V

Interaction Notes

Note 531

July 18, 1997

A Library of the Natural Frequency Responses for Rectangular Shaped Buried Plastic Mines

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Abstract

The dielectric mine problem is presented. An approximate approach for finding the natural frequencies of dielectric targets in a dielectric medium is presented. This approach is applied to (and demonstrated for) a dielectric, rectangular cavity. A region where this approach is "accurate" is shown. In the Appendix section a library of dielectric, rectangular shaped mines is presented. The library gives the name, country of origin, countries using, and electromagnetic information about 15 different plastic anti-personnel and anti-tank mines. A method for using this library to find the natural frequencies is presented and demonstrated. If this library proves effective it is suggested that the user input all of the data into software for real time (and easier) manipulation.

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Abstract

The dielectric mine problem is presented. An approximate approach for finding the natural frequencies of dielectric targets in a dielectric medium is presented. This approach is applied to (and demonstrated for) a dielectric, rectangular cavity. A region where this approach is "accurate" is shown. In the Appendix section a library of dielectric, rectangular shaped mines is presented. The library gives the name, country of origin, countries using, and electromagnetic information about 15 different plastic anti-personnel and anti-tank mines. A method for using this library to find the natural frequencies is presented and demonstrated. If this library proves effective it is suggested that the user input all of the data into software for real time (and easier) manipulation.

Contents

I. *Introduction*

Successfully detecting buried dielectric land mines has proven to be a technical nightmare. As I stated in Interaction Note 530 former war zone areas like Bosnia, Iraq, and Afghanistan have demonstrated just how lacking in technology we are in finding buried plastic targets. One common cry from the scientist and military personnel working on this problem has been to establish a library of the natural frequency responses for each and every plastic land mine. With such a library a user could know if what has been detected is of interest. In Interaction Note 530 a library of cylindrically shaped, plastic land mines was presented. It is the purpose of this note to establish a library for rectangular shaped, plastic land mines.

This task will be accomplished using Dr. Carl E. Baum's perturbation formula method for finding the signatures of dielectric targets in a dielectric medium [Baum, 1994]. This methodology will be applied to determine the pole functions for a dielectric rectangular cavity. An established "effective" region for using these "pole functions" will be presented. With this effective region one can decide whether or not to use the functions by simply entering in their soil conditions (ε and σ) and the thickness of their target into an equation and see if the result of this equation falls within an acceptable percent error [Worthy, 1997].

This note will supply the radar user with all of the tools needed to quickly find the pole locations of 15 different plastic anti-tank and anti-personnel mines. These are mines that are approximately rectangular in shape. The names, country of origin, countries known to use, and important electromagnetic information about each of these mines will be provided.

II. Definitions & Equations

Before we look at the perturbed functions its important that we define some necessary terms. The relative dielectric constant, ε_r, will be defined as

$$\varepsilon_r \equiv \frac{target}{medium} = \frac{\varepsilon_2}{\varepsilon_1}$$

The propagation constants for the medium, γ_1, and the target, γ_2, are

$$\gamma_1 = s\sqrt{\mu_o \varepsilon_1}\left[1 + \frac{\sigma_1}{s\varepsilon_1}\right]^{\frac{1}{2}}$$

and

$$\gamma_2 = s\sqrt{\mu_o \varepsilon_2}$$

where $s = \Omega + j\omega \equiv$ the complex frequency.

The ratios of these propagation constants, ξ, will be defined as:

$$\xi(s) \equiv \frac{\gamma_1}{\gamma_2} = \varepsilon_r^{-\frac{1}{2}}\left[1+\frac{\sigma_1}{s\varepsilon_1}\right]^{\frac{1}{2}}$$

We will define the natural frequencies as:

$$s_\alpha = s_{\alpha,0} + \Delta s_\alpha$$

Now we are ready for the perturbation formulas.

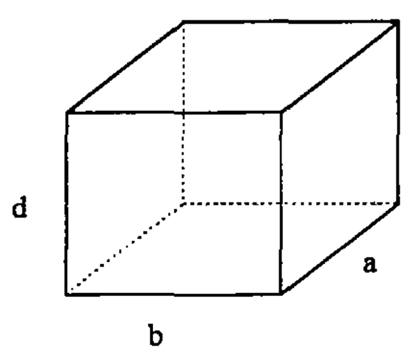
III. The Rectangular Cavity

An approximate method for finding the natural frequencies of dielectric targets in a dielectric medium was first presented by Dr. Carl Baum in 1994 (Interaction Note 504). In 1996 Dr. George Hanson developed a more general formula and applied it to a finite cylindrical shaped target (Interaction Note 520). In Interaction Note 529 I compared these formulas to the exact pole locations. In doing so an "effective" region for using this method was established [Worthy, 1997].

The general method for the perturbed functions involves solving for two terms $s_{\alpha,0}$, and Δs_α. The $s_{\alpha,0}$ term is found from the exact theoretical equations by applying the limiting case when $\xi \to \infty$. The Δs_α term is found from the following equation [Hanson, 1996]:

$$\Delta s_\alpha = \frac{-1}{2\xi(s_{\alpha,0})\sqrt{\mu_0\varepsilon_2}}\left[\frac{\oint_s |\vec{H}_0|^2\, ds}{\iint_v |\vec{H}_0|^2\, dv}\right]$$

The resonant frequency, $s_{\alpha,0}$, of a rectangular cavity has been found (and discussed) by Harrington, Dearholt, Collin, Kong, and others. Where for the rectangular cavity:



we have:

$$s_{\alpha,0} = \frac{j}{\sqrt{\mu_0 \varepsilon_2}} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2 \right]^{\frac{1}{2}}$$

Where $m = 0,1,2$, etc., $n = 0,1,2$, etc., and $p = 0,1,2$, etc.. Since these integers m, n, and p represent the half-wave periodicity in the y, x, and z directions respectively, "the lowest resonant frequency (the dominant mode) occurs when the integer associated with the smallest dimension is zero and the other two are unity" [Dearholt, 1973]. Therefore, if d is the smallest dimension then the most dominant frequency will take the following form.

$$s_{\alpha,0} = \frac{j}{\sqrt{\mu_0 \varepsilon_2}} \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 \right]^{\frac{1}{2}}$$

The next dominant mode depends upon the shape of the target. Where (assuming d is still the smallest dimension) if $a > b$ then the next dominant mode will be found by letting $m = 2$ and $n = 1$. However, if $a \cong b$ then the next dominant mode will be found by letting $m = 2$ and $n = 2$.

After applying the surface and volume integrals to our cavity (assuming that the thickness, d, was the smallest dimension) I found Δs_α to be:

$$\Delta s_\alpha = \frac{-1}{\xi(s_{\alpha,0})\sqrt{\mu_0 \varepsilon_2}} \left[\frac{2a^3 d + 2b^3 d + ab^3 + a^3 b}{abd\left(a^2 + b^2\right)} \right]$$

If, however, the width, b, had been the smallest dimension then we would use:

$$\Delta s_\alpha = \frac{-1}{\xi(s_{\alpha,0})\sqrt{\mu_0 \varepsilon_2}} \left[\frac{2a^3 b + 2d^3 b + ad^3 + a^3 d}{abd\left(a^2 + d^2\right)} \right]$$

From these two equations one could easily predict Δs_α for the case when the length, a, is the smallest dimension.

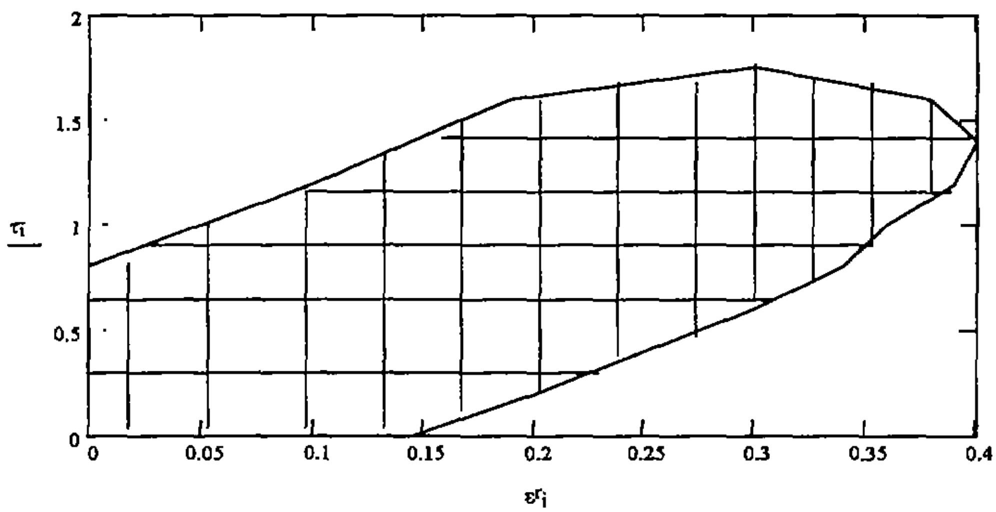
IV. The Effective Region

Now that we have established the perturbation functions it becomes important to know when they will give good results. In Interaction Note 529 I established a region that when given some values d, σ_1, ε_1, and ε_2 someone could check to see if using the perturbation functions would be accurate. I established two unit-less parameters, τ and ε_r, for graphing the effective region.

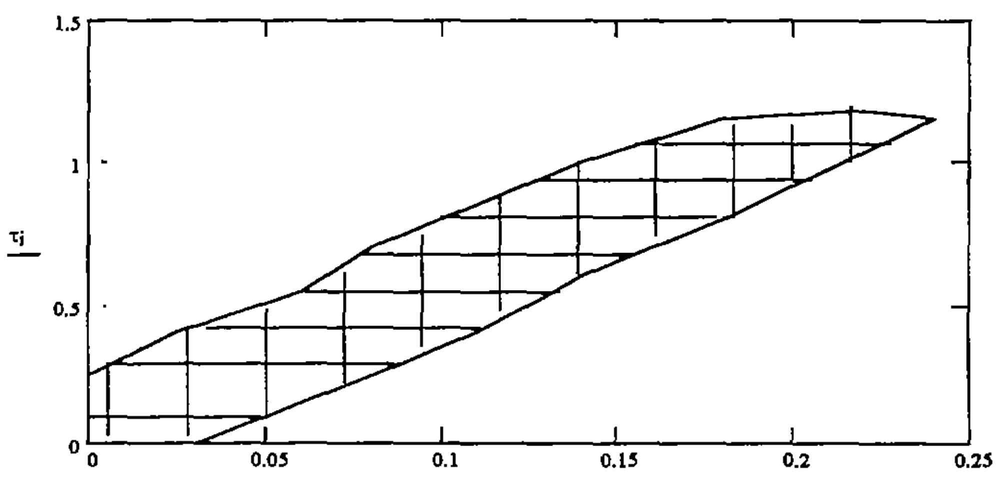
Where:

$$\tau = \frac{d\sigma_1\sqrt{\mu_o\varepsilon_2}}{\varepsilon_1} \quad \text{and} \quad \varepsilon_r = \frac{\varepsilon_2}{\varepsilon_1}$$

In the following graphs (below) everything inside the curves represents a "good" region. In the first graph the perturbed functions were less than 6% off for the region inside the curve, and in the second graph the perturbed functions were less than 2% off.



Graph 1: < 6% error for perturbed functions



Graph 2: < 2% error for perturbed functions

In order to use these graphs you must first know your soil conditions, σ_1 and ε_1. Then you will simply enter σ_1 and ε_1, and the d and ε_2 values for the mine into the equations for τ and ε_r. Finally, check these values to see if they fall into the "good" region on the graphs.

V. How the library works

A complete library of pole locations of dielectric mines in a dielectric medium would be an impossible task since there are infinite different possible values for σ_1 and ε_1 (even within the values for common soils and our effective region). Therefore, what I've done is compile a library of parameters for each land mine such that the user can quickly calculate the poles for the mine by simply inputting their particular soil conditions.

In the library I've listed the two most dominant $s_{\alpha,0}$ values for each mine. I've also listed a parameter CS such that:

$$CS = \frac{-1}{\sqrt{\mu_0 \varepsilon_2}}\left[\frac{2a^3b + 2d^3b + ad^3 + a^3d}{abd\left(a^2 + d^2\right)}\right]$$

Therefore:

$$\Delta s_\alpha = \frac{CS}{\xi(s_{\alpha,0})}$$

Hence, recalling that $s_\alpha = s_{\alpha,0} + \Delta s_\alpha$, in order to find the pole locations for a particular mine the user will first calculate

$$\xi(s_{\alpha,0}) = \varepsilon_r^{-\frac{1}{2}}\left[1 + \frac{\sigma_1}{s_{\alpha,0}\varepsilon_1}\right]^{\frac{1}{2}},$$

then input this value into the Δs_α equation above, and add this to the corresponding $s_{\alpha,0}$ value. Note that the CS equation used in the library is not necessarily the equation above. The CS equation used depended upon which dimension was the smallest (a, b, or d) in that particular mine (the CS equation above is for the case when d is the smallest).

Since repeatedly calculating these poles with this method can be a monotonous task (though much faster and easier than the exact method) the user might wish to enter the tabulated (library) values (along with the values for the cylindrical shaped mines from IN 530) into a math software program. This could potentially allow for real time identification (if a user had all of the library data entered in their computer).

VI. Example

As an example lets say you have radar of a suspected mine field laid by the Polish army. You've eliminated all the metallic mine choices. You've also used IN 530 to eliminate all of the possible cylindrically shaped mine choices. Unfortunately you still can't tell if the remaining "blips" on the screen are plastic mines or a child's toy. What do you do?

Assuming that you don't already have the library data in your computer, you should first (if you know the soil conditions) go to Appendix A. In Appendix A I have tabulated a list (with the help of a CD-ROM called "Mine Facts") of all the plastic rectangular shaped anti-personnel and anti-tank mines from all over the world. From this list you can see that the only plastic rectangular shaped land mine used by the Polish army is the anti-tank mine PT MI-BA II.

Next you will want to calculate the natural frequencies of this mine to see if it could possibly be the "blips" on your screen. To do this you must first see if the soil conditions will allow you to use the perturbation formulas (i.e. check the effective region). Lets say that the soil in the mine field has a permittivity of $\varepsilon_1 = 12\varepsilon_0$ and a conductivity of $\sigma_1 = 0.05$ S/m. From Appendix B we see that the thickness of the PT MI-BA II is 0.135m. This means that $\varepsilon r = 0.2083333333$ and $\tau = 0.335060166$. Upon looking at the "effective region" graphs (on page 6) we see that these values do lie within the < 6% error region.

So, lets see what Appendix B says about the PT MI-BA II:

B-II. Anti-Tank Mines

Mine	*a*	*b*	*d*	mode-1 $s_{\alpha,0}$ *(Rad/s)*	mode-2 $s_{\alpha,0}$ *(Rad/s)*	*CS (1/s)*
PT MI-BA II	.395m	.23m	.135m	$2.9968882 \times 10^9 j$	$3.9753727 \times 10^9 j$	-2.8788417×10^9

Now, recall the necessary equations:

$$\xi(s_{\alpha,0}) = \varepsilon_r^{-\frac{1}{2}}\left[1+\frac{\sigma_1}{s_{\alpha,0}\varepsilon_1}\right]^{\frac{1}{2}}$$

$$\Delta s_\alpha = \frac{CS}{\xi(s_{\alpha,0})}$$

$$s_\alpha = s_{\alpha,0} + \Delta s_\alpha$$

Calculating for mode-1 we find:

$\xi(s_{\alpha,0}) = 2.191160 - .0343982j$ per Rad, and
$\Delta s_\alpha = -1.3135198 \times 10^9 - 2.0620465 \times 10^7 j$ Rad/s.

Therefore, the most dominant pole will be:

$$s_\alpha = s_{\alpha,0} + \Delta s_\alpha = -1.3135198 \times 10^9 \pm 2.9762677 \times 10^9 j \text{ Rad/s}$$

Calculating for mode-2 we find:

$\xi(s_{\alpha,0}) = 2.1910437 - .0259329j$ per Rad, and
$\Delta s_\alpha = -1.3137293 \times 10^9 - 1.5549149 \times 10^7 j$ Rad/s.

Therefore, the next most dominant pole will be:

$$s_\alpha = s_{\alpha,0} + \Delta s_\alpha = -1.3137293 \times 10^9 \pm 3.9598236 \times 10^9 j \text{ Rad/s}$$

VII. Conclusions

The dielectric mine problem was presented. An approximate approach for finding the natural frequencies of dielectric, rectangular shaped targets in a dielectric medium was presented. A region where this approach is "accurate" was presented.

In the Appendix section a library of dielectric, rectangular shaped mines is presented. The library gives the name, country of origin, countries using, and electromagnetic information about 15 different anti-personnel and anti-tank mines.

A method for using this library is presented and demonstrated. If this library proves effective it is suggested that the user input all of the data into software for real time (and easier) manipulation.

Acknowledgments

I would to recognize Carl E. Baum for his guidance in this project. I would also like to most graciously thank AFOSR and Phillips Laboratory for their sponsorship in this work.

About the Appendices

Appendix A list all of the plastic land mines with an approximate uniform rectangular shape (which makes them applicable to our perturbation functions). For each mine a list of the country (countries) who manufacture the mine and the countries who use the mine is given.

Appendix B contains all of the $s_{\alpha,0}$, CS, and dimensional values for 13 different plastic anti-personnel mines. These mines are listed in alphabetical order.

Appendix A: Rectangular-Plastic-Mines List

A-I. Anti-Personnel Mines

Mine	*Made By*	*Counties Used By*
M-62	Hungary	Hungary, Cambodia
MI AP ID 48	France	France, Nigeria, Iceland
NR 15	France	Netherlands
PP-56	Former Yugoslavia	Former Yugoslavia
TM-200	Former Yugoslavia	Former Yugoslavia
TM-500	Former Yugoslavia	Former Yugoslavia

A-II. Anti-Tank Mines

Mine	*Made By*	*Counties Used By*
AT 3A	India	India
L9	United Kingdom	United Kingdom
M-19	USA, Chile, Iran, South Korea, Turkey	USA, Chile, South Korea, Angola, Zambia, Iran, Turkey
P2 MK3	Pakistan	Pakistan, Somalia, Eritrea, Ethiopia, Afghanistan
PT-56	Former Yugoslavia	Former Yugoslavia
PT MI-BA II	Former Czechoslovakia	Czech and Slovak Republics, Poland, South Africa, Namibia, Somalia, Eritrea, Ehiopia
TMA-2	Former Yugoslavia	Former Yugoslavia, Angola, Zambia, Namibia, South Africa
TMA-5	Former Yugoslavia	Former Yugoslavia, Angola, Zambia, Namibia, South Africa, Afghanistan
VS-AT4	Italy	Unknown

Appendix B: Pole Parameters for Rectangular, Plastic Mines

B-I. Anti-Personnel Mines

Mine	*a*	*b*	*d*	mode-1 $s_{\alpha,0}$ (Rad/s)	mode-2 $s_{\alpha,0}$ (Rad/s)	*CS* (1/s)
M-62	.187m	.05m	.065m	$9.7018622 \times 10^9 j$	$1.1160900 \times 10^{10} j$	-9.2158222×10^9
MI AP ID 48	.11m	.099m	.066m	$8.0947672 \times 10^9 j$	$1.6189534 \times 10^{10} j$	-6.5318004×10^9
NR 15	.095m	.088m	.065m	$9.2267250 \times 10^9 j$	$1.8453450 \times 10^{10} j$	-7.0795822×10^9
TM-200	.059m	.032m	.109m	$1.1480099 \times 10^{10} j$	$1.4878998 \times 10^{10} j$	$-1.1684382 \times 10^{10}$
TM-500	.07m	.05m	.108m	$1.0140540 \times 10^{10} j$	$1.3931592 \times 10^{10} j$	-8.6455364×10^9

B-II. Anti-Tank Mines

Mine	*a*	*b*	*d*	mode-1 $s_{\alpha,0}$ (Rad/s)	mode-2 $s_{\alpha,0}$ (Rad/s)	*CS* (1/s)
AT 3A	1.21m	.108m	.08m	$5.5373197 \times 10^9 j$	$5.6025831 \times 10^9 j$	-5.8559999×10^9
L-9	1.2m	.108m	.082m	$5.5376859 \times 10^9 j$	$5.6040308 \times 10^9 j$	-5.7977948×10^9
M-19	.332m	.332m	.094m	$2.5373313 \times 10^9 j$	$5.0746627 \times 10^9 j$	-3.1592777×10^9
P2 MK3	.262m	.262m	.12m	$3.2152443 \times 10^9 j$	$6.4304886 \times 10^9 j$	-3.0274123×10^9
PT MI-BA II	.395m	.23m	.135m	$2.9968882 \times 10^9 j$	$3.9753727 \times 10^9 j$	-2.8788417×10^9
TMA-2	.26m	.2m	.14m	$3.7575352 \times 10^9 j$	$5.4649105 \times 10^9 j$	-3.0877178×10^9
TMA-5	.312m	.275m	.113m	$2.8873346 \times 10^9 j$	$5.7746692 \times 10^9 j$	-2.9853724×10^9
VS-AT4	.28m	.188m	.104m	$3.8163538 \times 10^9 j$	$5.3048672 \times 10^9 j$	-3.6342654×10^9

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The information about the names of, country made in, countries used by, and dimensions of the mines in the Appendices came from "Mine Facts," a CD-ROM developed by the Essex Corporation for the Department of Defense in 1996.