Interaction Notes

Note 509

13 November 1994

Comparative System Response to Resonant and Unipolar Waveforms

Carl E. Baum

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Abstract

There are various possible incident-field temporal waveforms to which an electronic system may be exposed. Each of these will have its own degree of effectiveness for producing large signals deep within the system. This paper compares the response to damped sinusoidal waveforms, both far from a system resonance and matched to a system resonance, to that due to a decaying exponential waveform (a unipolar transient). The comparison is done on the basis of norms, both for the ∞-norm (peak) and 2-norm.

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1. Introduction

In maximizing the interaction of incident electromagnetic waves with electronic systems one is faced with a problem of great complexity. While it is extremely difficult to calculate the response from first principles, one can use electromagnetic theory to characterize the *form* that the response takes and rely on measurements to evaluate the parameters in the appropriate models, As discussed in [1,5] the appropriate model is the singularity expansion method (SEM) which explicitly exhibits the various resonances as poles in the complex-frequency plane. Particularly for signals which reach deep into the system to the circuit level there are, in general, many resonances associated not only with the exterior envelope of the system, but also with the transfer functions through cables, cavities, etc. to the interior. As discussed in [1,8] an incident plane wave has a typical or canonical response which rolls off below some frequency related to the largest dimensions of the system. Above some frequency (of the order of a GHz) related to the resonant dimensions of apertures, small antennas, and certain internal equipment, the response also rolls off for "backdoor" or unintended interaction paths.

It should be intuitively obvious that the maximum system response is usually achieved by selecting the incoming waveform as an approximate sinusoid of enough cycles to "ring up" the resonant response. This can be demonstrated by calculations as in [3,4]. A question of interest concerns what happens if one illuminates a system with a pulsed sinusoid which is not so tuned to a resonance. Perhaps one did not know what was the optimal choice of frequency and guessed incorrectly. In such an event is such a choice of waveform still appropriate, or does another temporal shape give stronger interaction with the system? So here let us consider a more impulsive-like waveform and compare the response to this waveform to that due to the pulsed sinusoidal waveform.

For comparing these responses, the various excitation and response waveforms are evaluated in norm sense [3,6] in order to assign simple positive scalars which can be readily compared. There are various possible norms that one might use. For present purposes we choose two cases of the usual p-norm. The ∞-norm corresponds to the peak of the waveform and is appropriate for failure due to voltage breakdown or upset due to the level comparable to or exceeding the normal operating signal level. The 2-norm is proportional to the square root of the energy in the waveform, appropriate for burnout, except in cases where the energy in the system (associated with power-supply voltage) is triggered by the response waveform to deposit energy into various electronic devices.

For simplicity the incident wave is represented by a voltage waveform going into a filter. Of course, the incident wave is an electric field with some direction of incidence and polarization [5], but for present purposes we consider these two parameters as fixed. (These can be separately optimized as in [5]). Then volts/meter is changed to volts by multiplication by some characteristic length to make the excitation and response units consistent. Then the output wave is related to the input wave via the filter as

$$
\begin{aligned}
&V^{(out)}(t) = T(t) \circ V^{(in)}(t)\,,\ \tilde{V}^{(out)}(s) = \tilde{T}(s)\tilde{V}^{(in)}(s) \\
&\qquad \sim \equiv \text{ Laplace transform (two-sided) over time} \\
&\qquad s \equiv \Omega + j\omega \equiv \text{ Laplace-transform variable or complex frequency} \\
&\qquad \circ \equiv \text{ convolution with respect to time} \\
&V^{(in)}(t) \equiv \text{ excitation or input voltage} \\
&V^{(out)}(t) \equiv \text{ response or output voltage} \\
&\tilde{T}(s) \equiv \text{ filter transfer function (representing system)} \\
&T(t) \circ \equiv \text{ filter transfer temporal operator}
\end{aligned}
\tag{1.1}
$$

The p-norm of a temporal waveform is given by [6]

$$
\|V(t)\|_p \equiv \left\{ \int_{-\infty}^{\infty} |V(t)|^p \, dt \right\}^{\frac{1}{p}} \quad , \quad 1 \le p < \infty \tag{1.2}
$$

with the special case of the ∞-norm as

$$
\|V(t)\|_\infty = \sup_{-\infty < t < \infty} |V(t)| \tag{1.3}
$$

which is simply regarded as a peak magnitude (least upper bound). The important 2-norm can be calculated using both temporal and complex-frequency representations via [6]

$$
\|V(t)\|_2 \equiv \left\{ \int_{-\infty}^{\infty} V^2(t) dt \right\}^{\frac{1}{2}} = \left\{ \frac{1}{2\pi j} \int_{Br} \tilde{V}(s)\tilde{V}(-s) ds \right\}^{\frac{1}{2}} \tag{1.4}
$$

with V(t) real and where the Bromwich (Br) contour is parallel to the $j\omega$ axis in the common strip of convergence of $\tilde{V}(s)$ and $\tilde{V}(-s)$.

The norm of the transfer operator is defined by

$$
\|T(t) \circ\| \equiv \sup_{V^{(in)}(t) \ne 0} \frac{\left\| V^{(out)}(t) \right\|}{\left\| V^{(in)}(t) \right\|} \tag{1.5}
$$

where the type of norm is yet to be specified and where $V^{(in)}(t)$ is not identically zero, at least "almost everywhere" or measure sense depending on the type of norm. This definition as a least upper bound of course insures the inequality

$$\left\|V^{(out)}(t)\right\| \le \|T(t)\circ\| \left\|V^{(in)}(t)\right\| \tag{1.6}$$

which gives a bound on the norm of the response. A general result for the ∞-norm of a convolution operation [6] is

$$\|T(t)\circ\|_\infty = \|T(t)\|_1 \equiv \int_{-\infty}^{\infty} |T(t)| dt \tag{1.7}$$

so that this norm can be evaluated in terms of the temporal function T(t) (as distinguished from the convolution operator). For the 2-norm we have [6]

$$\begin{aligned} \|T(t)\circ\|_2 &= \sup_{-\infty<\omega<\infty} \left|\tilde{T}(j\omega)\right| \equiv |T(j\omega)|_{\max} \\ &\equiv |T(j\omega_{\max})| \end{aligned} \tag{1.8}$$

2. Filter Model of the System

As discussed in [3] one can represent our filter by

$$\tilde{T}(s) = T_\infty + \sum_\beta T_\beta \left[s - s_\beta\right]^{-1} \tag{2.1}$$

If our voltage is interpreted as a wave (combining voltage and current) with a common reference impedance (resistance, say 50 Ω) for both input and output, then passivity requires

$$\left|\tilde{T}(j\omega)\right| \le 1 \text{ for all } \omega \text{ (real)} \tag{2.2}$$

implying

$$\begin{gathered} -1 \le T_\infty \le 1 \quad (T_\infty \text{ real}) \\ \left|\mathrm{T}_\beta\right|\left[-\mathrm{Re}\left[s_\beta\right]\right]^{-1} \le 1 \text{ for all } \beta \\ \mathrm{Re}\left[s_\beta\right] < 0 \text{ for all } \beta \end{gathered} \tag{2.3}$$

where, of course, the poles come in complex-conjugate pairs so that T(t) is real valued. Also we have (causality)

$$T(t) = 0 \quad \text{for } t < 0 \tag{2.4}$$

For present purposes let us take as our canonical filter a single pole pair plus a constant term as

$$\begin{gathered} \tilde{T}(s) = T_\infty + T_f\left[s - s_f\right]^{-1} + T_f^*\left[s - s_f^*\right]^{-1} \\ s_f \equiv \Omega_f + j\omega_f \,,\, \Omega_f < 0 \,,\, \omega_f > 0 \end{gathered} \tag{2.5}$$

Furthermore let us let the filter be highly resonant (high Q) given by

$$-\Omega_f << \omega_f \tag{2.6}$$

Here T_∞ can model the regions of ω for which $\tilde{T}(s)$ is small (like the valleys between the peaks (resonances)). There are usually many resonances in a real system. Here we take one at $s = s_f$ to study the effect of tuning (or lack of tuning) to a resonance.

Along the jω axis of the s plane the transfer function has a peak given by

$$\omega \equiv \omega_{\max} \simeq \omega_f$$

$$\tilde{T}(j\omega_f) = T_\infty + \frac{T_f}{-\Omega_f} + \frac{T_f^*}{j2\omega_f - \Omega_f} \simeq T_\infty + \frac{T_f}{-\Omega_f} \tag{2.7}$$

Let us assume that

$$\left|\frac{T_f}{-\Omega_f}\right| >> |T_\infty| \tag{2.8}$$

so that we have a significant resonance peak. So let us define

$$P \equiv \frac{1}{-\Omega_f}\frac{|T_f|}{|T_\infty|} >> 1 \tag{2.9}$$

giving

$$\frac{\left|\tilde{T}(j\omega_f)\right|}{|T_\infty|} \simeq P + 1 \simeq P \tag{2.10}$$

For present purposes we have the ∞-norm (for response peak) and 2-norm (proportional to square root of response energy). The ∞-norm is [3]

$$\begin{aligned}\|T(t)\circ\|_\infty &= \|T(t)\|_1 \\ &\simeq |T_\infty| + \frac{4}{\pi}\frac{|T_f|}{-\Omega_f}\end{aligned} \tag{2.11}$$

using the approximation in (2.6). Note that the 1-norm of the time-domain function

$$\begin{aligned}T(t) &= T_\infty\delta(t) + \left[T_f e^{s_f t} + T_f^* e^{s_f^* t}\right]u(t) \\ &= T_\infty\delta(t) + 2e^{\Omega_f t}\left|T_f\right|\cos\left(\omega_f t + \arg\left(T_f\right)\right)u(t)\end{aligned} \tag{2.12}$$

$$\|T(t)\|_1 \equiv \int_{0_-}^{\infty} |T(t)|\,dt$$

is used. The 2-norm is [3,6]

$$
\begin{aligned}
\|T(t)\circ\|_2 &= \left|\widetilde{T}(j\omega)\right|_{\max} = \left|\widetilde{T}(j\omega_{\max})\right| \\
&\simeq \left|\widetilde{T}(j\omega_f)\right| \simeq \frac{\left|T_f\right|}{-\Omega_f} = \left|T_\infty\right| P
\end{aligned}
\tag{2.13}
$$

3. Excitation 1: Damped Sinusoid

The first excitation waveform is taken as a damped sinusoid of the form

$$V_1^{(in)}(t) = \left[V_1 e^{s_1 t} + V_1^* e^{s_1^* t} \right] u(t)$$

$$\tilde{V}_1^{(in)}(s) = \frac{V_1}{s - s_1} + \frac{V_1^*}{s - s_1^*} \tag{3.1}$$

$$s_1 \equiv \Omega_1 + j\omega_1 \,, \Omega_1 \le 0 \,, \omega_1 > 0$$

Here we are concerned with the case

$$|\Omega_1| << \omega_1 \text{ (highly resonant)}$$

for which the norms under consideration are [3]

$$\left\| V_1^{(in)}(t) \right\|_\infty \simeq 2|V_1|$$

$$\left\| V_1^{(in)}(t) \right\|_2 = \left\{ -\mathrm{Re}\left[\frac{V_1^2}{s_1} \right] + \frac{|V_1|^2}{-\Omega_1} \right\}^{\frac{1}{2}} \tag{3.2}$$

$$\simeq |V_1|^2 [-\Omega_1]^{-\frac{1}{2}}$$

The ∞-norm is seen easily from (3.1). The 2-norm is found from (1.4) combined with contour deformation in the s plane to find the appropriate residues of $\tilde{V}^{(in)}(s)\tilde{V}^{(in)}(-s)$.

4. Excitation 2: Decaying Exponential

The second excitation waveform is taken as a decaying exponential of the form

$$
\begin{aligned}
&V_2^{(in)}(t) = V_2 e^{\Omega_2 t} u(t)\,,\, V_2 \text{ real} \\
&\tilde{V}_2^{(in)}(s) = V_2 [s - \Omega_2]^{-1} \\
&\qquad s_2 \equiv \Omega_2 \leq 0
\end{aligned}
\tag{4.1}
$$

This can model a narrow pulse ($|\Omega_2|$ large) or even a step input (for $\Omega_2 \to 0$. The norms under consideration are simply evaluated to give

$$
\begin{aligned}
&\left\| V_2^{(in)}(t) \right\|_\infty = |V_2| \\
&\left\| V_2^{(in)}(t) \right\|_2 = |V_2| [-2\Omega_2]^{-\frac{1}{2}}
\end{aligned}
\tag{4.2}
$$

5. Response 1

As developed in [3] using the residue theorem, the response to the first excitation is

$$
\begin{aligned}
V_1^{(out)}(t) &= \left[\widetilde{T}(s_1)V_1 e^{s_1 t} + T\left(s_1^*\right)e^{s_1^* t}\right]u(t) && \text{(excitation term)}\\
&\quad + \left[\widetilde{V}_1^{(in)}\left(s_f\right)T_f e^{s_f t} + \widetilde{V}_1^{(in)}\left(s_f^*\right)T_f^* e^{s_f^*}\right]u(t) && \text{(filter term)}\\
\widetilde{V}_1^{(out)}(s) &= \widetilde{T}(s_1)V_1[s-s_1]^{-1} + \widetilde{T}\left(s_1^*\right)V_1^*[s-s_1]^{-1} && \text{(excitation term)}\\
&\quad + \widetilde{V}_1^{(in)}\left(s_f\right)T_f\left[s-s_f\right]^{-1} + \widetilde{V}_1^{(in)}\left(s_f^*\right)T_f^*\left[s-s_f\right]^{-1} && \text{(filter term)}
\end{aligned}
\tag{5.1}
$$

where the terms are separated according to the poles of the excitation and those of the filter. Note that the above form assumes $s_1 \neq s_f$; otherwise second-order poles appear which can also be readily handled [6].

5.1 ∞-norm

Now let ω_1 be far from ω_f (i.e., excitation completely detuned from filter) with

$$
\left|\omega_1 - \omega_f\right| >> |\Omega_1|, \left|\Omega_f\right| \tag{5.2}
$$

Then we have

$$
\left\|V_1^{(out)}(t)\right\|_\infty \simeq 2\left|\widetilde{T}(s_1)\right||V_1| + 2\left|T_f\right|\left|\widetilde{V}_1^{(in)}\left(s_f\right)\right| \tag{5.3}
$$

Additionally constraining

$$
\begin{aligned}
|T_\infty| &>> \frac{\left|T_f\right|}{\left|\omega_1 - \omega_f\right|}\\
\left|\omega_1 - \omega_f\right| &>> \frac{T_f}{T_\infty} = \left|\Omega_f\right| P
\end{aligned}
\tag{5.4}
$$

(which is more severe than (5.2) since $P >> 1$) then we have

$$
\begin{aligned}
\left\|V_1^{(out)}(t)\right\|_\infty &\simeq 2|T_\infty||V_1| + 2\frac{\left|T_f\right|}{\left|\omega_1 - \omega_f\right|}|V_1|\\
&\simeq 2|T_\infty||V_1|
\end{aligned}
\tag{5.5}
$$

As one should expect, if the excitation frequency is far from the filter resonance, then only the constant term describing the filter away from the resonance is relevant.

Comparing the response to the excitation in ∞-norm sense we have

$$R_{1,0}^{(\infty)} \equiv \frac{\left\| V_1^{(out)}(t) \right\|_\infty}{\left\| V_1^{(in)}(t) \right\|_\infty} \simeq |T_\infty| \text{ for } \left|\omega_f - \omega_1\right| \text{ sufficiently large} \tag{5.6}$$

The subscript 0 is used to designate this detuned case.

Now consider the case that the excitation is closely matched to the filter so that

$$\omega_1 = \omega_f \tag{5.7}$$

with still some flexibility concerning the damping constants. In this case we have the close approach of two poles discussed in [6]. Keeping only the dominant terms in (5.1), and beginning with the s-plane form, we have

$$\begin{aligned}
\tilde{V}_1^{(out)}(s) &\simeq T_f V_1 \left[s - s_f\right]^{-1}\left[s - s_1\right]^{-1} + T_f^* V_1^* \left[s - s_f^*\right]^{-1}\left[s - s_1^*\right]^{-1} \\
V_1^{(out)}(t) &\simeq \left[T_f V_1 \frac{e^{s_f t} - e^{s_1 t}}{s_1 - s_f} _ T_f^* V_1 \frac{e^{s_f^* t} - e^{s_1^* t}}{s_1^* - s_f^*} \right] u(t) \\
&= \frac{e^{\Omega_f t} - e^{\Omega_1 t}}{\Omega_1 - \Omega_f}\left[T_f V_1 e^{j\omega_f t} + T_f^* V_1^* e^{-j\omega_f t}\right] u(t)
\end{aligned} \tag{5.8}$$

This last form consists of an envelope function times a rapidly varying oscillatory function, giving an ∞-norm (peak) as

$$\left\| V_1^{(out)}(t) \right\|_\infty \simeq \sup_{0 \le t < \infty} 2\left|T_f\right|\left|V_1\right| \left| \frac{e^{\Omega_f t} - e^{\Omega_1 t}}{\Omega_1 - \Omega_f} \right| \tag{5.9}$$

The peak of this envelope function is discussed in [2,7]. Considering the filter parameters as fixed this peak is maximized as

$$\begin{aligned}
&\Omega_1 = 0 \\
&\left\| V_1^{(out)}(t) \right\|_\infty \simeq \frac{2}{-\Omega_f}\left|T_f\right|\left|V_1\right|
\end{aligned} \tag{5.10}$$

Comparing the matched response to the excitation in ∞-norm sense gives

$$R_{1,m}^{(\infty)} \equiv \frac{\left\| V_1^{(out)}(t) \right\|_\infty}{\left\| V_1^{(in)}(t) \right\|_\infty} \simeq \frac{|T_f|}{-\Omega_f} = |T_\infty| P \quad \text{for } \omega_1 = \omega_f \text{ , } \Omega_1 = 0 \tag{5.11}$$

The subscript m is used to designate this matched case, where matched is defined as above. Comparing this result to the ∞-norm of the transfer operator in (2.11), one can note that for large P the result in (5.11) is lower by a factor of $\pi/4$. The difference is accounted for by the use of a sine-wave instead of square-wave excitation [4].

So for the ∞-norm the response for excitation matched to the filter is considerably greater than that for the response far from resonance by the factor

$$\frac{\left\| V_1^{(out)}(t) \right\|_\infty \Big|_{s_1 = j\omega_f}}{\left\| V_1^{(out)}(t) \right\|_\infty \Big|_{s_1 \text{ far from } s_f}} = \frac{R_{1,0}^{(\infty)}}{R_{1,m}^{(\infty)}} \simeq \frac{1}{-\Omega_f} \frac{|T_f|}{|T_\infty|} = P \tag{5.12}$$

This is as one would expect.

5.2 2-norm

As discussed in [3] the square of the 2-norm of the response is

$$\begin{aligned} \left\| V_1^{(out)}(t) \right\|_2^2 &= 2\,\mathrm{Re}\left[\tilde{T}(s_1) V_1 \tilde{T}(-s_1) \tilde{V}_1^{(in)}(-s_1)\right] \quad \text{(excitation term)} \\ &+ 2\,\mathrm{Re}\left[\tilde{V}_1^{(in)}(s_f) T_f \tilde{T}(-s_f) \tilde{V}_1^{(in)}(-s_f)\right] \quad \text{(filter term)} \end{aligned} \tag{5.13}$$

with the terms again separated according to the poles of the excitation and those of the filter.

For ω_1 far from ω_f in the sense of (5.2) and (5.4), we have for the excitation term

$$\begin{aligned} V_1^{(in)}(-s_1) &\simeq \frac{V_1^*}{-2\Omega_1} \\ \tilde{T}(s_1) &\simeq \tilde{T}(-s_1) \simeq T_\infty \end{aligned} \tag{5.14}$$

The filter term meanwhile has factors

$$\widetilde{V}_1^{(in)}(s_f) \simeq -j\left[\frac{V_1}{\omega_f - \omega_1} + \frac{V_1^*}{\omega_f + \omega_1}\right] \simeq \widetilde{V}_1^{(in)^*}(-s_f)$$
$$\widetilde{T}(-s_f) \simeq \frac{T_f^*}{-2\Omega_f} \tag{5.15}$$

giving

$$\left\|V_1^{(out)}(t)\right\|_2^2 \simeq \frac{T_\infty^2}{-\Omega_1}|V_1|^2 + \frac{|T_f|^2}{-\Omega_f}\left|\frac{V_1}{\omega_f - \omega_1} + \frac{V_1^*}{\omega_f + \omega_1}\right|^2 \tag{5.16}$$

Letting $|\Omega_1|$ be of the order of $|\Omega_f|$ or smaller, then (5.4) gives

$$\left\|V_1^{(out)}(t)\right\|_2 \simeq |T_\infty|[-\Omega_1]^{-\frac{1}{2}}|V_1| \tag{5.17}$$

Again only the constant term describing the filter away from the resonance is relevant.

Comparing the response to the excitation in 2-norm sense gives

$$R_{1,0}^{(2)} \equiv \frac{\left\|V_1^{(out)}(t)\right\|_2}{\left\|V_1^{(in)}(t)\right\|_2} \simeq |T_\infty| \text{ for } |\omega_f - \omega_1| \text{ sufficiently large} \tag{5.18}$$

just like the ∞-norm case in (5.6).

Next let the excitation be closely matched to the filter as in (5.7). As worked out in [3] with

$$\omega_1, \omega_f >> |\Omega_1|, |\Omega_2| \tag{5.19}$$

we have

$$\left\|\widetilde{V}_1^{(out)}(t)\right\|_2 \simeq \frac{|T_f|}{-\Omega_f - \Omega_1}\left[\frac{1}{-\Omega_1} + \frac{1}{-\Omega_f}\right]^{\frac{1}{2}}|V_1| \tag{5.20}$$

Comparing the matched response to the excitation in 2-norm sense gives

$$R_{1,m}^{(2)} \equiv \frac{\left\|V_1^{(out)}(t)\right\|_2}{\left\|V_1^{(in)}(t)\right\|_2} \simeq \frac{|T_f|}{-\Omega_f - \Omega_1}\left[1+\frac{\Omega_1}{\Omega_f}\right]^{\frac{1}{2}}$$

$$\rightarrow \frac{|T_f|}{-\Omega_f} = |T_\infty| P \quad \text{as } \Omega_1 \rightarrow 0 \tag{5.21}$$

$$\text{for } \omega_1 = \omega_f$$

Again this result is just like the ∞-norm case in (5.11). It also is equal to the 2-norm of the transfer operator as given in (2.13).

Again, now in 2-norm sense, the response for excitation matched to the filter exceeds that for the response far from resonance by the factor

$$\frac{\left.\left\|V_1^{(out)}(t)\right\|_2\right|_{s_1 = j\omega_f}}{\left.\left\|V_1^{(out)}(t)\right\|_2\right|_{s_1 \text{ far from } s_f}} = \frac{R_{1,m}^{(2)}}{R_{1,0}^{(2)}} \simeq \frac{1}{-\Omega_f}\frac{|T_f|}{|T_\infty|} = P \tag{5.22}$$

This is the same result as in (5.12) so in the senses of both norms there is an enhancement of a factor P in matching the excitation frequency ω_1 to the filter.

6. Response 2

Using the results of [3] the response to the second excitation is

$$
\begin{aligned}
V_2^{(out)}(t) &= \tilde{T}(\Omega_2)V_2 e^{\Omega_2 t}u(t) && \text{(excitation term)} \\
&\quad + \left[\tilde{V}_2^{(in)}(s_f)T_f e^{s_f t} + \tilde{V}_2^{(in)}(s_f^*)T_f^* e^{s_f^* t}\right]u(t) && \text{(filter term)} \\
&: \\
\tilde{V}_2^{(out)}(s) &= \tilde{T}(\Omega_2)V_2[s-\Omega_2]^{-1} && \text{(excitation term)} \\
&\quad + \tilde{V}_2^{(in)}(s_f)T_f[s-s_f]^{-1} + \tilde{V}_2^{(in)}(s_f^*)T_f^*[s-s_f]^{-1} && \text{(filter term)}
\end{aligned}
\tag{6.1}
$$

Again the terms are separated according to the poles of the excitation and those of the filter.

6.1 ∞-norm

With the filter resonance as in (2.6), and (like (5.4)) the constraint

$$
|T_\infty| >> \frac{|T_f|}{\omega_f}
\tag{6.2}
$$

we have

$$
\begin{aligned}
V_2^{(in)}(t) &\simeq T_\infty V_2 e^{\Omega_2 t}u(t) && \text{(excitation term)} \\
&\quad + 2\,\mathrm{Re}\left[V_2\left[j\omega_f - \Omega_2\right]^{-1} T_f e^{j\omega_f t}\right] e^{\Omega_f t}u(t) && \text{(response term)}
\end{aligned}
\tag{6.3}
$$

Noting that the response term consists of an envelope function times a rapidly varying oscillatory function, the ∞-norm is

$$
\begin{aligned}
\left\| V_2^{(out)}(t) \right\|_\infty &\simeq \sup_{0 \le t < \infty} \left[|T_\infty||V_2| e^{\Omega_2 t} + 2\frac{|T_f|}{|j\omega_f - \Omega_2|}|V_2| e^{\Omega_f t} \right] \\
&\simeq |T_\infty||V_2|
\end{aligned}
\tag{6.4}
$$

If (6.2) is relaxed then the relative sizes of Ω_2 and ω_1 need to be considered.

Comparing the response to the excitation in ∞-norm sense we have

$$
R_2^{(\infty)} \equiv \frac{\left\| V_2^{(out)}(t) \right\|_\infty}{\left\| V_2^{(in)}(t) \right\|_\infty} \simeq |T_\infty|
\tag{6.5}
$$

6.2 2-norm

Again using the results of [3] the square of the 2-norm of the response is

$$\left\|V_2^{(out)}(t)\right\|_2^2 = \widetilde{T}(\Omega_2)V_2\widetilde{T}(-\Omega_2)\widetilde{V}_2^{(in)}(-\Omega_2) \qquad \text{(excitation term)}$$
$$+2\,\mathrm{Re}\left[\widetilde{V}_2^{(in)}\left(s_f\right)T_f\widetilde{T}(-s_f)\widetilde{V}_2^{(in)}\left(-s_f\right)\right] \qquad \text{(filter term)} \tag{6.6}$$

With the restrictions the same as before we have for the excitation term

$$\widetilde{V}_2^{(in)}(-\Omega_2) = \frac{V_2}{-2\Omega_2}$$
$$\widetilde{T}(\Omega_2) \simeq T_\infty \simeq \widetilde{T}(-\Omega_2) \tag{6.7}$$

and for the filter term

$$\widetilde{V}_2^{(in)}\left(s_f\right) \simeq \frac{V_2}{j\omega_f - \Omega_2} \simeq \widetilde{V}_2^{(in)^*}\left(-s_f\right)$$
$$\widetilde{T}(-s_f) \simeq \frac{T_f^*}{-2\Omega_f} \tag{6.8}$$

giving

$$\left\|V_2^{(out)}(t)\right\|_2^2 \simeq T_\infty^2\left[-2\Omega_2\right]^{-1}V_2^2 + \left|T_f\right|^2\left|j\omega_f - \Omega_2\right|^{-2}\left[-\Omega_f\right]^{-1}V_2^2 \tag{6.9}$$

Comparing the response to the excitation in 2-norm sense gives

$$R_2^{(2)} \equiv \frac{\left\|V_1^{(out)}(t)\right\|_2}{\left\|V_1^{(in)}(t)\right\|_2}$$
$$\simeq \left|T_\infty\right|\left\{1 + 2\frac{\Omega_2}{\Omega_f}\left|j\omega_f - \Omega_2\right|^{-2}\frac{\left|T_f\right|^2}{T_\infty^2}\right\}^{\frac{1}{2}}$$
$$= \left|T_\infty\right|\left\{1 + 2\frac{\Omega_2\Omega_f}{\left|j\omega_f - \Omega_2\right|^2}P^2\right\}^{\frac{1}{2}} \tag{6.10}$$
$$\to \left|T_\infty\right| \quad \text{for } \Omega_2 \to 0 \quad \text{or } \Omega_2 \to -\infty$$

which is like the ∞-norm in (6.5). Note that for a fixed s_f characterizing the filter there can be some increase in this relative response by appropriate choice of Ω_2, giving

$$\Omega_2 = -\omega_f$$

$$\begin{aligned} R_2^{(2)} &= |T_\infty| \left\{ 1 + \frac{-\Omega_f}{\omega_f} p^2 \right\}^{\frac{1}{2}} \\ &= \left\{ T_\infty^2 + \frac{|T_f|^2}{\omega_f(-\Omega_f)} \right\}^{\frac{1}{2}} \end{aligned} \tag{6.11}$$

Whether or not this is significant depends on the relative size of the inequalities in (2.9) and (6.2).

7. Comparison of the Responses to the Two Excitation Waveforms

If the two excitation waveforms are taken to have the same norm, then one can compare the responses in the same norm sense to see which waveform produces the larger response (i.e., is more effective).

First, compare waveform 1 (damped sinusoid) for ω_1 far from ω_f (i.e., detuned) to waveform 2. For the ∞-norm (peak), (5.6) and (6.5) give

$$\frac{R_{1,0}^{(\infty)}}{R_2^{(\infty)}} \simeq 1 \tag{7.1}$$

so that both waveforms are equally effective here. For the 2-norm (5.18) and (6.10) give

$$\begin{aligned} \frac{R_{1,0}^{(2)}}{R_2^{(2)}} &\simeq \left\{ 1 + 2\frac{\Omega_2 \Omega_f}{\left| j\omega_f - \Omega_2 \right|^2} p^2 \right\}^{-\frac{1}{2}} \\ &\to 1 \ \text{ for } \Omega_2 \to 0 \text{ or } \Omega_2 \to -\infty \end{aligned} \tag{7.2}$$

If the width of the decaying exponential (waveform 2) is selected such that $-\Omega_2 \simeq \omega_f$, this latter waveform can be a little more effective than the damped sinusoid (detuned) depending on the size of $p^2 \Omega_f / \omega_f$ compared to unity (as in (6.11)).

Now let waveform 1 be closely tuned (matched) to the filter ($s_1 \simeq j\omega_f$). For the ∞-norm (5.11) and (6.5) give

$$\frac{R_{1,m}^{(\infty)}}{R_2^{(\infty)}} \simeq P >> 1 \tag{7.3}$$

with the expected result that the oscillatory waveform is much more effective. For the 2-norm (5.21) and (6.10) give

$$\begin{aligned} \frac{R_{1,m}^{(2)}}{R_2^{(2)}} &\simeq P \left\{ 1 + \frac{2\Omega_e \Omega_f}{\left| j\omega_f - \Omega_2 \right|^2} p^2 \right\}^{-\frac{1}{2}} \\ &\to P(>> 1) \ \text{ for } \Omega_2 \to 0 \ \text{ or } \ \Omega_2 \to -\infty \end{aligned} \tag{7.4}$$

Note that if Ω_2 is optimally chosen as in (6.11), then since $\left|\Omega_f\right| << \omega_f$ the ratio in (7.4) is still large compared to unity.

8. Concluding Remarks

In the senses of the ∞-norm (peak) and 2-norm (proportional to the square root of energy), then a damped sinusoidal waveform in general produces a response which is much larger than that due to a decaying exponential waveform, if the oscillatory waveform is tuned to a resonance peak of the system. If the damped sinusoid is detuned from the resonance, then the responses are comparable. For maximizing the response then an oscillatory waveform is in general preferable.

The present choices of waveforms are, of course, not exhaustive. Another interesting kind of waveform is a chirp in which the frequency is sweeped (sufficiently slowly) through the resonance of interest, thereby making the response less sensitive to accurate knowledge of the resonance frequency. The present comparisons are on the basis of comparable peaks or energies in the two incident waves, but there are other factors in the production of such waveforms which may also need to be considered.

The present discussion has been in the context of a simple kind of filter function which makes comparison of tuned and detuned responses relatively easy. Multiple resonances in the filter function can also be considered, but at a significant increase in complexity.

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