telecommunications operations at time, *t*, is

$$N_{DM} = N_{\max}\left(\frac{Q(t)}{Q_m}\right) \tag{56}$$

Thus,

$$\frac{N}{N_{\max}} = n \leq \frac{N_{DM}}{N_{\max}} = \left(\frac{Q(t)}{Q_m}\right) = q \tag{57}$$

Solving equations (52), (53) and (55) subject to initial conditions and the constraint of equation (57): $n : q$, provides a solution to the interdependency problem. Aside from the variability of the initial conditions, there is only one dimensionless parameter in the equations, and that is “ A ”.

Examination of the interdependency equations shows there are four dimensionless parameters that determine whether the combined systems will recover or collapse. These are: (1) $\hat{n}$, the fraction of telecommunication equipment required to provide a positive growth of power, (2) $n_0 = n(\tau = 0)$, the fraction of telecommunication equipment at the beginning of the joint power-telecomm recovery process, (3) $q_0 = q(\tau = 0)$, the fraction of power available at the beginning of the joint power-telecom recovery process, and (4), A, the dimensionless ratio defined by equation (50).

What do we mean by the “beginning of the joint power-recovery process”? We mean the time beginning *after* the abrupt change from backup power to dependence on the power grid. This is the time $t \geq t_{0+}$. Since power generation cannot change instantaneously, $Q(t_{0-}) = Q(t_{0+}) = Q(t_0)$, and

$$q_0 = \left(\frac{Q(t_0)}{Q_m}\right) \tag{58}$$

The starting level of telecom equipment is N_2, defined by equation (27), which is repeated here to emphasize its dependence on N_1, the amount of telecom equipment recovered during the backup power phase.

$$N_2 = N_1\left(\frac{Q(t_0)}{Q_m}\right) \tag{59}$$

We then have

$$n_0 = \frac{N_2}{N_{\max}} \tag{60}$$

The dimensionless constant: $A = C / KQ_m$, contains three parameters. We can show both analytically and numerically that recovery is associated with small values of *A* as opposed to large values of *A*. It’s interesting to explore how the parameters: K, C and Q_m affect the joint recovery.

First, lets consider *K*. Recall the *K* tells us how fast the telecom folks can fix their equipment. The dimensionality of, *K*, is $(\text{time})^{-1}$; let’s write

$$K = \frac{1}{\mathrm{T}_{tel}} \tag{61}$$

We define the recovery time constant for telecom equipment as T_{tel}. A large value of *K* means that T_{tel} is small, *A* is small, and recovery will be rapid for the same value of (C / Q_m).

We next identify the ratio associated with power recovery:

$$\mathrm{T}_{pwr} = \frac{Q_m}{C} \tag{62}$$

Inserting equations (61) and (62) into equation (50) gives the following interpretation of *A*:

$$A = \frac{T_{tel}}{T_{pwr}} \tag{63}$$

The initial conditions are $q(\tau = 0) = q_0$, and $n(\tau = 0) = n_0$. The first observation we make is that if $n_0 > \hat{n}$, the joint power-telecom system always recovers, although the time scale for recovery will depend on the parameters. If $n_0 > \hat{n}$, we see from equation (51b) that $\Gamma(\hat{n} - n)$ is negative and $-A\Gamma$ is therefore positive.

$$\left(\frac{dn}{d\tau}\right)_{\tau=0+} = 1 \tag{64}$$

$$\left(\frac{dq}{d\tau}\right)_{\tau=0+} = -A\Gamma(\hat{n} - n_0) > 0 \tag{65}$$

From equation (64) we see that *n* increases linearly with time, and $\Gamma(\hat{n} - n)$ becomes even more negative. From equation (52) this makes $(dq/d\tau)$ even larger than it was originally. And so we have a process in which both *n* and *q* continually increase. This is defined as *positive reinforcement*. This growth continues until $n = q$, and they both continue to recover at the same rate; or *q* reaches its terminal value: $q = 1.0$ first and then *n* catches up.

For the case in which $n = q$ before full recovery is reached, there is only one equation of motion: we replace *n* by *q* in equation (52) and get

$$\frac{dq}{d\tau} = -A\Gamma(\hat{n} - q) \tag{66}$$

Equation (84) applies from the moment $n = q$ until $q = 1.0$.

When $n_0 < \hat{n}$, the behavior of the system is more complex. Depending on the parameters the power-telecom system may either recover or collapse. Initially, $\Gamma(\hat{n} - n_0)$ is positive and the equations are

$$\left(\frac{dn}{d\tau}\right)_{\tau=0+} = 1 - \frac{n_0}{q_0} A\Gamma(\hat{n} - n_0) \tag{67}$$

$$\left(\frac{dq}{d\tau}\right)_{\tau=0+} = -A\Gamma(\hat{n} - n_0) \tag{68}$$

Because $\Gamma(\hat{n} - n_0)$ is positive, *q* begins to decrease. The term: $(n_0 A\Gamma(\hat{n} - n_0)/q_0)$ is positive, and acts to slow down the recovery of *n*. If $(n_0 A\Gamma(\hat{n} - n_0)/q_0) > 1$, the dimensionless telecom equipment variable, *n*, will also begin to decrease. This leads to a situation in which $\Gamma(\hat{n} - n)$ continuously increases, causing continuous decay and ultimate collapse of the system.

If the condition: $0 < (n_0 A\Gamma(\hat{n} - n_0)/q_0) < 1.0$ exists, we have a horserace. There is initially an increase in *n*. But is this growth sufficient to counteract the continuous decay in power? There's really no way to make a generalization. We must use specific models. For illustrative purposes we use a simple linear form for $\Gamma(\hat{n} - n)$. We have

$$\Gamma(\hat{n} - n) = \hat{n} - n \tag{69}$$

The dynamic equations then become

$$\frac{dq}{d\tau} = -A(\hat{n} - n) \tag{70}$$

$$\frac{dn}{d\tau} = 1 - \frac{n}{q} A(\hat{n} - n) \tag{71}$$

We have examined the behavior of $n(\tau)$ and $q(\tau)$ as a function of τ for a variety of values of $\hat{n}, n_0, q_0$ and *A*. Two examples are given in **Figure 5** and **6**.

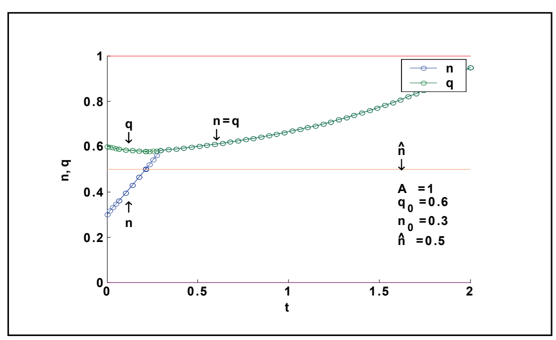

Figure 5. Dimensionless Power and Dimensionless Telecommunications Variables as a Function of

Dimensionless Time for $q_0 = 0.6, n_0 = 0.3, \hat{n} = 0.5$ and $A = 1.0$

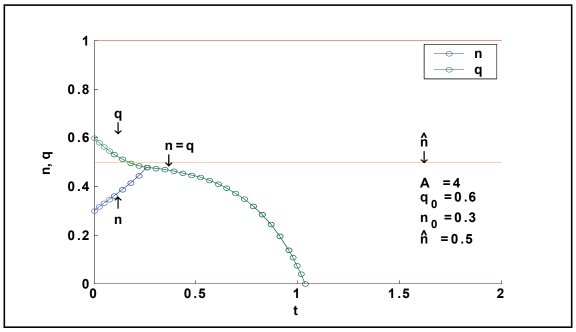

Figure 6. Dimensionless Power and Dimensionless Telecommunications Variables as a Function of Dimensionless Time for $q_0 = 0.6, n_0 = 0.3, \hat{n} = 0.5$ and $A = 4.0$

Figure 5 is the solution for $A = 1.0$. The initial counteractive term, $-(n_0 A\Gamma(\hat{n} - n_0)/q_0)$, is less than 1.0. As observed in this figure, *n* increases rapidly while *q* decreases relatively slowly, and at about $\tau = 0.25,\ n = \hat{n}$. At this point Γ passes through zero, subsequently becoming negative. When this happens, *q* begins to increase instead of decrease, and *n* increases faster than it did before $n = \hat{n}$. Shortly afterwards *n* reaches *q*, and they both recover subject to the constraint: $n = q$. Eventually, $n = q = 1.0$ and we have simultaneous full recovery of both systems (this recovery point is off the scale to the right in **Figure 5.**.

Figure 6 is where our luck runs out. This is the case where $A = 4.0$. The negative rate of change of *q* is now too fast to allow *n* to exceed $\hat{n}$. The combined system goes into complete collapse at around $\tau = 1.0$.

DISCUSSION

In the near future, the EPG may rely on public data networks for more than 50% of its real-time data needs. We have identified scenarios caused by HEMP where this level of dependence could lead to a condition of *negative reinforcement*—EPG and supporting telecommunication networks both collapse due to their mutual interdependence. Our theory that predicts negative reinforcement also predicts conditions for the opposite case, *mutual recovery*—both infrastructures recover when they are dynamically coupled. The scenario we examine is one where: (1) the EPG depends heavily on the real-time data supplied by the PDN, and (2) the backup power for telecommunications runs out, and it must rely completely on commercial power. When an explicit model of the EPG's dependence on data flow becomes available, we'll be able to predict recovery or collapse for specific coupled power-telecom systems in specific HEMP scenarios. It is also desirable to examine the case when there is a temporal distribution of back-up power outage times.

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BIOGRAPHIES

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