Interaction Notes

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Symmetry in Target Recognition

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Abstract

Symmetry has important consequences for target recognition. Rotation and reflection symmetry for targets near a ground or water surface not only help to identify the targets, but also aid in data processing to increase accuracy. While clutter (other scatterers) interfere with the target signatures, symmetry in the clutter can be used to suppress this interference.

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1. Introduction

Symmetry can play an important rule in radar target recognition. Symmetries in the target lead to symmetries in the scattered fields. This in turn places symmetry requirements on the radar antennas for polarization purity [1-3]. All of this is based on the group structure of the symmetries [9].

The target symmetries can be used in the processing of the scattering data to improve the resolution of the target signatures, including in the synthetic aperture-radar (SAR) sense. Clutter (from other scattering objects) is a large problem in target recognition. SAR focusing is often used to reduce the clutter. Time windowing can also be used for this purpose with a wideband pulse radar (or a radar with lots of frequencies and phase coherence). If the scatterers producing the clutter have symmetries, such can potentially be exploited for clutter reduction.

In the present context we consider two-dimensional symmetries, appropriate to targets on or near a ground (or water) surface. The radar is assumed to be far away from the target so that the incident wave can be approximated as a plane wave. This is appropriate to electromagnetic singularity identification (EMSI) for which the wavelengths of interest are of the order of the scatterer dimensions [5, 10] so as to excite the appropriate complex natural frequencies.

Without going into detail here, a very important property of target identification using complex natural frequencies (in the late-time response) is the fact of their independence of the incident wave parameters (direction of incidence and polarization). These are the locations of poles in the complex-frequency (or s) plane. The residues, however, are functions of the incident-wave parameters. One can then use the invariance of the natural frequencies to process the scattering under various incident conditions with the constraint that the natural frequencies must be the same in all returning waveforms, allowing for the possibility of zero residues for particular poles under special incidence conditions [8]. In the present paper we consider some of the implications of symmetry in conjunction with the above.

2. Continuous Rotation/Reflection Symmetry: $C_{\infty a} = O_2$

Let us first briefly review the special case of a body of revolution with axial (vertical) symmetry planes (infinitely many). In the presence of an assumed horizontal ground or water surface we can still have such symmetry provided the target rotation axis is orthogonal to this surface (i.e., is vertical).

The important result for such a target is that in the usual h, v radar coordinates there is no cross pol (hv) component in the scattered field (the vampire signature) [5]. This has been experimentally demonstrated [7, 11].

Referring to Fig. 2.1, if the radar moves on a track (variable x) of constant $(y, z) = (d, h)$ the elevation angle ψ varies as x varies in the form

$$\psi = \arctan\left(\frac{h}{\left[d^2 + x^2\right]^{1/2}}\right) \tag{2.1}$$

which is an even function of x. So we expect identical backscattering from x and $-x$, which can be used to improve signal-to-noise ratio. Of course the same natural frequencies appear for all ψ with residues a function of x.

Of course, one need not have a straight-line track. Referring to (2.1) we can keep ψ constant by appropriately varying h (i.e., z) as a function of x. Noting that the distance to the target is also a function of x one can correct the scattering for the r^{-1} variation of the fields to and from the target. The angle of the radar antenna(s) can also be adjusted to maintain a constant orientation to the target (spotlight SAR).

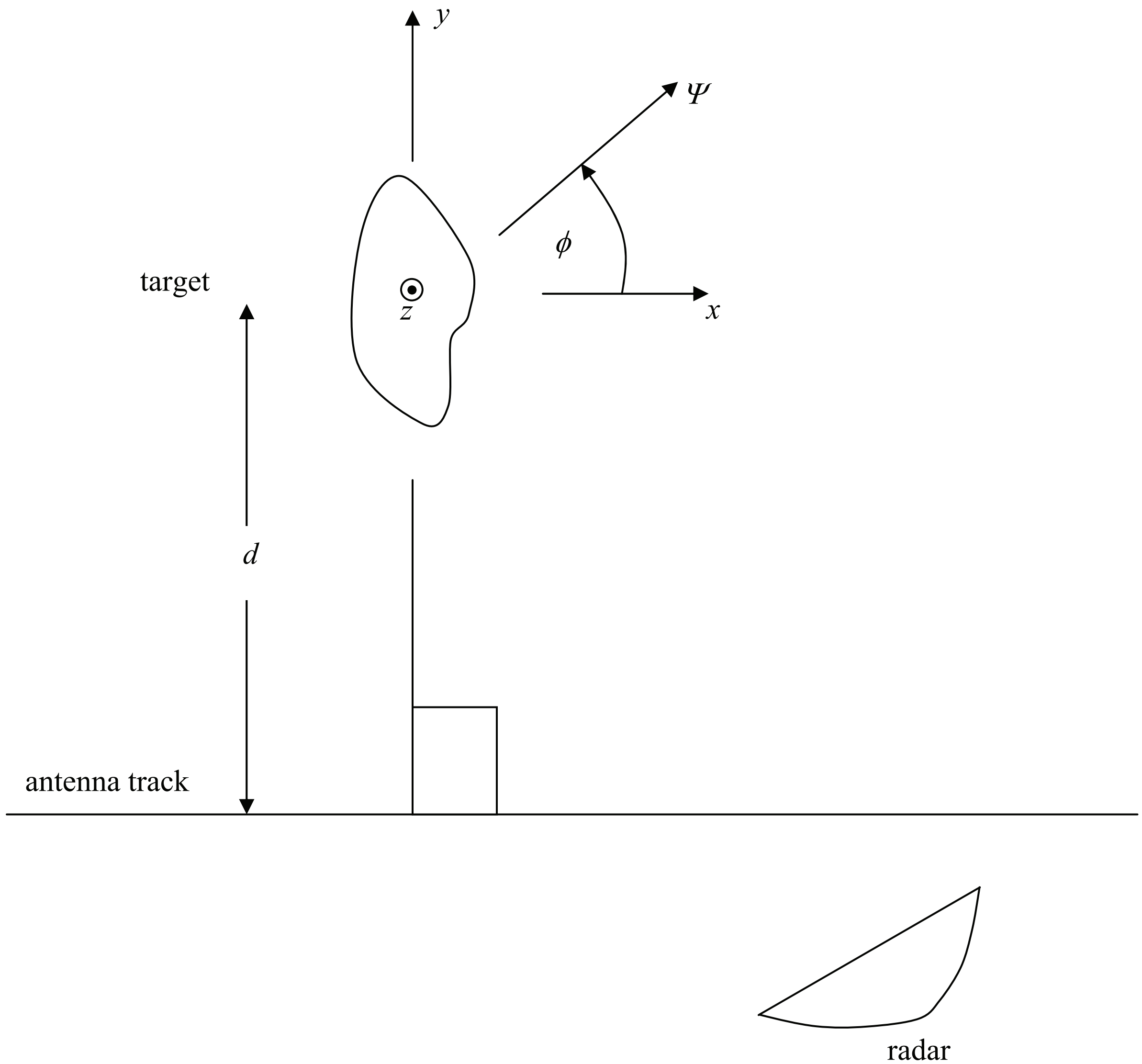

Fig. 2.1 Straight-Line Antenna Track Past Target

3. Discrete Rotation Symmetry C_N

Consider now some more complicated symmetries. For discrete rotation symmetry we have the group representation

$$
\begin{aligned}
\mathrm{C}_N &= \left\{ \left(C_{n,m}(\phi_\ell) \right) \middle| \ell = 1, 2, \ldots, N \right\} \\
\left(C_{n,m}(\phi_\ell) \right) &= \begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{pmatrix} = \exp\left(\phi \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right) \\
\phi_\ell &= \frac{2\pi\ell}{N} \\
\left(C_{n,m}(0) \right) &= \left(C_{n,m}(2\pi) \right) = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}
\end{aligned}
\tag{3.1}
$$

where we have used the two-dimensional form (since z does not enter). Our cylindrical coordinates (Ψ, ϕ) are related by

$$
\begin{aligned}
x &= \Psi \cos(\phi) \\
y &= \Psi \sin(\phi)
\end{aligned}
\tag{3.2}
$$

centered on the target. (See Fig. 2.1.)

This discrete rotation symmetry implies that, if one varies the look angle around the target by ϕ_1 or a multiple thereof, one should get the same backscatter (monostatic, or even bistatic if both transmitter and receiver are moved together by the same angle with respect to the target). For a detailed discussion of the symmetry properties of such a target, including its natural modes, see [6].

For our present discussion, note that as the radar follows a linear track the target signature is periodic in ϕ. Depending on the size of N one can encounter repeats of the same signature as one moves along the track (allowing for r^{-1}, elevation angle ψ, and radar orientation). Referring to Fig. 3.1 we can see that we need

$$
\phi_1 < \pi \tag{3.3}
$$

For such repeating signatures. This implies that

$$
N \geq 3 \tag{3.4}
$$

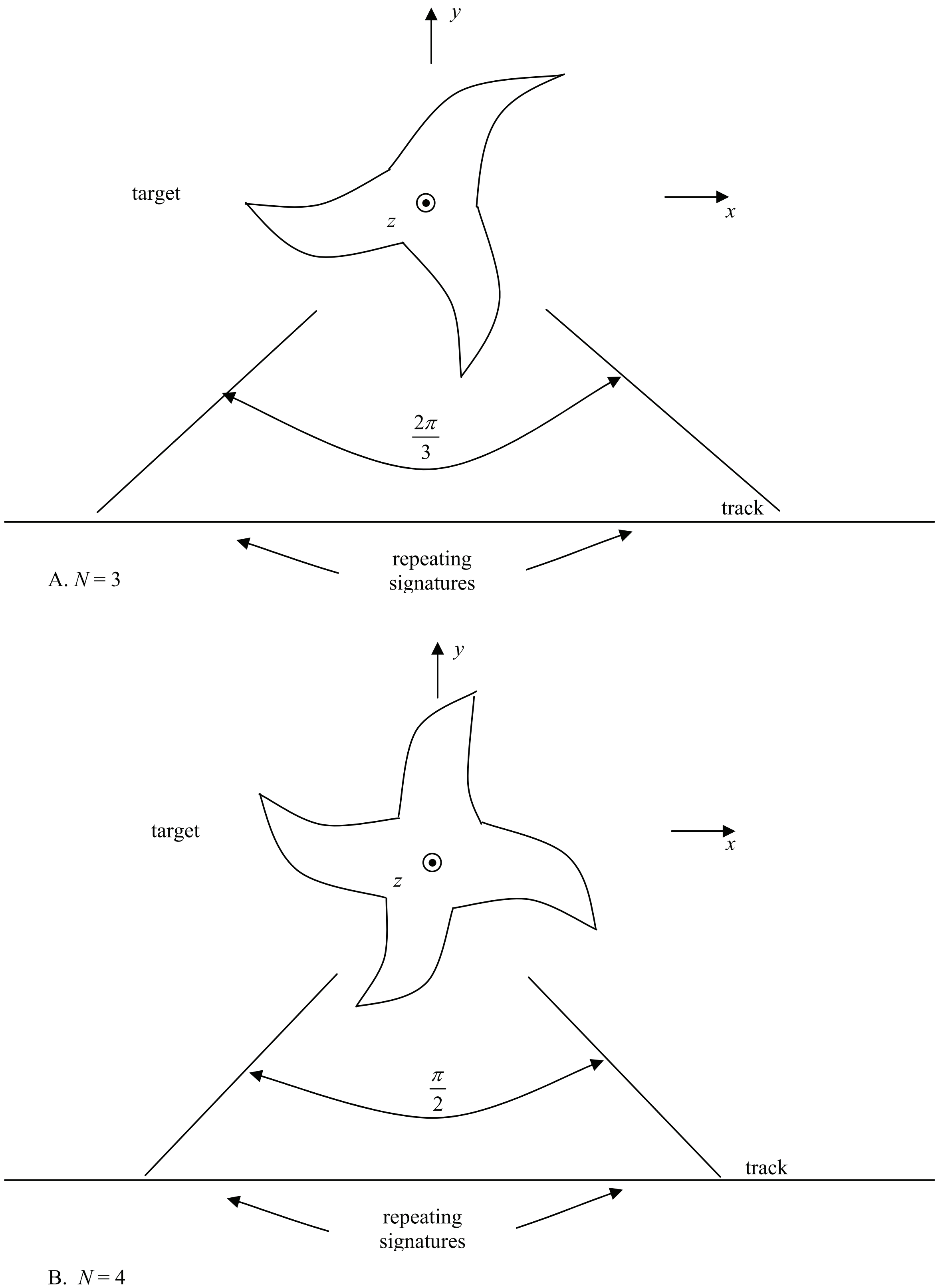

Fig. 3.1 Repetition of Signatures along Track for C_N Targets.

The number of times that the signature appears is

$$N_S = \left[\text{greatest interger } \le [N+1]/2\right] \tag{3.5}$$

So one can average over N_S signatures at each step along the track.

If one further requires that one only compares measurements for equal spacings of the radar from the *y* axis (i.e., *x* and –*x*), then there are only a small number of *x* values (angle $\phi_\ell/2$ away from the negative *y* axis) available for such comparisons. The number of such cases (position pairs) is the larget integer in [*N*+ 1]/4.

Of course, if one moves the radar on a circular track (constant radius Ψ) around the target, one has much more information for comparison.

4. Discrete Rotation/Reflection Symmetry C_{Na}

As in Section 2 we now adjoin axial symmetry planes, except that now there are only N of them. This adds a property to the scattering. As one looks at the target from equal angles on both sides of the symmetry plane one obtains equal scattering of a special form. For simplicity let $x = 0$ be such a symmetry plane (although any plane containing the z axis will do). Then we have

$$
\begin{aligned}
\mathrm{R}_x &= \{\overleftrightarrow{1}\ ,\ \overleftrightarrow{R}_x\} \\
\overleftrightarrow{R}_x &= \overleftrightarrow{1} - 2\,\vec{1}_x\,\vec{1}_x = -\,\vec{1}_x\,\vec{1}_x + \vec{1}_y\,\vec{1}_y + \vec{1}_z\,\vec{1}_z \\
&= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}
\tag{4.1}
$$

describing this symmetry [4, 9].

The incident and scattered fields can be decomposed into two parts designated as symmetric (sy) and antisymmetric (as) for which the electromagnetic fields behave as

$$
\begin{aligned}
\vec{E}_{\substack{sy\\as}}(\vec{r},t) &= \pm\,\overleftrightarrow{R}_x \cdot \vec{E}_{\substack{sy\\as}}(\vec{r}_m,t) \\
\vec{H}_{\substack{sy\\as}}(\vec{r},t) &= \mp\,\overleftrightarrow{R}_x \cdot \vec{H}_{\substack{sy\\as}}(\vec{r}_m,t)
\end{aligned}
\tag{4.2}
$$

$$
\vec{r}_m = \overleftrightarrow{R}_x \cdot \vec{r} \equiv \text{mirror coordinate}
$$

Interpreting this in terms of signals (voltages received) in a radar we have

$$
\begin{aligned}
V_{\mathrm{vv}}(\Delta\phi\,,t) &= V_{\mathrm{vv}}(-\Delta\phi\,,t) \\
V_{\mathrm{hh}}(\Delta\phi\,,t) &= V_{\mathrm{hh}}(-\Delta\phi\,,t) \\
V_{\mathrm{hv}}(\Delta\phi\,,t) &= V_{\mathrm{hv}}(\Delta\phi\,,t) = -V_{\mathrm{hv}}(-\Delta\phi\,,t)
\end{aligned}
\tag{4.3}
$$

where $\Delta\phi$ represents the angle away from the symmetry plane (for constant $r\,,\ \phi$). Note that the crosspol is zero for $\Delta\phi = 0$. This property applies with respect to all N symmetry planes.

The adjunction of axial symmetry planes gives a group with $2N$ elements as

$$
\mathrm{C}_{Na} = \left\{ \left(C_{n,m}(\phi_\ell)\right)\ ,\ \overleftrightarrow{R}_x \cdot \left(C_{n,m}(\phi_\ell)\right) \middle|\, \ell = 1,2,\ldots,N \right\}
\tag{4.4}
$$

As $N \to \infty$ we have the case of $\mathrm{C}_{\infty a} = \mathrm{O}_2$ in Section 2.

5. Translation Symmetry in One Dimension T_1

As discussed in [9] this has the group structure

$$T_1 = \left\{ T_1(\vec{\xi}) \middle| \vec{r} \rightarrow \vec{r} \ , \ \vec{\xi} \right\}$$
$$\vec{\xi} = a \, \vec{1}_x \tag{5.1}$$

where we have taken the periodicity with respect to the *x* coordinate.

If our target is finite in extent this may not be appropriate for modeling the target. However, it can model a certain kind of clutter (other scatterers). Consider the case in Fig. 5.1. Such a periodic clutter can model such things as fence posts, wall studs, etc. The target may be either in front of or behind the target (with respect to the radar).

Sans target, the radar scattering (monostatic or bistatic) will have period *d* as the radar is translated parallel to the *x* axis (constant *y, z* and orientation). One then has a return

$$V_0(x,t) = V_0(x + n\,a, t), \ n = 0, \pm 1, \ldots \tag{5.2}$$

One can average over some set of *n* for noise suppression to obtain a more accurate estimate of V_0. Note that the above form applies to all polarimetric radar channels (hh, vv, hv).

Accomplish the above measurements for *n* sufficiently far away from the target (at least far enough away to get the target scattering out of the eventual time window one will use for target identification (recognition)). As one varies *n* toward that *n* (say $n = 0$) for which the target is in an optimal position in the radar beam, the actual radar return $V(x,t)$ will be different from (5.2). Form

$$\Delta V(x,t) = V(x,t) - V_0(x,t) \tag{5.3}$$

with the V_0 estimate far from the target per (5.2). ΔV may then be used to obtain a more accurate estimate of the target signature. Note that while this procedure removes (approximately) the clutter scattering, it does not remove the multiple scattering between the clutter and the target (which should ideally be considerably smaller).

A special case of this translation symmetry has $a = 0$. Such might be the clutter of a uniform dielectric slab (wall). In this case V_0 is independent of *x*, making its estimate easier.

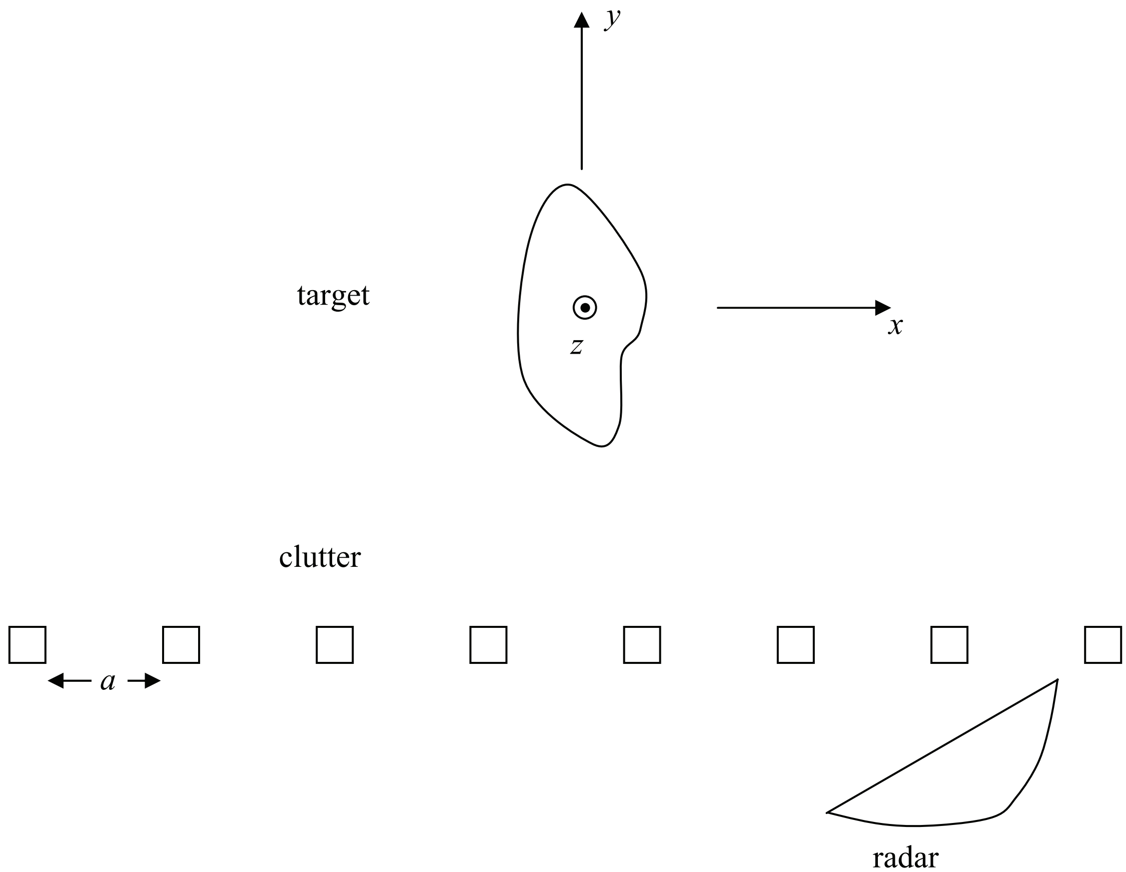

Fig. 5.1 Target in the Presence of Spatially Periodic Clutter.

6. Concluding Remarks

This paper has summarized the influence of target symmetry, in the presence of an earth or water surface, on the target scattering. Data processing and target identification is improved thereby. Less commonly recognized, symmetries in the clutter can also be utilized for clutter suppression, the case in point here being translation symmetry in one dimension.

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