Interaction Notes

Note 585

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Combining Polarimetry with SEM in Radar Backscattering for Target Identification

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Abstract

In identifying radar targets based on the poles (resonances) in the singularity expansion method (SEM), there is additional information to be gained from the residues. The polarizations of the substructure resonances can also be used as a target signature. In addition, the relative times of arrival of the various resonances at the radar can also be used as another way to construct a target image.

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1. Introduction

In identifying (classifying) radar targets the natural frequencies s_α (pole locations in the complex s-plane) have proven to be quite useful due to their aspect-independent property (independent of all incident-field parameters). This is but part of the singularity-expansion-method (SEM) description of electromagnetic scattering. Summarizing its general form in the case of an incident plane wave we have

$$\tilde{\vec{E}}^{(inc)}(\vec{r},s) = E_0 \tilde{f}(s)\, \vec{1}_p\, e^{-\gamma \vec{1}_i \bullet \vec{r}} \quad , \quad \vec{E}^{(inc)}(\vec{r},t) = E_0\, \vec{1}_p\, f\left(t - \frac{\vec{1}_i \bullet \vec{r}}{c}\right)$$

$$\tilde{\vec{H}}^{(inc)}(\vec{r},s) = \frac{E_0}{Z_0} \tilde{f}(s)\, \vec{1}_i \times \vec{1}_p\, e^{-\gamma \vec{1}_i \bullet \vec{r}} \quad , \quad \vec{H}^{(inc)}(\vec{r},t) = \frac{E_0}{Z_0} \vec{1}_i \times \vec{1}_p\, f\left(t - \frac{\vec{1}_i \bullet \vec{r}}{c}\right)$$

$\vec{1}_i \equiv$ direction of incidence, $\vec{1}_p \equiv$ polarization, $\vec{1}_i \bullet \vec{1}_p = 0$

$\sim \equiv$ Laplace transform (two-sided) over time t (1.1)

$s \equiv \Omega + j\omega \equiv$ Laplace-transform variable or complex frequency

$\gamma = \frac{s}{c} = s[\mu_0 \varepsilon_0]^{1/2} \equiv$ propagation constant (free space)

$Z_0 \equiv \left[\frac{\mu_0}{\varepsilon_0}\right]^{1/2} \equiv$ wave impdenace (free space)

$f(t) \equiv$ incident waveform

Taking $\vec{r} = \vec{0}$ (coordinate center) as some convenient position near the target the scattered far field takes the form

$$\tilde{\vec{E}}_f(\vec{r},s) = \frac{e^{-\gamma r}}{4\pi r} \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \bullet \tilde{\vec{E}}^{(inc)}(\vec{0},s) \quad , \quad \tilde{\vec{E}}_f(\vec{r},s) \bullet \vec{1}_o = 0$$

$$\vec{E}_f(\vec{r},t) = \frac{1}{4\pi r} \overleftrightarrow{\Lambda}(\vec{1}_o, \vec{1}_i; t) \overset{\circ}{\bullet} \vec{E}^{(inc)}\left(\vec{0}, t - \frac{\vec{1}_o \bullet \vec{r}}{c}\right)$$

$\circ \equiv$ convolution with respect to time

$\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) = \tilde{\overleftrightarrow{\Lambda}}^T(-\vec{1}_i, -\vec{1}_o; s) \equiv$ scattering dyadic (reciprocity)

$\vec{1}_o \equiv$ direction to observer (outgoing)

$$\vec{1}_o \bullet \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) = \vec{0} = \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \bullet \vec{1}_i \qquad (1.2)$$

For backscattering $(\overrightarrow{1}_o = -\overrightarrow{1}_i)$ this simplifies to

$$\tilde{\overleftrightarrow{\Lambda}}_b(\overrightarrow{1}_i,s) \equiv \tilde{\overleftrightarrow{\Lambda}}(-\overrightarrow{1}_i,\overrightarrow{1}_i;s) = \tilde{\overleftrightarrow{\Lambda}}_b^T(\overrightarrow{1}_i,s)$$

= complex symmetric dyadic (not Hermitian) (1.3)

$$\overleftrightarrow{\Lambda}_b(\overrightarrow{1}_i,t) = \overleftrightarrow{\Lambda}_b^T(\overrightarrow{1}_i,t)$$ = real symmetric dyadic (special case of Hermitian)

Note that in the case of backscattering this can be regarded as a 2 x 2 dyadic using transverse coordinates in the usual radar *h*, v (horizontal, vertical) coordinate system.

The SEM representation of the backscattering dyadic [9, 13] is given by

$$\tilde{\overleftrightarrow{\Lambda}}_b(\overrightarrow{1}_i,s) \equiv \sum_\alpha \frac{e^{-[s-s_\alpha]t_0}}{s-s_\alpha}\,\overrightarrow{c}_\alpha(\overrightarrow{1}_i)\,\overrightarrow{c}_\alpha(\overrightarrow{1}_i)$$

+ entire function

$$\overleftrightarrow{\Lambda}_b(\overrightarrow{1}_i,s) \equiv \sum_\alpha \overrightarrow{c}_\alpha(\overrightarrow{1}_i)\,\overrightarrow{c}_\alpha(\overrightarrow{1}_i)e^{s_\alpha t}u(t-t_0)$$

+ entire function (temporal form) (1.4)

$\overrightarrow{1}_0 \equiv$ turn-on time

$\mathrm{Re}(s_\alpha) < 0$ (natural frequencies)

The entire function is an early-time contribution with damped sinusoids applying at late times. The details of this are discussed in [2, 6] and need not concern us here. There are special cases for which second order poles appear, but this is neglected here.

While the terms in (1.3) can be regarded as experimental observables, they can also be calculated, for example, from an integral equation of the general form

$$\left\langle \tilde{\overleftrightarrow{Z}}(\overrightarrow{r},\overrightarrow{r}';s);\ \tilde{\overrightarrow{J}}(\overrightarrow{r}',s)\right\rangle = \tilde{\overrightarrow{E}}^{(inc)}(\overrightarrow{r},s) \quad (1.5)$$

involving volume or surface integration as appropriate. From this we have

$$\left\langle \overleftrightarrow{Z}(\vec{r},\vec{r}\,';s_\alpha);\ \vec{J}_\alpha(\vec{r}\,') \right\rangle = \vec{0} \tag{1.6}$$

$$\vec{j}_\alpha(\vec{r}\,') = \text{natural modes}$$

Taking the kernel as symmetric (E-field form) we have

$$w_\alpha^2 = -s_\alpha \mu_0 \left\langle \vec{j}_\alpha(\vec{r}); \frac{\partial}{\partial s} \overleftrightarrow{Z}(\vec{r},\vec{r}\,';s)\Big|_{s=s_\alpha}; \vec{j}_\alpha(\vec{r}\,') \right\rangle^{-1}$$

$$\vec{c}_\alpha(\vec{1}_i) = w_\alpha \left\langle \overleftrightarrow{1}_i e^{-\gamma_\alpha \vec{1}_i \cdot \vec{r}\,'}; \vec{j}_\alpha(\vec{r}\,') \right\rangle \tag{1.7}$$

$$\overleftrightarrow{1}_i = \overleftrightarrow{1} - \vec{1}_i \vec{1}_i \quad \text{(transverse dyadic)}$$

$$\gamma_\alpha \equiv \frac{s_\alpha}{c} \quad , \quad \vec{c}_\alpha(\vec{1}_i) \cdot \vec{1}_i = 0$$

While the natural frequencies s_α are very useful due to their aspect independence, one would like to use as much information in the scattered field as possible. For present purposes let us consider the pole residues.

2. Properties of SEM Pole Residues

Consider the physical properties, particularly with regard to polarization, of the SEM pole residues. For this purpose define

$$R_\alpha(\vec{1}_p, \vec{1}_m) \equiv \vec{1}_m \cdot \vec{c}_\alpha(\vec{1}_i)\vec{c}_\alpha(\vec{1}_i) \cdot \vec{1}_p$$

$$\vec{1}_m \equiv \text{direction (component) of scattered field measured} \tag{2.1}$$

Both $\vec{1}_p$ and $\vec{1}_m$ can be taken as $\vec{1}_h$ and $\vec{1}_v$ or some linear combination (real) of these. So our residues are comprised of two factors containing incident and scattered polarization. The above is for the case of nondegenerate natural frequencies, but this is easily extended to the degenerate case.

A fundamental concept concerning the properties of the residues is target symmetry. The geometrical symmetries of the target evidence themselves in the symmetries of the scattering dyadic [10, 12]. A simple example is a symmetry plane (say $y = 0$) with a group representation

$$R_y = \left\{\overleftrightarrow{1}, \overleftrightarrow{R}_y\right\}$$

$$\overleftrightarrow{R}_y = \overleftrightarrow{1} - 2\vec{1}_y\vec{1}_y = \vec{1}_x\vec{1}_x - \vec{1}_y\vec{1}_y + \vec{1}_z\vec{1}_z$$

$$\equiv \text{reflection dyadic} \tag{2.2}$$

$$\overleftrightarrow{1} \equiv \vec{1}_x\vec{1}_x + \vec{1}_y\vec{1}_y + \vec{1}_z\vec{1}_z \equiv \text{three-dimensional identity}$$

In such a case for $\vec{1}_i \cdot \vec{1}_y = 0$ (incidence parallel to the $x\,z$ plane) we have $\vec{c}_\alpha(\vec{1}_i)$ as parallel or perpendicular to this plane, i.e.,

$$\vec{c}_\alpha(\vec{1}_i) = \begin{cases} c_\alpha(\vec{1}_i)\,\vec{1}_y \\ \text{or} \\ c_\alpha(\vec{1}_i)\,\vec{1}_i \times (\vec{1}_i) \end{cases} \tag{2.4}$$

This property can be used to orient the resonance and hence the scatterer relative to the radar, but it requires the proper location of the radar on the symmetry plane.

The implications of such have been discussed in the context of mines (the vampire signature) in [5, 8, 13].

Bodies of revolution with axial symmetry planes extend the above, since at least one of these symmetry planes extends through the observer. In this case there are two $\vec{c}_\alpha$ to consider (with the same s_α, i.e., degeneracy) which are aligned parallel and perpendicular to this symmetry plane. This is particularly convenient if the symmetry plane is vertical, thereby fitting into the *h*, v radar coordinates with *no* crosspol in backscatter.

While previous considerations have concentrated on the symmetries of the total target, one can consider the properties of substructure resonances to the extent that they can be separated from the overall target scattering [7]. This can be accomplished in part by temporal isolation of the scattered signal from that of other "clutter" such as earth or water surface or other nearby large structures. (Such symmetries can be termed "partial symmetries" [3, 4, 11]).

3. Polarizations and Time Delays of Resonances in Backscatter

Figure 3.1 indicates a set of canonical target substructures that might be present on a larger object such as an aircraft, land vehicle, building, etc. If the target is situated on a reasonably flat earth we can regard this surface as the xy plane for our coordinates.

3.1 Substructure polarizations

For simplicity let the radar be directly above the target. Then our choice of h, v coordinates is ambiguous and we can use x, y coordinates for our radar. In Fig. 3.1 we have some thin-wire scatterers oriented in the three coordinate directions. The z-directed wire does not scatter the radar wave in x and y polarizations. However, the x- and y-polarized wires do interact with the radar beam. Suppose s_x and s_y are two corresponding natural frequencies. The fact that these two resonances are oriented 90° relative to each other is *itself* a *target signature*. Of course, the relative angle between these can be any angle in principle for this target signature.

Another substructure is shown as a body of revolution (with axis parallel to the z axis). This backscatters equally in both x and y polarizations (with no crosspol).

Yet another substructure is indicated with a symmetry plane perpendicular to $\vec{1}_y$. In this case the scattering dyadic is diagonal in this coordinate system with separate natural frequencies for each of the two polarizations.

3.2 Substructure relative elevations

Consulting Fig. 3.1 we can see more information potentially available for target recognition. In particular the target substructure resonances arrive back at the radar at various times. The time difference of arrival of say s_1 and s_2 located at elevations h_1 and h_2 have a time difference of arrival of approximately $[h_1 - h_2]/(2c)$ from which $h_1 - h_2$ can be inferred. This is *also* a *target signature*. This extends to all the substructure resonances. Of course, there is some ambiguity as to exactly when a complex resonances begins in time. (This is an analog of the Heisenberg uncertainty principle in quantum mechanics.)

3.3 Combining resonances with multiple directions of incidence: resonance imaging

The discussion of the previous subsection can be extended if we have multiple directions of incidence (and backscattering). In a conceptually simple case let us suppose that we have three mutually perpendicular directions of incidence. Then generalizing from a set of heights h_α to the coordinate positions $\vec{r}_\alpha$ we can determine the

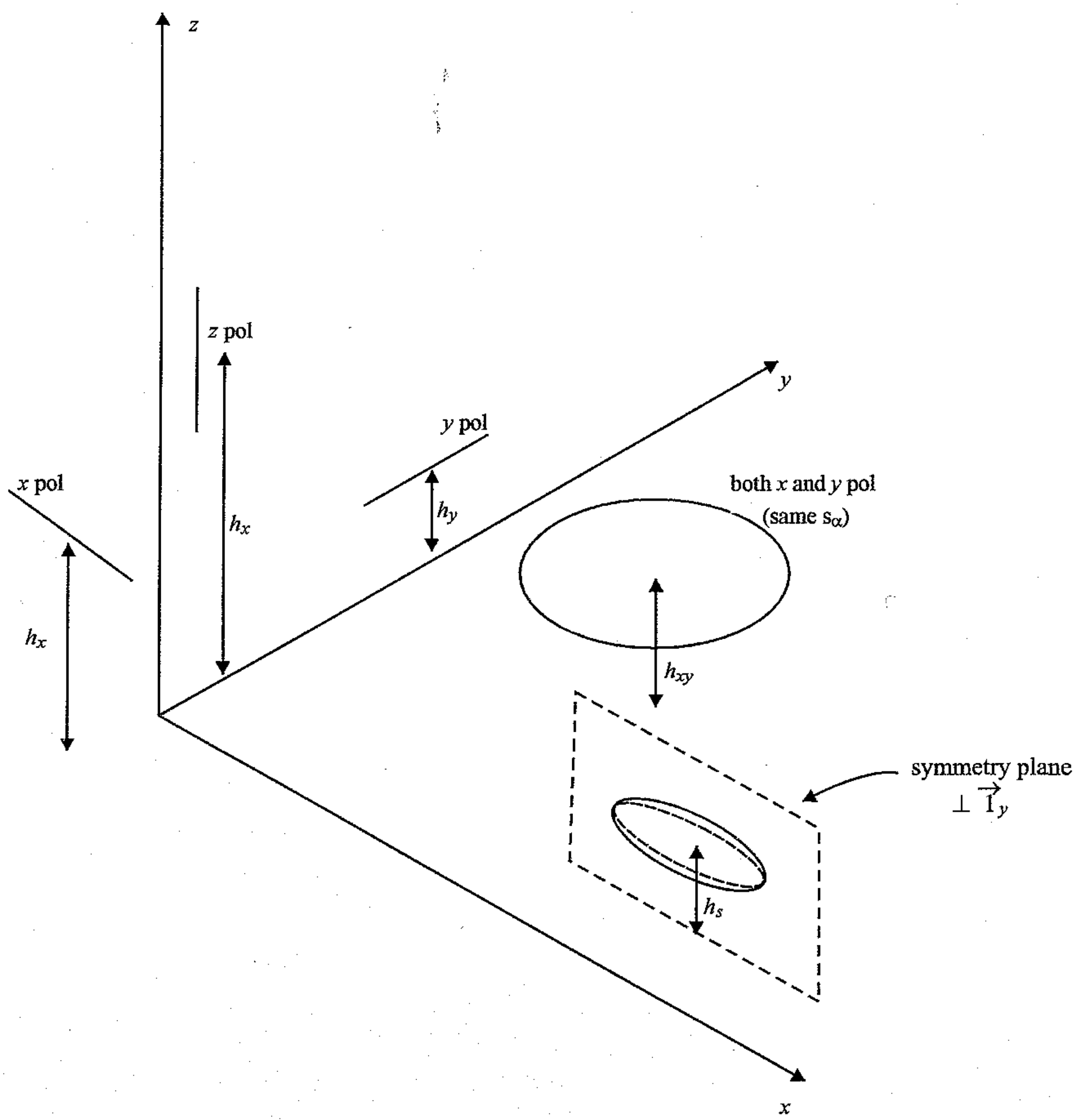

Fig. 3.1 Target Substructures with Simple Polarization Properties

vector, say $\vec{r}_1 - \vec{r}_2$, from the three orthogonal components, thereby giving the relative positions of the resonant centers in space. This gives a kind of *image* of the target. For a set of nonorthogonal incidence directions one can still determine the relative positions via matrix inversion.

Note that as one varies the direction of incidence some of the target substructures may be shadowed by the larger target structures. One needs to allow for this when constructing the image.

3.4 Including ground reflection

With a ground (or water) surface present under the target, the ground reflection also influences the measurements. Consider the geometry in Fig. 3.2. Again assuming an approximately flat ground surface we have a ground reflection of the incident plane wave, this reflected wave also scattering from target substructures. As indicated in the figure the first scattered signal in the direction $\vec{1}_o$ with

$$\vec{1}_o \cdot \vec{1}_z = \cos(\theta) \tag{3.1}$$

follows the reverse path of the incident ray.

The second signal follows two directions on the path involving a single ground reflection as well as scattering from the target [1]. Due to reciprocity this second signal has equal contributions from both directions on the path, effectively doubling the signal. However, there is also loss associated with the ground reflection. There is an extra path length $\ell_2 - \ell_1$ delaying the signal after the first signal a time t_2 as

$$\begin{aligned} ct_2 &= \ell_2 - \ell_1 = \frac{h}{\sin(\theta)} - \ell_2 \cos(\pi - 2\theta) \\ &= \frac{h}{\sin(\theta)}\left[1+\cos(2\theta)\right] \\ &= h\,\frac{2\cos^2(\theta)}{\sin(\theta)} \end{aligned} \tag{3.2}$$

From a measurement of t_2, then h can be determined. By extension this applies to the various substructures to determine the various h_n, thereby aiding in the resonant imaging.

There is also a third signal associated with two ground reflections which can also be thought of as scattering from the target image. This arrives yet later, and has yet more loss due to the two ground reflections.

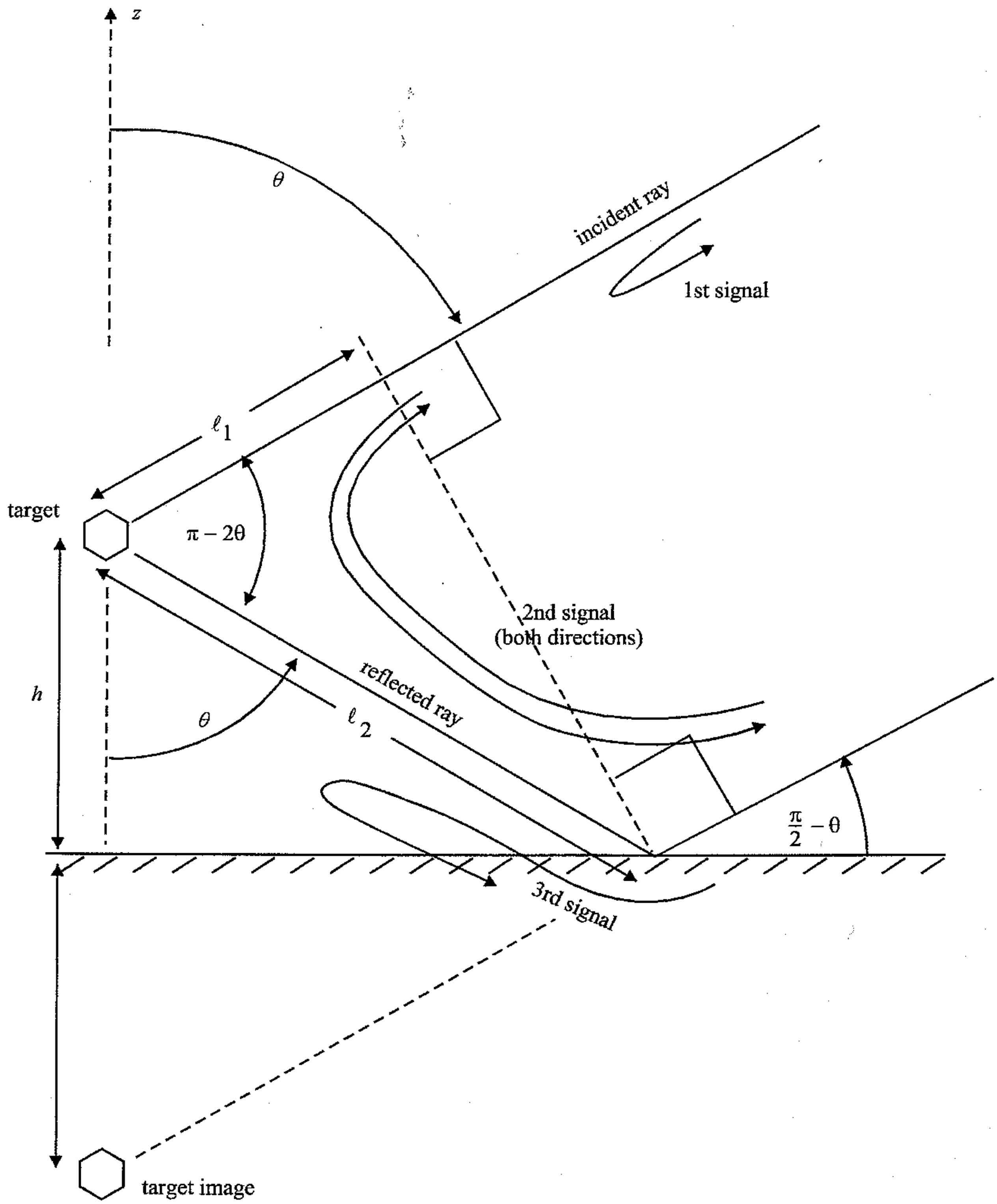

Fig. 3.2 Target or Target Substructure Above Ground or Water Surface

4. Concluding Remarks

So we see that the pole residues contain information useful for target identification. The relative polarizations of the target-substructure resonances are target signatures. The time delays between the substructure resonances (including delays due to ground reflection) also allow one to form a target image.

References

1. C. E. Baum, "Airframes as Antennas", Sensor and Simulation Note 381, May 1995.

2. C. E. Baum, "Representation of Surface Current Density and Far Scattering in EEM and SEM with Entire Functions", Interaction Note 486, February 1992; ch. 13, pp. 273-316, in P. P. Delsanto and A. W. Saenz (eds.), *New perspectives on Problems in Classical and Quantum Physics, Part II, Acoustic Propagation and Scattering, Electromagnetic Scattering*, Gordon and Breach, 1998.

3. C. E. Baum, "Transforms and Symmetries in Target Identification", Interaction Note 496, May 1993.

4. C. E. Baum, "Transforms of Frequency Spectra in Target Identification", Interaction Note 497, July 1993.

5. C. E. Baum, "Symmetry in Electromagnetic Scattering as a Target Discriminant", Interaction Note 523, October 1996; pp. 295-307, in H. Mott and W. Boerner (eds.), *Wideband Interferometric Sensing and Imaging Polarimetry*, July 1997, Proc. SPIE, Vol. 3120.

6. C. E. Baum, "An Observation Concerning the Entire Function in SEM Scattering", Interaction Note 567, April 2001.

7. C. E. Baum, "Substructure SEM", Interaction Note 570, June 2001.

8. L. Carin, R. Kapoor, and C. E. Baum, "Polarimetric Imaging of Buried Landmines", IEEE Trans. Geoscience and Remote Sensing, 1998, pp. 1985-1988.

9. C. E. Baum, E. J. Rothwell, K.-M. Chen, and D. P. Nyquist, "The Singularity Expansion Method and Its Application to target Identification:, Proc. IEEE, 1991, pp. 1481-1492.

10. C. E. Baum and H. N. Kritikos, "Symmetry in Electromagnetics", ch. 1, pp. 1-90, in C. E. Baum and H. N. Kritikos, *Electromagnetic Symmetry*, Taylor & Francis, 1995.

11. C. E. Baum, "Symmetry and Transforms of Waveforms and Waveform Spectra in Target Identification", ch. 7, pp. 309-343, in C. E. Baum and H. N. Kritikos (eds.), *Electromagnetic Symmetry*, Taylor & Francis, 1995.

12. C. E. Baum, "Target Symmetry and the Scattering Dyadic", ch. 4, pp. 204-236, in D. H. Werner and R Mittra (eds.), *Frontiers in Electromagnetics*, IEEE Press., 2000.

13. C. E. Baum and L. Carin, "Singularity Expansion method, Symmetry and Target Identification", ch. 1.6.3, pp. 431-447, in R. Pike and P. Sabatier, *Scattering*, Academic Press, 2002.