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Magnetic-Singularity Identification of Nonsymmetrical Targets

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Abstract

For nonsymmetrical targets the unit vectors characterizing the pole residues and the constant-dyadic part of the magnetic-polarizability dyadic do not in general line up as mutually parallel or perpendicular. For magnetic-singularity identification of such highly (but not perfectly) conducting targets one can still fit the magnetic-singularity representation of the target (in a target library) to measured data by optimal choice of six real parameters. Three are rotation angles (Euler angles) for target orientation, two are angles from the observer to the target, and one is the distance from the target. Besides target identification, target location and orientation are also estimated.

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1. Introduction

In magnetic-singularity identification (MSI) of highly (but not perfectly) conducting targets (of finite size) we have the general form of the magnetic-polarizability dyadic as [1-4]

$$
\begin{aligned}
\tilde{\overleftrightarrow{M}}(s) &= \tilde{\overleftrightarrow{M}}(\infty) + \sum_{\alpha} M_{\alpha} \overrightarrow{M}_{\alpha} \overrightarrow{M}_{\alpha} [s-s_{\alpha}]^{-1} \\
&= \tilde{\overleftrightarrow{M}}(0) + -\sum_{\alpha} M_{\alpha} \overrightarrow{M}_{\alpha} \overrightarrow{M}_{\alpha} \frac{s}{s_{\alpha}} [s-s_{\alpha}]^{-1} \\
\overrightarrow{M}_{\alpha} \cdot \overrightarrow{M}_{\alpha} &= 1, \quad \overrightarrow{M}_{\alpha} = \text{real unit vector for } \alpha\text{th mode} \\
M_{\alpha} &= \text{real scalar} \\
s_{\alpha} &< 0 \ \text{(all negative real natural frequencies)} \\
\tilde{\overleftrightarrow{M}}\binom{\infty}{0} &= \sum_{\upsilon=1}^{3} M_{\upsilon}^{\binom{\infty}{0}} \overrightarrow{M}_{\upsilon}^{\binom{\infty}{0}} \overrightarrow{M}_{\upsilon}^{\binom{\infty}{0}} \\
&\equiv \begin{cases} \text{entire function (constant dyadic for } s=\infty) \\ \text{DC response (for } s = 0) \end{cases} \\
\overrightarrow{M}_{\upsilon_1}^{\binom{\infty}{0}} \cdot M_{\upsilon_2}^{\binom{\infty}{0}} &= 1_{\upsilon_1,\upsilon_2} \ \text{(orthonormal)} \\
\overrightarrow{M}_{\upsilon}^{\binom{\infty}{0}} &\equiv \text{real eigenvalues (not necessarily distinct)} \begin{cases} \text{nonpositive for } s = \infty \\ \text{nonnegative for } s = 0 \end{cases} \\
s &= \Omega + j\omega \ \text{complex frequency or Laplace - transform variable} \\
\sim &\equiv \text{Laplace transform (two - sided)}
\end{aligned}
\tag{1.1}
$$

This magnetic-polarizability dyadic can also be expressed in time-domain (as a convolution operator) in various forms appropriate to delta-function and step-function response.

In [3, 4] we found that symmetries in the target could be used to simplify the discrimination of such targets. The symmetries make the various $\overrightarrow{M}_{\alpha}$ line up according to axes and planes of symmetry. This separates the response into scalar functions of s (each containing a subset of the poles) times a common angular (aspect) dependence. This allows one to constrain appropriate residues and eigenvalues in the identification process and reduce the number of allowed arbitrary scalar parameters for fitting the measured data to the taret signatures in the target library.

What now if there are no special geometrical symmetries (rotation and reflection) in the target? One can still use the natural frequency set as an identifier. However, what about the scalar M_α parts of the residues? Can these be constrained as part of the identification process?

For present purposes we use coordinates as in Fig. 1.1 where

$$\begin{aligned} \vec{r} &= (x, y, z) \equiv \text{coordinates for target stored in library} \\ \vec{r}' &= (x', y', z') \equiv \text{coordinates based on observer location and chosen orientation} \end{aligned} \tag{1.2}$$

The target is located at (roughly centered on) the coordinate origin in the $\vec{r}$ system which we can also designate as $\vec{r}'_0$ in the $\vec{r}'$ system. Note that the various coordinate axes are not in general parallel between the two coordinate systems since the target orientation is assumed not to be known by the observer a priori.

The observer at $\vec{r}' = \vec{0}$ senses the scattered magnetic field which is written as [3, 4]

$$\begin{aligned} \tilde{H}^{(sc)}(s) &= \vec{1}_s \cdot \vec{\tilde{H}}^{(sc)}(s) \\ &= \frac{\tilde{m}^{(inc)}(s)}{16\pi^2 R^6} \vec{1}_s \cdot [3\, \vec{1}_R \vec{1}_R - \overleftrightarrow{1}] \cdot \overleftrightarrow{\tilde{M}}(s) \cdot [3\, \vec{1}_R \vec{1}_R - \overleftrightarrow{1}] \cdot \vec{1}_c \\ \vec{1}_s &\equiv \text{orientation of sensor coil} \\ \tilde{m}^{(inc)}(s)\, \vec{1}_c &\equiv \text{magnetic moment of transmitter coil} \\ \vec{1}_c &\equiv \text{orientation of transmitter coil} \\ R &= |\vec{r}'| = \text{distance of target from observer} \\ \vec{1}_R &= -\frac{\vec{r}'_0}{R} = \text{direction from target to observer} \end{aligned} \tag{1.3}$$

Our problem is then to use the scattered magnetic field (either for many frequencies or for a time-domain pulse) to identify the target at unknown location $\vec{r}'_0$ with unknown orientation with respect to the observer. While Fig. 1.1 shows the example of a target buried below the ground surface, this need not be the case. It could be present on some other nonmagnetic body with low (or no) conductivity.

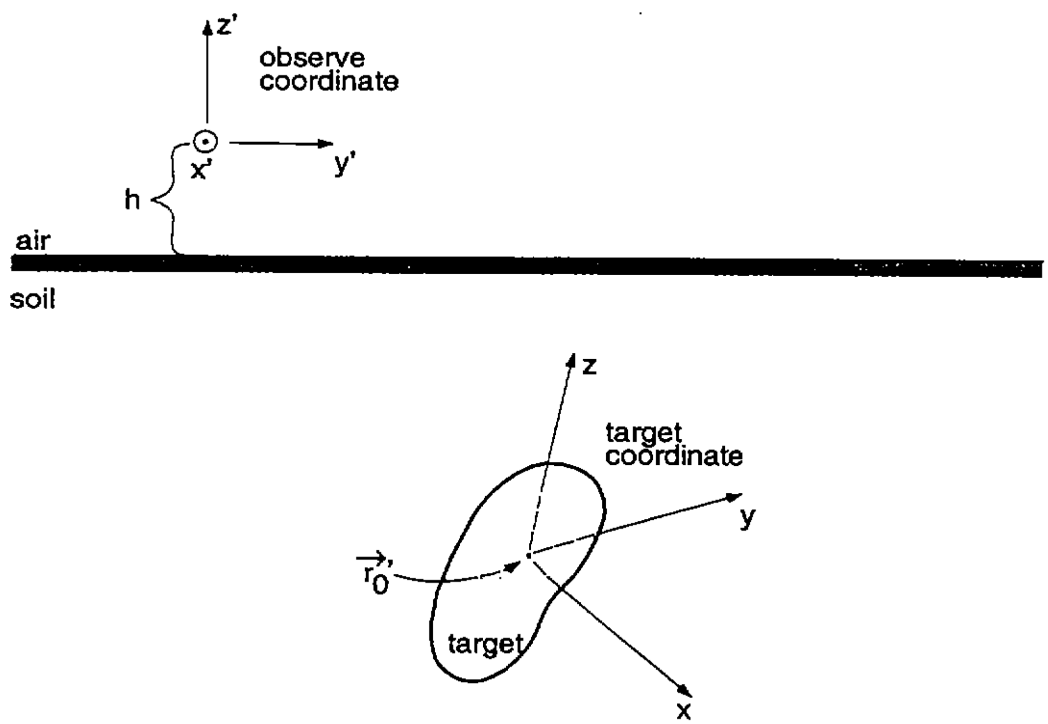

Fig. 1.1 Measurement Coordinates.

2. Symmetries in Magnetic-Polarizability Dyadic

Even with no point symmetries (rotation/reflection) [6] applying to the target, the magnetic-polarizability dyadic has certain symmetries. Since we have assumed that the target is made of reciprocal media the constitutive-parameter dyadics must be symmetric (equal to their transpose) as must we have

$$\overleftrightarrow{M}(s)^T = \overleftrightarrow{M}(s) \tag{2.1}$$

reducing the number of distinct elements in a 3 x 3 dyadic from nine to six.

Geometric symmetries in the target are characterized by orthogonal dyadics $\overleftrightarrow{G}_\ell$ (corresponding to group elements) which transform coordinates and constitutive parameters (say generically $\overleftrightarrow{p}(\vec{r})$) which transform as [2]

$$\begin{aligned} \vec{r}^{(2)} &= \overleftrightarrow{G}_\ell \cdot \vec{r}^{(1)} \\ \overleftrightarrow{p}(\vec{r}^{(2)}) &= \overleftrightarrow{G}_\ell \cdot \overleftrightarrow{p}(\vec{r}^{(1)}) \overleftrightarrow{G}_\ell^T \\ \overleftrightarrow{G}_\ell^T &= \overleftrightarrow{G}_\ell^{-1} \quad \text{(orthogonal real dyadic)} \end{aligned} \tag{2.2}$$

While the target geometry may have no symmetry the magnetic-polarizability dyadic still has inversion symmetry

$$\begin{aligned} &I = \{(1),\ (I)\} \ \text{inversion group} \\ &(1) \to \overleftrightarrow{1} = \vec{1}_x \vec{1}_x + \vec{1}_y \vec{1}_y + \vec{1}_z \vec{1}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &(I) \to -\overleftrightarrow{1} \\ &\left\{ \overleftrightarrow{1},\ -\overleftrightarrow{1} \right\} = \text{group representation} \end{aligned} \tag{2.3}$$

as can be seen from

$$(-\overleftrightarrow{1}) \cdot \overleftrightarrow{M}(s) \cdot (-\overleftrightarrow{1}) = \overleftrightarrow{M}(s) \tag{2.4}$$

This is a consequence of the fact that $\overleftrightarrow{M}(s)$ is not a function of $\vec{r}$.

3. Rotation of Target Coordinates

For a target in our library with $\overleftrightarrow{M}(s)$ expressed in terms of (x, y, z) coordinates, we need to rotate it to express it in terms of (x', y', z') coordinates. Since the location $\vec{r}'_0$ is not included here we need only rotate the target coordinates to align the coordinate directions (unit vectors).

We have rotation matrices describing rotations about the coordinate axes by an angle ψ as [5, 6]

$$\begin{aligned}
\left(C_{n,m}^{(x)}(\psi)\right) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\psi) & -\sin(\psi) \\ 0 & \sin(\psi) & \cos(\psi) \end{pmatrix} = \text{positive rotation by } \psi \text{ around } \vec{1}_x \\
\left(C_{n,m}^{(y)}(\psi)\right) &= \begin{pmatrix} \cos(\psi) & 0 & \sin(\psi) \\ 0 & 1 & 0 \\ -\sin(\psi) & 0 & \cos(\psi) \end{pmatrix} = \text{positive rotation by } \psi \text{ around } \vec{1}_y \\
\left(C_{n,m}^{(z)}(\psi)\right) &= \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \text{positive rotation by } \psi \text{ around } \vec{1}_z
\end{aligned} \tag{3.1}$$

Each of these is an orthogonal matrix and is a group element in $O_2^+(= SO(2))$ for continuous rotations about the indicated axes (with $0 \le \psi < 2\pi$, or any real 2π interval). This does not imply that our target has such symmetry.

Every proper rotation (no reflections) in three dimensional Euclidean space can be represented in terms of what are called Euler angles [5]. These can take various forms. For present purposes we can define

$$\left(O_{n,m}^{(+)}(\psi_1, \psi_2, \psi_3)\right) = \left(C_{n,m}^{(z)}(\psi_3)\right) \cdot \left(C_{n,m}^{(y)}(\psi_2)\right) \cdot \left(C_{n,m}^{(z)}(\psi_1)\right) \tag{3.2}$$

corresponding to successive rotations by ψ_1, ψ_2, and ψ_3 using axial rotations in (3.1). Note that

$$\left(C_{n,m}^{(\xi)}(\psi)\right)^{-1} = \left(C_{n,m}^{(\xi)}(\psi)\right)^{T} = \left(C_{n,m}^{(\xi)}(-\psi)\right) \quad \text{for } \xi = x, y, z \tag{3.3}$$

from which we find

$$\left(O_{n,m}^{(+)}(\psi_1,\psi_2,\psi_3)\right)^{-1} = \left(C_{n,m}^{(z)}(\psi_1)\right)^{-1}\cdot\left(C_{n,m}^{(y)}(\psi_2)\right)^{-1}\cdot\left(C_{n,m}^{(z)}(\psi_3)\right)^{-1}$$
$$= \left(C_{n,m}^{(z)}(\psi_1)\right)^{T}\cdot\left(C_{n,m}^{(y)}(\psi_2)\right)^{T}\cdot\left(C_{n,m}^{(z)}(\psi_3)\right)^{T}$$
$$= \left(O_{n,m}^{(+)}(\psi_1,\psi_2,\psi_3)\right)^{T} \tag{3.4}$$
$$= \left(C_{n,m}^{(z)}(-\psi_1)\right)\cdot\left(C_{n,m}^{(y)}(-\psi_2)\right)\cdot\left(C_{n,m}^{(z)}(-\psi_3)\right)$$
$$= \left(O_{n,m}^{(+)}(-\psi_3,-\psi_2,-\psi_1)\right)$$
$$0 \le \psi_n < 2\pi \quad , \quad n = 1,2,3$$

This is then an orthogonal matrix and is a group element in $O_3^+(=SO(3))$. It can be written as a single matrix as

$$\left(O_{n,m}^{(+)}(\psi_1,\psi_2,\psi_3)\right) = \begin{pmatrix} [\cos(\psi_3)\cos(\psi_2)\cos(\psi_1) - \sin(\psi_3)\sin(\psi_1)] & [-\cos(\psi_3)\cos(\psi_2)\sin(\psi_1) - \sin(\psi_3)\cos(\psi_1)] & [\cos(\psi_3)\sin(\psi_2)] \\ [\sin(\psi_3)\cos(\psi_2)\cos(\psi_1) + \cos(\psi_3)\sin(\psi_1)] & [-\sin(\psi_3)\cos(\psi_2)\sin(\psi_1) - \sin(\psi_3)\cos(\psi_1)] & [\sin(\psi_3)\sin(\psi_2)] \\ [-\sin(\psi_2)\cos(\psi_1)] & [\sin(\psi_2)\sin(\psi_1)] & [\cos(\psi_2)] \end{pmatrix} \tag{3.5}$$

This matrix can be used rotate the $\vec{r}$ coordinates to be parallel to the corresponding $\vec{r}'$ coordinates, or conversely. In order to express an orientation in the $\vec{r}$ system in terms of $\vec{r}'$ directions one reverses the process so that

$$\vec{1}_{r'} = \left(O_{n,m}^{(+)}(\psi_1,\psi_2,\psi_3)\right)^{T}\cdot\vec{1}_r \tag{3.6}$$

The magnetic polarizability dyadic is then expressed in the $\vec{r}'$ system as

$$\overleftrightarrow{M}'(s) = \left(O_{n,m}^{(+)}(\psi_1,\psi_2,\psi_3)\right)^{T}\cdot\overleftrightarrow{M}'(s)\cdot\left(O_{n,m}^{(+)}(\psi_1,\psi_2,\psi_3)\right)$$

$\equiv$ magnetic-polarizability dyadic expressed in terms of $\vec{r}'$ coordinates (3.7)

This can be applied termwise to the vectors in (1.1) as

$$\vec{M}'_{\alpha} = \left(O^{(+)}_{n,m}(\psi_1,\psi_2,\psi_3)\right)^T \cdot \vec{M}_{\alpha}$$
$$\vec{M}'^{\binom{\infty}{0}}_{\upsilon} = \left(O^{(+)}_{n,m}(\psi_1,\psi_2,\psi_3)\right)^T \cdot \vec{M}^{\binom{\infty}{0}}_{\upsilon} \tag{3.8}$$

This gives us three real scalar parameters to vary each over a 2π interval to attempt to fit the $\vec{M}_{\alpha}$ of a library target to the measured data.

Related to the inversion symmetry of $\overleftrightarrow{M}$ in (2.3) we can also note that $(O^{(+)}_{n,m})$ can be replaced equally well by $-(O^{(+)}_{n,m})$ in the above formulae. If the library target has geometrical symmetries we may expect there to be multiple choices of the angles in optimally fitting the library target to the data.

4. Position of Target Relative to Observer

Returning to Fig. 1.1, we also need to consider the position $\vec{r}'_0$ of the target with respect to the observer. This is present in (1.3) in R and $\vec{1}_R$ which we need to fit the data to the targets in the library.

For this purpose we have the usual cylindrical (Ψ', ϕ', z') and spherical (r', θ', ϕ') coordinates related as

$$x' = \Psi'\cos(\phi') \quad , \quad y' = \Psi'\sin(\phi')$$
$$\Psi' = r'\sin(\theta') \quad , \quad z' = r'\cos(\theta') \tag{94.1)}$$

with unit vectors

$$\begin{aligned}
\vec{1}_{\Psi'} &= \cos(\phi')\,\vec{1}_{x'} + \sin(\phi')\,\vec{1}_{y'} \\
\vec{1}_{\phi'} &= -\sin(\phi')\,\vec{1}_{x'} + \cos(\phi')\,\vec{1}_{y'} \\
\vec{1}_{r'} &= \sin(\theta')\,\vec{1}_{\Psi'} + \cos(\theta')\,\vec{1}_{z'} \\
&= \sin(\theta')\cos(\phi')\,\vec{1}_{x'} + \sin(\theta')\sin(\phi')\,\vec{1}_{y'} + \cos(\theta')\,\vec{1}_{z'}
\end{aligned} \tag{4.2}$$

Since $\vec{1}_R$ points from the target to the observer we have

$$\begin{aligned}
\vec{1}_R &= -\,\vec{1}_{r'_0} \\
&= -\left[\sin(\theta'_0)\cos(\phi'_0)\,\vec{1}_{x'} + \sin(\theta'_0)\sin(\phi'_0)\,\vec{1}_{y'} + \cos(\theta'_0)\,\vec{1}_{z'}\right] \\
\vec{1}_R\,\vec{1}_R &= \vec{1}_{r'_0}\,\vec{1}_{r'_0}
\end{aligned} \tag{4.3}$$

this gives two parameters θ'_0 and ϕ'_0 for fitting data to library targets. If we know something about the target location a priori (e.g. buried in Fig. 1.1 so that $\pi/2 < \theta'_0 \leq \pi$) then the range of these angles can be limited from the general case of $0 \leq \theta'_0 \leq \pi$ and $0 \leq \phi'_0 < 2\pi$.

There is one more scaling parameter, the coefficient in (1.3), i.e.

$$\tilde{P}_6(s) = \frac{\tilde{m}^{(inc)}(s)}{16\pi^2 R^6} \tag{4.4}$$

Knowing $\tilde{m}^{(inc)}(s)$ (the transmitter strength) then knowledge of $\tilde{P}_6$ gives

$$R = \left[\frac{\tilde{m}^{(inc)}(s)}{16\pi^2 \tilde{P}_6(s)}\right] \qquad \text{(real and positive)} \tag{4.5}$$

Since $R > 0$ we can use R as a real scaling parameter in fitting the data to the library targets. Such is more constraining than varying $\tilde{P}_6(j\omega)$ over the complex numbers.

5. Concluding Remarks

Summarizing, for general nonsymmetric targets, there are six real parameters for use in fitting target magnetic-singularity signatures to measured data:

3 for target orientation: ψ_1, ψ_2, ψ_3

2 for direction from observer: θ_0', ϕ_0'

1 for distance from observer: R

The angles have a finite range while R has a semi-infinite range. In practice the range of R is limited by the distance at which a target can be detected. If one is successful in identifying such a target then estimates of target location and orientation are also obtained. (In [3] this is also obtained for a class of symmetrical targets.)

By comparison targets with a 3-fold (or greater) symmetry axis (C_N for $N \geq 3$) can be characterized by only two real scaling parameters without simultaneously obtaining target orientation and location [4]. The six-parameter fitting for nonsymmetrical targets makes it generally easier to fit a library target signature than does a two-parameter fitting. Perhaps this drawback can be compensated by the inclusion of more poles (greater bandwidth) in the representation of the target in the library and in the data collected.

In (1.3) there are two measurement parameters $\vec{1}_c$ (transmitter coil orientation) and $\vec{1}_s$ (sensor coil orientation) which can be varied to obtain more than one broadband measurement. By judicious choices of this pair of unit vectors, say utilizing all three of the coordinate directions in the $\vec{r}\,'$ system, one can obtain six independent measurements (noting reciprocity). Requiring that the library target parameters fit the data with the same set of the six parameters (i.e., only one value of each of the six parameters used for fitting all the data) should make the target discrimination more robust. The actual target, after all, does not change its orientation and location as one uses various combinations of $\vec{1}_c$ and $\vec{1}_s$.

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