Interaction Notes

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16 December 1995

Scattering Dyadic for Self-Dual Target

Carl E. Baum
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Abstract

One of the symmetries one can impose on a target is that of self duality, i.e., invariance to interchange of electric and magnetic parameters. With this symmetry the scattering dyadic describing the scattering of electromagnetic waves attains special properties. These include rotation of the scattered field by the same angle the incident field is rotated, and zero backscattering for a three-fold or higher rotation axis aligned along the direction of incidence. Various point symmetries of the target also introduce simplifications (symmetries) in the scattering dyadic.

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1. Introduction

A recent paper [5] has considered the effect of target symmetries, specifically the point symmetry groups (rotation and reflection) on the symmetries in the scattering dyadic. The commonly applicable symmetry of reciprocity is also assumed there. The present paper extends the results to the case of a self-dual scatterer (symmetry on interchange of electric and magnetic parameters).

As indicated in fig. 1.1, let there by a scatterer contained in a volume V bounded by a surface S of finite linear dimensions. The scatterer is characterized by constitutive parameter dyadics

$$\begin{aligned} \tilde{\overleftrightarrow{\varepsilon}}(\vec{r},s) &\equiv \text{permittivity} \\ \tilde{\overleftrightarrow{\mu}}(\vec{r},s) &\equiv \text{permittivity} \\ \sim &\equiv \text{Laplace transform (two sided) over time} \\ s &\equiv \Omega + j\omega \equiv \text{Laplace-transform variable or complex frequency} \end{aligned} \tag{1.1}$$

One can also have conductivity, but for present purposes, this can be included in the permittivity.

The incident wave is taken as a plane wave

$$\begin{aligned} &\tilde{\vec{E}}^{(inc)}(\vec{r},s) = E_0\,\tilde{f}(s)\,\vec{1}_p\,e^{-\tilde{\gamma}\,\vec{1}_i\cdot\vec{r}}\;,\;\vec{E}^{(inc)}(\vec{r},t) = E_0\,f\left(t-\frac{\vec{1}_i\cdot\vec{r}}{c}\right)\vec{1}_p \\ &\vec{H}^{(inc)}(\vec{r},t) = \frac{1}{Z_0}\vec{1}_i\times\vec{E}^{(inc)}(\vec{r},t)\;,\;\tilde{\vec{E}}^{(inc)}(\vec{r},t)\cdot\vec{1}_i = 0 \\ &\qquad c \equiv [\mu_0\varepsilon_0]^{-\frac{1}{2}}\;,\;Z_0 \equiv \left[\frac{\mu_0}{\varepsilon_0}\right]^{\frac{1}{2}}\;,\;\tilde{\gamma} \equiv \frac{s}{c} \\ &\qquad \vec{1}_i \equiv \text{direction of incidence} \\ &\qquad \vec{1}_p \equiv \text{polarization}\;,\;\vec{1}_i\cdot\vec{1}_p = 0 \end{aligned} \tag{1.2}$$

The scattered field (superscript sc) is given in the far-field limit by

$$\begin{aligned} \tilde{\vec{E}}(\vec{r},s) &= \frac{e^{-\tilde{\gamma}r}}{4\pi r}\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o,\vec{1}_i;s)\cdot\tilde{\vec{E}}^{(inc)}(\vec{0},s) \\ \vec{E}_f(\vec{r},t) &= \frac{1}{4\pi r}\overleftrightarrow{\Lambda}(\vec{1}_o,\vec{1}_i;t)\circ\vec{E}^{(inc)}\left(\vec{0},t-\frac{r}{c}\right) \end{aligned}$$

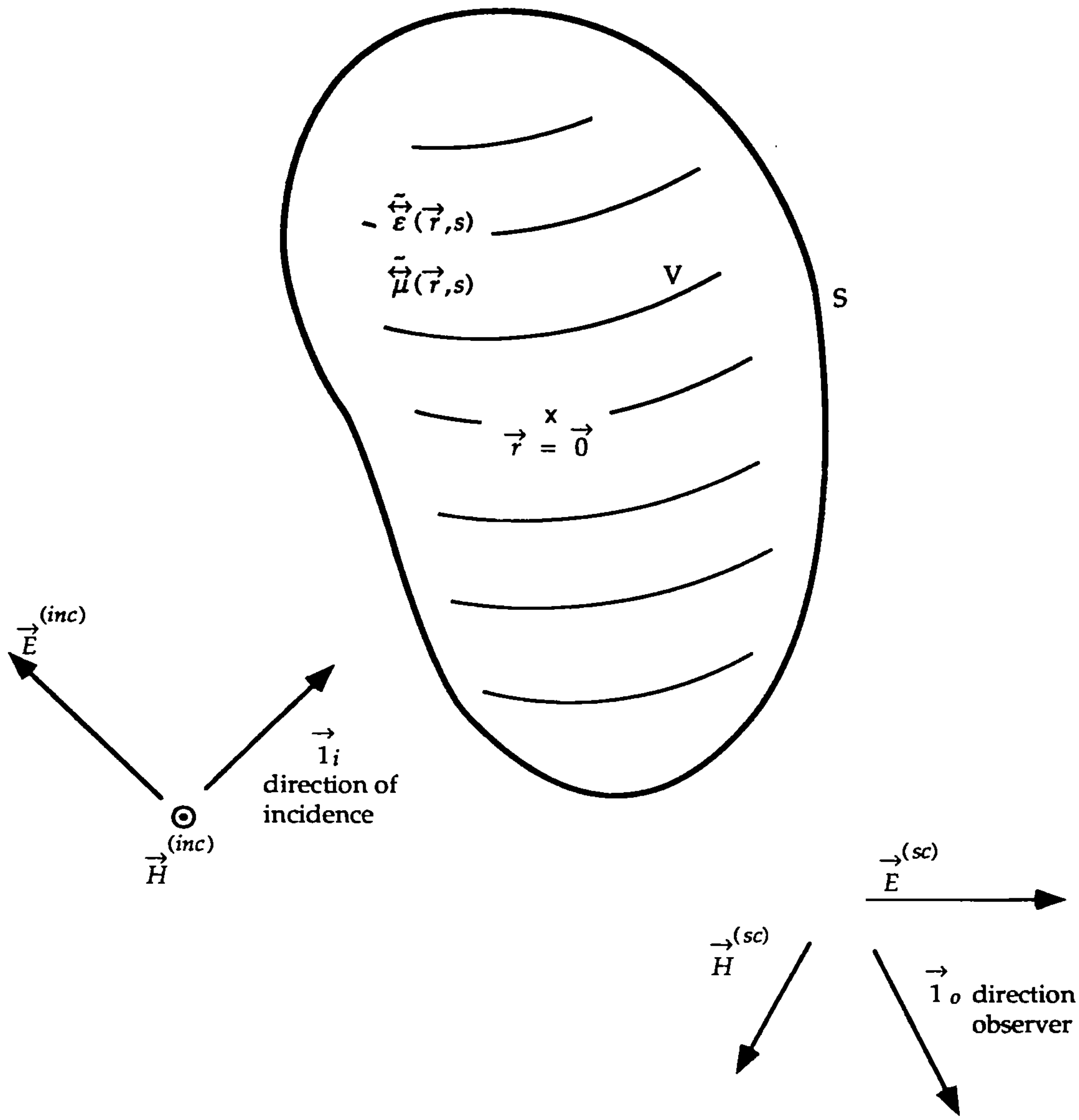

Fig. 1.1. Scattering of Incident Plane Wave

$$\vec{H}_f(\vec{r},t) = \frac{1}{Z_o}\vec{1}_o \times \vec{E}_f(\vec{r},t) \quad , \quad \vec{E}_f(\vec{r},t) \cdot \vec{1}_o = 0$$

$$\begin{aligned}
\vec{r} &= r\,\vec{1}_o \ , \ \vec{1}_o \equiv \text{direction to observer} \\
\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) &\equiv \text{scattering dyadic} \\
\overleftrightarrow{\Lambda}(\vec{1}_o, \vec{1}_i; t) \circ &\equiv \text{scattering dyadic operator} \\
\circ &\equiv \text{convolution with respect to time}
\end{aligned} \tag{1.3}$$

where $\vec{r} = \vec{0}$ is appropriately centered near or inside the target. This scattering dyadic contains all the properties of the target that one can observe in a radar.

A common symmetry which is sometimes assumed (and often applicable) is reciprocity as

$$\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) = \tilde{\overleftrightarrow{\Lambda}}^T(-\vec{1}_i, -\vec{1}_o; s) \tag{1.4}$$

giving a symmetry on interchange of transmitter and receiver. A sufficient condition for this is that the constitutive-parameter dyadics be symmetric. For backscattering this gives

$$\begin{aligned}
\vec{1}_o &= -\vec{1}_i \\
\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i; s) &\equiv \tilde{\overleftrightarrow{\Lambda}}(-\vec{1}_i, -\vec{1}_i; s) = \tilde{\overleftrightarrow{\Lambda}}_b^T(\vec{1}_i, s)
\end{aligned} \tag{1.5}$$

Note that the scattering dyadic is transverse to both incidence and scattering directions as

$$\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \cdot \vec{1}_i = \vec{0} = \vec{1}_i \cdot \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \tag{1.6}$$

due to the transverse nature of the incident and scattered far fields.

For later use, define some dyadics associated with incidence and scattering directions as

$$\begin{aligned}
\overleftrightarrow{1}_i &\equiv \overleftrightarrow{1} - \vec{1}_i\,\vec{1}_i \equiv \text{dyadic transverse to } \vec{1}_i \\
\overleftrightarrow{1}_i &\equiv \overleftrightarrow{1} - \vec{1}_o\,\vec{1}_o \equiv \text{dyadic transverse to } \vec{1}_o \\
\overleftrightarrow{1} &\equiv \vec{1}_x\,\vec{1}_x + \vec{1}_y\,\vec{1}_y + \vec{1}_z\,\vec{1}_z \equiv \text{identity dyadic} \\
\overleftrightarrow{\tau}_i &\equiv \vec{1}_i \times \overleftrightarrow{1}_i = \overleftrightarrow{1}_i \times \vec{1}_i = -\overleftrightarrow{\tau}_i^T
\end{aligned}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ in } \vec{1}_h, \vec{1}_v, -\vec{1}_i \text{ coordinates}$$

$= -\pi/2$ rotation in h, v plane

(observer looking in $\vec{1}_i$ direction)

$$\overleftrightarrow{\tau}_o \equiv \vec{1}_o \times \overleftrightarrow{1}_o = \overleftrightarrow{1}_o \times \vec{1}_o = -\overleftrightarrow{\tau}_o^T \text{ with similar interpretation with respect to } \overleftrightarrow{1}_o \text{ direction} \tag{1.7}$$

$$\overleftrightarrow{\tau}_i^2 = -\overleftrightarrow{1}_i \quad , \quad \overleftrightarrow{\tau}_i^2 = -\overleftrightarrow{1}_o$$

$$\overleftrightarrow{\tau}_i \cdot \overleftrightarrow{\tau}_i^T = \overleftrightarrow{1}_i \quad , \quad \overleftrightarrow{\tau}_o \cdot \overleftrightarrow{\tau}_o^T = \overleftrightarrow{1}_o \quad \text{(two dimensional inverses)}$$

In terms of the standard h, v radar coordinates we have (with $\vec{1}_n$, $\vec{1}_v$, $-\vec{1}_i$ as a right-handed system)

$$\begin{aligned} \overleftrightarrow{1}_i &= \vec{1}_h \vec{1}_h + \vec{1}_v \vec{1}_v \\ \overleftrightarrow{\tau}_i &= \vec{1}_h \vec{1}_v - \vec{1}_v \vec{1}_h \end{aligned} \tag{1.8}$$

Consider an angle ψ for positive rotation in the usual mathematical convention (counterclockwise) in the h, v plane

$$\begin{aligned} \overleftrightarrow{C}_i(\psi) &= \overleftrightarrow{1}_i \cos(\psi) - \overleftrightarrow{\tau}_i \sin(\psi) + \vec{1}_i \vec{1}_i \\ \overleftrightarrow{C}_i(-\psi) &= \overleftrightarrow{C}_i^T(\psi) = \overleftrightarrow{C}_i^{-1}(\psi) \\ \overleftrightarrow{C}_i(\psi) \cdot \overleftrightarrow{C}_i(-\psi) &= \overleftrightarrow{1} \text{ (identity)} \end{aligned} \tag{1.9}$$

Similarly for the scattering direction we have

$$\overleftrightarrow{C}_o(\psi) = \overleftrightarrow{1}_o \cos(\psi) - \overleftrightarrow{\tau}_o \sin(\psi) + \vec{1}_o \vec{1}_o \tag{1.10}$$

with similar properties. Then in the $\vec{1}_h, \vec{1}_v, -\vec{1}_i$ coordinates we have

$$\overleftrightarrow{C}_i(\psi) = \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1.11}$$

as an explicit form of the rotation. A similar form can be written for the scattering. Note that for backscattering we have

$$\vec{1}_o = -\vec{1}_i \quad , \quad \overleftrightarrow{1}_o = \overleftrightarrow{1}_i \quad , \quad \overleftrightarrow{\tau}_o = -\overleftrightarrow{\tau}_i$$
$$\overleftrightarrow{C}_i(\psi) = \overleftrightarrow{C}_o(-\psi) \tag{1.12}$$

so that in backscattering both conventions merge with attention to signs.

2. Duality

By duality is meant the symmetry in the Maxwell equations on the interchange of electric and magnetic parameters [8]. Using the combined field and current density

$$\begin{aligned}
\vec{E}_q(\vec{r},t) &= \vec{E}(\vec{r},t) + j\,q\,Z_0\,\vec{H}(\vec{r},t) \\
\vec{J}_q(\vec{r},t) &= \vec{J}_e(\vec{r},t) + j\frac{q}{Z_0}\vec{J}_h(\vec{r},t) \\
q &= \pm 1 \equiv \text{separation index}
\end{aligned} \tag{2.1}$$

the combined Maxwell equation is

$$\left[\nabla\times \; -j\frac{q}{c}\frac{\partial}{\partial t}\right]\vec{E}_q(\vec{r},t) = j\;q\;Z_0\;\vec{J}_q(\vec{r},t) \tag{2.2}$$

The electric and magnetic current densities can here be interpreted as

$$\begin{aligned}
\tilde{\vec{J}}_e(\vec{r},s) &= s\left[\tilde{\overleftrightarrow{\varepsilon}}(\vec{r},s) - \varepsilon_0\overleftrightarrow{1}\right]\cdot\tilde{\vec{E}}(\vec{r},s) \\
\tilde{\vec{J}}_h(\vec{r},s) &= s\left[\tilde{\overleftrightarrow{\mu}}(\vec{r},s) - \mu_0\overleftrightarrow{1}\right]\cdot\tilde{\vec{H}}(\vec{r},s)
\end{aligned} \tag{2.3}$$

Source currents can also be included in (2.3) which in this form include electric and magnetic polarization currents, conductivity having been lumped in with the permittivity.

The duality transformation is just

$$\begin{aligned}
&\vec{E}_q^{(d)}(\vec{r},t) = -q\,j\,\vec{E}_q(\vec{r},t) \\
&\vec{E}^{(d)}(\vec{r},t) = Z_0\,\vec{H}(\vec{r},t) \;\;,\;\; \vec{H}^{(d)}(\vec{r},t) = -\frac{1}{Z_0}\vec{E}(\vec{r},t)
\end{aligned} \tag{2.4}$$

Applying this twice gives the negative of the original fields. The identity is recovered by four applications which makes this like C_4 symmetry in the complex plane [8]. Applying this to the current densities gives

$$
\begin{aligned}
\vec{J}_q^{(d)}(\vec{r},t) &= -q\,j\,\vec{J}_q(\vec{r},t) \\
\vec{J}_e^{(d)}(\vec{r},t) &= \frac{1}{Z_0}\vec{J}_h(\vec{r},t) \;, \quad \vec{J}_h(\vec{r},t) = -Z_0\,\vec{J}_e(\vec{r},t)
\end{aligned}
\tag{2.5}
$$

where we have

$$
\begin{aligned}
\tilde{\vec{J}}_e^{(d)}(\vec{r},s) &= s\left[\tilde{\overleftrightarrow{\varepsilon}}^{(d)}(\vec{r},s) - \varepsilon_0\,\overleftrightarrow{1}\right]\cdot\tilde{\vec{E}}^{(d)}(\vec{r},s) \\
\tilde{\vec{J}}_h^{(d)}(\vec{r},s) &= s\left[\tilde{\overleftrightarrow{\mu}}^{(d)}(\vec{r},s) - \mu_0\,\overleftrightarrow{1}\right]\cdot\tilde{\vec{H}}^{(d)}(\vec{r},s)
\end{aligned}
\tag{2.6}
$$

Comparing the dual to the original current densities gives

$$
\frac{\tilde{\overleftrightarrow{\mu}}^{(d)}(\vec{r},s)}{\mu_0} = \frac{\tilde{\overleftrightarrow{\varepsilon}}(\vec{r},s)}{\varepsilon_0} \;, \quad \frac{\tilde{\overleftrightarrow{\varepsilon}}^{(d)}}{\varepsilon_0} = \frac{\tilde{\overleftrightarrow{\mu}}(\vec{r},s)}{\mu_0}
\tag{2.7}
$$

as the relation of the dual target to the original one. From this we conclude that for a target to be self dual we need

$$
\frac{\tilde{\overleftrightarrow{\mu}}(\vec{r},s)}{\mu_0} = \frac{\tilde{\overleftrightarrow{\varepsilon}}(\vec{r},s)}{\varepsilon_0}
\tag{2.8}
$$

Scalar permeability and permittivity are merely a special case of this. Note that reciprocity (symmetric dyadics) has not been assumed for this result.

3. Scattering by Self-Dual Target

From (1.3) scattering from the dual target is given by

$$
\begin{aligned}
\tilde{\vec{E}}_f^{(d)}(\vec{r},s) &= \frac{e^{-\gamma r}}{4\pi r}\,\tilde{\overleftrightarrow{\Lambda}}^{(d)}(\vec{1}_o,\vec{1}_i;s)\cdot\tilde{\vec{E}}^{(inc,d)}(\vec{0},s)\\
\tilde{\vec{H}}^{(inc,d)}(\vec{r},t) &= \frac{1}{Z_0}\,\vec{1}_i\times\vec{E}^{(inc,d)}(\vec{0},t) = \frac{1}{Z_0}\,\overleftrightarrow{\tau}_i\cdot\vec{E}^{(inc,d)}(\vec{0},t)\\
\tilde{\vec{H}}_f^{(d)}(\vec{r},t) &= \frac{1}{Z_0}\,\vec{1}_o\times\vec{E}_f^{(d)}(\vec{r},t) = \frac{1}{Z_0}\,\overleftrightarrow{\tau}_o\cdot\vec{E}_f(\vec{r},t)
\end{aligned}
\tag{3.1}
$$

Relating the dual to the original fields as

$$
\begin{aligned}
\tilde{\vec{E}}^{(inc,d)}(\vec{0},t) &= Z_0\,\vec{H}^{(inc)}(\vec{r},t) = \vec{1}_i\times\vec{E}^{(inc)}(\vec{r},t) = \overleftrightarrow{\tau}_i\cdot\vec{E}^{(inc)}(\vec{r},t)\\
\tilde{\vec{E}}_t^{(d)}(\vec{0},t) &= Z_0\,\vec{H}_f(\vec{r},t) = \vec{1}_o\times\vec{E}_f(\vec{r},t) = \overleftrightarrow{\tau}_o\cdot\vec{E}_f(\vec{r},t)
\end{aligned}
\tag{3.2}
$$

and substituting into the dual scattering equation we have

$$
\tilde{\vec{E}}_f(\vec{r},s) = -\frac{e^{-\gamma r}}{4\pi r}\,\overleftrightarrow{\tau}_o\cdot\tilde{\overleftrightarrow{\Lambda}}^{(d)}(\vec{1}_o,\vec{1}_i;s)\cdot\overleftrightarrow{\tau}_i\cdot\tilde{\vec{E}}^{(inc)}(\vec{0},s)
\tag{3.3}
$$

Comparing this to (1.3) we have

$$
\begin{aligned}
\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o,\vec{1}_i;s) &= -\,\overleftrightarrow{\tau}_o\cdot\tilde{\overleftrightarrow{\Lambda}}^{(d)}(\vec{1}_o,\vec{1}_i;s)\cdot\overleftrightarrow{\tau}_i\\
&= -\,\vec{1}_o\times\tilde{\overleftrightarrow{\Lambda}}^{(d)}(\vec{1}_o,\vec{1}_i;s)\times\vec{1}_i\\
\overleftrightarrow{\Lambda}^{(d)}(\vec{1}_o,\vec{1}_i;s) &= -\,\overleftrightarrow{\tau}_o\cdot\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o,\vec{1}_i;s)\cdot\overleftrightarrow{\tau}_i\\
&= -\,\vec{1}_o\times\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o,\vec{1}_i;s)\times\vec{1}_i
\end{aligned}
\tag{3.4}
$$

For a self-dual scatterer (as in (2.8)) we then have a restricted form of the scattering dyadic obeying

$$
\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o,\vec{1}_i;s) = -\,\overleftrightarrow{\tau}_o\cdot\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o,\vec{1}_i;s)\cdot\overleftrightarrow{\tau}_i
\tag{3.5}
$$

This is an interesting kind of symmetry which makes the scattering invariant to rotation of the fields about the directions of incidence and scattering. From the basic scattering equation (1.3) we have

$$\begin{aligned}
\overleftrightarrow{\tau}_o \cdot \tilde{\vec{E}}_f(\vec{r},s) &= \frac{e^{-\tilde{\gamma}r}}{4\pi r} \overleftrightarrow{\tau}_o \cdot \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_i, \vec{1}_i; s) \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s) \\
&= -\frac{e^{-\tilde{\gamma}r}}{4\pi r} \overleftrightarrow{\tau}_o \cdot \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \cdot \overleftrightarrow{\tau}_i^{\,2} \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s) \\
&= \frac{e^{-\tilde{\gamma}r}}{4\pi r} \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \cdot \overleftrightarrow{\tau}_i \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s)
\end{aligned} \tag{3.6}$$

Weighting this by -sin(ψ) and adding to (1.3) weighted by cos (ψ) gives

$$\overleftrightarrow{C}_o(\psi) \cdot \tilde{\vec{E}}_f(\vec{r},s) = \frac{e^{-\tilde{\gamma}r}}{4\pi r} \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \cdot \overleftrightarrow{C}_i(\psi) \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s) \tag{3.7}$$

where the rotation-by-angle-ψ dyadics are in (1.9) and (1.10). So rotating the incident field by ψ (about $\vec{1}_i$) results in rotation of the far scattered field by the same angle ψ (about $\vec{1}_o$). The waveforms (incident and scattered) are invariant to this transformation. Applying this to the scattering dyadic itself, dot multiply (3.7) by $\overleftrightarrow{C}_o(-\psi)$ giving

$$\begin{aligned}
\tilde{\vec{E}}_f(\vec{r},s) &= \frac{e^{-\tilde{\gamma}r}}{4\pi r} \overleftrightarrow{C}_o(-\psi) \cdot \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \cdot \overleftrightarrow{C}_i(\psi) \cdot \tilde{\vec{E}}^{(inc)}(\vec{0},s) \\
\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) &= \overleftrightarrow{C}_o(-\psi) \cdot \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \cdot \overleftrightarrow{C}_i(\psi)
\end{aligned} \tag{3.8}$$

This is a generalization of (3.5) to arbitrary rotation angles. Note that

$$\overleftrightarrow{C}_i\left(-\frac{\pi}{2}\right) = \overleftrightarrow{\tau}_i + \vec{1}_i\,\vec{1}_i \quad , \quad \overleftrightarrow{C}_o\left(\frac{\pi}{2}\right) = -\overleftrightarrow{\tau}_o + \vec{1}_i\,\vec{1}_i \tag{3.9}$$

giving (3.5) as a special case.

4. Backscattering by Self-Dual Target

Applying (3.5) to backscattering for a self-dual scatterer we have

$$\vec{1}_o = -\vec{1}_i \quad , \quad \overleftrightarrow{1}_o = \overleftrightarrow{1}_i \quad , \quad \overleftrightarrow{\tau}_o = -\overleftrightarrow{\tau}_i$$
$$\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) \equiv \tilde{\overleftrightarrow{\Lambda}}(-\vec{1}_i,\vec{1}_i;s) = \overleftrightarrow{\tau}_i \cdot \tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) \cdot \overleftrightarrow{\tau}_i \tag{4.1}$$

In terms of the more general rotation in (3.8) we have

$$\overleftrightarrow{C}_o(-\psi) = \overleftrightarrow{C}_i(\psi)$$
$$\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) = \overleftrightarrow{C}_i(\psi) \cdot \tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_o,\vec{1}_i;s) \cdot \overleftrightarrow{C}_i(\psi) \tag{4.2}$$

Note that in backscattering rotation about $\vec{1}_o$ is reversed when referred to $\vec{1}_i$. Also note in (4.2) that reciprocity has not yet been assumed.

Writing out the components of the backscattering dyadic we have, referred to the $\vec{1}_h, \vec{1}_v, -\vec{1}_i$ coordinate directions,

$$\begin{aligned}
\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) &= \tilde{\Lambda}_{b_{h,h}} \vec{1}_h \vec{1}_h + \tilde{\Lambda}_{b_{h,v}} \vec{1}_h \vec{1}_v + \tilde{\Lambda}_{b_{v,h}} \vec{1}_v \vec{1}_h + \tilde{\Lambda}_{b_{v,v}} \vec{1}_v \vec{1}_v \\
&= \vec{1}_i \times \tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) \times \vec{1}_i \\
&= -\tilde{\Lambda}_{b_{h,h}} \vec{1}_v \vec{1}_v + \tilde{\Lambda}_{b_{h,v}} \vec{1}_v \vec{1}_h + \tilde{\Lambda}_{b_{v,h}} \vec{1}_h \vec{1}_v - \tilde{\Lambda}_{b_{v,v}} \vec{1}_h \vec{1}_h \\
\tilde{\Lambda}_{b_{h,h}} &= -\tilde{\Lambda}_{b_{v,v}} \quad , \quad \tilde{\Lambda}_{b_{h,v}} = \tilde{\Lambda}_{b_{v,h}} \\
\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) &= \tilde{\overleftrightarrow{\Lambda}}_{b_{h,h}}(\vec{1}_i,s)\left[\vec{1}_h \vec{1}_h - \vec{1}_v \vec{1}_v\right] = \tilde{\overleftrightarrow{\Lambda}}_{b_{h,v}}(\vec{1}_i,s)\left[\vec{1}_h \vec{1}_v + \vec{1}_v \vec{1}_h\right] \\
&= \begin{pmatrix} \tilde{\overleftrightarrow{\Lambda}}_{b_{h,h}}(\vec{1}_i,s) & \tilde{\overleftrightarrow{\Lambda}}_{b_{h,v}}(\vec{1}_i,s) & 0 \\ \tilde{\overleftrightarrow{\Lambda}}_{b_{h,v}}(\vec{1}_i,s) & -\tilde{\overleftrightarrow{\Lambda}}_{b_{h,h}}(\vec{1}_i,s) & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_i,s) &= \tilde{\overleftrightarrow{\Lambda}}_b^{T}(\vec{1}_i,s) \quad \text{(symmetric)}
\end{aligned} \tag{4.3}$$

Note that the backscattering dyadic is symmetric even though reciprocity has not been assumed. Furthermore, this dyadic has only two independent components.

As in [5], let us now consider some implications of geometrical symmetries. Consider the case of an axial symmetry plane (containing $\vec{1}_i$) which we take without loss of generality as containing $\vec{1}_h$. Such reflection is described by the group and dyadic representation

$$
\begin{gathered}
R_h = \{(1)\ ,\ (R_h)\} \\
(1) \to \overleftrightarrow{1}\ ,\ (R_h) \to \overleftrightarrow{R}_h = \overleftrightarrow{1} - \vec{1}_h\vec{1}_h = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{gathered}
\tag{4.4}
$$

Applying this to the backscattering dyadic as

$$
\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) = \overleftrightarrow{R}_h \cdot \tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) \cdot \overleftrightarrow{R}_h
\tag{4.5}
$$

gives

$$
\begin{gathered}
\tilde{\Lambda}_{b_{h,v}}(\vec{1}_i,s) = \tilde{\Lambda}_{b_{v,h}}(\vec{1}_i,s) = 0 \\
\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) = \tilde{\overleftrightarrow{\Lambda}}_{b_{h,h}}(\vec{1}_i,s) = \begin{pmatrix} \tilde{\Lambda}_{b_{h,h}}(\vec{1}_i,s) & 0 & 0 \\ 0 & -\tilde{\Lambda}_{b_{h,h}}(\vec{1}_i,s) & 0 \\ 0 & 0 & 0 \end{pmatrix}
\end{gathered}
\tag{4.6}
$$

with now only one independent component. However, noting the minus sign in the v, v location, this is *not* proportional to $\vec{1}_i$. Note that this dyadic is also invariant with respect to another axial plane containing $\vec{1}_v$ as

$$
\begin{gathered}
R_v = \{(1)\ ,\ (R_h)\}\ ,\ \overleftrightarrow{R}_v = \overleftrightarrow{1} - \vec{1}_v\vec{1}_v = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) = \overleftrightarrow{R}_v \cdot \tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i,s) \cdot \overleftrightarrow{R}_v
\end{gathered}
\tag{4.7}
$$

This backscattering dyadic is then invariant to transformation by the direct product group

$$
C_{2a} = R_h \otimes R_v
\tag{4.8}
$$

which is also described as a two-fold rotation axis with two axial symmetry planes.

Now consider rotation about the $-\vec{1}_i$ axis. Such an N-fold rotation is described by

$$
\begin{aligned}
C_N &= \left\{ (C_N)_\ell \,\middle|\, \ell = 1, 2, \ldots, N \right\} \\
(C_N) &= (C_N)_1^\ell \equiv \text{rotation by } \phi_\ell = \frac{2\pi\ell}{N} \\
(C_N)_1^N &= (C_N)_N = (1)
\end{aligned}
\tag{4.9}
$$

This has a dyadic (matrix) representation

$$
\begin{aligned}
\overleftrightarrow{C}_i(\phi_\ell) &= \overleftrightarrow{1}_i \cos(\phi_\ell) - \overleftrightarrow{\tau}_i \sin(\phi_\ell) + \vec{1}_i \vec{1}_i \\
&= \begin{pmatrix} \cos(\phi_\ell) & -\sin(\phi_\ell) & 0 \\ \sin(\phi_\ell) & \cos(\phi_\ell) & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
\overleftrightarrow{C}_i(\phi_\ell) &= \overleftrightarrow{C}_i^{\,\ell}(\phi_1) \;,\quad \overleftrightarrow{C}_i^{\,N}(\phi_1) = \overleftrightarrow{1} \;,\quad \overleftrightarrow{C}_i^{\,-1}(\phi_\ell) = \overleftrightarrow{C}_i(-\phi_\ell)
\end{aligned}
\tag{4.10}
$$

As discussed in [1, 7, 8] backscattering for targets with such symmetry has the form

$$
\begin{aligned}
\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i, s) &= \tilde{\Lambda}_{b_{1,1}}(\vec{1}_i, s)\, \overleftrightarrow{1}_i + \tilde{\Lambda}_{b_{1,2}}(\vec{1}_i, s)\, \overleftrightarrow{\tau}_i \\
&= \begin{pmatrix} \tilde{\Lambda}_{b_{1,1}}(\vec{1}_i, s) & \tilde{\Lambda}_{b_{1,2}}(\vec{1}_i, s) & 0 \\ -\tilde{\Lambda}_{b_{1,2}}(\vec{1}_i, s) & \tilde{\Lambda}_{b_{1,1}}(\vec{1}_i, s) & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{for } N \geq 3
\end{aligned}
\tag{4.11}
$$

where reciprocity has not yet been assumed. (With reciprocity the coefficient of $\overleftrightarrow{\tau}_i$ is zero.) For $N = 2$ the above result is not obtained and a general transverse $\tilde{\overleftrightarrow{\Lambda}}_b$ (four non-zero components) is invariant to C_2 transformation which is equivalent to a sign reversal of the incident and scattered field, a consequence of linearity.

Comparing (4.11) to (4.3) the unique solution is

$$\tilde{\overleftrightarrow{\Lambda}}_b(\vec{1}_i, s) = \overleftrightarrow{0} \qquad \text{for } N \geq 3 \tag{4.12}$$

i.e., zero backscattering. This is a generalization of the result in [6] where zero backscattering is derived for the case of $N = 4$, invariance to rotation by $\pi/4$. Now the result is also generalized from scalar to dyadic permittivity and permeability.

5. Forward Scattering by Self-Dual Target

Applying (3.5) to forward scattering from a self-dual target we have

$$\begin{aligned}&\vec{1}_o = \vec{1}_i \quad , \quad \vec{1}_o = \vec{1}_i \quad , \quad \overleftrightarrow{\tau}_o = \overleftrightarrow{\tau}_i \\ &\tilde{\overleftrightarrow{\Lambda}}_f(\vec{1}_i,s) \equiv \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_i,\vec{1}_i;s) = -\overleftrightarrow{\tau}_i \cdot \tilde{\overleftrightarrow{\Lambda}}_f(\vec{1}_i,s)\cdot \overleftrightarrow{\tau}_i\end{aligned} \tag{5.1}$$

From (3.8) we have the more general rotation

$$\begin{aligned}\tilde{\overleftrightarrow{\Lambda}}_f(\vec{1}_i,s) &= \overleftrightarrow{C}_i(-\psi)\cdot\tilde{\overleftrightarrow{\Lambda}}_f(\vec{1}_i,s)\cdot\overleftrightarrow{C}_i(\psi) \\ &= \overleftrightarrow{C}_i^T(\psi)\cdot\tilde{\overleftrightarrow{\Lambda}}_f(\vec{1}_i,s)\cdot\overleftrightarrow{C}_i(\psi)\end{aligned} \tag{5.2}$$

so that the forward scattering dyadic is invariant to rotation about $\vec{1}_i$ (independent of any symmetries in the target other than self duality). Writing out the components as in (4.3) we have

$$\begin{aligned}\tilde{\overleftrightarrow{\Lambda}}_f(\vec{1}_i,s) &= \tilde{\Lambda}_{f_{h,h}}\vec{1}_h\vec{1}_h + \tilde{\Lambda}_{f_{h,v}}\vec{1}_h\vec{1}_v + \tilde{\Lambda}_{f_{v,h}}\vec{1}_v\vec{1}_h + \tilde{\Lambda}_{f_{v,v}}\vec{1}_v\vec{1}_v \\ &= -\vec{1}_i\times\tilde{\overleftrightarrow{\Lambda}}_f(\vec{1}_i,s)\times\vec{1}_i \\ &= \tilde{\Lambda}_{f_{h,h}}\vec{1}_v\vec{1}_v - \tilde{\Lambda}_{f_{h,v}}\vec{1}_v\vec{1}_h - \tilde{\Lambda}_{f_{v,h}}\vec{1}_h\vec{1}_v + \tilde{\Lambda}_{f_{v,v}}\vec{1}_h\vec{1}_h \\ \tilde{\Lambda}_{f_{h,h}} &= \tilde{\Lambda}_{f_{v,v}} \quad , \quad \tilde{\Lambda}_{f_{h,v}} = -\tilde{\Lambda}_{f_{v,h}} \\ \tilde{\overleftrightarrow{\Lambda}}_f(\vec{1}_i,s) &= \tilde{\Lambda}_{f_{h,h}}(\vec{1}_i,s)\overleftrightarrow{1}_i + \tilde{\Lambda}_{f_{h,v}}(\vec{1}_i,s)\,\overleftrightarrow{\tau}_i \\ &= \begin{pmatrix}\tilde{\Lambda}_{f_{h,h}}(\vec{1}_i,s) & \tilde{\Lambda}_{f_{h,v}}(\vec{1}_i,s) & 0 \\ -\tilde{\Lambda}_{f_{h,v}}(\vec{1}_i,s) & \tilde{\Lambda}_{f_{h,h}}(\vec{1}_i,s) & 0 \\ 0 & 0 & 0\end{pmatrix}\end{aligned} \tag{5.3}$$

which is a combination of the transverse identity and a rotation. The forward scattering dyadic has only two independent components.

If the target has an axial symmetry plane parallel to $\vec{1}_h$, then the reflection described by (4.4) applied to the forward scattering dyadic gives [5]

$$\tilde{\Lambda}_{f_{h,v}}(\vec{1}_i,s) = \tilde{\Lambda}_{f_{v,h}}(\vec{1}_i,s) = 0$$
$$\tilde{\overleftrightarrow{\Lambda}}_f(\vec{1}_i,s) = \tilde{\Lambda}_f(\vec{1}_i,s)\,\overleftrightarrow{1}_i \tag{5.4}$$

which is symmetric even though reciprocity has not been invoked. Furthermore, there is no change in the polarization from incident to scattered field. This forward-scattering dyadic has $C_{\infty a}$ (or O_2) symmetry. (rotation by any angle with infinitely many axial symmetry planes).

Considering $\vec{1}_i$ as an N-fold rotation axis, one can apply $\overleftrightarrow{C}_i(\phi_\ell)$ as in (4.10). However, note that this is just a special case of the general property of the forward scattering dyadic in (5.2). So C_N symmetry here adds no new property to the forward scattering dyadic.

Inversion symmetry is described by

$$I = \{(1)\ ,\ (I)\}$$
$$(1) \rightarrow \overleftrightarrow{1}\ ,\ (I) \rightarrow -\overleftrightarrow{1} \tag{5.5}$$

Applying this to the backscattering dyadic for a target with this symmetry gives

$$-\overleftrightarrow{1} \cdot \vec{1}_i = -\vec{1}_i$$
$$\tilde{\overleftrightarrow{\Lambda}}_f(\vec{1}_i,s) = \tilde{\overleftrightarrow{\Lambda}}_f(-\vec{1}_i,s) \tag{5.6}$$

which with (5.3) gives

$$\tilde{\Lambda}_{f_{h,h}}(\vec{1}_i,s) = \tilde{\Lambda}_{f_{v,v}}(-\vec{1}_i,s)\ ,\ \tilde{\Lambda}_{f_{h,v}}(\vec{1}_i,s) = \tilde{\Lambda}_{f_{h,v}}(-\vec{1}_i,s) \tag{5.7}$$

Remember, however, that on inversion we have

$$(-\overleftrightarrow{1}) \cdot \vec{1}_i = -\vec{1}_i\ \ ,\ \overleftrightarrow{\tau}_{-i} = -\overleftrightarrow{\tau}_i \tag{5.8}$$

and one of the h and v directions reverses. While inversion produces the above form for any $\vec{1}_i$, various other symmetries which reverse $\vec{1}_i$ (including reflection R_t through a transverse symmetry plane, as well as rotation reflection and dihedral D_N symmetry with this principal axis) produce similar results.

Consider now the implication of reciprocity. From (1.4) we have

$$\begin{aligned}
\tilde{\overleftrightarrow{\Lambda}}_f(-\vec{1}_i,s) &= \tilde{\overleftrightarrow{\Lambda}}_f^T(\vec{1}_i,s) \\
&= \tilde{\Lambda}_{b_{h,h}}(\vec{1}_i,s)\,\overleftrightarrow{1}_i - \tilde{\Lambda}_{b_{h,v}}(\vec{1}_i,s)\,\overleftrightarrow{\tau}_i \\
&= \tilde{\Lambda}_{b_{h,h}}(-\vec{1}_i,s)\,\overleftrightarrow{1}_i - \tilde{\Lambda}_{b_{h,v}}(-\vec{1}_i,s)\,\overleftrightarrow{\tau}_i \\
\tilde{\Lambda}_{b_{h,h}}(-\vec{1}_i,s) &= \tilde{\Lambda}_{b_{h,h}}(\vec{1}_i,s)\ ,\ \tilde{\Lambda}_{b_{h,v}}(-\vec{1}_i,s) = -\tilde{\Lambda}_{b_{h,v}}(\vec{1}_i,s)
\end{aligned} \tag{5.9}$$

which is a kind of invariance to direction reversal. Note the change of sign on $\overleftrightarrow{\tau}_i$ on direction reversal, and the change of the h direction or v direction.

Now combine inversion symmetry from (5.6) with reciprocity to give

$$\begin{aligned}
\tilde{\overleftrightarrow{\Lambda}}_f(\vec{1}_i,s) &= \tilde{\overleftrightarrow{\Lambda}}_f^T(\vec{1}_i,s) = \tilde{\Lambda}_b(\vec{1}_i,s)\,\overleftrightarrow{1}_i \\
&= \tilde{\overleftrightarrow{\Lambda}}_f(-\vec{1}_i,s) = \tilde{\Lambda}_b(-\vec{1}_i,s)\,\overleftrightarrow{1}_i
\end{aligned} \tag{5.10}$$

The coefficient of $\overleftrightarrow{\tau}_i$ vanishes and the scattering dyadic is proportional to the transverse dyadic with a coefficient which is invariant to reversal of $\vec{1}_i$. Similar results hold for other symmetries which reverse $\vec{1}_i$ for particular choices of $\vec{1}_i$.

6. Low-Frequency Scattering by Self-Dual Target

For low frequencies the scattering is dominated by the induced electric and magnetic dipole moments via the respective polarizabilities [2, 3, 5] as

$$\tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) = \tilde{\gamma}^2 \left[-\overleftrightarrow{1}_o \cdot \tilde{\overleftrightarrow{P}}(s) \cdot \overleftrightarrow{1}_i + \overleftrightarrow{\tau}_i \cdot \tilde{\overleftrightarrow{M}}(s) \cdot \overleftrightarrow{\tau}_i \right] \text{ as } s \to 0 \tag{6.1}$$

From (3.4) the dual scattering dyadic is

$$\begin{aligned} \tilde{\overleftrightarrow{\Lambda}}^{(d)}(\vec{1}_o, \vec{1}_i; s) &= \overleftrightarrow{\tau}_o \cdot \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) \cdot \overleftrightarrow{\tau}_o \\ &= \tilde{\gamma}^2 \left[-\overleftrightarrow{1}_o \cdot \tilde{\overleftrightarrow{P}}^{(d)}(s) \cdot \overleftrightarrow{1}_i + \overleftrightarrow{\tau}_o \cdot \tilde{\overleftrightarrow{M}}^{(d)}(s) \cdot \overleftrightarrow{\tau}_i \right] \text{ as } s \to 0 \end{aligned} \tag{6.2}$$

from which we identify

$$\tilde{\overleftrightarrow{P}}^{(d)}(s) = \tilde{\overleftrightarrow{M}}(s) \quad , \quad \tilde{\overleftrightarrow{M}}^{(d)}(s) = \tilde{\overleftrightarrow{P}}(s) \tag{6.3}$$

This should be apparent from the interchanged roles of $\overleftrightarrow{\varepsilon}$ and $\overleftrightarrow{\mu}$ in (2.7) together with the interchange of electric and magnetic fields. For a self-dual target then evidently

$$\begin{aligned} \tilde{\overleftrightarrow{P}}(s) &= \tilde{\overleftrightarrow{M}}(s) \\ \tilde{\overleftrightarrow{\Lambda}}(\vec{1}_o, \vec{1}_i; s) &= \tilde{\gamma}^2 \left[-\overleftrightarrow{1}_o \cdot \tilde{\overleftrightarrow{P}}(s) \cdot \overleftrightarrow{1}_i + \overleftrightarrow{\tau}_o \cdot \tilde{\overleftrightarrow{P}}(s) \cdot \overleftrightarrow{\tau}_i \right] \text{ as } s \to 0 \\ &\equiv \tilde{\gamma}^2 \, \tilde{\overleftrightarrow{X}}(\vec{1}_o, \vec{1}_i; s) \text{ as } s \to 0 \end{aligned} \tag{6.4}$$

Note a general symmetry on reversing both directions of incidence and scattering [5], giving

$$\tilde{\overleftrightarrow{X}}(-\vec{1}_o, -\vec{1}_i; s) = \tilde{\overleftrightarrow{X}}(\vec{1}_o, \vec{1}_i; s) \tag{6.5}$$

For backscattering this becomes

$$\tilde{\overleftrightarrow{X}}_b(\vec{1}_i,s) \equiv \tilde{\overleftrightarrow{X}}(-\vec{1}_i,\vec{1}_i;s) = -\overleftrightarrow{1}_i \cdot \tilde{\overleftrightarrow{P}}(s) \cdot \overleftrightarrow{1}_i - \overleftrightarrow{\tau}_i \cdot \tilde{\overleftrightarrow{P}}(s) \cdot \overleftrightarrow{\tau}_i$$
$$= \left[\tilde{P}_{v,v}(s) - \tilde{P}_{h,h}(s)\right]\left[\vec{1}_h\vec{1}_h - \vec{1}_v\vec{1}_v\right] - \left[\tilde{P}_{h,v}(s) + \tilde{P}_{v,h}(s)\right]\left[\vec{1}_h\vec{1}_v + \vec{1}_v\vec{1}_h\right] \tag{6.6}$$

This exhibits the fact that the backscattering dyadic is symmetric without invoking reciprocity.

In forward scattering we have

$$\tilde{\overleftrightarrow{X}}_f(\vec{1}_i,s) \equiv \tilde{\overleftrightarrow{X}}(\vec{1}_i,\vec{1}_i;s) = -\overleftrightarrow{1}_i \cdot \tilde{\overleftrightarrow{P}}(s) \cdot \overleftrightarrow{1}_i + \overleftrightarrow{\tau}_i \cdot \tilde{\overleftrightarrow{P}}(s) \cdot \overleftrightarrow{\tau}_i$$
$$= -\left[\tilde{P}_{h,h}(s) + \tilde{P}_{v,v}(s)\right]\overleftrightarrow{1}_i - \left[\tilde{P}_{h,v}(s) - \tilde{P}_{v,h}(s)\right]\overleftrightarrow{\tau}_i \tag{6.7}$$

consistent with the general form in (5.3). Invoking reciprocity gives

$$\overleftrightarrow{P}(s) = \overleftrightarrow{P}^T(s) \quad , \quad \overleftrightarrow{M}(s) = \overleftrightarrow{M}^T(s)$$
$$\tilde{\overleftrightarrow{X}}_f(\vec{1}_i,s) = \tilde{\overleftrightarrow{X}}_f^T(\vec{1}_i,s) = -\left[\tilde{P}_{h,h}(s) + \tilde{P}_{v,v}(s)\right]\overleftrightarrow{1}_i$$
$$= \tilde{\overleftrightarrow{X}}_f(-\vec{1}_i,s) \tag{6.8}$$

Sections 4 and 5 consider the influence of geometrical symmetries on backscattering and forward scattering, respectively. For low frequencies these results also apply. Of course, it is the symmetries in the polarizabilities, whether or not they come from geometrical symmetries that are important. Various geometrical symmetries (with reciprocity) induce simplifications (i.e., symmetries) in the forms of the polarizability dyadics as tabulated in [4, 5]. A special case is that of a generalized spherical (O_3), tetrahedral (T), octahedral (O), or icosahedral (Y) symmetry (including reciprocity), each of which gives

$$\tilde{\overleftrightarrow{P}}(s) = \tilde{P}(s)\overleftrightarrow{1}$$
$$\tilde{\overleftrightarrow{X}}(\vec{1}_o,\vec{1}_i;s) = \tilde{P}(s)\left[-\overleftrightarrow{1}_o \cdot \overleftrightarrow{1}_i + \overleftrightarrow{\tau}_o \cdot \overleftrightarrow{\tau}_i\right]$$
$$\tilde{\overleftrightarrow{X}}_b(\vec{1}_i,s) = \overleftrightarrow{0}$$
$$\tilde{\overleftrightarrow{X}}_f(\vec{1}_i,s) = -2\tilde{P}(s)\overleftrightarrow{1}_i \tag{6.9}$$

Note that the forward scattering is polarization independent and independent of the orientation of the target. For a spherical target, this is immediately apparent, but for the regular polyhedra this relies on the fact that the multiple rotation axes make the polarizabilities (related to dipole moments, not including the higher order multipoles) invariant to all rotations and reflections.

7. Concluding Remarks

Self duality gives some special properties to target scattering. For general angles of incidence and scattering, rotation of the incident field produces an equal rotation of the scattered field. For backscattering, this means that the scattered field is rotated with sense opposite to the incident field when both are referred to the same direction, i.e., the direction of incidence. If, in addition, the target has a three-fold or higher rotation axis parallel to the direction of incidence, the backscattering is zero. When combined with various geometrical symmetries and/or reciprocity in the target, the scattering dyadic attains various simplifications (symmetries) as well.

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