Interaction Notes

Note 513

February 1994

BACK DOOR COUPLING OF RF (MICROWAVE) ENERGY TO SPACECRAFT INTERIOR CABLING

William J. Karzas
Consultant for
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ABSTRACT

The chain of physical processes which lead from exterior RF (microwave) illuminating flux to the power induced in spacecraft interior cabling (and thus conducted into subsystems) is described. A sequence of physically derived algorithms for computation is presented. For conditions appropriate to randomized RF energy in overmoded spacecraft cavities (roughly, that interior dimensions >> RF wavelengths), the result is simple and requires little knowledge of cable parameters and their detailed geometric configurations. For convenience, an effective cable coupling cross section is defined by which the exterior flux is multiplied to get power induced in a cable.

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Objective

We wish to compute the energy coupled into subsystems in the interior of a spacecraft illuminated with narrow-band microwave RF radiation. We start from a given Poynting vector describing the incident radiation of frequency f:

Incident Flux

$\mathbf{S}(\omega)$ = watts/ meter^2 in range $\omega \rightarrow \omega + d\omega$, where $\omega = 2\pi f$, is a vector pointing from the illuminating source (earth) to the space craft.

Spacecraft RF Characterization

For the spacecraft we assign an arbitrary axis, (say along a line of symmetry if it exists) and locate all the apertures, assigning each an index " i " and a vector cross section $\boldsymbol{\sigma}_i$ depending on the relevant parameters (major dimension "a_i", shape factor "s_i" , aperture resonant frequency "f_{0i}"). The vector direction "$\mathbf{n}_i$" points toward the interior of the spacecraft.

$$\boldsymbol{\sigma}_i(\omega) = s_i\, a_i^2 \left(\frac{f}{f_{0i}}\right)^4 \mathbf{n}_i \quad , \quad f \le f_{0i},$$

$$= s_i\, a_i^2 \left(\frac{f_{0i}}{f}\right)^2 \mathbf{n}_i \quad , \quad f \ge f_{0i}$$

$$= \sigma_i(\omega)\ \mathbf{n}_i \quad .$$

Power Reaching Spacecraft Interior

The power penetrating the spacecraft through the apertures is "$P_{in}(\omega)$". We have three choices for this, depending on what we know about the orientation of the spacecraft:

If the craft is oriented and stabilized,

$$P_{in}(\omega) = \sum_i \mathbf{S}\cdot\mathbf{n}_i\, \sigma_i \quad u[\,\mathbf{S}\cdot\mathbf{n}_i\,]$$

If the craft is tumbling or of unknown orientation, we average, so that (one-half the time any aperture will be shielded, and $< \cos > = 1/2$):

$$P_{in}(\omega) = \Sigma_i <\mathbf{S}\cdot\mathbf{n}_i\, \sigma_i\; u[\,\mathbf{S}\cdot\mathbf{n}_i\,]> = 1/4\, \Sigma_i\, |\mathbf{S}|\sigma_i$$

For a worst case estimate,

$$P_{in}(\omega) = \Sigma_i\, |\mathbf{S}|\sigma_i \;.$$

(The step function "$u[\,\mathbf{S}\cdot\mathbf{n}_i\,]$" allows only unshadowed apertures to be counted.)

Energy in Spacecraft Cavity

By definition of the "quality factor" "Q" , the energy "W_{tot}" inside the cavity satisfies

$$\frac{d\, W_{tot}}{d\, t} + \frac{\omega}{2\,\pi\, Q}\, W_{tot} = P_{in}(\omega\,,t), \quad \text{so that}$$

$$W_{tot} \approx \frac{2\,\pi\, Q}{\omega}\, P_{in}(\omega) = \frac{Q}{f}\, P_{in} \quad \text{if the incident pulse length is} \gg \frac{1}{f}\;,$$

$$\approx P_{in} \times (\text{ incident pulse length}) \quad \text{otherwise .}$$

For convenience we define $\overline{W}$ as one-third the average energy density.

$$\overline{W} \equiv \frac{W_{tot}}{3\,\mathcal{V}}\;, \quad \text{with } \mathcal{V} \text{ the spacecraft interior volume.}$$

Statistical Description of Interior EM Fields

If the cavity is "overmoded" (many modes in the narrow bandwidth of the illumination, or equivalently, cavity dimensions are large compared to incident wavelength) the energy will be "randomized" and distributed probabilistically with equal amounts in each of three orthogonal directions.* In particular, the energy density $\mathcal{W}_s$ in each direction is exponentially distributed (χ^2) with p.d.f.

$$p(\mathcal{W}_s)\,d\mathcal{W}_s = \frac{e^{-\frac{\mathcal{W}_s}{\overline{W}}}}{\overline{W}}\, d\mathcal{W}_s \ .$$

In a like manner, the electric field is randomly distributed. In any particular direction "s" , each of the two cavity eigenfunctions (analogous to two polarizations) of the electric field, $\mathcal{E}_{s,i}$, is normally distributed:

$$p(\mathcal{E}_{s,i})\, d\,\mathcal{E}_{s,i} = \frac{e^{-\frac{\mathcal{E}_{s,i}^2}{2\overline{E^2}}}}{\sqrt{2\pi\overline{E^2}}}\, d\,\mathcal{E}_{s,i} \ .$$

Here, $\frac{1}{2}\varepsilon_0\overline{E^2} \equiv \frac{1}{2}\overline{W}$, with equipartition of energy between the two eigenfunctions.

Correlation of EM Fields

Electric fields in different directions are independently distributed (i.e., uncorrelated). Electric fields in the same direction but at different locations, for the same eigenfunction, are correlated for relatively close separation. Specifically, following Lehman's work:

With $k \equiv \frac{\omega}{c} = \frac{2\pi}{\lambda}$,

$$\frac{<\mathcal{E}_{s,i}(\mathbf{x})\,\mathcal{E}_{s,i}(\mathbf{x}')>}{\overline{E^2}} \equiv \kappa(\mathbf{x},\mathbf{x}') = \frac{\sin k\,|\mathbf{x}-\mathbf{x}'|}{k\,|\mathbf{x}-\mathbf{x}'|} \ .$$

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* The exponential distribution for RF power has been observed in many experiments with various cavities. A careful theoretical derivation has recently been given by T. H. Lehman, "A Statistical Theory of Electromagnetic Fields in Complex Cavities", Interaction Note 494, May 1994. Our description of the statistical properties of the fields follows this work.

Where necessary, we can approximate the correlation coefficient as:

$$\kappa(\mathbf{x}, \mathbf{x}') \cong 1, \quad |\mathbf{x} - \mathbf{x}'| < \frac{\lambda}{2}$$

$$\cong 0, \quad \text{otherwise.}$$

Thus, the joint probability distribution for electric fields at different locations is:

$$p[\mathcal{E}_{s,i}(\mathbf{x}), \mathcal{E}_{s,i}(\mathbf{x}')] = \frac{1}{2\pi\overline{E^2}\sqrt{1-\kappa^2}} \exp\left[-\frac{\mathcal{E}_{s,i}^2 - 2\kappa\,\mathcal{E}_{s,i}\,\mathcal{E}'_{s,i} + \mathcal{E}'^2_{s,i}}{2(1-\kappa^2)\,\overline{E^2}}\right]$$

where $\kappa \equiv \kappa(\mathbf{x}, \mathbf{x}')$ and $\mathcal{E}_{s,i} \equiv \mathcal{E}_{s,i}(\mathbf{x})$; $\mathcal{E}_{s,i}' \equiv \mathcal{E}_{s,i}(\mathbf{x}')$.

Using the [one, zero] approximation for the correlation coefficient,

$$p[\mathcal{E}_{s,i}(\mathbf{x}), \mathcal{E}_{s,i}(\mathbf{x}')] \cong \frac{e^{-\frac{\mathcal{E}_{s,i}^2 + \mathcal{E}'^2_{s,i}}{2\overline{E^2}}}}{2\pi\overline{E^2}}, \quad |\mathbf{x} - \mathbf{x}'| > \frac{\lambda}{2}$$

$$\cong \delta[\mathcal{E}_{s,i} - \mathcal{E}'_{s,i}]\,\frac{e^{-\frac{\mathcal{E}_{s,i}^2}{2\overline{E^2}}}}{\sqrt{2\pi\overline{E^2}}}, \quad |\mathbf{x} - \mathbf{x}'| < \frac{\lambda}{2}$$

We note that

$$\langle \mathcal{E}_{s,i}(\mathbf{x})\, \mathcal{E}_{s,i}(\mathbf{x}') \rangle \cong 0, \quad |\mathbf{x} - \mathbf{x}'| > \frac{\lambda}{2}$$

$$\cong \overline{E^2}, \quad |\mathbf{x} - \mathbf{x}'| < \frac{\lambda}{2}$$

In what follows we may use this property to simplify our computation.

To get the total averaged electric field in a particular direction, we combine, incoherently, the contribution of both eigenfunctions and look at the magnitude:

$\mathcal{E}_s = \sqrt{\mathcal{E}_{s,1}^2 + \mathcal{E}_{s,2}^2}$, which has a Rayleigh distribution, with p.d.f.

$$p(\mathcal{E}_s) = \frac{\mathcal{E}_s}{\overline{E^2}} \, e^{-\frac{\mathcal{E}_s^2}{\overline{E^2}}} .$$

Coupling to Cabling and Subsytems

The coupling of the RF energy to subsystems is dominantly by means of the currents and voltages induced on inter-box cabling and thereby conducted into subsytems, rather than by direct field penetration into boxes. Thus we next compute the cable interaction.

We might use transmission-line formalism for this purpose; however, the wavelengths and geometries involved do not meet the conditions for this to be valid. Instead, we use an approach which has been used for lines above ground interacting with high-altitude EMP and does not require that wavelenths be large compared to separations. We shall first consider a semi-infinite line and later modify the result for real segment lengths.

Voltage Induced in Cables

For an illuminated line above a ground plane, the open-circuit voltage induced at the end is given (in the frequency domain) by:

$V_{oc}(\omega) \approx \int_{-\infty}^{0} ds \, e^{\gamma(\omega) s} \, \mathcal{E}_s(s, \omega)$, where s is measured along the line.

$\gamma(\omega)$ is the complex propagation constant for the line-ground plane configuration. For idealized perfectly conducting boundaries we would have $\gamma(\omega) = \gamma_0 = j k_0 = j \omega/c$; for realistic conductors, $\gamma(\omega) = \gamma_0 \mathcal{H}(j\omega)$, where

$$\mathcal{H}(j\,\omega) = \left[1 + \frac{\ln\left\{ \dfrac{1 + h\sqrt{j\,\omega\,\mu_0\,\sigma_g}}{h\sqrt{j\,\omega\,\mu_0\,\sigma_g}} \right\} + \dfrac{1}{a_{eff}\sqrt{j\,\omega\,\mu_0\,\sigma_w}}}{\ln \dfrac{2\,h}{a_{eff}}} \right]^{\frac{1}{2}} .$$

Here h is the average height of the bundle above a ground plane, σ_g the ground plane conductivity (generally, that for aluminum), σ_w the wire conductivity (that for copper) and a_{eff} is an equivalent radius for the multiwire bundle. For a bundle of diameter d_c made up of n wires of radius a_w,

$$a_{eff} \approx \sqrt[n]{a_w\, d_c^{\,n-1}}$$

and the common-mode characteristic impedance of a mutiwire cable-ground plane configuration is

$$Z_c = \frac{\eta_0}{2\,\pi}\, \ln \frac{2\,h}{a_{eff}} ,$$

where η_0 is the impedance of free space (377 ohms).

Probability Distribution of Induced Voltage

In general, the p.d.f. for a new function of a random variable $y = f(x)$ is given by

$$p\,(y) = \int dx\; p\,(x)\;\delta[\,y - f\,(x)\,] ,$$ with δ the Dirac delta-function.

Thus, the p.d.f for V_{oc} is given by

$$p\,(V_{oc}) = \int d\,\mathcal{E}_s\; p_{\mathcal{E}}(\mathcal{E}_s)\;\delta\left[\,V_{oc} - \int_{-\infty}^{0} d\,s\; e^{\gamma\, s}\, \mathcal{E}_s(s)\,\right]$$

To evaluate the integral we note that it can be expressed as a sum of random variables. If they were independent, and there were many of them, we could use the central limit theorem and get a normal distribution for V_{oc} . A problem arises, however, when the random variables in the sum are correlated; the ensuing computation is unmanageably complicated. We avoid this with an approximate solution which should be quite accurate.

An Approximate Solution

If we use the "one, zero" approximation for the correlation coefficient, the various terms in the integral (sum) <u>are</u> independent if separated by more than one wavelength. So we shall approximate the integral by a sum of segments of length λ :

$$\int_{-\infty}^{0} ds\ e^{\gamma(\omega)\, s}\, \mathcal{E}_s(s, \omega) \approx \sum_{n=1}^{n=N=\frac{L}{\lambda}} \lambda\, e^{-\gamma \lambda n}\, \mathcal{E}_s(s_n, \omega) .$$

Each of the N random variables is $\lambda\ e^{-\gamma \lambda n}\ \mathcal{E}_s(\omega)$, which has mean zero and variance

$$\sigma_n^2 = \lambda^2 \left| e^{-2\gamma\lambda n} \right| < \mathcal{E}_s^2 > = \lambda^2\ e^{-2\,\mathrm{Re}\gamma\,\lambda n}\ \overline{E^2} .$$

Then, by the central limit theorem (or, since each of the random variables is normally distributed) the p.d.f for the sum for each polarization is normal with zero mean and with variance $= \Sigma\, \sigma_n{}^2$. That is,

$$p(V_{oc,i}) = \frac{e^{-\frac{V_{oc,i}^2}{2\sigma_V^2}}}{\sqrt{2\pi\sigma_V^2}} , \quad \text{with } \sigma_V^2 = \sum_{n=1}^{n=N=\frac{L}{\lambda}} \sigma_n .$$

or

$$\sigma_V^2 = \sum_{n=1}^{n=N=\frac{L}{\lambda}} e^{-2\,\mathrm{Re}\gamma\,\lambda n}\ \lambda^2 < \mathcal{E}_s^2 > = \frac{1 - e^{-2\,\mathrm{Re}\gamma\,\lambda N}}{1 - e^{-2\,\mathrm{Re}\gamma\,\lambda}}\ \lambda^2\ \overline{E^2}$$

$$= \frac{1 - e^{-2\,\mathrm{Re}\gamma\, L}}{1 - e^{-2\,\mathrm{Re}\gamma\,\lambda}}\ \lambda^2\ \overline{E^2} \approx \frac{L}{\lambda}\ \lambda^2\ \overline{E^2} = L\,\lambda\,\overline{E^2} .$$

Here we have used the fact that for typical spacecraft, the propagation attenuation distance is very much longer than any cable lengths, so that $(\mathrm{Re}\,\gamma)L \ll 1$. (The evaluation of the relevant cable parameters is given in the appendix.) This approximation, which is equivalent to omitting the exponential factor from the integral from the start , produces a tight upper bounding. Substituting for σ_V ,

$$p(V_{oc,i}) = \frac{e^{-\frac{V_{oc,i}^2}{2\overline{E^2}L\lambda}}}{\sqrt{2\pi\overline{E^2}L\lambda}} .$$

Power Induced in Cable

Since the voltage V has zero mean, and because we are interested in it anyway, we give the distribution for the power flowing to the cable end (if terminated in a matched impedance); for this we incoherently add the contributions from both polarizations. That is,

$$P = P_1 + P_2 = \frac{V_{oc,1}^2 + V_{oc,2}^2}{Z_c}$$, with Z_c the characteristic impedance as defined earlier.

The p.d.f. for this cable power flow is:

$$p(P) = \int dV_{oc,1}\int dV_{oc,2}\, p(V_{oc,1})\, p(V_{oc,2})\, \delta\left[P - \frac{V_{oc,1}^2 + V_{oc,2}^2}{Z_c}\right]$$

$$= \int_0^\infty dy_1\int_0^\infty dy_2\, Z_c\, \delta[Z_c P - (y_1 + y_2)]\, \frac{e^{-\frac{y_1+y_2}{2\sigma_V^2}}}{2\pi\sigma_V^2\sqrt{y_1 y_2}} ,$$

where $y_1, y_2 = V_{oc,1}, V_{oc,2}$.

Thus,

$$p(P) = \frac{Z_c\, e^{-\frac{Z_c P}{2\sigma_V^2}}}{2\pi\sigma_V^2} \int_0^{Z_c.P} \frac{dy_1}{\sqrt{y_1(1-y_1)}} = \frac{Z_c}{2\sigma_V^2}\, e^{-\frac{Z_c P}{2\sigma_V^2}}$$

We note that this is distributed exponentially in the same manner as the RF energy density in the cavity.

$$p(P_{cable}) = \frac{e^{-\frac{P_{cable}}{\overline{P}}}}{\overline{P}}$$, where the mean power $\overline{P}$ is given by

$$\overline{P} = \frac{2\sigma_V^2}{Z_c} = \frac{4\pi\lambda L \eta_o \overline{E^2}}{\ell n \frac{2h}{a_{eff}}} = \frac{4\pi\lambda L \eta_o W_{tot}}{\ell n \frac{2h}{a_{eff}}\, \varepsilon_o\, 3\mathcal{V}}$$

$$= \frac{4\pi\lambda L \eta_o Q P_{in}}{3\varepsilon_o f \mathcal{V}\, \ell n \frac{2h}{a_{eff}}} = \frac{4\pi c\lambda L Q P_{in}}{3 f \mathcal{V}\, \ell n \frac{2h}{a_{eff}}}$$

$$= \frac{4\pi c\lambda L Q}{3 f \mathcal{V}\, \ell n \frac{2h}{a_{eff}}} \frac{\sum_{apertures} \mathbf{S}\bullet \mathbf{n}_i\, \sigma_i}{|\mathbf{S}|} |\mathbf{S}|$$

or

$$= \frac{4\pi c\lambda L Q}{3 f \mathcal{V}\, \ell n \frac{2h}{a_{eff}}} \frac{\sum_{apertures} \sigma_i}{4} |\mathbf{S}|$$

or

$$= \frac{4\pi c\lambda L Q}{3 f \mathcal{V}\, \ell n \frac{2h}{a_{eff}}} \sum_{apertures} \sigma_i\, |\mathbf{S}| ,$$

depending on the choices originally made (see above). As before, $\mathcal{V}$ is the spacecraft interior volume.

Effective Cable Coupling Cross Section

We can express this mean cable-coupled power in terms of an effective cross section for the incident flux:

$$\overline{P} = \sigma_{\text{eff: cable}} \; |\mathbf{S}| \quad , \text{ where}$$

$$\sigma_{\text{eff: cable}} = \frac{4\pi\, c\lambda\, L\, Q}{3\, f\, \mathcal{V}\, \ln\frac{2h}{a_{\text{eff}}}} \; \frac{\sum_{\text{apertures}} \mathbf{S}\cdot \mathbf{n}_i\, \sigma_i}{|\mathbf{S}|}$$

or

$$= \frac{4\pi\, c\lambda\, L\, Q}{3\, f\, \mathcal{V}\, \ln\frac{2h}{a_{\text{eff}}}} \; \frac{\sum_{\text{apertures}} \sigma_i}{4}$$

or

$$= \frac{4\pi\, c\lambda\, L\, Q}{3\, f\, \mathcal{V}\, \ln\frac{2h}{a_{\text{eff}}}} \sum_{\text{apertures}} \sigma_i \; .$$

This is the final result we have sought. We note that very little information about the cabling is needed; the length and rough dimensions for cable size and stand off are all that is required. (The characteristic impedance depends only logarithmically on the latter dimensions; thus details are not necessary.)

SUMMARY

We list the sequence of physical processes which lead from the exterior incident RF (microwave) flux to interior coupled power in cables, and indicate the quantities computed and used in back door estimates.

Incident Flux	$S(\omega)$	
Spacecraft Back Door Apertures	$\sigma_i(\omega)$, n_i	
Power Penetrating Through Back Door	$P_{in}(\omega)$	
Mean Energy in Cavity	$W_{tot}(\omega)$	
Distribution of Cavity Energy	$p(W)$:	Exponential
Distribution of Electric Field Components	$p(\mathcal{E}_{s,i})$:	Normal
Correlation of Electric Fields	$< \mathcal{E}_s(\mathbf{x}) \mathcal{E}_s(\mathbf{x}') >$	
Distribution of Total E-Field Magnitude	$p(\mathcal{E}_s)$:	Rayleigh
Cable-Ground Propagation Parameters	$\gamma(\omega)$, Z_c	
Distribution of Cable Induced Voltage	$p(V_{oc})$:	Normal
Distribution of Cable Power Flow	$p(P_{cable})$:	Exponential
Equivalent Cable Coupling Cross Section	$< P_{cable} > = \sigma_{eff:\ cable} \mathbf{S}$	

APPENDIX

NUMERICAL EVALUATION OF SPACECRAFT CABLE PARAMETERS

First we numerically evaluate the most significant cable parameter -- the propagation constant for a cable-ground system. From above, $\gamma(\omega)$ is the complex propagation constant for the line-ground plane configuration. For idealized perfectly conducting boundaries we would have $\gamma(\omega) = \gamma_0 = j\,k_0 = j\,\omega/c$; for realistic conductors, $\gamma(\omega) = \gamma_0\,\mathcal{H}(j\,\omega)$, where

$$\mathcal{H}(j\,\omega) = \left[1 + \frac{\ell n \left\{ \dfrac{1 + h\sqrt{j\,\omega\,\mu_0\,\sigma_g}}{h\sqrt{j\,\omega\,\mu_0\,\sigma_g}} \right\} + \dfrac{1}{a_{eff}\sqrt{j\,\omega\,\mu_0\,\sigma_w}}}{\ell n \dfrac{2\,h}{a_{eff}}} \right]^{\frac{1}{2}} .$$

Here h is the average height of the bundle above a ground plane, σ_g the ground plane conductivity (generally, that for aluminum), σ_w the wire conductivity (that for copper). and a_{eff} an effective cable bundle radius defined previously.

With appropriate values for the metal conductivities:

Copper conductivity	=	5.5×10^7 mho/m
Aluminum conductivity	=	3.5×10^7 mho/m

we find,

$$\frac{1}{\mathrm{Re}\,\gamma} \approx \frac{123\; h\; a_{eff}}{\sqrt{f}\;(h + 1.25\,a_{eff})} \text{ meters, with } f \text{ in GHz, } h,\, a_{eff} \text{ in cm.}$$

and, $Z_c \approx \dfrac{\eta_0}{2\pi}\,\ell n \dfrac{2\,h}{a_{eff}} = 60\;\ell n \dfrac{2\,h}{a_{eff}}$ ohms, which ranges between 42 and 180 ohms as $\dfrac{h}{a_{eff}}$ goes from 1 to 10.

Using the following values as typical:

Frequency = 10 GHz	=	10^{10} Hz
Effective cable radius	=	1 cm = .01 m
Height above ground plane	=	2 cm = .02 m

we find, $1/\text{Re}\,\gamma$ = 240 meters. This is very much longer than expected cable runs, so that attenuation of the induced signals will be due to other than ohmic loss; this justifies the approximations made earlier. For these same values, Z_c = 83 ohms.