Interaction Notes

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Second Time Integral of the Impulse Response for Enhancing the Late-Time Target Response for Target Identification

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Abstract

There is a dynamic-range problem in using the late-time response, containing the natural-frequency information, for target identification. This can be mitigated by analog filtering of the data before digitization. A special filter of interest is the second time integral of the impulse response because of its special physical properties.

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1. Introduction

A general problem, concerning target identification via the late-time aspect-independent complex natural resonant frequencies, is the presence of the large early-time response. It would be advantageous to enhance the late-time response relative to the early-time response. In a general sense this implies some kind of frequency-dependent filter which attenuates the high-frequency parts of the waveform while providing little or no attenuation to the complex resonant frequencies of interest. Such a low-pass filter (passive and/or active) is the subject of this paper.

A desirable feature of such a filter is the temporal separation of early- and late-time responses. Furthermore we would like to be able to still use time gating (windowing) to remove some of the clutter in the signal returned to the radar. So we do not wish to introduce significant dispersion into the scattering data.

There are other problems associated with the signal including the response(s) of the radar antenna(s) and propagation characteristics (e.g., multipath). For present purposes we neglect such problems and assume that appropriate deconvolution, gating, etc., have been applied to give the target delta-function response.

2. Multiple Integration of the Target Delta-Function Response

Consider first a related problem, low-frequency radiation from an antenna and its implication for pulse radiation. In [1] we have observed that, if a step-like voltage pulse is applied to an electric dipole, the low frequency content of the radiation is proportional to s ($= \Omega + j\omega$, the Laplace transform variable or complex frequency). This in turn implies that the radiated pulse must have at least one zero crossing. If the late-time voltage pulse is allowed to decay to zero, then the radiated pulse must have at least two zero crossings.

Consider now the far-field scattering problem. Let the incident wave be like a delta-function, i.e., one sided, decaying back to zero with finite, nonzero area (time integral). The low-frequency content of the incident field is then just the nonzero area. The scatterer is characterized at low frequencies by the induced electric and magnetic dipoles ($\vec{p}$ and $\vec{m}$) which are also constant vectors (in the low-frequency limit). The far field scattered by such dipoles is proportional to $s^2 \vec{p}$ and $s^2 \vec{m}$. If we multiply the Laplace transform of the far scattered field by s^2, the resulting low-frequency far scattered field is proportional to $\vec{p}$ and $\vec{m}$. In time domain this is a double time integral which goes to zero at late time since the area (the low-frequency limit of the Fourier transform) must be finite. (Note that the waveform is damped due to the radiation of electromagnetic energy.)

In the singularity-expansion-method (SEM) representation of the scattered field [2] we have the delta-function response

$$\vec{E}_f(t) = \frac{1}{r}\sum_{\alpha} \vec{R}_\alpha \, e^{s_\alpha [t-r/c]} u(t-r/c)$$
$$+ \text{ entire function (temporal form, early time)} \tag{2.1}$$

The second time integral then takes the general form

$$\int_{-\infty}^{t}\int_{-\infty}^{t'} \vec{E}_f(t'')dt''dt' = \frac{1}{r}\sum_{\alpha} \frac{\vec{R}_\alpha}{s_\alpha^2} e^{s_\alpha [t-r/c]} u(t-r/c)$$
$$+ \text{ new entire function} \tag{2.2}$$

Again the damped sinusoids going to zero implies that the entire-function contribution must also be zero at late times. The waveform is also zero before some time, applying as well to the multiple integrals. Here we have taken $t = 0$ as the beginning ot the late-time response (or $t - r/c$ in the far field).

From a frequency-domain point of view we have enhanced the low frequencies by changing the residues $\vec{R}_\alpha$ to $\vec{R}_\alpha s_\alpha^{-2}$. If the delta-function response at early time looks like an impulse, the first integral is a step function and the second integral is a ramp function, thereby suppressing it considerably. (At this point we can note that the early-time ramp response has also been considered by other authors [4].)

Noting that clutter in the radar return is just the scattering from other "targets", the second time integral has the same properties as above. This clutter is then not significantly stretched in time, allowing windowing about the target response to be used as before.

3. Analog Second Time Integral

In analyzing the scattering data one may wish to perform the temporal integration by analog means before digitizing the data. This is because digitizing a small signal (late-time portion) in the presence of a large signal (early-time response) can result in digitization errors due to the large dynamic range to be covered. By the use of passive or active analog integration the early-time signal and high-frequency noise can be suppressed before digitization, allowing one more accurately to represent the late-time data.

As examples let us consider some simple passive-integrator circuits. Fig. 3.1A shows an RC type, assuming the input signal is on a 50 Ω transmission line. The first integrator has a 50 Ω input impedance Z_{in}. The approximate first integral is the voltage across C_1. The integrating resistance is large compared to 50 Ω (say 5kΩ). For the second stage we need a large value resistor to sample the voltage on C_1 and integrate again as the voltage on C_2. In turn this voltage is sampled by a very large input impedance to the recorder.

A more ideal form of integrator is the RLC type, based on what is called an all-pass network. Consider the first stage in Fig. 3.2B and constrain

$$\tau_1 \equiv \frac{L_1}{R} = R\,C_1 \quad , \quad R = 50\,\Omega \tag{3.1}$$

Then if the input impedance to the second stage is 50 Ω, the transfer function of this integrator is

$$\tilde{T}_1(s) = \frac{\tilde{V}_{out}^{(1)}(s)}{\tilde{V}_{in}^{(1)}(s)} = \left[1 + s\tau_1\right]^{-1} \tag{3.2}$$

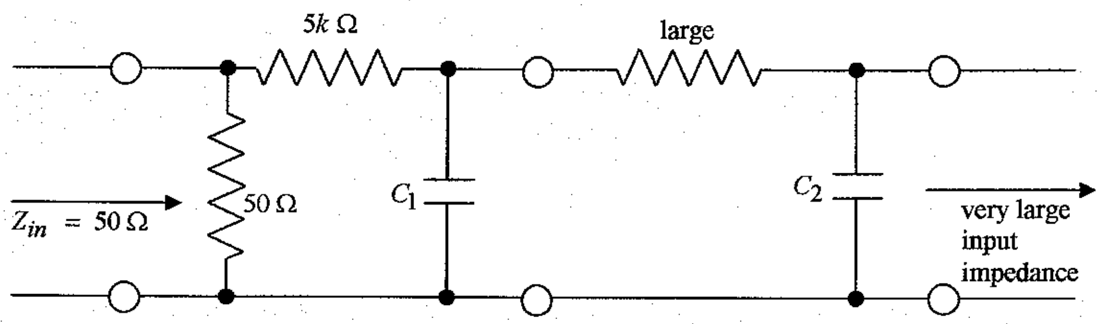
with iinput impedance

$$Z_{in} = 50\,\Omega \tag{3.3}$$

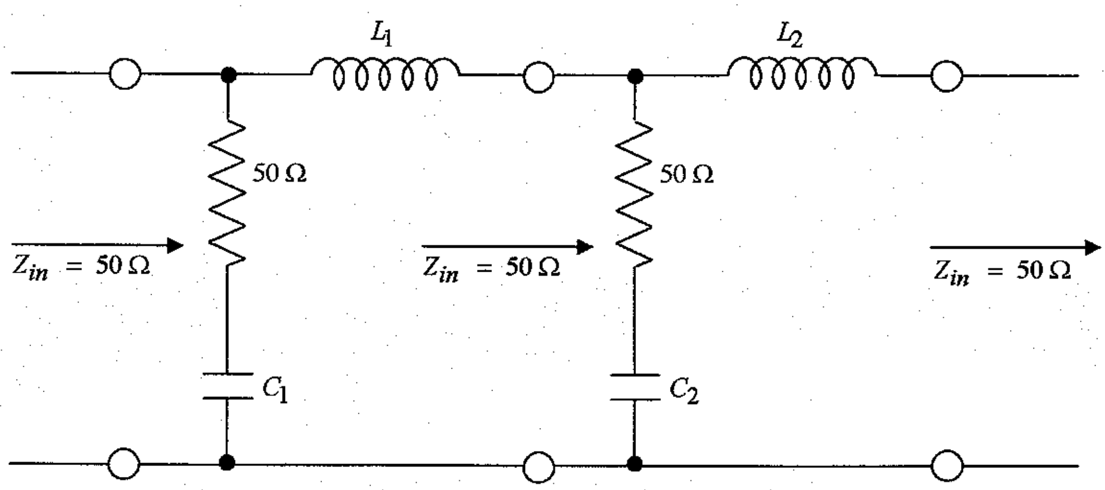
for all frequencies. Similarly for the second stage we require

$$\tau_2 \equiv \frac{L_2}{R} = R\,C_2 \tag{3.4}$$

giving an input impedance of 50 Ω provided it is terminated in 50 Ω (which might be a 50 Ω cable). The transfer function of the second integrator is



A. RC



B. LRC

Fig. 3.1 Cascaded Passive Integrators

$$\tilde{T}_2(s) = \frac{\tilde{V}_{out}^{(2)}(s)}{\tilde{V}_{in}^{(2)}(s)} = [1 + s\tau_2]^{-1} \tag{3.5}$$

where

$$\tilde{V}_{in}^{(2)}(s) = \tilde{V}_{out}^{(1)}(s) \tag{3.6}$$

The transfer function of the double integrator is then

$$\begin{aligned} \tilde{T}(s) &= \tilde{T}_1(s)\tilde{T}_2(s) = [1 + s\tau_1]^{-1}[1 + s\tau_2]^{-1} \\ &= \frac{\tilde{V}_{out}^{(2)}(s)}{\tilde{V}_{in}^{(1)}(s)} \end{aligned} \tag{3.7}$$

Rearranging we have

$$s^{-2}\tilde{V}_{in}(s) = \left[\tau_1 + s^{-1}\right]\left[\tau_2 + s^{-1}\right]\tilde{V}_{out}(s) \tag{3.8}$$

which in time domain is

$$\begin{aligned} &\int_{-\infty}^{t}\int_{-\infty}^{t'} V_{in}(t'')\,dt''dt' \\ &= \left[\tau_1\delta(t) + u(t)\right] \circ \left[\tau_2\delta(t) + u(t)\right] \circ V_{out}(t) \\ &= \left[\tau_1\,\tau_2\delta(t) + [\tau_1+\tau_2]u(t) + tu(t)\right] \circ V_{out}(t) \\ &= \tau_1\,\tau_2\,V_{out}(t) + [\tau_1+\tau_2]\int_{-\infty}^{t} V_{out}(t')\,dt' + \int_{-\infty}^{t}\int_{-\infty}^{t'} V_{out}(t'')\,dt''dt' \end{aligned} \tag{3.9}$$

$\circ \equiv$ convolution with respect to time.

This is another example of "analytic deconvolution" to add to those in [3].

This gives a simple correction technique to obtain the second time integral of $\tilde{V}_{in}^{(1)}(t)$. This removes the requirement that times of interest be short compared to τ_1 and τ_2. So we can choose τ_1 and τ_2 such that the complex resonances of interest are not significantly attenuated before digitizing the signal, while significantly attenuating the higher frequencies.

4. Concluding Remarks

There are various possible filters one may apply to the scattering data to emphasize the important information for target identification. Here we have explored the second time integral of the delta-function response because of its special properties in emphasizing the late-time response without introducing dispersion of the early-time data into the late-time regime. Having applied such a filter, the various natural-frequency identification techniques such as E-pulse, matrix pencil, etc., are still applicable.

In implementing the second time integral there is some advantage in using analog filters before digitizing the data. This can reduce the impact of digitization errors in the late-time data, effectively increasing the dynamic range.

References

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