Interaction Notes

Note 581

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Including Apertures and Cavities in the BLT Formalism

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Abstract

In the BLT formalism, based on the electromagnetic topology of a complex system, black boxes (junctions) and multiconductor transmission lines (tubes, especially uniform ones) have long been included. This paper discusses some features of including apertures and cavities in the formalism. This involves the definition of appropriate voltages and currents for inclusion in the scattering matrices.

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1. Introduction

In the quantitative aspects of electromagnetic topology one uses various forms of the BLT equation (organized according to the topology) [2-5, 7]. Fundamental to this is the use of scattering matrices to separate the variables for each topological piece in a form which readily allows the pieces to be incorporated into a description of the entire electromagnetic system. In special cases chain matrices can be used to collapse certain parts of the system into a more compact representation with the results then converted back to appropriate scattering matrices [6].

In its original BLT1 form [2] the special case of uniform multiconductor transmission lines (MTLs) were treated as special "tubes" connecting junctions (general multiport linear and reciprocal "black boxes") giving

$$\left[\left((1_{n,m})_{u,v}\right) - \left((\tilde{S}_{n,m}(s))_{u,v}\right) \odot \left((\tilde{\Gamma}_{n,m}(s))_{u,w}\right)\right] \odot \left((\tilde{V}_n(0,s))_u\right) = \left((\tilde{S}_{n,m}(s))_{u,v}\right) \odot \left(\left(\tilde{V}_n^{(s)}\right)_{u,v}\right)$$

$\left((\tilde{S}_{n,m}(s))_{u,v}\right) \equiv$ scattering matrix of vth wave into uth wave at some junction

$$(\tilde{\Gamma}_{n,m}(s))_{u,u} = e^{-(\gamma_{n,m}(s))_u L_u}$$

$\equiv$ delay matrix for uth wave (supermatrix being block diagonal)

$$(\tilde{\gamma}_{n,m}(s))_u = \left[(\tilde{Z}'_{n,m}(s))_u \cdot (\tilde{Y}'_{n,m}(s))_u\right]^{1/2}$$

$\equiv$ propagation matrix ($N_u \times N_u$) for the uth wave

$$(\tilde{V}_n(z_u,s))_u = (\tilde{V}_n(z_u,s))_u + (\tilde{Z}_{c_{n,m}}(s))_u \cdot (\tilde{I}_n(z_u),s)$$

$\equiv$ combined voltage

$z_u \equiv$ coordinate from 0 to L_u in direction of wave propagation (2 different values of u for the two waves)

$$(\tilde{Z}_{c_{n,m}}(s))_u = (\tilde{\gamma}_{c_{n,m}}(s))_u \cdot (\tilde{Y}'_{n,m}(s))_n^{-1} = (\tilde{\gamma}_{c_{n,m}}(s))_u^{-1} \cdot (\tilde{Z}'_{n,m}(s))$$

$\equiv$ characteristic-impedance matrix for uth wave (1.1)

$$\left(\tilde{V}_n^{(s)}(s)\right)_u = \int_0^{L_u} e^{-(\gamma_{n,m}(s))[L_u - z'_u]} \cdot \left(\tilde{V}_n^{(s)}(z'_u,s)\right)_u dz'_u$$

$\equiv$ source vector for uth wave

$$\left(\tilde{V}_n^{(s)}(z_u,s)\right)_u = \left(\tilde{V}_n^{(s)}(z_u,s)\right) + \left(\tilde{Z}_{c_{n,m}}(s)\right)_u \cdot \left(\tilde{I}_n^{(s)}(z_u,s)\right)$$

$\equiv$ combined distributed source for uth wave

Much more detail concerning these terms can be found in the references.

As discussed in [5] one can redefine any tube as a junction (a multiport "black box") with sources at the ports. However, increasing the number of junctions in the network increases the size of the supermatrices. For nonuniform MTLs (NMTLs) there is a special form of NBLT equation which preserves the supermatrix size but introduces additional terms into the equation [5]. For completeness, if all tubes are replaced by junctions the BLT2 equation takes the form as above with the delay matrices reduced to identities and the sources reduced to discrete sources at the various ports. There is also a special form, the BLT3, in which the BLT1 is manipulated into a special form using geometric matrix series appropriate for early-time calculations [7].

Practical implementation to date has used the BLT1 from [8]. While the electromagnetic topological description is quite general, certain pieces of real systems have not yet been practically included in the calculations. In particular the present paper is intended to discuss some aspects of apertures and cavities as pieces to be included in the BLT formalism.

2. Approach to the Problem

For cavities with apertures and cables consider the illustration in Fig. 2.1. The cavity might have any three-dimensional shape. It might have any number of apertures, with one shown here with aperture surface S_a. There may be any number of cables (single conductors or MTLs) entering or passing through the cavity. Here one is shown penetrating the cavity wall at boundary surfaces S_{c1} and S_{c2}. There is also a surface S_{cc} separating the cable from the cavity. It surrounds the cable except for a part consisting of the cavity wall (assumed perfectly conducting) serving as the reference conductor in transmission-line theory. There are situations in which the cable may pass through the cable away from the wall for which S_{cc} would completely surround the cable, but such is not considered here.

By the electromagnetic uniqueness theorem we need know only appropriate fields on the closed cavity boundary, including the penetrations, to specify the problem. Our goal is appropriately to specify the fields on the aforementioned surfaces in the form of scattering matrices which fit into the BLT formalism. We defer the problem of construction of the cavity Green function and concentrate on the general form of the scattering matrices which link the fields on the two sides of the surfaces. It is these surfaces themselves that we wish to characterize.

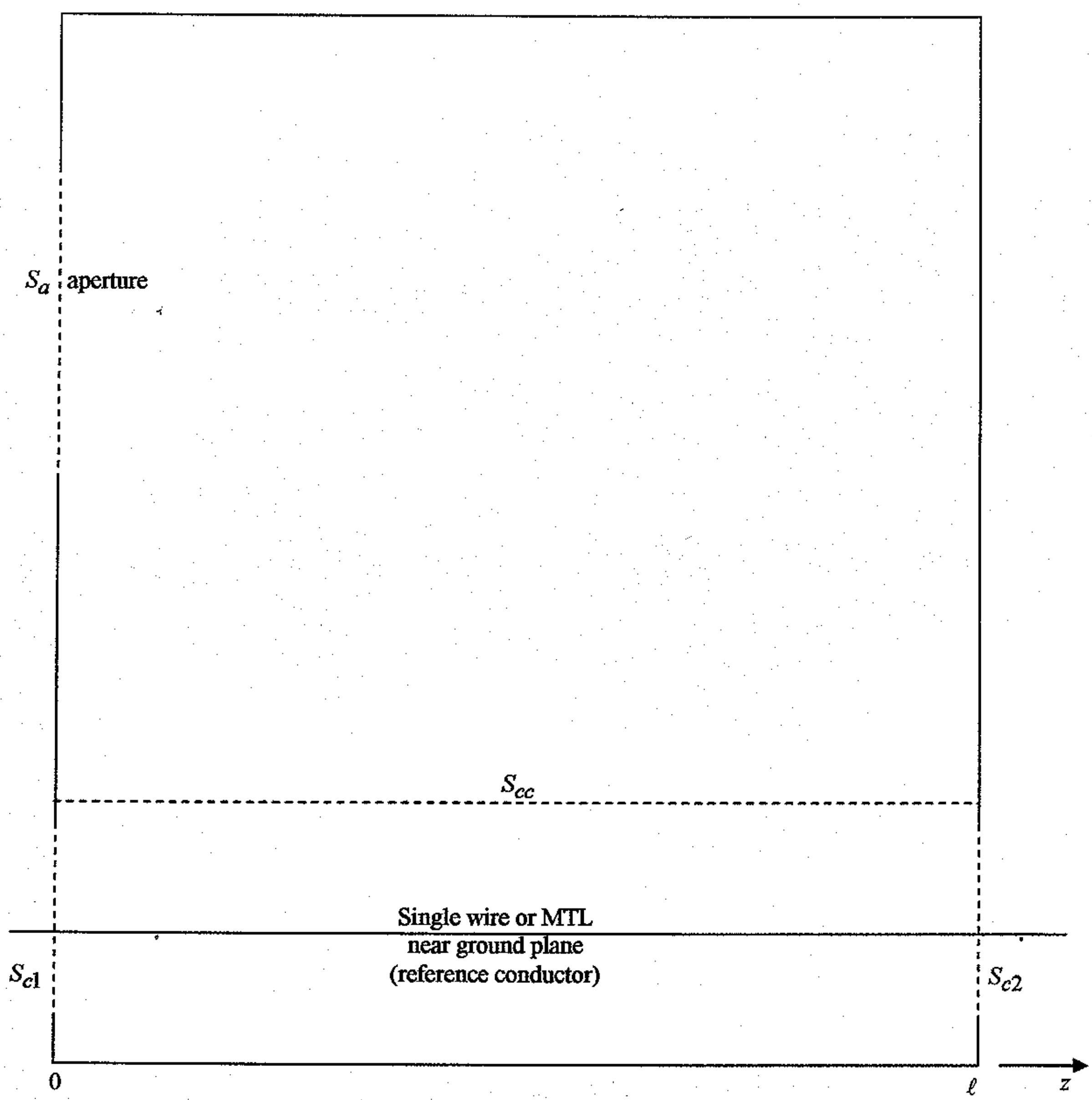

Fig. 2.1 Cavity with Aperture and Cable

3. Aperture Scattering Matrix

To construct a scattering matrix we need to define incoming and outgoing waves. As illustrated in Fig. 3.1 we can think of the aperture as being a degenerate case of a cavity with the two sides of the aperture, S_{a1} and S_{a2}, coming together as S_a. Since it is the tangential components of the fields, $\vec{E}_t$ and $\vec{H}_t$, that must be specified on a surface, we need to think of these in terms of incoming and outgoing waves on both sides of the aperture.

Consider dividing up the aperture into N patches which we take as rectangular for simplicity of discussion as indicated in Fig. 3.2. Other shapes (e.g., triangular) with special basis functions on them may be more useful numerically. Special treatment may also be given to patches adjacent to the boundary of S_a (edge conditions). The present choice is merely for illustration of the concept.

Associated with each patch there are two polarizations, two incoming wave directions, and two outgoing wave directions, giving 8N variables to consider. The scattering matrix then relates the 4N incoming waves to the 4N outgoing waves giving a 4N x 4N matrix.

The two side-lengths of the rectangular patch are $\ell_{n;p}$

$\ell_{n;1}\,\ell_{n;2}$ = patch area

$p = 1, 2$ polarization index

There are two outward pointing normals

$$\vec{1}_q \text{ for } q = 1,2$$

$$q \equiv \text{side index} \tag{3.2}$$

$$\vec{1}_2 = -\vec{1}_1$$

As indicated in Fig. 3.1. For outgoing waves we need $\vec{E}_t \times \vec{H}_t$ to be in the direction of $\vec{1}_q$. Note the reversal on passing through S_a.

In the BLT formalism combined voltages are constructed from voltages and currents. For consistency one would like to define such variables for an aperture. For outgoing waves associate a transmission line with each polarization. Refer to Fig. 3.2 and think of the direction out of the page $\vec{1}_z = \vec{1}_x \times \vec{1}_y$ as the outward direction on the visible side of S_a. We can imagine a transmission line corresponding to $\vec{E}_t$ parallel to $\vec{1}_x$ with

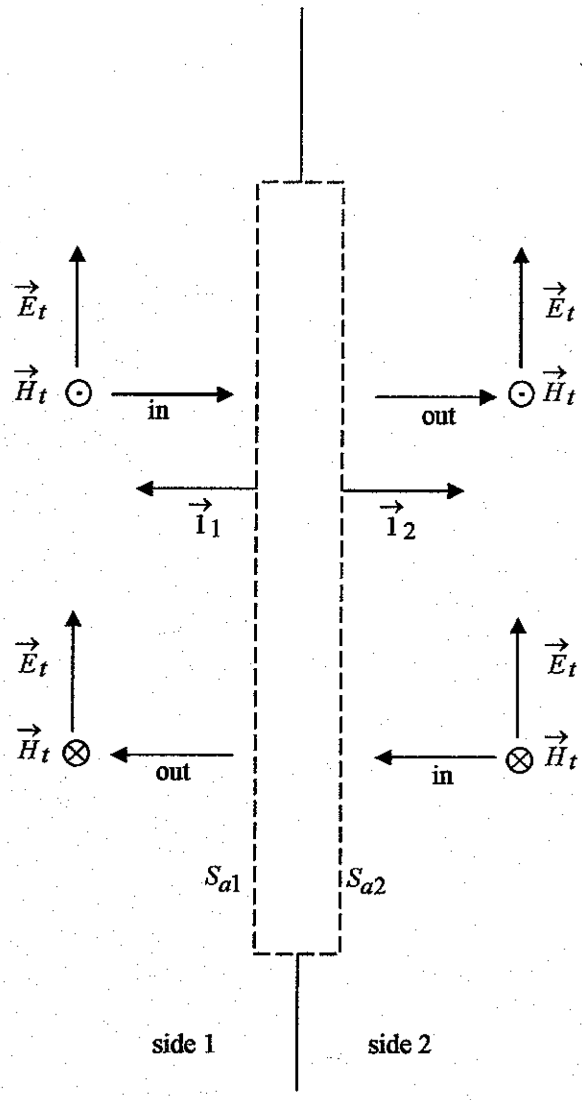

Fig. 3.1 Tangential Fields on Both Sides of Aperture

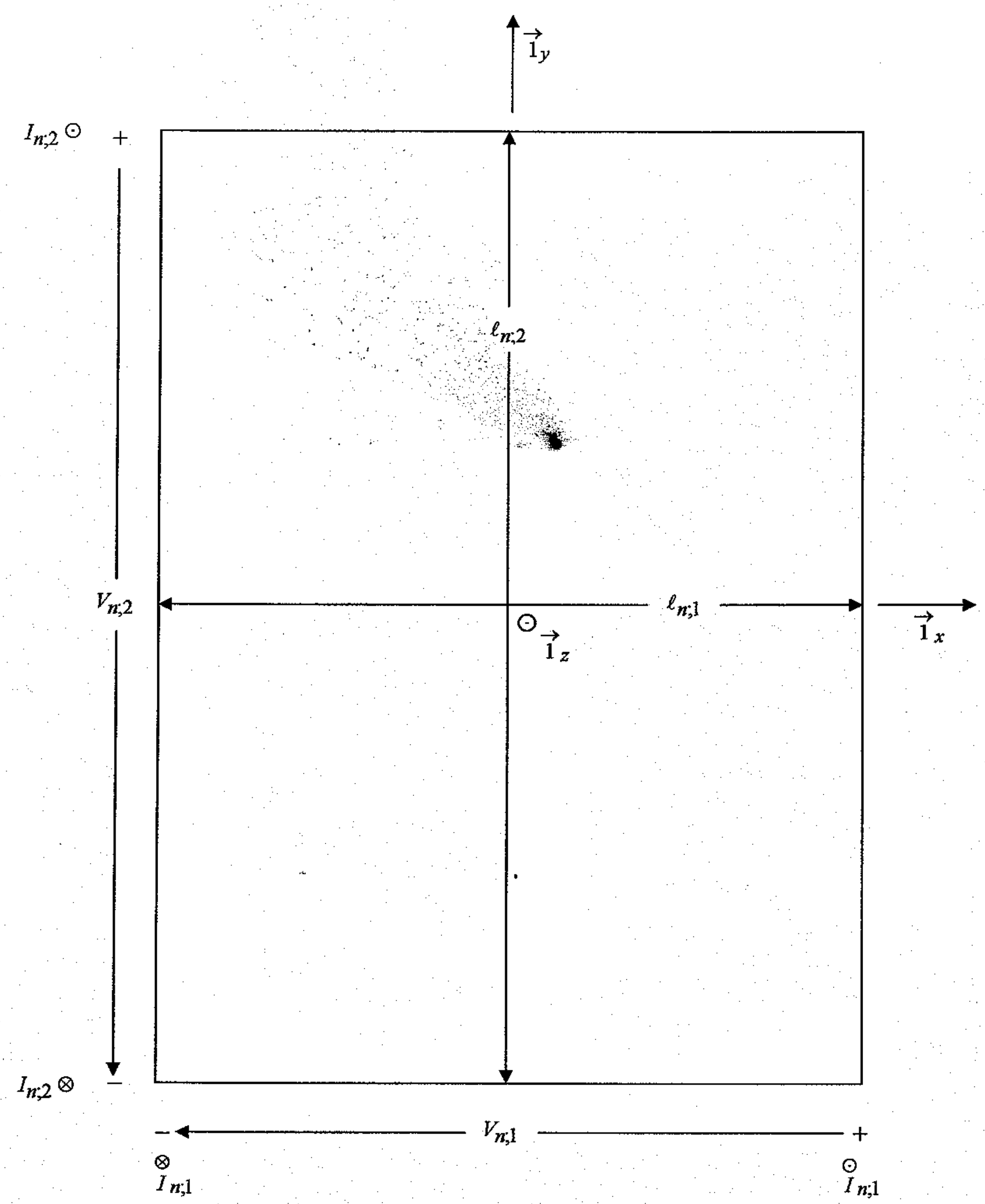

Fig. 3.2 One Rectangular Patch of Aperture

$$
\begin{aligned}
V_{n;1} &= -\vec{1}_x \cdot \vec{E}_t \, \ell_{n;1} \\
I_{n;1} &= -\vec{1}_y \cdot \vec{H}_t \, \ell_{n;2} \\
Z_{c_{n;1}} &= Z_0 \frac{\ell_{n;1}}{\ell_{n;2}} = \text{characteristic impedance of hypothetical transmission line} \\
Z_0 &\equiv \text{wave impedance of medium} \\
&= \left[\frac{\mu_0}{\varepsilon_0}\right]^{1/2} \text{ for free space}
\end{aligned}
\tag{3.3}
$$

Similarly we have

$$
\begin{aligned}
V_{n;2} &= -\vec{1}_y \cdot \vec{E}_t \, \ell_{n;2} \\
I_{n;2} &= +\vec{1}_x \cdot \vec{H}_t \, \ell_{n;1} \\
Z_{c_{n;2}} &= Z_0 \frac{\ell_{n;2}}{\ell_{n;1}} \\
Z_{c_{n;1}} Z_{c_{n;2}} &= Z_0^2
\end{aligned}
\tag{3.4}
$$

Note that if the above is for side 1, then for side 2 there is a sign reversal in either $\vec{1}_x$ or $\vec{1}_y$ (but not both) for referencing outgoing waves in the $\vec{1}_2$ direction. While here $I_{n,p}$ is visualized as current on a transmission line, it can also be regarded as current on the aperture in the direction of voltage increase (negative of electric field) giving outward propagating power.

Now define combined-voltage waves as

$$
V_{n;p}^{\binom{out}{in}} = V_{n;p} \pm Z_{c_{n;p}} I_{n;p} \tag{3.5}
$$

Here we see the significance of Z_0 in the normalization. For a plane wave propagating normal to the aperture in the high-frequency limit, it passes through the aperture, the incoming wave on one side becoming the outgoing wave on the other side (and conversely). Of course, if the wave is not propagating perpendicular to the aperture the choice of Z_0 for normalization may not be optimal, but it should serve as some kind of average. Note in (3.5) that if both outgoing and incoming waves are known then $V_{n;p}$ and $I_{n;p}$ are easily recovered.

Matching $\vec{E}_t$ and $\vec{H}_t$ through the aperture (boundary conditions) gives

$$\begin{aligned} V_{n;p;1} &= V_{n;p;2} \\ I_{n;p;1} &= I_{n;p;2} \end{aligned} \tag{3.6}$$

where the subscript $q = 1, 2$ distinguishes the two sides. With (3.5) we now have

$$\begin{aligned} V^{(out)}_{n;p;1} &= V^{(in)}_{n;p;2} \\ V^{(out)}_{n;p;2} &= V^{(in)}_{n;p;1} \end{aligned} \tag{3.7}$$

For fixed n and p this can be written as (ranging over q, q')

$$\begin{aligned} \left(V^{(out)}_{n;p;q}\right) &= \left(S_{n;p;q;q'}\right) \cdot \left(V^{(in)}_{n;p;q}\right) \\ \left(S_{n;p;q;q'}\right) &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{aligned} \tag{3.8}$$

This elementary scattering matrix corresponds to one polarization on one patch.

Rearrange the $n;p;q$ index set by defining a single index

$$\ell = n;p;q \quad , \quad \ell = 1, 2, \cdots, 4N \tag{3.9}$$

by running

$$\begin{aligned} \ell &= 1, \cdots, N \quad \text{for} \quad p;q = 1, 1 \\ &= N+1, \cdots, 2N \quad \text{for} \quad p;q = 2, 1 \\ &= 2N+1, \cdots, 3N \quad \text{for} \quad p;q = 1, 2 \\ &= 3N+1, \cdots, 4N \quad \text{for} \quad p;q = 2, 2 \end{aligned} \tag{3.10}$$

Then we have

$$\left(V^{(out)}_{\ell}\right) = \left(S_{\ell,\ell'}\right) \cdot \left(V^{(in)}_{\ell}\right)$$

$$
\left(S_{\ell,\ell'}\right) = \begin{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & & & \mathbf{0} \\ & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & & \\ & & \ddots & \\ \mathbf{0} & & & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{pmatrix} \tag{3.11}
$$

giving a block-diagonal scattering matrix (4N x 4N) with determinant +1.

There are other forms that the connection of the two aperture sides can take. We have a chain matrix

$$
\begin{aligned}
&\left.\begin{pmatrix} V_v \\ I_v \end{pmatrix}\right|_{q=1} = \left(C_{v,v'}\right) \cdot \left.\begin{pmatrix} V_v \\ -I_v \end{pmatrix}\right|_{q=2} \\
&\left(C_{v,v'}\right) = \left(1_{v,v'}\right) \qquad \text{(determinant } +1\text{)}
\end{aligned} \tag{3.12}
$$

where the index v also ranges from 1 to 4N but from 1 to 2N for voltages and 2N+1 to 4N for currents. In this form the chain matrix is like a special case of the propagation matrix in (1.1) for zero tube length.

As we can see the representation for the aperture is rather simple in this formalism. This is because we have separated out the Green functions for each of the two sides, with the various geometries that may exist there, from that of the aperture itself. Essentially this representation of the aperture is just the connection of fields through the aperture. It can however, be treated as a separate entity in the BLT formalism for organizing the entire system. This, in turn, will need to be combined with the scattering matrices representing the volumes on the two aperture sides.

Some work has been done concerning an aperture connecting two regions [9]. In this case Green functions are used to generate admittance matrices for the two regions of the aperture. Of course, admittance matrices can be converted to scattering matrices. In the general problem each region may be connected to various other regions by apertures. In such cases scattering matrices are needed to fit into the general BLT formalism for large systems of such regions.

Here we have used simple rectangular patches with uniform basis functions for the fields for simplicity of description. More optimal choices for numerical accuracy may well be appropriate. In any case, we need associated definitions of voltages, currents, and waves.

4. Boundary Dividing Cable from Volume

Turning our attention to the cable near the wall in Fig. 2.1, let us consider the division of the cable and cavity volumes by a boundary surface S_{cc}. Figure 4.1 shows a cross section. This boundary can take various shapes, the hemicylindrical one (radius a) being a convenient one.

Here let us consider what are sometimes called entire-domain basis functions. For this purpose we can use the form of the electromagnetic fields in cylindrical (Ψ, ϕ, z) coordinates as found in various texts and papers, e.g., [1]. Assuming a lossless, dispersionless dielectric medium (e.g., free space) we have

$$\begin{aligned} \gamma &= s[\mu\varepsilon]^{1/2} \equiv \text{propagation constant} \\ Z_0 &= \left[\frac{\mu}{\varepsilon}\right]^{1/2} \equiv \text{wave impedance} \\ s &= \Omega + j\omega \equiv \text{Laplace-transform variable (with respect to time } t\text{) or complex frequency} \end{aligned} \tag{4.1}$$

If we specialize s to $j\omega$ the propagation constant becomes

$$\gamma = jk \quad , \; k = \omega[\mu\varepsilon]^{1/2} \tag{4.2}$$

We will find this convenient for present purposes. The reader will note that the more general Laplace form uses modified Bessel functions (I_n and K_n).

One begins with solutions of the scalar wave equation

$$\begin{aligned} &\Xi^{(n)}\left(m, \zeta_1, {}^{e}_{o}\right) \equiv F_m^{(n)}(k\Psi\zeta_2)e^{-jkz\zeta_1}\begin{Bmatrix}\cos(m\phi)\\ \sin(m\phi)\end{Bmatrix} \\ &\zeta_1^2 + \zeta_2^2 = 1 \\ &n = \begin{cases} 1 \Rightarrow J_m(k\Psi\zeta_2) \\ 2 \Rightarrow Y_m(k\Psi\zeta_2) \\ 3 \Rightarrow H_m^{(1)}(k\Psi\zeta_2) \\ 4 \Rightarrow H_m^{(2)}(k\Psi\zeta_2) \end{cases} \end{aligned} \tag{4.3}$$

From these, three sets of solutions to the vector wave equation are obtained as

$$
\begin{aligned}
\vec{L}^{(n)}\left(m,\zeta_1,{}^e_o\right) &\equiv \frac{1}{k}\nabla\Xi^{(n)}\left(m,\zeta_1,{}^e_o\right) \\
\vec{M}^{(n)}\left(m,\zeta_1,{}^e_o\right) &\equiv \frac{1}{k}\nabla\times\left[\Xi^{(n)}\left(m,\zeta_1,{}^e_o\right)\vec{1}_z\right] \\
&= \vec{L}^{(n)}\left(m,\zeta_1,{}^e_o\right)\times\vec{1}_z = \frac{1}{k}\nabla\times\vec{N}^{(n)}\left(m,\zeta_1,{}^e_o\right) \\
\vec{N}^{(n)}\left(m,\zeta_1,{}^e_o\right) &\equiv \frac{1}{k}\nabla\times\vec{M}^{(n)}\left(m,\zeta_1,{}^e_o\right) \\
&= \frac{\partial}{\partial(kz)}\vec{L}^{(n)}\left(m,\zeta_1,{}^e_o\right) + \Xi^{(n)}\left(m,\zeta_1,{}^e_o\right)\vec{1}_z
\end{aligned}
\tag{4.4}
$$

By various choices of n (including linear combinations), noting that only two of these are linearly independent, the various types of waves (incoming, outgoing, etc.) can be constructed. Here we need only the $\vec{N}$ and $\vec{M}$ functions since source regions (charge) are excluded (zero divergence).

From these functions we can find solutions of the Maxwell equations in the form

$$
\begin{aligned}
\vec{E}^{(n)}\left(m,\zeta_1,{}^e_o\right) &= E_1\vec{M}^{(n)}\left(m,\zeta_1,{}^e_o\right) + E_2\vec{N}^{(n)}\left(m,\zeta_1,{}^e_o\right) \\
\vec{H}^{(n)}\left(m,\zeta_1,{}^e_o\right) &= j\frac{E_1}{Z_0}\vec{N}^{(n)}\left(m,\zeta_1,{}^e_o\right) + j\frac{E_2}{Z_0}\vec{M}^{(n)}\left(m,\zeta_1,{}^e_o\right)
\end{aligned}
\tag{4.5}
$$

Note the pairing of the $\vec{M}$ and $\vec{N}$ functions between $\vec{E}$ and $\vec{H}$.

It is the variables (z,ϕ) in these functions which give us an appropriate form of the basis functions on S_{cc}. Note that only z and ϕ components are used (tangential E and H). For present purposes we assume that a is electrically small, so that only $m = 0, 1$ are of interest, implying only constants, $\cos(\phi)$, or $\sin(\phi)$ in the expansions. The cable length ℓ is assumed $>> a$, and not in general electrically small. The $\vec{M}$ functions representing $\vec{E}$ correspond to H (or TE with respect to z) modes, with associated $\vec{N}$ functions for $\vec{H}$. The $\vec{N}$ functions for $\vec{E}$ correspond to E (or TM) modes with associated $\vec{M}$ functions for $\vec{H}$.

Writing out the wave functions on S_{cc} we have for tangential components

$$
\begin{aligned}
M_{\phi}^{(n)}\left(1,\zeta_1,{}^{e}_{o}\right) &= -\zeta_2 F_1^{(\ell)'}(k\Psi\zeta_2)e^{-jkz\zeta_1}\begin{Bmatrix}\cos(\phi)\\ \sin(\phi)\end{Bmatrix}\\
M_{z}^{(n)}\left(1,\zeta_1,{}^{e}_{o}\right) &= 0\\
N_{\phi}^{(n)}\left(1,\zeta_1,{}^{e}_{o}\right) &= -j\zeta_1\frac{F_1^{(n)}(k\Psi\zeta_2)}{k\Psi}e^{-jkz\zeta_1}\begin{Bmatrix}-\sin(\phi)\\ \cos(\phi)\end{Bmatrix}\\
N_{2}^{(n)}\left(1,\zeta_1,{}^{e}_{o}\right) &= -\zeta_2^2 F_1^{(n)}(k\Psi\zeta_2)e^{-jkz\zeta_1}\begin{Bmatrix}\cos(\phi)\\ \sin(\phi)\end{Bmatrix}
\end{aligned}
\tag{4.6}
$$

where a prime on the Bessel functions indicates a derivative with respect to the argument.

For H modes we have for tangential fields on S_{cc} (noting $E_\Psi, E_z, H_\phi = 0$ on $\phi = 0, \pi$) for $m = 0$

$$
\begin{aligned}
E_\phi &\propto e^{-jkz\zeta_1}\\
E_z &- 0\\
H_\phi &= 0\\
H_z &\propto e^{-jkz\zeta_1}\cos(\phi)
\end{aligned}
\tag{4.7}
$$

Inside S_{cc} this corresponds to a uniform H_z with E_ϕ (small for small a) encircling it. For $m = 1$ we have

$$
\begin{aligned}
E_\phi &\propto e^{-jkz\zeta_1}\cos(\phi)\\
E_z &= 0\\
H_\phi &= 0\\
H_z &\propto e^{-jkz\zeta_1}\cos(\phi)
\end{aligned}
\tag{4.8}
$$

This corresponds to a uniform vertical E (i.e., E_y) inside S_{cc}. The sign reversal on H_z shows that this term is small.

For E modes we have for tangential fields on S_{cc} for $m = 0$

$$
\begin{aligned}
E_\phi &= 0\\
E_z &= 0\\
H_\phi &= 0\\
H_z &= 0
\end{aligned}
\tag{4.9}
$$

For $m = 1$ we have

$$
\begin{aligned}
E_\phi &\propto e^{-jkz\zeta_1}\cos(\phi) \\
E_z &\propto e^{-jkz\zeta_1}\sin(\phi) \\
H_\phi &\propto e^{-jkz\zeta_1}\sin(\phi) \\
H_z &= 0
\end{aligned}
\tag{4.10}
$$

This corresponds to a locally uniform E_y plus E_z and H_x proportional to y inside S_{cc}.

Now we have a set of basic functions appropriate to S_{cc}. These match directly to wave expansions outside S_{cc}. Inside S_{cc} these correspond to quasi static fields, in the absence of conductors and insulators (cables) inside S_{cc}. However, we can think of these as incident fields from which we can calculate coupling parameters appropriate to the cables. Since we want these basis functions to take the form of voltages and currents, then we can scale them by an appropriate length, say ℓ, so that E fields go into voltages, and H fields into currents.

As seen above these basic function involve exponentials in $jkz\zeta_1$. An arbitrary distribution with respect to z can be constructed from this by Fourier transforms over z. For some finite length ℓ of cable (e.g., as in Fig. 2.1) a Fourier series may be more appropriate using terms like $\cos(\nu\pi z/\ell)$ and $\sin(\nu\pi z/\ell)$ for integer ν. Converting these basis functions to the equivalent MTL sources we have for the BLT cable formalism

$$
\begin{aligned}
\left(\tilde{V}_n^{(s)'}(z_u,s)\right)_u &= \left(\tilde{V}_n^{(s)'}(z_u,s)\right)_u + \left(\tilde{Z}_{c_{n,m}}(s)\right)_u \cdot \left(\tilde{I}_n^{(s)'}(z_u,s)\right) \\
0 &\le z_u \le \ell
\end{aligned}
\tag{4.11}
$$

where for one u value $z_u = z$ (increases to right) and for a second u value (to give the two wave directions) $z_u = \ell - z$. Positive current convention is for increasing z_u. This in turn goes into integrals of the form (for uniform MTLs)

$$
\begin{aligned}
\left(\tilde{V}_n^{(s)}(s)\right)_u &= \int_0^{L_u} e^{-\left(\tilde{\gamma}_{n,m}(s)\right)_u [L_u - z'_u]} \cdot \left(\tilde{V}_n^{(s)'}(z_u,s)\right)_u \, dz_u \\
L_u &= \ell
\end{aligned}
\tag{4.12}
$$

Diagonalizing the propagation matrix we have [2]

$$
\begin{aligned}
\left(\tilde{\gamma}_{n,m}(s)\right)_u &= \sum_{\beta=1}^{N} \tilde{\gamma}_\beta(s) \left(\tilde{\mathrm{v}}_{c_n}(s)\right)_\beta \left(\tilde{i}_{c_n}(s)\right)_\beta \\
e^{-\left(\gamma_{n,m}(s)\right)_u [L_u - z_u]} &= \sum_{\beta=1}^{N} e^{-\tilde{\gamma}_\beta(s)[L_u - z_u]} \left(\tilde{\mathrm{v}}_{c_n}(s)\right)_\beta \left(\tilde{i}_{c_n}(s)\right)_\beta
\end{aligned}
\tag{4.13}
$$

Then (4.12) splits into N scalar integrals of the form

$$
\begin{aligned}
\tilde{V}_\beta^{(s)}(s) &= \int_0^{L_u} e^{-\tilde{\gamma}_\beta(s)[L_u - z_u']} \left[a_{u;\beta} e^{j\nu\pi z_u'/L_u} + b_{u;\beta} e^{j\nu\pi z_u'/L_u} \right] dz_u' \\
&= \left[a_{u;\beta} e^{-\tilde{\gamma}_\beta(s) L_u} \left[\tilde{\gamma}_\beta(s) + j\frac{\nu\pi}{L_u} \right]^{-1} e^{\tilde{\gamma}_\beta(s) z_u' + j\nu\pi z_u'/L_u} \right. \\
&\quad \left. + b_{u;\beta} e^{-\tilde{\gamma}_\beta(s) L_u} \left[\tilde{\gamma}_\beta(s) - j\frac{\nu\pi}{L_u} \right]^{-1} e^{\tilde{\gamma}_\beta(s) z_u' + j\nu\pi z_u'/L_u} \right]_0^{L_u} \\
&= a_{u;\beta} \left[\tilde{\gamma}_\beta(s) + j\frac{\nu\pi}{L_u} \right]^{-1} \left[e^{j\nu\pi} - e^{-\tilde{\gamma}_\beta(s) L_u} \right] \\
&\quad + b_{u;\beta} \left[\tilde{\gamma}_\beta(s) - j\frac{\nu\pi}{L_u} \right]^{-1} \left[e^{-j\nu\pi} - e^{-\tilde{\gamma}_\beta(s) L_u} \right]
\end{aligned}
\tag{4.14}
$$

Thus the z integrals for a uniform MTL are all *analytic.*

Now what we need is to relate the $a_{u;\beta}$ and $b_{u;\beta}$ to the sources in (4.11) decomposed via the eigenmodes in (4.13) into the β th components. This requires a separate calculation. Referring to Fig. 4.1 we can see how to approach this. The voltage source per unit length is found from the E mode for $m = 1$ in (4.10) on a plane of constant z. This is just $-hE_z$ if h is a single wire above a ground plane. Equivalently, this is also $-j\omega\mu h H_x$. This generalizes to N wires if they are not closely packed so that the magnetic distortion near the open-circuited wires does not couple significantly to adjacent wires. With a assumed small compared to ℓ we can first-order neglect the z variation (small ν) in the computation.

For the current source per unit length the computation is a little more complicated. We can obtain the open-circuit voltage on an incremental length of cable from E_y corresponding to (4.8) for H modes and (4.10) for E modes by a computation of the form $-h\,E_y$, except that now dielectrics (if any) need to be included the computation. Then with the capacitance-per-unit-length matrix $(C'_{n,m})$ we can convert the open-circuit voltages to transverse current-per-unit length source $(\tilde{I}'(s))$ via $j\omega(C'_{n,m})$. So one needs to compute $(C'_{n,m})$ (or its inverse), including the effects of dielectrics.

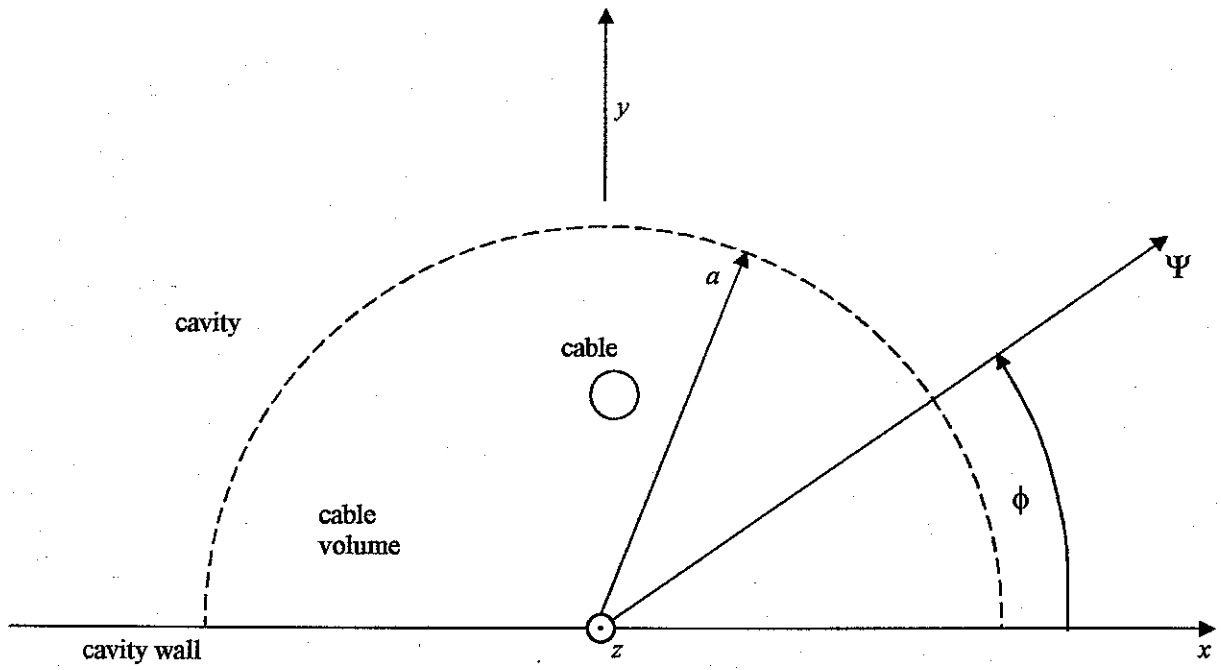

Fig. 4.1 Cross Section of Cable Near Cavity Wall

5. Concluding Remarks

This paper explores some possible techniques for including apertures and cavities including MTLs as special junctions with appropriate scattering matrices in the BLT formalism. There are various other improvements that can likely be made. Having separated the cavity boundaries with nonzero tangential electric field from the problem, this leaves the problem of construction of appropriate Green functions for the various volumes (including cavities and the external geometry).

The present choice of separating boundaries is but one possible choice; others need to be investigated. For example, one might not use S_{cc} as a boundary, but instead use S_{c1} and S_{c2}. In this case the cable is treated as part of the cavity. This of course, changes the cavity Green function. Perhaps this will be the subject of a future paper.

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