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Direct Construction of a ξ-Pulse from Natural Frequencies and Evaluation of the Late-Time Residuals

Carl E. Baum

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Abstract

This paper considers the direct construction of a pulse to annihilate the late-time response of a target from the a priori knowledge of the target natural frequencies as in a target library. In evaluating the late-time residual after application of such a ξ-pulse one needs to know how large in some sense is the late-time residual to determine whether the ξ-pulse for a given target is matched to a particular radar return and thereby identify that target class. This can be aided by application of window Fourier/Laplace and wavelet transforms to the late-time residual.

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1. Introduction

An important type of target identification utilizes the natural frequencies of the target, these being poles in the complex-frequency s-plane as expressed in the singularity expansion method (SEM) [11]. Since the location of these natural frequencies s_α in the s-plane is aspect independent (independent of the properties of the incident field such as direction of incidence and polarization), this simplifies the identification problem to the comparison of the target pole patterns stored in some target library. There is also information in the pole residues (e.g., coupling vectors), but that is another matter.

In order to implement the pole pattern as a target discrimination scheme one needs some scheme to find the s_α in experimental data (scattered fields), either explicitly or implicitly. Some of the early schemes used what is referred to as the Prony method in which one fits a sum of dumped sinusoids to a waveform. This had various limitations which have been overcome to some degree by what is referred to as a matrix pencil method [12]. These and related techniques are explicit in that they determine a set of s_α from one or more waveforms which are then compared to the target library.

In contrast, implicit methods do not generate a set of s_α from the data, but utilize predetermined sets of s_α for various targets which are then manipulated with the data in some way which transforms it to a form in which the discrimination can be more readily performed. Those used to date have used temporal functions with two-sided Laplace transforms with zeros corresponding to the s_α (poles) of preselected targets. Convolving these with the target impulse response removes (annihilates) the late-time response for the properly chosen target. These go by various names such as K-pulse [9] and E-pulse [10] and are discussed in [11]. In the present paper we go further into this latter type of target identification.

2. Form of the Response for Electromagnetic Singularity Identification (EMSI)

Consider an incident plane wave of the form

$$\tilde{\vec{E}}^{(inc)}(\vec{r},s) = E_0\,\tilde{f}(s)\,\vec{1}_p\,e^{-\gamma\vec{1}_i\cdot\vec{r}} \quad , \quad \vec{E}^{(inc)}(\vec{r},t) = E_0\,\vec{1}_p\,f\left(t-\frac{\vec{1}_i\cdot\vec{r}}{c}\right)$$

$$\begin{aligned}
&\vec{1}_i \equiv \text{direction of incidence}, \quad \vec{1}_p \equiv \text{polarization} \\
&\gamma = \frac{s}{c} \equiv \text{propagation constant}, \; c \equiv [\mu\varepsilon]^{-1/2} \\
&\sim\; \equiv \text{two-sided Laplace transform (over time } t\text{)} \\
&s \equiv \Omega + j\omega \equiv \text{complex frequency or Laplace-transform variable}
\end{aligned} \tag{2.1}$$

where the medium of interest is considered lossless and dispersionless (e.g., free space). The far scattered field is given by

$$\begin{aligned}
\tilde{\vec{E}}_f(\vec{r},s) &= \frac{e^{-\gamma r}}{4\pi r}\,\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o,\vec{1}_i;s)\cdot\tilde{\vec{E}}^{(inc)}(\vec{0},s) \\
\vec{E}_f(\vec{r},t) &= \frac{1}{4\pi r}\,\overleftrightarrow{\Lambda}(\vec{1}_o,\vec{1}_i;s)\circ\vec{E}^{(inc)}\left(\vec{0},t-\frac{\vec{1}_o\cdot\vec{r}}{c}\right) \\
\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o,\vec{1}_i;s) &\equiv \text{scattering dyadic} \\
\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o,\vec{1}_i;s)\cdot\vec{1}_i &= \vec{0} = \vec{1}_o\cdot\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o,\vec{1}_i;s) \\
\vec{1}_o &\equiv \text{direction to observer} \\
\circ &\equiv \text{convolution with respect to time}
\end{aligned} \tag{2.2}$$

The coordinate origin ($\vec{r} = \vec{0}$) is chosen for convenience near or even within the target (such as the center of the minimum circumscribing sphere [3]). We also often have the case from reciprocity

$$\begin{aligned}
\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o,\vec{1}_i;s) &= \tilde{\overleftrightarrow{\Lambda}}^T(-\vec{1}_i,\vec{1}_o;s) \\
\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) &\equiv \tilde{\overleftrightarrow{\Lambda}}^T(-\vec{1}_i,\vec{1}_i;s) \equiv \tilde{\overleftrightarrow{\Lambda}}_b^T(\vec{1}_i,s) \text{ (symmetric in backscattering)}
\end{aligned} \tag{2.3}$$

As discussed in [3] the scattering dyadic can be written in SEM form (class 1)

$$
\begin{aligned}
\overleftrightarrow{\Lambda}(\vec{1}_o,\vec{1}_i,s) &= \sum_\alpha \frac{e^{-(s-s_\alpha)t_i}}{s-s_\alpha}\,\vec{c}_\alpha(-\vec{1}_o)\,\vec{c}_\alpha(\vec{1}_i) \\
&\quad + \text{possible entire function} \\
\overleftrightarrow{\Lambda}_b(\vec{1}_i,s) &= \sum_\alpha \frac{e^{-(s-s_\alpha)t_i}}{s-s_\alpha}\,\vec{c}_\alpha(\vec{1}_i)\,\vec{c}_\alpha(\vec{1}_i) \\
&\quad + \text{entire function} \\
\overleftrightarrow{\Lambda}_b(\vec{1}_i,t) &= \sum_\alpha u(t-t_i)\,e^{s_\alpha t}\,\vec{c}_\alpha(\vec{1}_i)\,\vec{c}_\alpha(\vec{1}_i) \\
&\quad + \text{entire function} \\
t_i &\equiv \text{initial time or turn-on time}
\end{aligned}
\tag{2.4}
$$

For the backscattering dyadic (in contradistinction to the currents induced on the scatterer) an entire function (no singularities in the finite s-plane) is definitely required. In time domain (meaning inverse Laplace transform of the entire function) this is a time-limited function related to transit times on the body. Note that the analysis in [3] is for the case of finite-size perfectly conducting scatterers (also allowing certain types of sheet-impedance loading).

Defining

$$
\begin{aligned}
L_0 &\equiv \text{maximum linear dimension of target} \\
t_0 &\equiv \frac{L_0}{c}
\end{aligned}
\tag{2.4}
$$

it is shown in [3] that in a worst-case sense for backscattering there is a time window of length t_0, beginning with the first return from the target, for which the pole series does not converge, and hence cannot be used as a valid representation in this time window. One can also define a discrimination time t_d as a minimum time for a target identification based on the time after the first signal return that information from everywhere on the target can reach the observer. For a "transparent" target, defined such that such information can travel in straight lines (allowing some attenuation) at the speed of light, c, we have for backscattering

$$
t_d = 2\,t_0 = 2\frac{L_0}{c} \tag{2.5}
$$

So even for times greater than t_0 there is some ambiguity in the discrimination process in the sense that different shapes (impedances, etc.) can be ascribed to the target, implying a different set of s_α and associated modes, but giving the same temporal response up to a time t_d.

For non-transparent targets such as perfectly conducting targets, the time required to observe the far side of the target in backscattering is increased so that (2.5) is rewritten as

$$t_d \geq 2t_0 \tag{2.6}$$

i.e., as a bound. Given a particular target one can of course calculate this minimum discrimination time maximized over all directions of incidence. Furthermore t_d might be indexed as t_{dm} , i.e., a separate value for each mth target type.

Another important case for EMSI with wavelengths of the order of target dimensions concerns targets buried in soil. This introduces some new difficulties in the identification problem for both perfectly conducting [4] and dielectric [7] targets. For not-too-conducting soil one can operate in the "high-frequency window" and utilize some approximate scaling relationships to allow for the dependence of the s_α on the external-medium parameters. This complicates the calculation of the t_d and even introduces branch cuts for which one should allow in the discrimination scheme.

While the present discussion has been in the context of the electromagnetic fields, the concepts can also be applied in the context of acoustic and elastodynamic fields [8]. For acoustic singularity identification (ASI), particularly if the response is dominated by p-waves in the external medium (e.g.,water), similar considerations concerning discrimination time t_d apply.

3. Form of the Response for Magnetic Singularity Identification (MSI)

For highly, but not perfectly, conducting targets there is a set of natural frequencies s_α which are based on diffusion of fields in the target, and scale as proportional to $(\mu\sigma d^2)^{-1}$ where d is some characteristic linear dimension of the target and σ is the target conductivity (high as in the case of typical metals) [5,6]. The external medium is assumed to have permeability μ_0 and conductivity typical of soil or water, and as such does not significantly affect the target response in the low-frequency range. The target permeability may be μ_0 or some higher permeability (e.g., iron), and this also affects the natural frequencies.

The incident field for this purpose would generally not be a plane wave as in (2.1), but the near magnetic field of one or more loops. Skin depths in the external medium for the low frequencies of interest are assumed large compared to all distances of interest. It is then the induced magnetic dipole moment $\vec{m}(t)$ that one uses for the waveform to be fed into the target-discrimination process. This is related to the magnetic-polarizability dyadic of the target via

$$\tilde{\vec{m}}(s) = \tilde{\overleftrightarrow{M}}(s) \cdot \tilde{\vec{H}}^{(inc)}(s) \tag{3.1}$$

where the incident magnetic field is evaluated at some convenient location in the vicinity of the target. The scattered magnetic field is

$$\begin{aligned} \tilde{\vec{H}}^{(sc)}(\vec{r},s) &= \frac{1}{4\pi r^3}\left[3\,\vec{1}_r\,\vec{1}_r - \overleftrightarrow{1}\right]\cdot\tilde{\overleftrightarrow{M}}(s)\cdot\tilde{\vec{H}}^{(inc)}(s) \\ \overleftrightarrow{1} &\equiv \vec{1}_x\,\vec{1}_x + \vec{1}_y\,\vec{1}_y + \vec{1}_z\,\vec{1}_z \equiv \text{identity dyadic} \\ \vec{1}_r &= \frac{\vec{r}}{r}\ , \quad r = |\vec{r}| \end{aligned} \tag{3.2}$$

where $\vec{r}$ is the observer position measured with respect to the target.

The general form for the magnetic polarizability dyadic is [5,6]

$$\begin{aligned} \tilde{\overleftrightarrow{M}}(s) &= \tilde{\overleftrightarrow{M}}(\infty) + \sum_\alpha M_\alpha\,\vec{M}_\alpha\,\vec{M}_\alpha\,\left[s - s_\alpha\right]^{-1} \\ \overleftrightarrow{M}(t) &= \tilde{\overleftrightarrow{M}}(\infty)\,\delta(t) + \sum_\alpha M\,\vec{M}_\alpha\,\vec{M}_\alpha\,e^{s_\alpha t}u(t) \end{aligned}$$

$s_\alpha < 0$, i.e., real and negative

$\vec{M}_\alpha \cdot \vec{M}_\alpha = 1$, $\vec{M}_\alpha$ = real vector , M_α = real scalar

$$\overset{\leftrightarrow}{\tilde{M}}_\alpha(\infty) = \sum_{\upsilon=1}^{3} M_\upsilon^{(\infty)} \vec{M}_\upsilon^{(\infty)} \vec{M}_\upsilon^{(\infty)}$$

$$\vec{M}_{\upsilon_1}^{(\infty)} \cdot \vec{M}_{\upsilon_2}^{(\infty)} = 1_{\upsilon_1,\upsilon_2} \text{ (orthonormal eigenvectors)} \tag{3.3}$$

$M_\upsilon^{(\infty)} \equiv$ real eigenvalues (non positive, not necessarily distinct)

$\vec{M}_\upsilon^{(\infty)}$ = real eigenvectors (three)

This is the form for the delta-function response. One can also use the step response which takes the form

$$\frac{1}{s}\overset{\leftrightarrow}{\tilde{M}}(s) = \frac{1}{s}\overset{\leftrightarrow}{\tilde{M}}(0) + \sum_\alpha \frac{M_\alpha}{s_\alpha} \vec{M}_\alpha \vec{M}_\alpha [s - s_\alpha]^{-1}$$

$$\int_{-\infty}^{t} \overset{\leftrightarrow}{M}(t')\, dt' = \overset{\leftrightarrow}{\tilde{M}}(0)\, u(t) + \sum_\alpha \frac{M_\alpha}{s_\alpha} \vec{M}_\alpha \vec{M}_\alpha\, e^{s_\alpha t}\, u(t)$$

$$\overset{\leftrightarrow}{\tilde{M}}(0) = \sum_{\upsilon=1}^{3} M_\upsilon^{(0)} \vec{M}_\upsilon^{(0)} \vec{M}_\upsilon^{(0)}$$

$$\vec{M}_{\upsilon_1}^{(0)} \cdot \vec{M}_{\upsilon_2}^{(0)} = 1_{\upsilon_1,\upsilon_2} \text{ (orthonormal eigenvectors)} \tag{3.4}$$

$M_\upsilon^{(0)} \equiv$ real eigenvalues (non negative, not necessarily distinct)

$\vec{M}_\upsilon^{(0)}$ = real eigenvectors (three)

The DC (zero frequency) polarizability applies only to permeable scatterers characterized by $\overset{\leftrightarrow}{\mu}(\vec{r})$ different from $\mu_o \overset{\leftrightarrow}{1}$ (free space, or more generally, the external medium).

Returning to the subject of discrimination time t_d discussed in the previous section, in the present context (MSI) the problem is considerably simplified. On the present time scale (s_α^{-1} associated with diffusion in the target) transit times across the target (as well as to and from the target) are neglected. Se we can write

$$t_d = 0_+ \tag{3.5}$$

i.e., just a little larger than zero. In (3.3) the entire function contribution is $\delta(t)$, but in practice will have some small non-zero width consistent with (3.5). One can also use the step response in (3.4) for the discrimination which has no entire-function contribution, but has an additional (and useful) pole at $s = 0$.

4. A Simple Way of Constructing of a ξ-Pulse

Consider a sampling pulse $f_s(t)$ which is to be convolved with the data. This can be a filter, say to remove high-frequency noise, or whatever. For convenience this can be normalized as

$$\int_{-\infty}^{\infty} f_s(t)dt = 1 \tag{4.1}$$

unless one wishes this pulse to have zero area. This can be used as a basis function, such as a subsectional basis function on an interval from 0 to T_0 as

$$f_s(t) = \begin{cases} T_0^{-1} & \text{for } 0 < t \leq T_0 \\ 0 & \text{otherwise} \end{cases} \tag{4.2}$$

Here T_0 would be a sampling interval and such a basis function is often used with K/E pulses.

Consider an elementary pulse (an annihilation pulse)

$$\begin{aligned} a_\ell(t) &= \delta(t) - e^{s_\ell T_0}\,\delta(t-T_0) \\ \tilde{a}_\ell(s) &= 1 - e^{[s_\ell - s]T_0} \\ \tilde{a}_\ell(s_\ell) &= 0 \end{aligned} \tag{4.3}$$

where $T_0 > 0$ is any convenient time (including possibly a sampling interval). Convolving this with an elementary target response $e^{s_\ell t}u(t)$ for any single pole gives zero after time T_0. i.e.

$$a_\ell(t) \circ \left[e^{s_\ell t}u(t)\right] = e^{s_\ell t}\left[u(t) - u(t-T_0)\right] \tag{4.4}$$

This is fine for a single pole with s_ℓ real (as is the case for all poles in MSI). In a more general case (e.g., EMSI and ASI) s_ℓ can be complex. One can generalize one's thinking to allow complex temporal waveforms, or recognize that such cases come in complex-conjugate pairs. In such a case we can associate

$$s_{\ell'} = s_\ell^* \tag{4.5}$$

and then form

$$\tilde{a}_\ell(s)\,\tilde{a}_{\ell'}(s) = \left[1-e^{[s_\ell - s]T_0}\right]\left[1-e^{[s_{\ell'} - s]T_0}\right]\left[1-e^{[s_{\ell'} - s]T_0}\right] \tag{4.6}$$

which can be used to annihilate real valued waveforms of the form $\left[a\,e^{s_\ell t} + a^* e^{s_{\ell'} t}\right]u(t)$ after a time $2\,T_0$.

Using this procedure one can construct a pulse with zeros at

$$s = s_\ell \quad \text{for} \quad \ell = 1, 2, \ldots, L \tag{4.7}$$

for L even or odd, including both conjugate pairs and individual poles on the negative Re[s] axis of the s-plane. Defining for convenience

$$S_\ell \equiv s_\ell\, T_0 \quad , \quad S \equiv s\,T_0 \quad , \quad X_\ell \equiv e^{S_\ell} \quad , \quad X \equiv e^{S} \tag{4.8}$$

we have

$$\begin{aligned}
\tilde{A}(s) &= \prod_{\ell=1}^{L} \tilde{a}_\ell(s) = \prod_{\ell=1}^{L}\left[1 - X_\ell\, X^{-1}\right] \\
&= \sum_{n=1}^{L} A_n X^{-n} \\
A(t) &= \underset{\ell=1}{\overset{L}{O}}\; a_\ell(t) = \underset{\ell=1}{\overset{L}{O}}\left[\delta(t) - X_\ell \delta(t - T_0)\right] \\
&= \sum_{n=0}^{L} A_n \delta(t - \ell T_0)
\end{aligned} \tag{4.9}$$

Properly including conjugate pole pairs, all the A_n are real.

These coefficients can be explicitly calculated as

$$
\begin{aligned}
A_0 &= 1 \\
A_1 &= \sum_{\ell=1}^{L} X_\ell \\
A_2 &= \sum_{\substack{\text{all pairs} \\ \ell^{(1)}=\ell^{(2)}}} X_{\ell^{(1)}} X_{\ell^{(2)}} \text{ i.e., sum over all pairs with distinct indices} \\
A_n &= \sum_{\substack{\text{all distinct} \\ \text{ntuplets}}} X_{\ell^{(1)}} X_{\ell^{(2)}} \ldots X_{\ell^{(n)}} \text{ i.e., sum over all ntuplets with } n \text{ distinct indices} \\
A_L &= \sum_{\ell=1}^{N} X_\ell
\end{aligned}
\tag{4.10}
$$

The products are the combination of L objects taken n at a time without repetition [13 (Appendix C)]. All such products are summed for each n. So we have all the required coefficients to construct A(t) for a given set of natural frequencies without inverting a matrix. Furthermore, all of the A_n are real since $A(t)$ is real, even though the X_ℓ can in general be complex, since they are included in conjugate pairs. Note that if we set all the $X_\ell = 1$ we obtain the binomial coefficients for the A_n.

Now recall our sampling pulse $f_s(t)$ to form

$$
\begin{aligned}
\xi(t) &= f_s(t) \circ A(t) = \sum_{n=0}^{L} A_n \, f_s(t - nT_0) \\
\tilde{\xi}s(s) &= f_s(t) \circ A(t) = \sum_{n=0}^{L} A_n \, f_s(t - nT_0)
\end{aligned}
\tag{4.11}
$$

where we have chosen the symbol ξ (pronounced ksee) for this late-time-annihilating waveform (not being sure of whether to use K or E). Note in this form that the A_n are independent of the choice of $f_s(t)$ which can be chosen for various useful properties.

Relating this form to previous considerations [11], enforce the L zeros as

$$
\tilde{A}(s_\ell) = \sum_{n=0}^{L} A_n \; X_\ell^{-n} = 0 \;\text{ for }\; \ell = 1, 2, \ldots, L \tag{4.12}
$$

giving the matrix $(L \times (L+1))$ equation

$$\begin{pmatrix} X_1^0 & X_1^{-1} & \cdots & X_1^{-L} \\ X_2^0 & X_2^{-1} & \cdots & X_2^{-L} \\ \vdots & \vdots & \cdots & \vdots \\ X_L^0 & X_L^{-1} & \cdots & X_L^{-L} \end{pmatrix} \cdot \begin{pmatrix} A_0 \\ A_1 \\ \vdots \\ A_L \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{4.13}$$

which, by moving the first column (all elements as well as A_0 being 1), gives

$$\begin{pmatrix} X_1^{-1} & \cdots & \cdots & X_1^{-L} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ X_L^{-1} & \cdots & \cdots & X_L^{-1} \end{pmatrix} \cdot \begin{pmatrix} A_1 \\ \vdots \\ \vdots \\ A_L \end{pmatrix} = \begin{pmatrix} 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix} \tag{4.14}$$

so that the matrix is now square ($L \times L$).

Solving for the A_ℓ requires that this last matrix have an inverse. This means that we must have L independent rows and L independent columns. This matrix (of the Vandermonde type) then must have distinct X_ℓ. For real S_ℓ (as for MSI in Section 3) this requires that all the S_ℓ be distinct. Turning this around, under what conditions do distinct S_ℓ give non-distinct X_ℓ, i.e.

$$\begin{aligned} e^{S_{\ell 1}} &= e^{S_{\ell 2}} \\ \mathrm{Re}\left[S_{\ell_1}\right] &= \mathrm{Re}\left[S_{\ell_2}\right] \quad \text{(same damping)} \\ \mathrm{Im}\left[S_{\ell_1}\right] &= \mathrm{Im}\left[S_{\ell_2}\right] + 2m\pi \quad (m = \text{any integer}) \end{aligned} \tag{4.15}$$

Noting that a complex pole pair has

$$\Omega_{\ell_1} + j\omega_{\ell_1} = \Omega_{\ell_2} - j\omega_{\ell_2} \tag{4.16}$$

and applying (4.15) to the case gives

$$\begin{aligned} \omega_{\ell_1} T_0 &= \omega_{\ell_2} T_0 + 2m\pi \\ \omega_{\ell_1} T_0 &= -\omega_{\ell_2} T_0 = m\pi \\ T_0 &= \frac{m\pi}{\omega_{\ell_1}} \end{aligned} \tag{4.17}$$

The smallest T_0 implies $m = 1$ and maximum ω_ℓ. This relates to the "natural" E-pulse if we have L poles = $2N$ poles or N pole pairs (neglecting poles in the negative real s-axis), and the duration of $A(t)$ is just (for $L + 1$ delta functions)

$$LT_0 = 2NT_0 = \frac{2N\pi}{\max\limits_{\ell} \omega_\ell} \tag{4.18}$$

This ties our present development to previous considerations [11]. Note, however, that in (4.12) if we have, say, equal X_{ℓ_1} and X_{ℓ_2}, then there are only L (not $L + 1$) equations when one goes to the matrix in (4.13). So the A_n are still uniquely determined as in (4.10) whether or not all the X_ℓ are distinct. The "natural" E-pulse occurs when an $a_\ell(t)$ as in (4.3) annihilates more than one pole, indicating that there are two values of ℓ giving the same $a_\ell(t)$, one thereby being redundant.

5. Application of ξ-Pulse

The ξ-Pulse has a time-duration

$$t_\xi = LT_0 + t_s$$

$$t_s \equiv \text{time duration of } f_s(t) \quad \text{(e.g., } T_0 \text{ for subsectional basis function)} \tag{5.1}$$

When applied to a target impulse-response waveform containing the target natural frequencies s_α as its zeros, the late-time response becomes zero (except for noise). The time at which this occurs depends on the target discrimination time and the duration of the ξ-pulse. The ξ-pulse has a duration proportional to L, the number of natural frequencies used in its construction. While targets generally have an infinite number of natural frequencies, only a finite number, the important natural frequencies are used in practice. So it is only an approximation to regard the late-time response to be zero after the application of the ξ-pulse due to this truncation. Of course, it is necessary to have such a truncation of the ξ-pulse so that it does not have an infinite duration (for fixed $T_0 > 0$) and thereby remove the concept of a late-time region with zero response.

The ξ-pulse need not only have zeros at target natural frequencies; others can be added for convenience. A transient radar will transmit some incident waveform which is not a perfect delta function (finite height, non-zero width, zero time integral (antennas not radiating DC), etc.), or the waveform might be quite different from the delta function. The target impulse response can be recovered by deconvolution techniques, but of course only approximately so. An alternate technique would be to construct a ξ-pulse which anihilates the late-time portions of the transmit and receive impulse responses, and combine this with the ξ-pulses for the individual targets. The ξ-pulse can also incorporate some noise filtering by choice of $f_s(t)$, to remove high frequencies beyond some frequency of interest. Inclusion of a zero at $s = 0$ in the ξ-pulse also serves as a low-frequency filter.

Sections 2 and 3 have considered the target discrimination time t_d. Times (after initial backscatter) need to be greater than this to establish the particular set of s_α characteristic of a particular target. For EMSI this is bounded below by $2t_0$ where t_0 is the transit time for the target maximum linear dimension. For MSI this is a negligible time compared to the diffusion time through the target. For some set of N targets we have N sets of s_α and N corresponding ξ-pulses which we can designate $\xi_n(t)$. Corresponding to this we have durations t_{ξ_n} as in (5.1). Then we can define

$$T_n \equiv t_{\xi_n} + t_d \tag{5.2}$$

giving a time interval (after first signal return with worst case aspect) after which the $\xi_n(t)$ will annihilate the late-time of the target impulse response. We can also find the longest such time as

$$T_{\max} = \max_{1 \le n \le N} T_n \tag{5.3}$$

Let $f_m(t)$ be a target scattered waveform which has been processed to remove the radar transfer functions (to some extent), and which presumably contains noise. Then form for this mth target ($m = 1, 2, ..., M$)

$$\begin{aligned} F_{n,m}(t) &\equiv \xi_n(t) \circ f_m(t) \\ \tilde{F}_{n,m}(s) &= \tilde{\xi}_n(s)\,\tilde{f}_m(s) \end{aligned} \tag{5.4}$$

As illustrated conceptually in fig. 5.1 we have some waveform which continues on beyond T_m. When operated on by $\xi_n(t)$ the late-time behavior ($t > T_m$) is dramatically different depending on whether or not $n = m$.

One way that one may wish to quantify the judgment of which target is present is by forming

$$\begin{aligned} U_{n,m}^{(0)} &= \left[\frac{\int_{T_n}^{t_n^{(\max)}} F_{n,m}^2(t)\,dt}{\int_0^{t_n^{(\max)}} F_{n,m}^2(t)\,dt} \right]^{\frac{1}{2}} \\ &= \frac{\left\| F_{n,m}(t)\left[u(t-T_n) - u\left(t - t_n^{(\max)}\right)\right]\right\|_{2,t}}{\left\| F_{n,m}(t)\left[1 - u\left(t - t_n^{(\max)}\right)\right]\right\|_{2,t}} \end{aligned} \tag{5.5}$$

which is a kind of 2-norm evaluation of the residual (energy related). Since $f_m(t)$ is assumed to begin at $t = 0$ for present purposes, negative times are excluded from the integrals. We also need to truncate for times greater than $t_n^{(\max)}$ to avoid an infinite value due to integrating the squared noise out to ∞. An optimal choice for $t_n^{(\max)}$ concerns when the residual from $f_m(t)$ (after filtering by $\xi_n(t)$) becomes less than the remaining noise (as filtered). The choice of T_n is as previously discussed. Then we find

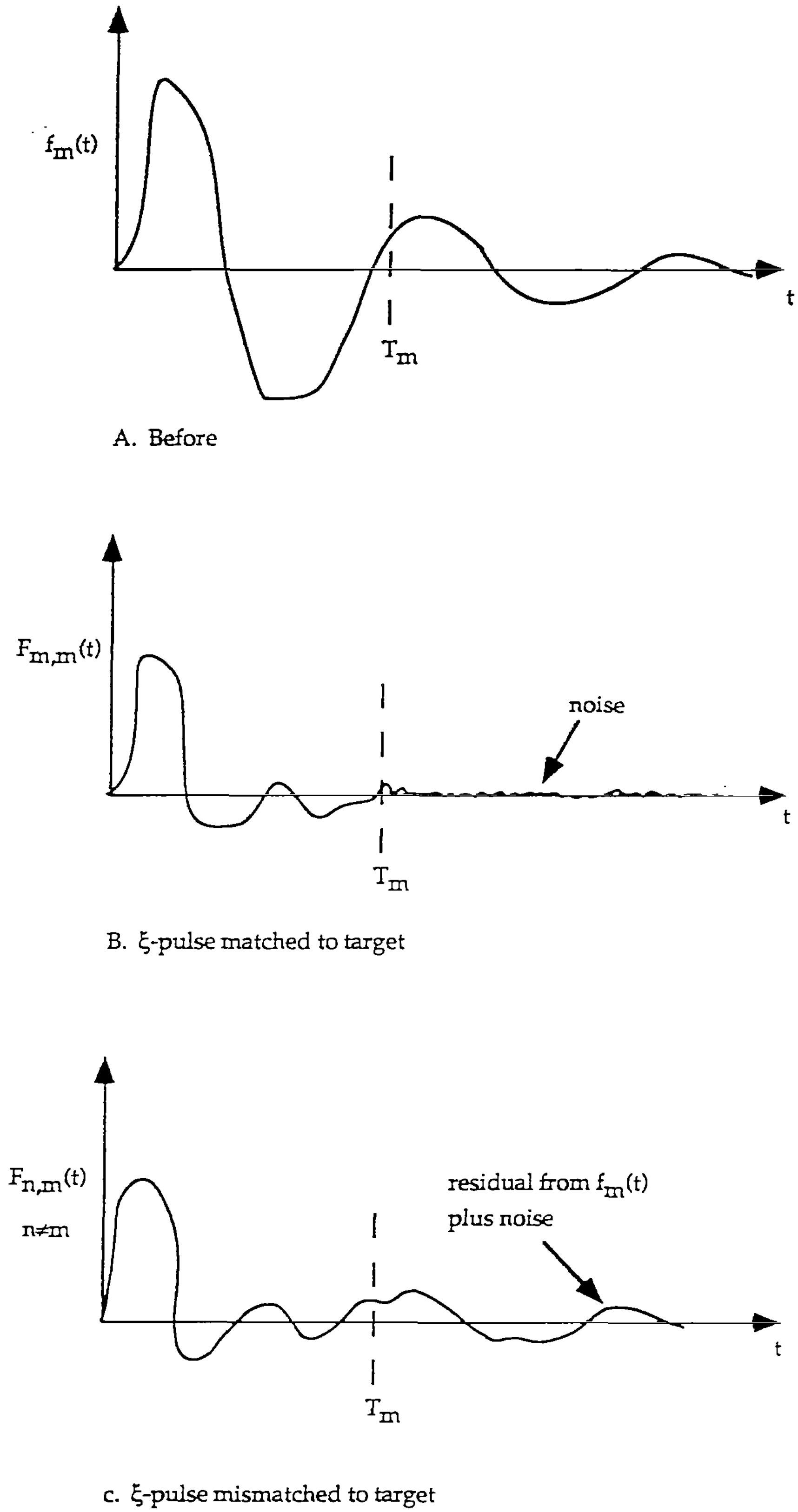


Fig. 5.1. Target Waveforms Before and After Application of ξ-Pulse (Merely Ilustrative)

$$
\begin{aligned}
U^{(0)}_{\min,m} &\equiv \min_n \; U^{(0)}_{n,m} \\
n_{\min} &\equiv \text{value of } n \text{ achieving this minimum} \\
&\equiv \text{value assigned to } m
\end{aligned}
\tag{5.6}
$$

If there are other m approaching this minimum. i.e., other $U^{(0)}_{n,m}/U^{(0)}_{\min,m}$ not too much greater than one, then there is some uncertainty in our choice of m for the correct target. At this point one may wish to introduce probabilistic estimates.

An alternate definition of $U_{n,m}$ might be

$$
U^{(1)}_{n,m} = \frac{\left\| F_{n,m}(t)\, e^{-\frac{t}{t_w}}\, u(t-T_n) \right\|_{2,t}}{\left\| F_{n,m}(t)\, e^{-\frac{t}{t_w}}\, u(t) \right\|_{2,t}}
\tag{5.7}
$$

In this formulation the late-time contribution is not abruptly cut off, but is weighted by a decaying exponential with a time constant (width) t_w. This allows one to weight the late time less as one goes out in time and only noise is left. One might choose t_w as comparable to the decay constant of the longest decaying pole (largest $-\Omega_\alpha^{-1}$) of significance. Yet another definition is

$$
U^{(2)}_{n,m} = \frac{\left\| F_{n,m}(t)\, e^{-\frac{t-T_n}{t_w}}\, u(t-T_n) \right\|_{2,t}}{\left\| F_{n,m}(t)\, e^{-\frac{t}{t_w}}\, u(t) \right\|_{2,t}}
\tag{5.8}
$$

This gives a stronger weight to the late-time regime used for the target discrimination.

6. Use of Window Laplace/Fourier Transform to Evaluate Late-Time Residuals

Instead of considering the "total energy" remaining in the late-time interval as in (5.5), one can look more in detail in this time interval by various transforms. The window-Laplace and wavelet transforms use a triwave-transform kernel [14] of the form

$$\frac{1}{t_w} g\left(\frac{t-t_s}{t_w}\right) e^{-st} \;, \quad t_w > 0 \;, \quad t_s \text{ real}$$

$$\begin{aligned} s &\equiv \text{complex frequency} \\ t_s &\equiv \text{time shift} \\ t_w &\equiv \text{time dilation (width of window or wavelet)} \end{aligned} \tag{6.1}$$

An interesting form to choose for this for present purposes is

$$\begin{aligned} g(\zeta) &= e^{-\zeta} u(\zeta) \;, \quad \zeta \text{ real} \\ \int_{-\infty}^{\infty} g(\zeta)\, d\zeta &= 1 \end{aligned} \tag{6.2}$$

With this kind of normalized window function this is appropriate to think of as a window Laplace transform.

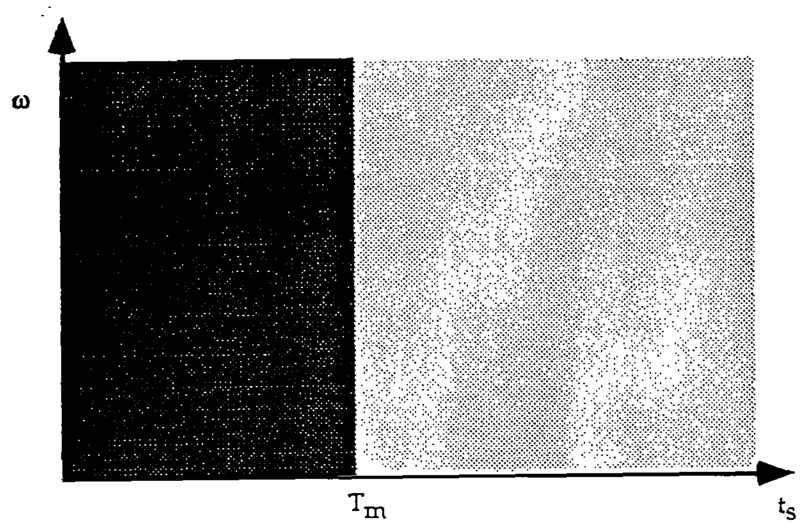
Apply this to the filtered waveforms as

$$\tilde{F}_{n,m}(s,t_s,t_w) \equiv \int_{-\infty}^{\infty} F_{n,m}(t) \frac{1}{t_w} e^{-\frac{t-t_s}{t_w}} u(t-t_s)\, e^{-st} dt \tag{6.3}$$

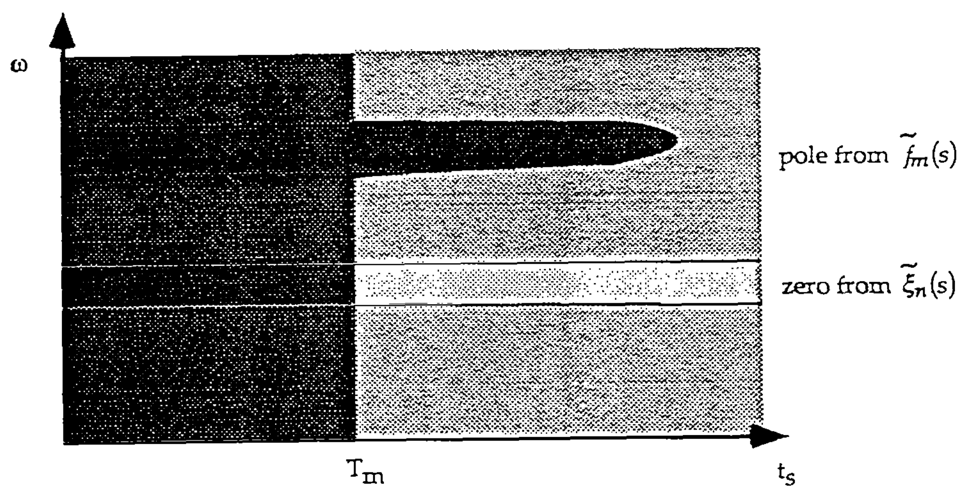
As one increases ts from zero, we first have the contribution from the large response for all targets. As t_s passes T_m the contribution to the integral in (6.3) should drop for $n = m$ by comparison to other $n \neq m$. Setting

$$s \equiv j\omega \tag{6.4}$$

we can have the phase-space diagrams as illustrated in fig. 6.1 where we plot in a rough illustrative sense the magnitude of the transform versus both ω and t_s (for some selected t_w). For $t_s > T_m$ and the correct choice $n = m$ we have noise, whereas for $n \neq m$ we have poles from



A. $\left|\tilde{F}_{m,m}(j\omega, t_s, t_w)\right|$ for correct target



B. $\left|\tilde{F}_{n,m}(j\omega, t_s, t_w)\right|$ for incorrect target

Fig. 6.1. Phase Space for Target Discrimination (Merely Illustrative)

the target and zeros from the filter appearing with the noise. This may be a more sensitive discriminant than a simple 2-norm of the late-time, but one would still like to reduce this information to a few numbers (ultimately from a computer) that distill this phase-space information, say by looking at peaks (and perhaps valleys) in the late time.

There is still the question of normalization for better comparison of the $F_{n,m}(t)$. One choice might be the 2-norm denominator in (5.5). For present purposes one might define a normalized transform related to (6.3) as

$$\tilde{F}^{(n)}_{n,m}(s,t_s,t_w) = \left\| F_{n,m}(t)\frac{1}{t_w}e^{-\frac{t}{t_w}}u(t)\right\|_1^{-1} \tilde{F}_{n,m}(s,t_s,t_w)$$
$$\left\| F_{n,m}(t)\frac{1}{t_w}e^{-\frac{t}{t_w}}u(t)\right\|_{1,t} \equiv \int_{-\infty}^{\infty} \left|F_{n,m}(t)\right|\frac{1}{t_w}e^{-\frac{t}{t_w}}u(t)\,dt \tag{6.5}$$

The 1-norm is used to make the result dimensionless, but other norms could also be used. A norm is desirable to avoid a possible zero denominator. Note the use of the same exponential decay in the denominator (avoiding a discrete late-time cut off) as in the numerator.

The window width t_w is a parameter available to us to choose for optimal results. It should be comparable to or somewhat larger than the decay time of the dominant target resonances (those of significant amplitude which last for the longest times). However, t_w should not be too large so as to reduce the influence of the late-time noise (when the target information is negligible).

At this point let us note that this phase-space view of the late time may be more appropriate in the case of EMSI for which we have large $j\omega_\alpha$ parts of s_α which can carry over in late time. For MSI with only Ω_α parts of s_α, resolution along the $j\omega$ axis may not be too useful. Note that if we set $s = 0$ in (6.3) and (6.5) then the normalized transform in (6.5) begins to look like (5.8) except that different norms are used the present numerator is a linear operation on the filtered waveform. With $s = 0$, this leaves two parameters t_w and t_s which one might also use. This can be viewed as a wavelet transform with t_w^{-1} (frequency like) for the vertical axis and t_s for the horizontal axis to give the phase space [14].

Additional properties of the transformed filtered waveform can be found by manipulating (6.3) as

$$
\begin{aligned}
\tilde{F}_{n,m}(s,t_s,t_w) &= \frac{e^{-st_s}}{t_w}\int_{-\infty}^{\infty} F_{n,m}(t)\,u(t-t_s)\,e^{-\left[s+t_w^{-1}\right][t-t_s]}dt \\
&= \frac{e^{-st_s}}{t_w}\int_{-\infty}^{\infty} F_{n,m}(t+t_s)\,u(t)\,e^{-\left[s+t_w^{-1}\right]t}dt
\end{aligned}
\tag{6.6}
$$

This effectively replaces s by $s+t_w^{-1}$ in the transform. With $s = j\omega$ this is like looking off in the right half s-plane a distance t_w^{-1}. As the sampling pulse width t_w is made larger and larger the Fourier transform ($s = j\omega$) of the late time (after t_s) is pushed closer and closer to the $j\omega$ axis. The factor $e^{-j\omega t_s}$ in front is a phase shift with magnitude one.

At this point we can note a connection between the window-Laplace/Fourier transform and the time-domain form. Considering the energy-related 2-norm the Parseval theorem gives

$$
\begin{aligned}
\left\|\tilde{F}_{n,m}(j\omega,t_s,t_w)\right\|_{2,\omega} &= \left[\int_{-\infty}^{\infty}\left|\tilde{F}_{n,m}(j\omega,t_s,t_w)\right|^2 d\omega\right]^{1/2} \\
&= \left[2\pi\int_{-\infty}^{\infty}\left[F_{n,m}(t)\frac{1}{t_w}e^{-\frac{t-t_s}{t_w}}u(t-t_s)\right]^2 dt\right] \\
&= [2\pi]^{1/2}\left\|F_{n,m}(t)\frac{1}{t_w}e^{-\frac{t-t_s}{t_w}}u(t-t_s)\right\|_{2,t}
\end{aligned}
\tag{6.7}
$$

So integrating the magnitude squared of the transform over all ω gives the integral of the square of the late-time ($t_s = T_m$) weighted filtered waveform as in (5.8). In this form 2-norms are appropriate, including the normalization.

7. Generalization to Dyadic Waveforms

Depending on the design of the radar, one can have more than one waveform from a given target in backscatter. One can, of course, use multiple copies of the same waveform (stacking, same polarization). Here, however, one may have two polarizations (h,v radar coordinates) in transmission combined with two in reception to establish the 2 x 2 scattering dyadic (only components transverse to $\vec{1}_i$) for EMSI as in (2.2). Since the natural frequencies s_α are aspect independent (i.e., are the same for all directions of incidence and polarizations), then the four waveforms inherent in the scattering dyadic operator all have the same s_α set. So form

$$\begin{aligned} \overleftrightarrow{F}_{n,m}(t) &\equiv \xi_n(t) \circ \overleftrightarrow{\Lambda}_{bm}(\vec{1}_i, t) \\ \tilde{\overleftrightarrow{F}}_{n,m}(s) &= \tilde{\xi}_n(s)\, \tilde{\overleftrightarrow{\Lambda}}_{bm}(\vec{1}_i, s) \end{aligned} \tag{7.1}$$

with subscript m for the scattering dyadic operator (impulse response) of the mth target (subscript m) analogous to (5.4). Then the norms used in (5.5), (5.7), and (5.8) can be readily generalized to dyadic functions [1, 2]. Furthermore, the transforms used in Section 6, being scaler linear operations which commute with dyadics, can also be directly applied to (7.1). In phase space (as in figure 6.1) the magnitude of the transform is readily replaced by the 2-norm in matrix sense. While (7.1) is defined in terms of backscattering, it is readily generalized to multistatic scattering.

For MSI as in (3.3) or (3.4) we deal with SEM representation of a 3 x 3 magnetic-polarizability dyadic (in general symmetric). Depending on the type of experimental data available one may have some or all of the components of this dyadic available for analysis. So form (to the extent components are available)

$$\overleftrightarrow{F}_{n,m}(t) = \begin{cases} \xi_n(t) \circ \overleftrightarrow{M}(t) \\ or \\ \xi_n(t) \circ \displaystyle\int_{-\infty}^{t} \overleftrightarrow{M}(t')\, dt' \end{cases} \tag{7.2}$$

similar to (7.1). The late-time evaluation including transforms then proceeds as discussed previously.

8.0 Concluding Remarks

There is still a lot to be considered. Noise is an obvious limitation. To supress the effects of noise one can filter out frequencies outside the range of interest. By enhancing the transmit waveforms in the frequency range of interest one can increase the signal-to-noise ratio for the frequencies. The receive waveforms then need to be deconvolved to produce an approximate delta-function response (or some other convenient form such as step-function response).

Another subject of interest is a possible adaptive ξ-pulse. In the case of metal targets (approximated as perfectly conducting) buried in soil there are scaling relationships for the natural frequencies [4]. If one knows the ε and σ of the soil (say from measurement using the same radar) then one can alter the A_n coefficients by the scaling relationship. Another approach would consider the approximate affine scaling of the s_α (in a simple soil) which preserves the pattern of the s_α in the s-plane, and iteratively adjust ε and σ to achieve the late-time annihilation. For dielectric targets in the soil the problem is yet more complicated, but some asymptotic formulas for the s_α movement in the s-plane may be helpful [7]. This problem of the movement of the s_α in the s-plane due to the variability of the external-medium constitutive parameters is a significant one, perhaps indicating that a quite different approach to the target identification (say explicit instead of implicit) may be useful.

With all of these problems to be addressed (as well as others as yet not identified) there is clearly much to be done, both theoretically and experimentally.

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