Interaction Notes

Note 523

15 October 1996

Symmetry in Electromagnetic Scattering as a Target Discriminant

Carl E. Baum
Phillips Laboratory

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Symmetry in a target gives related properties to the electromagnetic scattering. Emphasizing targets on or below the surface of the ground (or water), this paper considers symmetry in the context of magnetic singularity identification (utilizing low-frequency diffusion in metal targets) and electromagnetic singularity identification (utilizing a ground-penetrating radar with wavelengths of the order of the dimensions of the metal and/or dielectric targets). It is found that the presence of various target symmetries can be detected and that this can potentially be used as a target discriminant (a first-order cut) for concentrating ones attention on more interesting targets (for further analysis and/or destruction).

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Symmetry in a target gives related properties to the electromagnetic scattering. Emphasizing targets on or below the surface of the ground (or water), this paper considers symmetry in the context of magnetic singularity identification (utilizing low-frequency diffusion in metal targets) and electromagnetic singularity identification (utilizing a ground-penetrating radar with wavelengths of the order of the dimensions of the metal and/or dielectric targets). It is found that the presence of various target symmetries can be detected and that this can potentially be used as a target discriminant (a first-order cut) for concentrating ones attention on more interesting targets (for further analysis and/or destruction).

1. Introduction

Much is now understood concerning the influence of geometric symmetries (and associated groups) on electromagnetic interaction with and scattering from an object [14]. In target identification the singularity expansion method (SEM) utilizes the poles and associated natural modes in the scattering, these being organized according to the symmetries [2, 3, 6, 8, 11-13]. While the natural-frequency sets form useful target identifiers, what about the symmetries themselves? This is the subject of the present paper.

As a working hypothesis let us associate symmetry in target shape with the assumption that this object was made by human beings. There are various reasons for this association including ease of construction, simplicity of analysis (by the designer/fabricator), and performance characteristics (e.g., aerodynamic/hydrodynamic properties, pattern of detonation products, etc.). So as a general (but not-necessarily-always-followed) rule let us consider

$$\begin{aligned} &\text{symmetry in target} \Leftrightarrow \text{made by human beings} \\ &\qquad \Leftrightarrow \text{interest in target for further consideration or destruction} \end{aligned} \tag{1.1}$$

Note the two-way implication (equivalence) so that lack of symmetry is used to drop a target from further interest. Such a rule can be used as a sifting technique, or first step in target classification, selecting some targets for further analysis of their signatures [12,13]. It is then important to understand the circumstances under which this rule can be used reliably so that targets of interest are not excluded.

Of course symmetry can also appear in objects not made by human beings (e.g., crystal lattices, certain animal shells). Some of these can be ruled out by noting that only macroscopic shapes are important here, not microscopic crystal structure. Furthermore, we have a general range of sizes to consider for our targets of interest. As we shall see there is also the degree of symmetry to be considered, i.e., the richness of the structure of the various point symmetry groups appropriate to our finite-size targets of interest.

While the rule (1.1) can be applied to general radar targets (e.g., aircraft and missiles), our present concern is with targets on or below the surface of the ground or water, i.e., unexploded ordnance (UXO) and mines [10,15]. This leads one to consider the implications of symmetry in the context of magnetic singularity identification (MSI) associated with the natural frequencies of metal, and dielectric targets as measured by ground penetrating radar (GPR).

2. Magnetic Singularity Identification (MSI)

Utilizing the near magnetic field of loops one can excite the diffusion natural frequencies and measure the resulting magnetic-polarizability dyadic which takes the form [5, 15]

$$\tilde{\overleftrightarrow{M}}(s) = \tilde{\overleftrightarrow{M}}(\infty) + \sum_{\alpha} M_\alpha \vec{M}_\alpha \vec{M}_\alpha \left[s - s_\alpha\right]^{-1}$$

$$\frac{1}{s}\tilde{\overleftrightarrow{M}}(s) = \frac{1}{s}\tilde{\overleftrightarrow{M}}(0) + \sum_{\alpha} \frac{M_\alpha}{s_\alpha} \vec{M}_\alpha \vec{M}_\alpha \left[s - s_\alpha\right]^{-1}$$

$$\vec{M}_\alpha \cdot \vec{M}_\alpha = 1, \quad \vec{M}_\alpha \equiv \text{ real unit vector for } \alpha\text{th mode}$$

$$\vec{M}_\alpha \equiv \text{ real scalar, } s_\alpha < 0 \text{ (all negative real natural frequencies)}$$

$$\tilde{\overleftrightarrow{M}}(\infty) = \sum_{\upsilon=1}^{3} M_\upsilon^{(\infty)} \vec{M}_\upsilon^{(\infty)} \vec{M}_\upsilon^{(\infty)}$$

$$\vec{M}_\upsilon^{(\infty)} \equiv \text{ real eigenvectors (three)}$$

$$\vec{M}_{\upsilon_1}^{(\infty)} \cdot \vec{M}_{\upsilon_2}^{(\infty)} = 1_{\upsilon_1,\upsilon_2} \text{ (orthonormal)}$$

$$M_\upsilon^{(\infty)} \equiv \text{ real eigenvalues (non positive, not necessarily distinct)}$$

$$\tilde{\overleftrightarrow{M}}(0) = \sum_{\upsilon=1}^{3} M_\upsilon^{(0)} \vec{M}_\upsilon^{(0)} \vec{M}_\upsilon^{(0)} \tag{2.1}$$

$$\vec{M}_\upsilon^{(0)} \equiv \text{ real eigenvectors (three)}$$

$$\vec{M}_{\upsilon_1}^{(0)} \cdot \vec{M}_{\upsilon_2}^{(0)} = 1_{\upsilon_1,\upsilon_2} \text{ (orthonormal)}$$

$$M_\upsilon^{(0)} \equiv \text{ real eigenvalues (non negative, not necessary distinct)}$$

$$1_{\upsilon_1,\upsilon_2} = \begin{cases} 1 \text{ for } \upsilon_1 = \upsilon_2 \\ 0 \text{ for } \upsilon_1 \neq \upsilon_2 \end{cases}$$

$$s = \Omega + j\omega \equiv \text{ complex frequency or two-sided Laplace-transform variable}$$

Note that in the physical approximations used to derive these formulas transit times across the target and to and from the loops through the external medium are negligible. Furthermore, the external medium (such as soil or water) has no significant influence provided its permeability μ is the same as μ_0, the permeability of free space.

Consider now table 2.1, taken from [6, 15]. This summarizes the implications of the various point symmetry groups [14] on the form taken by the magnetic polarizability dyadic. As we can see, as we increase the symmetry in the target (moving down the table) the form taken by $\overleftrightarrow{M}(s)$ becomes more and more simple. The various unit vectors $\vec{M}_\alpha$ in (2.1) begin to align according to the various symmetry planes and axes. This requires that they line up as

$$\vec{M}_{\alpha_1} \cdot \vec{M}_{\alpha_2} = \begin{cases} 0 & \text{(perpendicular)} \\ \pm & \text{(parallel or antiparallel)} \end{cases} \tag{2.2}$$

Note that one can always replace any $\vec{M}_\alpha$ by $-\vec{M}_\alpha$ since they appear in the form of dyadic (outer) products with themselves. In an appropriate Cartesian system they can each be represented by only one of the unit vectors $\vec{1}_x, \vec{1}_y, \vec{1}_z$, i.e.,

$$\vec{M}_\alpha = \vec{1}_x \text{ or } \vec{1}_y \text{ or } \vec{1}_z \tag{2.3}$$

Table 2.1. Decomposition of Magnetic Polarizability Dyadic According to Target Point Symmetries

Form of $\tilde{\overleftrightarrow{M}}(s)$	Symmetry Types (Groups)
$\tilde{M}_z(s)\vec{1}_z\vec{1}_z + \tilde{\overleftrightarrow{M}}_t(s)$ $\left(\tilde{\overleftrightarrow{M}}_t(s)\cdot\vec{1}_z = \vec{0}\right)$	R_z (single symmetry plane) C_2 (2-fold rotation axis)
$\tilde{M}_z(s)\vec{1}_z\vec{1}_z + \tilde{M}_x(s)\vec{1}_x\vec{1}_x + \tilde{M}_y(s)\vec{1}_y\vec{1}_y$	$C_{2a} = R_x \otimes R_y$ (two axial symmetry planes) D_2 (three 2-fold rotation axes)
$\tilde{M}_z(s)\vec{1}_z\vec{1}_z + \tilde{M}_t(s)\vec{1}_z$ $\left(\overleftrightarrow{1}_z = \overleftrightarrow{1} - \vec{1}_z\vec{1}_z \Rightarrow \text{double degeneracy}\right)$	C_N for $N \geq 3$ (N-fold rotation axis) S_N for N even and $N \geq 4$ (N-fold rotation-reflection axis) D_{2d} (three 2-fold rotation axes plus diagonal symmetry planes)
$\tilde{M}(s)\overleftrightarrow{1}$ $\left(\overleftrightarrow{1} \Rightarrow \text{triple degeneracy}\right)$	O_3 (generalized sphere) T, O, Y (regular polyhedra)

which one being dependent on the natural-model index α. This result is achieved provided there is sufficient symmetry (second or lower form in the table) such as the common occurrence of two (or more) axial symmetry planes, or sufficient rotation symmetry.

Note that the third form down the table covers cases of double degeneracy. In such cases the components transverse to $\vec{1}_z$ are equal, indicating that there are two natural modes with each "transverse" natural frequency. This fact, if detected in the measurement, can of course be used to categorize the target by the degree of symmetry. While this makes the choice of $\vec{1}_x$ and $\vec{1}_y$ ambiguous, one can still choose them perpendicular to $\vec{1}_z$ (and to each other) in any convenient way to satisfy (2.3). The fourth form in the table gives cases of triple degeneracy, indicating that there are three natural modes for each natural frequency. Such degeneracy is in principle detectable. Any choice of a right-handed set of three unit vectors is acceptable in such cases. Remember that these results pertain to the dominant magnetic-dipole scattering. Higher order magnetic multipoles will have generally different properties.

In order to implement this symmetry-detection scheme it should be clear that one needs three-axis transmission and reception to obtain the 3×3 dyadic $\overleftrightarrow{M}(s)$. One can use reciprocity to reduce the number of components to be measured to six. This does, however, require a more elaborate set of coils than for just measuring natural frequencies.

3. Electromagnetic Singularity Identification (EMSI)

Consider now the case of a more classical ground-penetrating radar (GPR). In this case we are concerned with target resonant frequencies with wavelengths in the air and ground of the order of the target dimensions. The targets may be metal and/or dielectric. In the case of metal targets it is the external resonances that are of concern [4, 15]. In the case of dielectric targets there are both internal and external resonances [7, 15].

As discussed in [1] there can be various approaches to transmitting fields to (and receiving fields from) targets at or below the ground surface. For present purposes let the antenna(s) be above the ground sending/receiving a beam to/from the target. As indicated in fig. 3.1 let there be a target of interest near the ground surface (on, below, or partially buried). This ground surface S_g is assumed flat with surface normal $\vec{1}_z$. The usual h, v radar coordinates are assumed with $\vec{1}_h // S_g$ and with $(\vec{1}_h, \vec{1}_v, -\vec{1}_i)$ forming a right-handed system with $\vec{1}_i$ being the direction of incidence for the wave from the transmitting antenna. This gives us the two polarizations ("horizontal" and "vertical") for the electric field. For backscattering (the case of present interest) we have propagation direction $\vec{1}_o = -\vec{1}_i$ and can use the same h,v coordinates.

Beginning the symmetry discussion let there be a symmetry plane P (reflection symmetry) containing $\vec{1}_z$ and the antenna(s). Ideally all antennas should have such a vertical symmetry plane so that the fields are transmitted and received, in pure h and v senses. This could be one antenna with both polarization capabilities, or two antennas located on the same symmetry plane (one above the other). If the distance r to the target is sufficiently large then the transmit and receive antennas can be placed side by side due to the small difference in azimuth with respect to the target, this making only a small error. The ground (soil) should also have the same symmetry plane. As a practical matter, since the antennas are to be positioned at an arbitrary azimuth with respect to the target, this means that we are assuming translation symmetry (with respect to both x and y) for the ground, but allow z variation of the constitutive parameters (layering). If the constitutive parameters are not scalar (i.e., dyadic) similar symmetry constraints are required.

The target scatters fields given in the far field by a scattering dyadic as [8]

$$\tilde{\vec{E}}_f = \frac{e^{-\gamma r}}{4\pi r}\,\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s)\cdot\tilde{\vec{E}}^{(inc)}(\vec{0}, s) \quad , \quad \gamma = \frac{s}{c}$$

$$\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \equiv \text{scattering dyadic} \tag{3.1}$$

$$\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s)\cdot\vec{1}_i = \vec{0} = \vec{1}_o\cdot\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s)$$

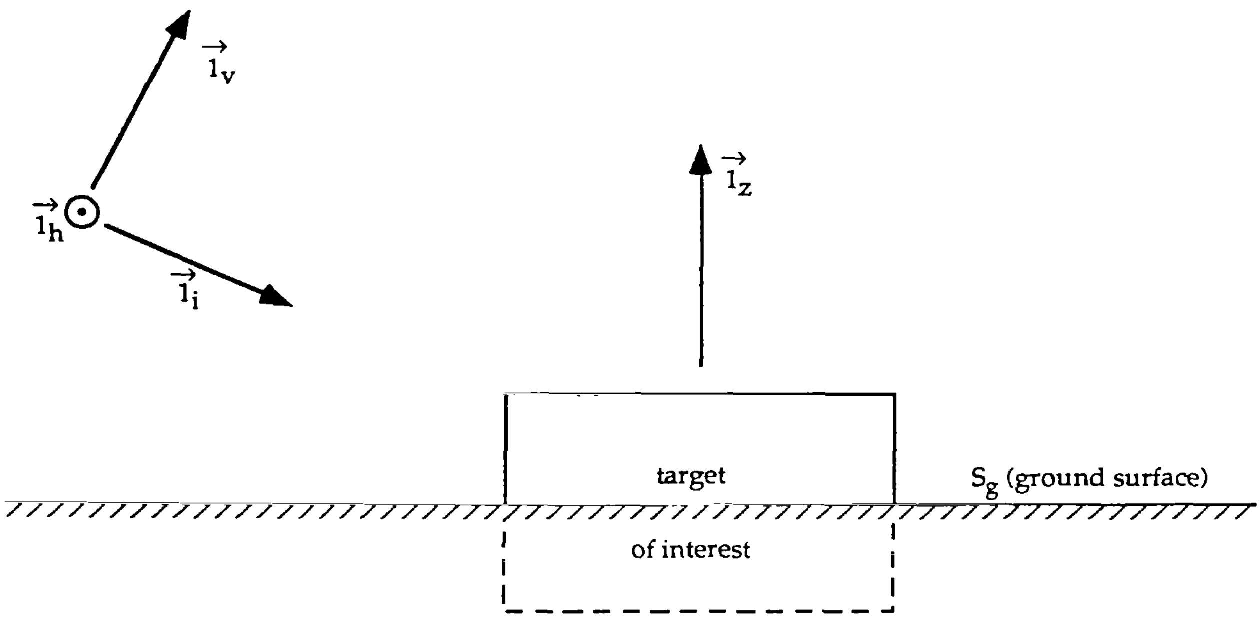


Fig. 3.1. Target and Radar Coordinates

which for backscattering reduces to

$$\vec{1}_o = -\vec{1}_i$$

$$\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) \equiv \tilde{\overleftrightarrow{\Lambda}}(-\vec{1}_i,\vec{1}_i;s) = \tilde{\overleftrightarrow{\Lambda}}_b^T(\vec{1}_i,s) \text{ (reciprocity)} \tag{3.2}$$

The coordinate origin ($\vec{r} = \vec{0}$) is taken near (or even inside) the target. Due to the lack of longitudinal components we have

$$\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) = \begin{pmatrix} \tilde{\Lambda}_{b_{h,h}}(\vec{1}_i,s) & \tilde{\Lambda}_{b_{h,v}}(\vec{1}_i,s) & 0 \\ \tilde{\Lambda}_{b_{v,h}}(\vec{1}_i,s) & \tilde{\Lambda}_{b_{v,v}}(\vec{1}_i,s) & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.3}$$

$$\tilde{\Lambda}_{b_{v,h}}(\vec{1}_i,s) = \tilde{\Lambda}_{b_{h,v}}(\vec{1}_i,s)$$

leaving only three components to consider in backscatter. There is also a wave scattered from the ground, but as long as $\vec{1}_i$ is not too near $-\vec{1}_z$ (antennas not above the target) this is scattered away from the target in a direction $\vec{1}_i - 2(\vec{1}_i \cdot \vec{1}_z)\vec{1}_z$.

Turning our attention now to the target let us consider its symmetry properties. As discussed in [8] there are symmetry properties that appear in the backscattering dyadic as summarized in table 3.1. Moving down the symmetry table go to the third entry. For R_a symmetry let us now assume that the vertical symmetry plane P previously discussed (for the antennas and soil) is also a symmetry plane for the target. Then the backscattering dyadic takes the simpler form in the *h,v* coordinates as

$$\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) = \begin{pmatrix} \tilde{\Lambda}_{b_{h,h}}(\vec{1}_i,s) & 0 & 0 \\ 0 & \tilde{\Lambda}_{b_{v,v}}(\vec{1}_i,s) & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.4}$$

$$\tilde{\Lambda}_{b_{v,h}}(\vec{1}_i,s) = 0 = \tilde{\Lambda}_{b_{h,v}}(\vec{1}_i,s)$$

i.e., the cross polarization is zero. This is readily detectable, and applies to pulse radars (as well as narrowband radars) since this property is frequency independent.

Table 3.1. Point Symmetry Groups (Rotation and Reflection) for Back-Scattering Dyadic for Reciprocal Target

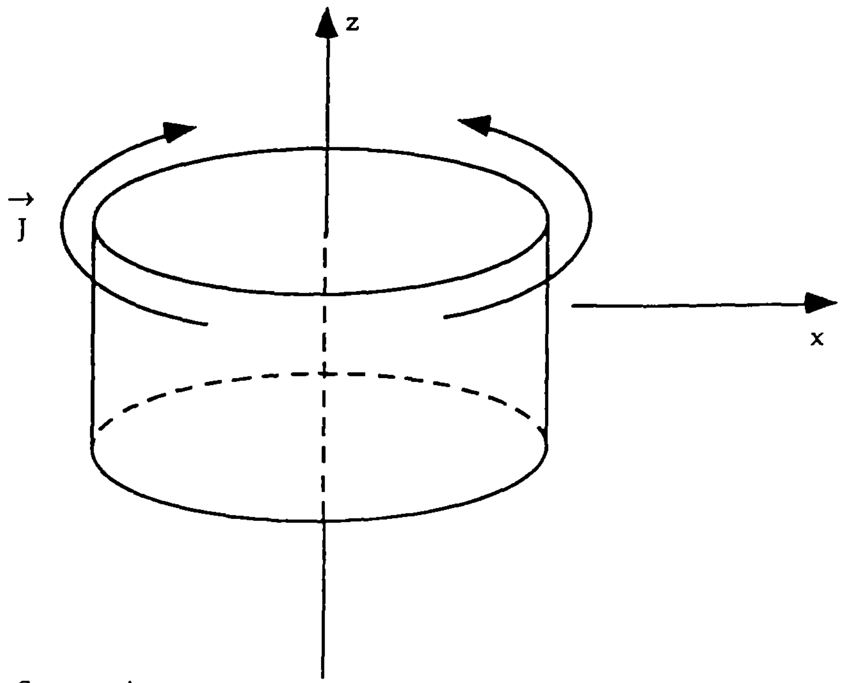
Symmetry in Target (transverse to $\vec{1}_i$)	Form of $\tilde{\overleftrightarrow{\Lambda}}_b$	Symmetry in $\tilde{\overleftrightarrow{\Lambda}}_b$
C_1 (no symmetry)	$\tilde{\overleftrightarrow{\Lambda}}_b = \tilde{\overleftrightarrow{\Lambda}}_b^T$	C_2 (two-fold axis $\vec{1}_i$)
C_2 (two-fold axis $\vec{1}_i$)	$\tilde{\overleftrightarrow{\Lambda}}_b = \tilde{\overleftrightarrow{\Lambda}}_b^T$	C_2 (two-fold axis $\vec{1}_i$)
R_a (single axial symmetry plane	$\tilde{\overleftrightarrow{\Lambda}}_b$ diagonal when referred to axial symmetry plane or perpendicular axial plane	C_{2a} (two-fold axis $\vec{1}_i$ with two axial symmetry planes)
C_N for $N \geq 3$ (N-fold axis $\vec{1}_i$)	$\tilde{\Lambda}_b \quad \overleftrightarrow{1}_i$	$C_{\infty a} = O_2$ (continuous rotation axis $\vec{1}_i$ with all axial planes as symmetry planes)

Discovering a target as in fig. 3.1, one does not expect *a priori* that the antenna/ground symmetry plane P is aligned with such a symmetry plane in the target. One could move the antenna(s) around the target in an attempt to discover the target symmetry plane. However, if the target has an infinite number of such symmetry planes as in $C_{\infty a}$ symmetry then the above results apply for every azimuthal position of the antennas around the target. This symmetry group corresponds to invariance to rotation by an arbitrary azimuthal angle ϕ as well as every symmetry plane being now an axial plane (a) containing the symmetry axis with orientation

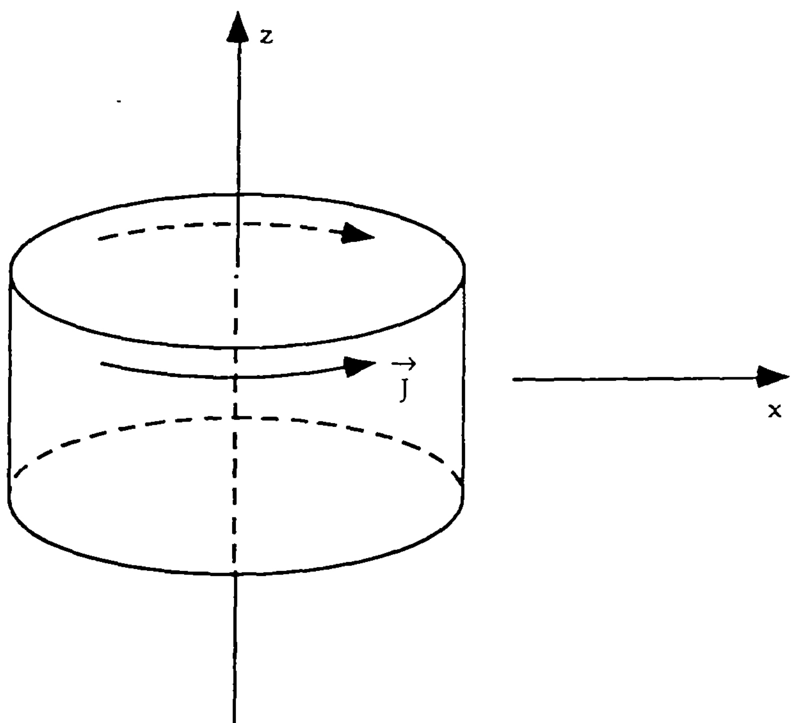
$$\vec{1}_{\infty a} = \vec{1}_z \tag{3.5}$$

Per our previous discussion the ground is assumed to also possess such symmetry, now including any depression or cavity in the ground occupied by the target. While there is a strong scattering from the ground (as discussed previously) the target scattering (including the effect of the local ground) can still be described by the foregoing scattering dyadic.

Another property of the $C_{\infty a}$ symmetry is the sorting of the natural frequencies due to modal degeneracy [2, 3, 11, 14]. The various natural modes have their ϕ dependence (in the usual (Ψ,ϕ,z) cylindrical coordinate system) as $\cos(m\phi)$ and $\sin(m\phi)$. For $m \geq 1$ there are two natural modes for each natural frequency s_α as illustrated in fig. 3.2 for the case of $m = 1$. Note that all modes can be categorized as symmetric or antisymmetric (not both) by their reflection properties with respect to the symmetry plane P (which we take as the yz plane or $x = 0$ plane). In this case the two modes differ only



A. Symmetric



B. Antisymmetric

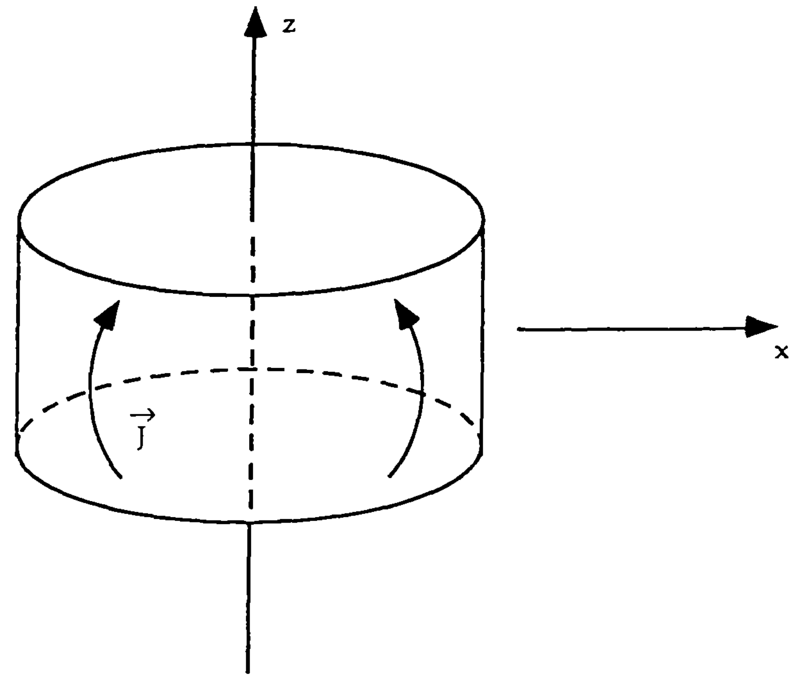
Fig. 3.2. Modal Degeneracy for $C_{\infty a}$ Targets for $m = 1$

by a rotation. In fig. 3.2 $\vec{J}$ indicates the pattern of the current density, including polarization current density in the case of dielectric targets. Both $\tilde{\Lambda}_{h,h}(\vec{1}_i,s)$ and $\tilde{\Lambda}_{v,v}(\vec{1}_i,s)$ will contain these same natural frequencies, except possibly for certain angles determined by $\vec{1}_i$. In the case of $m = 0$ there is not this type of degeneracy. Instead, as illustrated in fig. 3.3, the antisymmetric modes have current density in the form $J\,\vec{1}_\phi$ independent of ϕ; this implies zero divergence and, hence, H modes. The symmetric modes have current density perpendicular to $\vec{1}_\phi$ independent of ϕ; this implies a generally non-zero divergence and we can think of these as E modes.

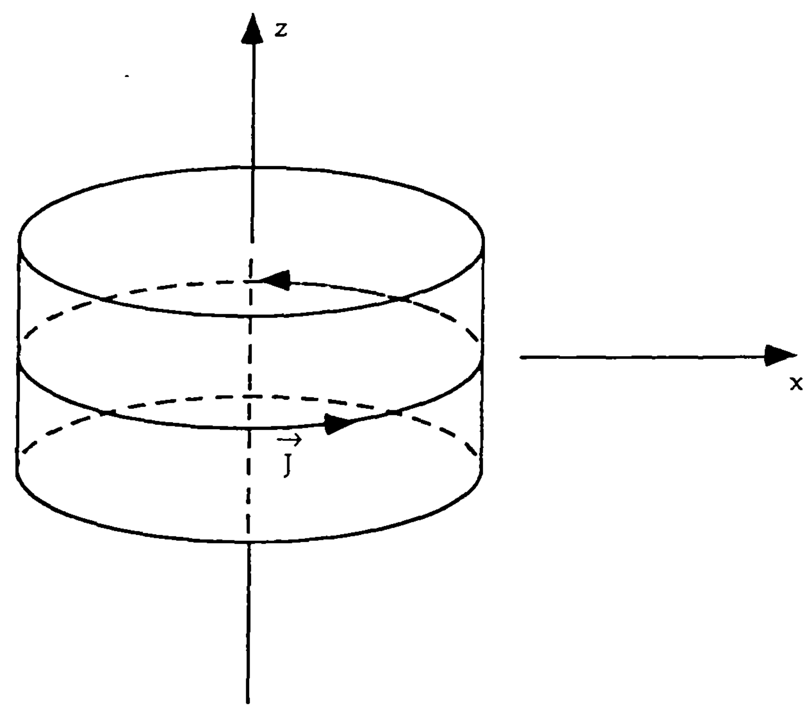
So for the case of $C_{\infty a}$ symmetry, not only do we have no cross polarization in the scattering, but also we have a separation of the natural frequencies according to m, the mode rotation index. For $m = 0$ we have certain natural frequencies that appear only in symmetric form (v,v scattering) or only in antisymmetric form (h,h scattering). For $m \geq 1$ the natural frequencies appear in both v,v and h,h scattering, but with generally different pole residues (amplitudes). While the presence of the ground shifts the natural frequencies away from their free-space or other convenient reference values, this sorting according to symmetry still applies, the actual change in the s_α not being important in this symmetry consideration. All these properties apply to a common target shape like a finite-length right circular cylinder of metal or dielectric resting on the ground surface or buried provided the $C_{\infty a}$ symmetry is maintained. Of course other more general target shapes adhering to $C_{\infty a}$ symmetry also have these properties.

Another possible symmetry of interest is C_N symmetry for $N \geq 3$. As indicated in the fourth entry in table 3.1, if the rotation axis is parallel to the direction of incidence ($\vec{1}_i = -\vec{1}_z$), there is no depolarization in backscattering even if there is no symmetry plane in the target. (This can apply to shapes like propellers [3, 11, 14]). This also implies a two-fold degeneracy of the natural modes, those that are excited in this axial illumination. In more general illumination conditions (off axis) as in fig. 3.1 these modes will still be present, so, except for those behaving like $m = 0$ modes in fig. 3.3, there are two modes for each natural frequency (a detectable condition). However, the absence of a vertical symmetry plane P in the measurement means that there is in general an h,v component in the scattering and (3.4) does not apply. So part, but not all, of the properties in the case of $C_{\infty a}$ symmetry apply in this case of lesser symmetry.

While the discussion in this section has been conducted on the basis of backscattering, the results can be applied to bistatic scattering as well. In the case, however, of $C_{\infty a}$ symmetry, for the no-cross-polarization result one can require that both antennas and target have a common vertical symmetry plane so that a common $\vec{1}_h$ applies to both transmit and receive antennas.



A. Symmetric (E mode)



B. Antisymmetric (H mode)

Fig. 3.3. Modes for $C_{\infty a}$ Targets for $m = 0$

4. Concluding Remarks

Symmetry can be used as a sifting technique to sort targets into those that are not interesting (clutter) and those that are of interest for further analysis and/or action. Of course symmetry is not perfectly maintained in the ground, the target, or even the antennas. So one will need to quantify how much asymmetry is tolerable for this technique to be useful in various applications (terrain, target, etc.). For example, how well do the $\vec{M}_\alpha$ line up as in (2.3), and how small should be cross-polarized $\tilde{\Lambda}_{b_{h,v}}$ (in (3.3) and (3.4)) be compared to $\tilde{\Lambda}_{b_{h,h}}$ and $\tilde{\Lambda}_{v,v}$?

Symmetry has been applied here in the contexts of MSI and EMSI. In the case of acoustic singularity identification (ASI) [9, 10, 15], if we only have pressure (p) waves in the far field (as in water) we have only a longitudinal component and hence no polarization on which to rely. The development in this paper has found that polarization (or more generally orientation of various vectors) is of great assistance in discovering target symmetry. If transverse shear (s) waves (besides p waves) are supported by the external medium then we have more general elastodynamic scattering and we can have in principle acoustic/elastodynamic scattering with polarization information also available for symmetry detection.

References

1. C.E. Baum, Antennas for Transmitting Fast Transients Through an Air/Earth Interface, Sensor and Simulation Note 375, December 1994.
2. C.E. Baum, Scattering, Reciprocity, Symmetry, EEM, and SEM, Interaction Note 475, May 1989.
3. C.E. Baum, SEM Backscattering, Interaction Note 476, July 1989.
4. C.E. Baum, The SEM Representation of Scattering from Perfectly Conducting Targets in Simple Lossy Media, Interaction Note 492, April 1993.
5. C.E. Baum, Low-Frequency Near-Field Magnetic Scattering from Highly, But Not Perfectly, Conducting Bodies, Interaction Note 499, November 1993.
6. C.E. Baum, The Magnetic Polarizability Dyadic and Point Symmetry, Interaction Note 502, May 1994.
7. C.E. Baum, Concerning the Identification of Buried Dielectric Targets, Interaction Note 504, July 1994.
8. C.E. Baum, Target Symmetry and the Scattering Dyadic, Interaction Note 507, September 1994.
9. C.E. Baum, The SEM Representation of Acoustic and Elastodynamic Scattering, Interaction Note 512, June 1955.
10. C.E. Baum, Discrimination of Buried Targets Via the Singularity Expansion, Interaction Note 521, August 1996.
11. C.E. Baum, SEM and EEM Scattering Matrices and Time-Domain Scatterer Polarization in the Scattering Residue Matrix, pp. 427-486, in W.-M. Boerner et al. (eds.), *Direct and Inverse Methods in Radar Polarimetry,* Kluwer Academic Publishers, Dordrecht, 1992.
12. C.E. Baum, Signature-Based Target Identification and Pattern Recognition, IEEE Antennas and Propagation Magazine, June 1994, pp. 44-51.
13. C.E. Baum, Concepts in Transient/Broadband Electromagnetic Target Identification, pp. 515-525, in L. Carin and L.B. Felsen (eds.), *Ultra-Wideband, Short-Pulse Electromagnetics 2,* Plenum Press, 1995.
14. C.E. Baum and H.N. Kritikos, Symmetry in Electromagnetics, Ch, 1, pp. 1-90, in C.E. Baum and H.N. Kritikos (eds.), *Electromagnetic Symmetry,* Taylor & Francis, 1995.
15. C.E. Baum (ed.), *Detection and Identification of Visually Obscured Targets,* Taylor & Francis (in publication).