Interaction Notes

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Desirable Antenna Characteristics for Late-Time Target Identification

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Abstract

All antennas, being limited in their low-frequency response, cannot radiate only an approximate delta function. The remaining part(s) of the pulse (to give net zero area, or complete time integral) can appear before and/or after the delta function. For late-time target identification we find here that these other parts are better placed before rather than after the approximate delta function.

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1. Introduction

An antenna is required to send an interrogating electromagnetic wave at a target. An antenna (a different antenna or the same antenna) is required to receive the scattered wave from the target. In both cases the antenna characteristics mix with the target response function to complicate the target identification. What, then, can we do to simplify this problem? The antennas can be designed to give the best possible transient response (or, equivalently, transfer function).

As we know, antennas cannot radiate DC (to the far field). As such, within physical limits [2] an antenna cannot radiate a pure delta function (impulse). Exciting the radiating antenna with a step function, the antenna will radiate a wave with at least one zero crossing. The area (complete time integral) of the radiated waveform must be zero [1]. In reception, the antenna response is the time derivative of its response in transmission [3]. Thus in reception the response to a delta-function wave must also have at least one zero crossing.

An important class of antennas for use with transients is that of impulse radiating antennas (IRAs) [2], which come in at least three types: reflector, lens (including small-angle TEM horns), and array. An important characteristic is that they radiate an approximate delta function when driven by a step-function voltage. Even then, however, the foregoing limitations concerning zero area still apply. The same applies to their receiving response to a delta function.

This leads to the question discussed in this paper. Which of these types of IRAs is most suited for use in late-time target identification?

While the antenna characteristics are, in general, vector-valued convolution operators, and the scatterer (target) characteristics are, in general, dyadic-valued convolution operators, let us simplify the problem for our analysis. Consider only a single polarization in radiation, scattering and reception. This simplifies the problem to a scalar one. Let the target delta-function response be

$$f(t) = f_E(t) + f_L(t)$$

$$f_E(t) = \begin{cases} 0 & \text{for } t<0 \\ 0 & \text{for } t \geq t_L \end{cases} \quad \text{(early-time response, entire function, temporal form)} \tag{1.1}$$

$$f_L(t) = 0 \text{ for } t < t_L \quad \text{(late-time response)}$$

The late-time is characterized by the ability to describe the response by a set of damped sinusoids (only) [7, 8]. For present illustration we take a single damped sinusoid for the late-time response as

$$f_L(t) = \left[R_\alpha e^{s_\alpha[t-t_L]} + R_\alpha^* e^{s_\alpha^*[t-t_L]}\right] u\left(t-t_L\right) \tag{1.2}$$

While R_α is in general a complex number, this can be considered as a phase factor and we might take

$$f_L(t) = 2 R_\alpha e^{\mathrm{Re}(s_\alpha)[t-t]} \cos\left(\mathrm{Im}(s_\alpha[t-t_L])u(t-t_L)\right) \tag{1.3}$$

For illustration, but $\sin(Im(s_\alpha[t-t_L])$ would do as well.

While (1.1) characterizes the target response to a delta-function wave, it has been shown that it applies to the time integral (step response) and second time integral (ramp response) as well [9]. This results from the fact that the target delta-function goes as s^2 as $s \to 0$ where

$\sim$ ≡ two-sided Laplace transform

$s = \Omega + j\omega$ = Laplace-transform variable or complex frequency

s_α ≡ natural frequency

$*$ ≡ complex conjugate

R_α ≡ pole residue

This assures that

$$f_L(t) \to 0 \text{ as } t \to \infty \tag{1.5}$$

For delta, step, and ramp responses. The entire-function $f_E(t)$, being time-limited must go to zero as $t \to \infty$ faster than any exponential (including, in general, a step to zero).

For later use we can write (1.1) as

$$f_\delta(t) = f_{\delta E} + f_{\delta L}(t) \tag{1.6}$$

As the target delta-function response. We then also have

$$f_u(t) = f_{uE} + f_{uL} = \int_{0_-}^{t} f_\delta(t')\,dt' =$$

$$f_{uE}(t) = \int_{0_-}^{t_\ell} f_{\delta E}(t')\,dt' \qquad (1.7)$$

$$f_{uL}(t) = \int_{t_\ell}^{t} f_{\delta L}(t')\,dt'$$

as the step-function response (also going to zero at late time).

2. Ideal Reflector- and Lens-IRA Temporal Responses

Figure 1.1 illustrates the ideal reflector- and lens-IRA responses which are described mathematically by

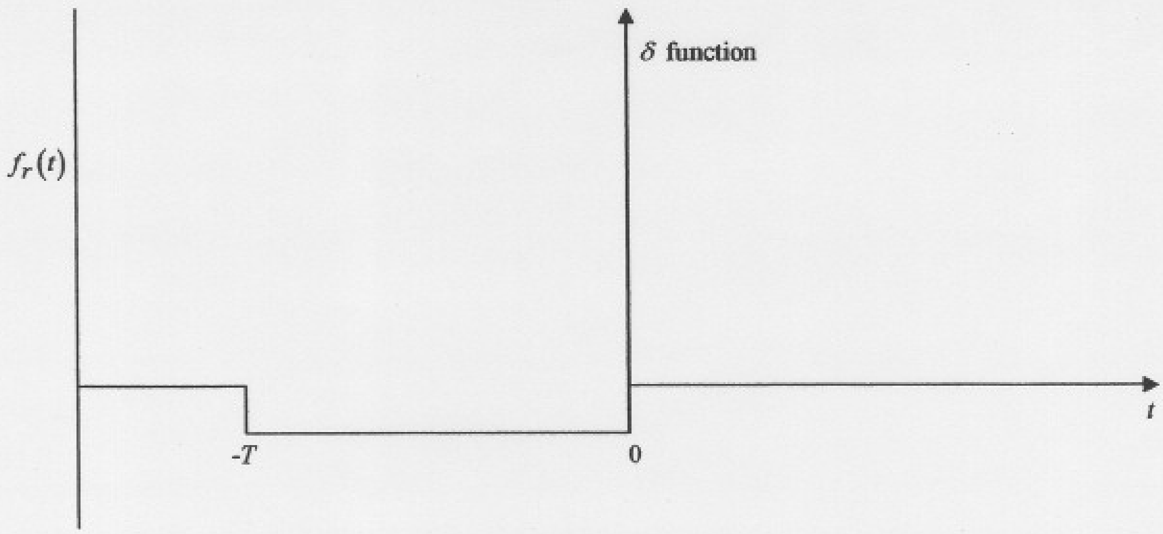
$$\begin{aligned} f_r(t) &= \underbrace{\delta(t)}_{\substack{\text{main}\\ \text{event}}} \underbrace{-\frac{1}{T}\left[u(t+T) - u(t)\right]}_{\text{prepulse}} \quad \text{(reflector IRA)} \\ f_\ell(t) &= \underbrace{\delta(t)}_{\substack{\text{main}\\ \text{event}}} \underbrace{-\frac{1}{T}\left[u(t) - u(t-T)\right]}_{\text{postpulse}} \quad \text{(lens IRA)} \end{aligned} \tag{2.1}$$

Here, for convenience, the *main event* (the delta-function part) has been placed at $t = 0$. The reflector IRA then also has a prepulse [2, 4], and the lens IRA has a postpulse [5], both consisting of a difference of step functions. The complete time integral of both functions in (2.1) is zero. For present purposes these functions are for step response in transmission and impulse response in reception.

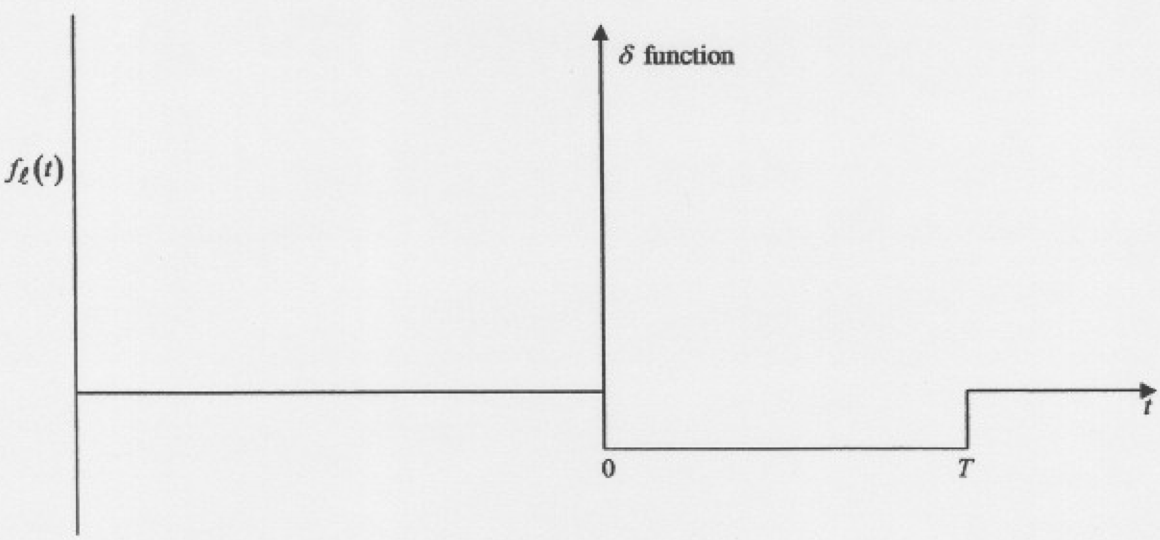
The widths of both prepulse and postpulse are taken as the same T for convenience, but they need not be the same. For the reflector IRA T is 2 F/c where F is the focal length of the paraboloidal reflector and c is the speed of light. For the lens IRA (including small-angle TEM horns) T is twice the transit time on the TEM horn. Array IRAs are similar to lens IRAs in that the delta function comes first [6]. To give zero area, then a postpulse is required, but the details of this waveshape have not been studied.

One way to remove the effects of the prepulse and postpulse is deconvolution. This can be accomplished in various ways, such as by transformation into frequency domain, division of the target scattering by the antenna-response spectrum, and inverse transformation back to time domain. Suppose, however, that one does not use such deconvolution, for whatever reason, such as simplicity, or the presence of nonlinear elements (such as a TR switch) in the system. How, then, do the prepulse and postpulse affect the late-time target identification?

While both transmit and receive antennas enter into the total system response, let us consider just one of these antennas. Applying the results twice accounts for both.



A. Reflector IRA



B. Lens IRA

Fig. 2.1 Ideal IRA Responses

3. Convolution of Target Response with Reflector IRA

Convolving the reflector-IRA response with the target delta-function response gives

$$
\begin{aligned}
f_{\delta r}(t) &= f_r(t) \circ f_\delta(t) = f_\delta(t) + \frac{1}{T}\left[f_u(t) - f_u(t+T)\right] \\
&= f_{\delta E}(t) + f_{\delta L}(t) + \frac{1}{T}\left[f_{uE}(t) + f_{uL}(t) - f_{uE}(t+T) - f_{uL}(t+T)\right]
\end{aligned}
\tag{3.1}
$$

$\circ \equiv$ convolution with respect to time, t

Here we see the desired response $f_\delta(t)$. At the same time its integral $T^{-1}f_u(t)$ appears. In the late time, $t > t_L$, both terms have the same damped sinusoid with complex frequency s_α. The residue (amplitude) is changed but the s_α is unchanged.

The next term $-T^{-1}f_u(t+T)$ begins at $t = -T$ and the late-time portion begins at $t = -T + t_L$. The late-time portion of this waveform begins a time T before that of the previous two terms.

Let us now consider what happens after the "main event", namely the delta-function part. Going out to time t_ℓ for late-time identification we have three terms to consider. The first two (considered above) give a single damped sinusoid with the same s_α. The third term has its late-time start at $-T + t_\ell$. At time t_L this term contributes also a damped sinusoid of the same natural frequency s_α, but reduced in amplitude and shifted in phase by the factor

$$
e^{s_\alpha T} = e^{\mathrm{Re}(s_\alpha)T}\left[\cos(\omega_\alpha T) + j\sin(\omega_\alpha T)\right]
\tag{3.2}
$$

Together with the conjugate. Since $\mathrm{Re}[s_\alpha]T < 1$ the amplitude is significantly reduced at time t_L after the main event. If we begin our late-time identification at this time the third term does not interfere with the identification, merely adding some small contribution to the residue.

4. Convolution of Target Response with Lens IRA

Convolving the lens-IRA response with the target delta-function response gives

$$\begin{aligned} f_{\delta\ell}(t) &= f_\ell(t) \circ f_\delta(t) \\ &= f_\delta(t) - \frac{1}{T}\left[f_u(t) - f_u(t-T)\right] \\ &= f_{\delta E}(t) + f_{\delta\ell}(t) - \frac{1}{T}\left[f_{uE}(t) + f_{uL}(t) - f_{uE}(t-T) - f_{uL}(t-T)\right] \end{aligned} \tag{4.1}$$

Now the desired response $f_{\delta\ell}(t)$ appears at the same time as its integral $-T^{-1}f_u(t)$ appears. This part is not a problem.

The next term $T^{-1}f_u(t-T)$ begins at $t = +T$ and the late-time portion begins a time T *after* that of the previous two terms. *This is a problem.*

Now after the main event, if we begin our late-time identification at time T_L, we have the third term mixing into the time window we are trying to analyze in an undesirable way. First $T^{-1}f_{uE}(t-T)$ intrudes into this window from time T_L to T_L+T, confusing the identification since we have more than our original damped sinusoid during this part of the late time. Second $T^{-1}f_{uL}(t-T)$ begins a damped sinusoid of complex frequency s_α at a time T_{L+T}, mixing with the "main" damped sinusoid beginning at time T_L. If one attempts to match these two damped sinusoids with a single damped sinusoid beginning at time T_L, then error is introduced in the estimation of s_α.

To estimate this latter error let us consider the combination

$$e^{s_\alpha[t-T_L]}u(t-T_L) + a\ e^{s_\alpha[t-T_L-T]}u(t-T_L-T) \tag{4.2}$$

At $t = T_L$ this has amplitude 1. At $t = T_L + T$ this has amplitude

$$e^{s_\alpha T} + a \qquad (a \text{ in general small and complex}) \tag{4.3}$$

Attempting to fit this with a damped sinusoid of complex frequency at the two points gives

$$
\begin{aligned}
e^{s'_\alpha T} &= e^{s_\alpha T} + a \\
s'_\alpha T &= \ell n\left(e^{s_\alpha T} + a\right) = s_\alpha T \;\ell n\left(1 + a e^{-s_\alpha T}\right) \\
&\cong s_\alpha T \, a \, e^{-s_\alpha T} \text{ for small } a\,e^{-s_\alpha T}
\end{aligned} \tag{4.4}
$$

However, $e^{-s_\alpha T}$ can be quite large, in which case

$$
s'_\alpha T = \ell n(a) \ell n\left(1 + \frac{e^{s_\alpha T}}{a}\right) \simeq \frac{\ell n(a)}{a} e^{s_\alpha T} \text{ for large } a\,e^{-s_\alpha T} \tag{4.5}
$$

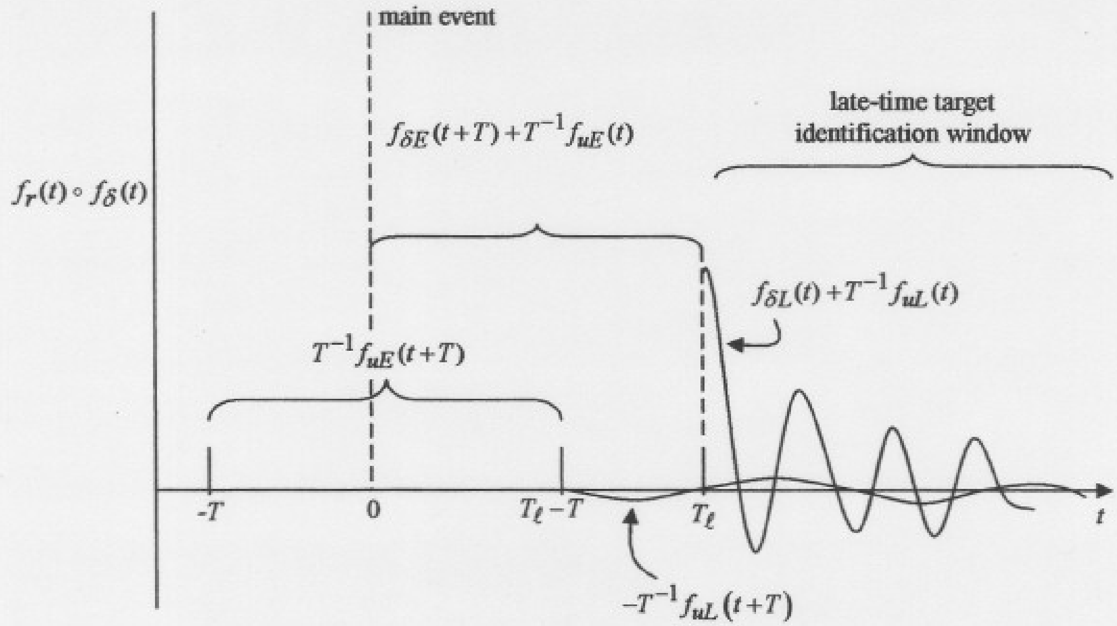
Thus significant errors can occur. Of course, the situation is more complicated since one is attempting to fit (4.2) over an interval, not just two points.

To avoid these errors, one can redefine the target-identification window to begin at $t = T_L + T$. Hwoever, the response to the main event has decayed to smaller values at this later time, leading to potential signal-to-noise-ratio and signal-to-clutter-ratio problems.

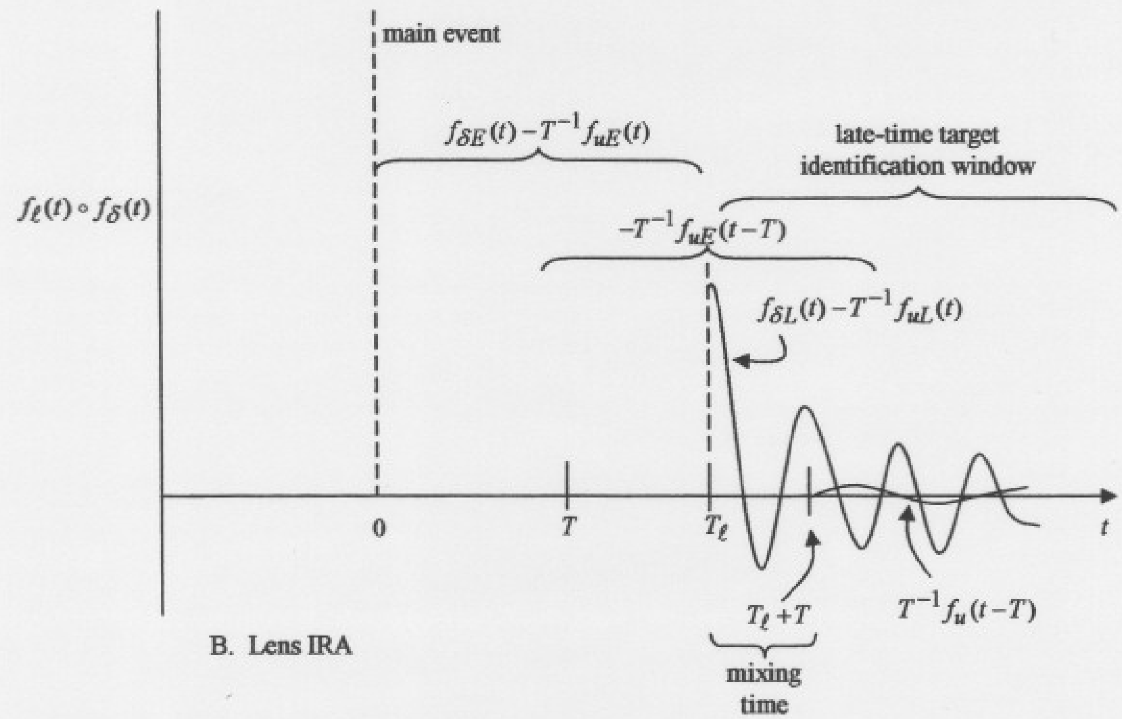
5. Comparison of Responses

Figure 5.1 gives a graphical depiction of the responses which we have previously discussed. This shows the damped sinusoid with complex frequency s_α being the only signal for analysis for $t > t_L$ in the case of the ideal reflector IRA.

For the lens IRA, on the other hand, we have a portion of the late-time window (the mixing time) from T_L to $T_L + T$ with other signals, confusing the identification. The example has $T < T_L$. For $T > T_L$ we still have the problem, except more severely due to the extention of unwanted signals yet more into the late time response from the main event.



A. Reflector IRA



B. Lens IRA

Fig. 5.1 Comparison of Responses

6. Concluding Remarks

The analysis here is somewhat simplified due to the ideal-antenna assumptions. Furthermore, the parameter, T, need not be the same for the various antennas one might consider.

While the discussion here is in terms of a single antenna (in transmission or reception), the results apply to the combination of transmission and reception, whether with two antennas (bistatic) or a single antenna (monostatic). Merely apply the previous analysis twice.

The basic lesson here is that, for late-time target identification, it is better to have the "bad" or "unwanted" parts of the antenna response before instead of after the main event (approximate delta function).

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