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THE THEORY OF ELECTROMAGNETIC INTERFERENCE CONTROL

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ABSTRACT

In order to control electromagnetic interference in complex electronic systems one needs some way to organize the problem such that it is tractable. Electromagnetic topology allows one to identify (or construct) a set of surfaces which limit the penetration of this interference. It then organizes the interaction equations in a way which identifies a set of controlling variables at the subshields. Introducing norms one can bound the response in terms of transfer functions and special nonlinear protection devices at the subshields.

1. INTRODUCTION

In considering the problem of electromagnetic interaction with electronic systems one is often faced with an overwhelming problem of complexity. In dealing with a wide variety of frequencies and even transient waveforms for the incident electromagnetic fields one is often concerned with paths of electromagnetic propagation into the system that are unintentional, i.e., are not designed to be there to transmit or receive desired signals. As a result one may be confronted with hundreds or thousands of potential signal penetration paths into the system. As a practical matter many of these paths are not identified in an a priori sense, the number of possibilities being so large.

So the question is: What can one do to make this problem of electromagnetic interference control more tractable? One would like to have some way to dramatically reduce the number of things (variables) requiring attention. These variables should be controlling variables in the sense that appropriate choice of the values of these controls all the signal levels of concern propagating into the system. Note that such signals need not be made to be of precise amplitudes, but only be bounded as being less than some values chosen in appropriate senses.

Such a concept is that of electromagnetic topology introduced in [1]. One might also consider EM topology as a generalized shielding theory. It is based on the concept that EM fields (and voltages and currents) in a volume are determined uniquely (for passive, linear media) by the boundary values on the closed surface surrounding this volume [38]. One can find some of the beginnings of this over a century ago in the context of protection against lightning [30].

In this paper the basic concepts of EM topology and the associated use of norms for bounding are summarized. First there is the qualitative (or descriptive) topology involving the appropriate division of space into volumes and boundary surfaces. This topology is used to organize the equations of signal propagation (the BLT equation). This leads to the good-shielding approximation which is in turn simplified in a bounding sense by the use of norms. The important parameters concern the transmission of signals through the penetrations at subshields, involving both linear and nonlinear protection elements. The responses at the various penetrations are combined to give an overall bound for signals in the associated volume (sublayer).

2. QUALITATIVE EM TOPOLOTY

In qualitative EM topology three-dimensional Euclidean space is divided into a set of volumes. Advancing to a hierarchical EM topology think of a set of nested surfaces (shields) dividing space into a set of layers with layer index λ

$$\lambda = 1, 2, \cdots, \lambda_{max}$$

$$\lambda = \begin{cases} 1 \text{ for outside layer} \\ \lambda_{max} \text{ for innermost layer} \end{cases} \tag{2.1}$$

$$V_\lambda = \lambda\text{th layer}$$

$$S_{\lambda;\,\lambda+1} \equiv \text{shield separating } V_\lambda \text{ and } V_{\lambda+1}$$

This is next generalized by the concepts of sublayers and subshields as indicated in fig. 2.1A. Here one of the layers ($\lambda = 3$ in the example) is divided into separated volumes (sublayers) with external subshield (proper subshield) boundaries, this process is continued as much as desired. The sublayer index is taken as ℓ with λ and ℓ indices now appearing on the topological entities. One can also carry the division to the level of elementary volumes and surfaces by further arbitrary division of sublayers, but this aspect will not be considered here [7, 21].

Corresponding to the EM topology there is a dual graph: the interaction sequence diagram. As indicated in fig. 2.1B this keeps track of the signal transport from one layer to another through the intervening subshields. At the sublayer level of decomposition this graph is a tree which means that the path from one sublayer to another is unique. Note that in the graph there are symbols for both sublayers and subshields which is useful later when we need the scattering matrices for both these entities. As indicated there are waves travelling in the "1" and "2" directions on each edge of the graph. Conceptually in each sublayer one can think of coupling ($\mu = 1$) for sources in the sublayer, propagation ($\mu = 2$) through the sublayer, and penetration ($\mu = 3$) to the next sublayer(s) [3, 21, 40].

From the interaction sequence diagram it is easy to see that one can relabel sublayers so that any particular one is "outside". This corresponds to inversion of coordinates about an origin within that particular sublayer. With this in mind the later use of the good-shielding approximation will apply to signal transport from any sublayer to any other sublayer. This points out an important use of the sublayer concept in that it applies not only to protection from external sources, but from

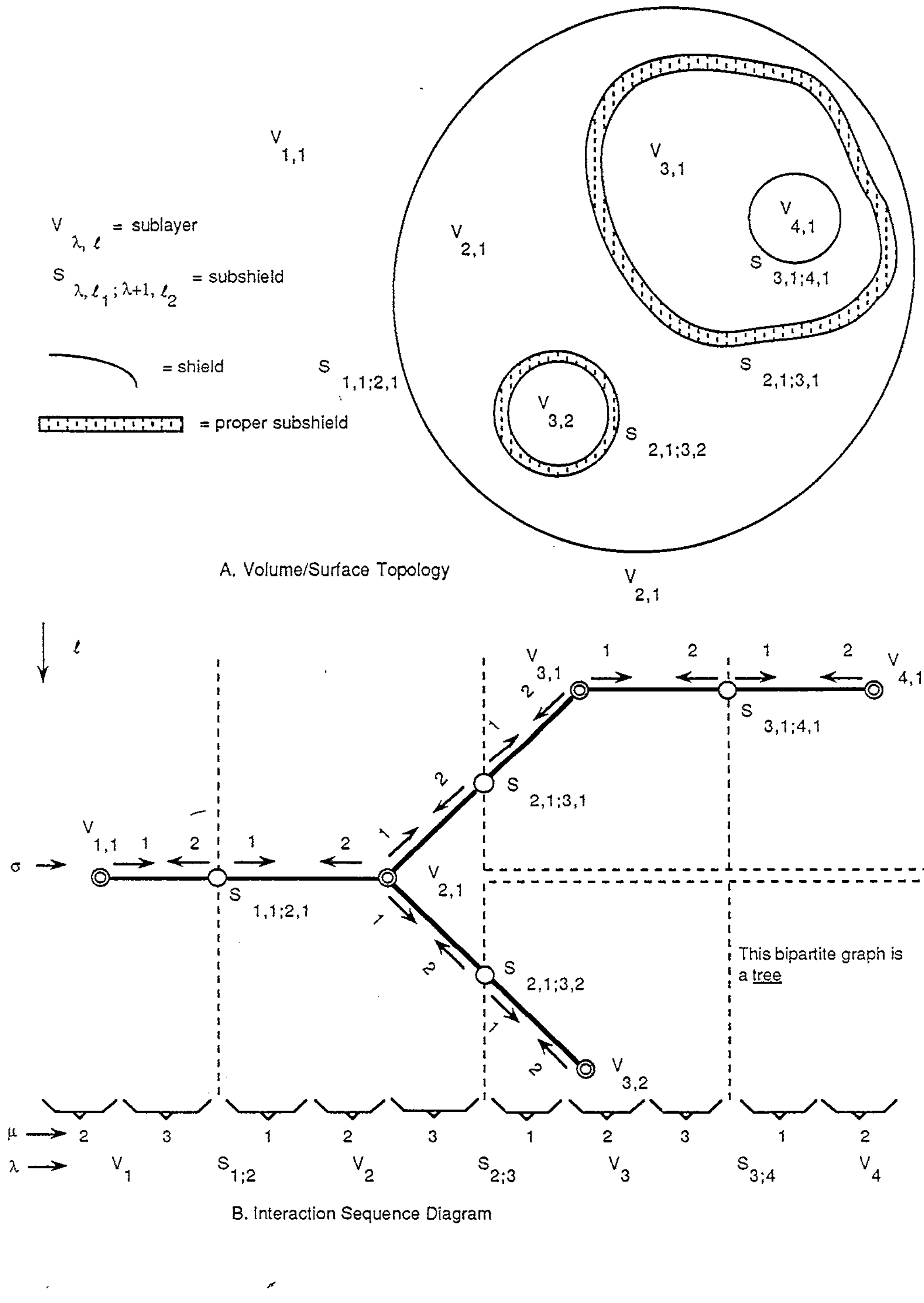

Fig. 2.1. Sublayers and Subshields in Hierarchical Topology.

internal ones as well. For qualitative EM topology one can define a relative shielding order as [9]

$$R_{\lambda_1,\ell_1;\lambda_2,\ell_2} \equiv \sum_{P_{\lambda_1,\ell_1;\lambda_2,\ell_2}} 1 = \text{positive integer}$$

$$\equiv \text{relative shielding order between } V_{\lambda_1,\ell_1} \text{ and } V_{\lambda_2,\ell_2} \tag{2.2}$$

$$P_{\lambda_1,\ell_1;\lambda_2,\ell_2} \equiv \text{path from } V_{\lambda_1,\ell_1} \text{ to } V_{\lambda_2,\ell_2}$$

$$R_{\lambda_1,\ell_1;\lambda_2,\ell_2} = R_{\lambda_2,\ell_2;\lambda_1,\ell_1} \quad \text{(path reversal symmetry)}$$

Defining a set of primary sublayers (as say those containing noise sources or equipment to be protected from interference), one can attempt to synthesize an EM topology with a desired set of relative shielding orders (a symmetric matrix of non-negative integers). This leads to some interesting results such as the uniform-relative-shielding-order theorem which states that if we have three or more distinct primary sublayers with uniform relative shielding order R among all pairs of these, then R = even and the smallest non-trivial R = 2. These results are generalized in [26] to various relative-shielding-order matrices and the various possible EM topologies of this type are exhibited up to a maximum relative shielding order (R_{max}) of 5.

One can also find implications of the topology of subshields relevant to low-frequency shielding [10]. Here the issue is the single or multiple connectedness of the surface, single connected subshields being better under these circumstances. The definition of proper grounding schemes is also formalized by EM topology [6, 10, 23].

3. BLT EQUATION AND GOOD-SHIELDING APPROXIMATION

The original form of the BLT equation for transmission-line networks includes delay terms for propagation along the tubes (multiconductor transmission lines) as well as distributed sources along the tubes [5]. In applying this the tube lengths are shrunk to zero and now represent the transmission of signals from one volume to another. The junctions (e.g., electronic boxes) now become the volumes (in N-port representation). The BLT equation for EM topology becomes [7, 21].

$$\left(\left(\tilde{I}_{n,m}(s)\right)_{u,v}\right) \odot \left(\left(\tilde{V}_n(s)\right)_u\right) = \left(\left(\tilde{V}_n^{(s)}(s)\right)_u\right)$$

$$\left(\left(\tilde{I}_{n,m}(s)\right)_{u,v}\right) = \left(\left(1_{n,m}\right)_{u,v}\right) - \left(\left(\tilde{S}_{n,m}(s)\right)_{u,v}\right)$$

$$\equiv \text{interaction supermatrix}$$

$$\left(\left(\tilde{S}_{n,m}(s)\right)_{u,v}\right) \equiv \text{scattering supermatrix}$$

$$\left(\left(\tilde{V}_n^{(s)}(s)\right)_u\right) \equiv \left(\left(S_{n,m}(s)\right)_{u,v}\right) \odot \left(\left(\tilde{V}_{s_n}(s)\right)_u\right) \tag{3.1}$$

$\equiv$ equivalent source supervector

The combined voltage is a linear combination of voltage and current as

$$\left(\left(\tilde{V}_n(s)\right)u\right) = \left(\left(\tilde{V}_n^{(0)}(s)\right)_u\right) + \left(\left(\tilde{Z}_{n,m}(s)\right)_{u,v}\right) \odot \left(\left(\tilde{I}_n^{(0)}(s)\right)_u\right) \tag{3.2}$$

$\equiv$ combined voltage supervector

where the voltage and current supervectors are combined by a convenient impedance supermatrix. As discussed in [15] a useful and convenient choice is

$$\left(\left(\tilde{Z}_{n,m}(s)\right)_{u,v}\right) \equiv Z\left(\left(1_{n,m}\right)_{u,v}\right)$$

$$\left(\left(1_{n,m}\right)_{u,v}\right) \equiv \text{identity supermatrix} \tag{3.3}$$

$$Z \equiv \text{constant resistance} > 0$$

where Z may be chosen based on some typical impedance in the system (e.g. 50 Ω). Note that currents are taken as positive in the wave directions (as in fig. 2.1B) so that there are two directions on each graphical edge for the two directions of propagation.

At this point note that supermatrices and supervectors are partitioned matrices and vectors (i.e. matrices of matrices, etc.). This partitioning can be accomplished to any number of levels of partition. For this purpose we have [21]

$\odot$ $\equiv$ generalized dot product (contraction)

$$\left(\left(\cdots\left(A_{n_\nu,m_\nu}\right)_{n_{\nu-1},m_{\nu-1}}\cdots\right)_{n_1,m_1}\right) \odot \left(\left(\cdots\left(B_{n_\nu,m_\nu}\right)_{n_{\nu-1},m_{\nu-1}}\cdots\right)_{n_1,m_1}\right)$$

$$= \left(\left(\cdots\left(C_{n_\nu,m_\nu}\right)_{n_{\nu-1},m_{\nu-1}}\cdots\right)_{n_1,m_1}\right) \tag{3.4}$$

$$C_{n_\nu,m_\nu;\cdots;n_1,m_1} = \sum_{n_1'=1}^{N_1}\cdots\sum_{n_\nu'=1}^{N_\nu(n_1,n_2,\cdots,n_{\nu-1})} A_{n_\nu,n_\nu';\cdots;n_1,n_1'}B_{n_\nu',m_\nu;\cdots;n_1',m_1}$$

which is just the successive application of the dot product to second indices of the blocks of the first supermatrix and first indices of blocks of the second supermatrix. For this to be meaningful the columns of the first must be partitioned the same as rows of the second. For addition the partitioning must be the same for both matrices. Partitioning rows and columns the same way (for square matrices) gives what is termed symmetric compatible order. By simple extension the above applies to supervectors as well. As discussed in [21] one even can represent the supermatrix inverse in terms of the blocks resulting from the partitioning.

Note that the u, v indices (dummy variables) when in matrix or vector parentheses) represent the various topological indices, such as introduced in section 2. These have been introduced as [7, 21]

$$\lambda \equiv \text{layer index} \quad , \quad \mu = 1, 2, 3 \equiv \text{layer part index}$$

$$\ell = 1, 2, \cdots, \ell_{\text{max}}(\lambda) \equiv \text{sublayer index}$$

$$\tau = 1, 2, \cdots, \tau_{\text{max}}(\lambda, \ell) \equiv \text{elementary volume index (not used here)} \tag{3.5}$$

$$\sigma = 1, 2 \equiv \text{dual-wave index}$$

So that in general we might have five matrix partitions corresponding to these indices. Then we have

$$u = \{\sigma, \tau, \ell, \mu, \lambda\} \equiv \text{topological index set} \tag{3.6}$$

Concentrating our attention on the layer index write

$$w = \{\sigma, \tau, \ell, \mu\} \tag{3.7}$$

so the BLT equation is now written

$$\left(\left(\left(\tilde{I}_{n,m}(s)\right)_{w,w'}\right)_{\lambda,\lambda'}\right) \odot \left(\left(\left(\tilde{V}_n(s)\right)_w\right)_\lambda\right) = \left(\left(\left(\tilde{V}_n^{(s)}(s)\right)_w\right)_\lambda\right) \tag{3.8}$$

To obtain an approximate solution of the BLT equation note the interaction supermatrix is block tridiagonal, i.e., the topology (as in fig. 2.1) does not allow propagation directly from λ to λ +2 (or greater) without first going through λ + 1. Then let us assume that the off-diagonal blocks (corresponding to signal passage through shields) are small (or otherwise the shields are not very good at attenuating signals which is not very useful). Furthermore let

$$\left(\left(\tilde{V}_n^{(s)}(s)\right)_w\right)_\lambda = \left((0_n)_w\right) \text{ for } \lambda \neq 1 \tag{3.9}$$

(all sources on outside)

Then by Gaussian elimination with matrix coefficients and vector unknowns one can obtain the good-shielding approximation [7, 21]

$$\left(\left(\tilde{V}_n^{(s)}(s)\right)_w\right)_\lambda \quad \left\{\bigodot_{L=0}^{\lambda-2}\left[\left(\left(\tilde{S}_{n,m}(s)\right)_{w,w'}\right)_{\lambda-L,\,\lambda-1-L}\right]\right\} \odot \left(\left(\tilde{V}_n^{(s)}(s)\right)_w\right)_1 \tag{3.10}$$

This form requires that the penetrations have an impedance as in (3.3) (i.e. terminated) so that "reflections" (a special generalized sense) do not occur in the sublayers (which are also assumed passive). Other forms of (3.10) are also possible (involving inverse of diagonal blocks of the interaction matrix in the (λ - L, λ - L) position in the running product above) if the reflection condition is not met [7, 21].

The first benefit of the good-shielding approximation is the reduction in the matrix order to that of the blocks at λ level. Note now that the signals in the λth layer are represented by a product of matrices corresponding to the layers and shields in the topology, i.e. different factors correspond to different physical entities in the real system. Taking norms (special bounding procedure to be discussed later) gives a scalar equation as

$$\left\|\left(\left(\tilde{V}_n(s)\right)_w\right)_\lambda\right\| \le \left\{\prod_{L=0}^{\lambda-2}\left\|\left(\left(\tilde{S}_{n,m}(s)\right)_{w,w'}\right)_{\lambda-L,\,\lambda-L}\right\|\;\left\|\left(\left(\tilde{S}_{n,m}(s)\right)_{w,w'}\right)_{\lambda-L,\,\lambda-1-L}\right\|\right.$$
$$\left\|\left(\left(\tilde{V}_n^{(s)}(s)\right)_w\right)_1\right\| \tag{3.11}$$

In this form if we take s as jω this gives frequency-domain bounds. Including norms of convolution operators the above applies to time domain as well. One can often assume that the transmission through a layer is bounded in norm sense by 1 (especially for 2-norm [15]), in which case we have

$$\left\|\left(\left(\tilde{V}_n(s)\right)_w\right)_\lambda\right\| \le \left\{\prod_{L=0}^{\lambda-2}\left\|\left(\left(\tilde{S}_{n,m}(s)\right)_{w,w'}\right)_{\lambda-L,\lambda-1-L}\right\|\right\} \left\|\left(\left(\tilde{V}_n^{(s)}(s)\right)_w\right)_1\right\| \tag{3.12}$$

This is an ideal form to have the result in that the signals in the λth layer are bounded by the norm of the exterior excitation times a product of transmission factors, one for each shield. This is a logical quantitative form from which to define shielding effectiveness.

Since, by topological inversion, any sublayer can be transformed to the outside layer (an improper sublayer) these formulas can be applied to the attenuation of signals from any sublayer to any other sublayer. One merely consults the corresponding path in the interaction sequence diagram to find the subshields and corresponding subshield scattering matrices. The product of the norms of these gives a bound on the transmission between the two sublayers.

4. NORMS FOR BOUNDING RESPONSE

First consider vector norms, which are anything with the properties

$$\begin{aligned}
&\left\|(x_n)\right\| = 0 \ \textit{iff} \ (x_n) = (0_n) \\
&\left\|\alpha(x_n)\right\| = |\alpha| \left\|(x_n)\right\| \text{ , } \alpha \text{ a complex scalar} \\
&\left\|(x_n)\right\| + (y_n) \le \left\|(x_n)\right\| + (y_n) \\
&\left\|(x_n)\right\| \text{ depends continuously on } (x_n)
\end{aligned} \tag{4.1}$$

Matrix norms have the above properties with the addition of a product inequality

$$\left\|(A_{n,m}) \cdot (B_{n,m})\right\| \le \left\|(A_{n,m})\right\| \left\|(B_{n,m})\right\| \tag{4.2}$$

For our purposes matrix norms are defined as associated matrix norms with

$$\left\|(A_{n,m})\right\| \equiv \sup_{(x_n) \ne (0_n)} \frac{\left\|(A_{n,m}) \cdot (x_n)\right\|}{\left\|(x_n)\right\|} \tag{4.3}$$

so that the product inequality applies in a tight sense to matrix/vector products as well.

A commonly used norm is the p-norm which for N-component vectors is

$$\begin{aligned}
&\left\|(x_n)\right\|_p \equiv \left\{ \sum_{n=1}^{N} |x_n|^p \right\}^{\frac{1}{p}} \text{ for } p \ge 1 \\
&\left\|(x_n)\right\|_\infty = \max_n |x_n|
\end{aligned} \tag{4.4}$$

The associated matrix p-norms for special cases of interest are for N $\times$ M matrices

$$\begin{aligned}
&\left\|(A_{n,m})\right\|_1 = \max_{1 \le n \le N} \sum_{n=1}^{N} |A_{n,m}| \equiv \text{maximum column magnitude sum} \\
&\left\|(A_{n,m})\right\|_\infty = \max_{1 \le n \le N} \sum_{m=1}^{M} |A_{n,m}| \equiv \text{maximum row magnitude sum}
\end{aligned} \tag{4.5}$$

$$\left\|(A_{n,m})\right\|_2 = \left[\chi_{\max}\left((A_{n,m})^{\dagger} \cdot (A_{n,m})\right)\right]^{\frac{1}{2}}$$

$$\dagger = *T \quad \text{(conjugate transpose)}$$

with $\chi_{\max}$ as the maximum eigenvalue of the Hermitian matrix argument.

While the vector and matrix norms are appropriate for the frequency-domain parameters in the previous section, we need similar quantities for time-domain waveforms and associated convolution operators. The function norm is defined with the properties [28]

$$\|f(t)\| \begin{cases} = 0 & \textit{iff } f(t) \equiv 0 \text{ or has zero "measure"} \\ > 0 & \text{otherwise} \end{cases}$$

$$\|\alpha f(t)\| = |\alpha| \; \|f(t)\| \tag{4.6}$$

$$\|f_1(t) + f_2(t)\| \le \|f_1(t)\| + \|f_2(t)\|$$

Operator norms have these properties as well as

$$\|\Lambda(\Upsilon(\;))\| \le \|\Lambda(\;)\| \; \|\Upsilon(\;)\| \tag{4.7}$$

Again for our purposes we define these as associated operator norms with

$$\|\Lambda(\;)\| \equiv \sup_{\|f(t)\| \neq 0} \frac{\|\Lambda(f(t))\|}{\|f(t)\|} \tag{4.8}$$

The p-norm for time-domain waveforms is

$$\|f(t)\|_p \equiv \left\{ \int_{-\infty}^{\infty} |f(t)|^p \, dt \right\}^{\frac{1}{p}} \quad , \quad \|f(t)\|_\infty = \sup_t |f(t)| \tag{4.9}$$

where isolated values of f(t) are excluded by considering limits from both sides of points of concern. The important operator of concern is convolution designated g(t)o where

$$F(t) \equiv g(t) \, o \, f(t) = \int_{-\infty}^{\infty} g(t-t') \; f(t') \, dt' = \int_{-\infty}^{\infty} g(t') \; f(t-t') \, dt' \tag{4.10}$$

Note that convolution in time domain corresponds to multiplication in frequency domain (as in the good-shielding approximation in section 3). So the convolution norm is now

$$\|g(t)o\| \equiv \sup_{\|f(t)\| \neq 0} \frac{\|g(t)\, o\, f(t)\|}{\|f(t)\|} \tag{4.11}$$

Assuming g(t)o is causal we have the remarkable results [28]

$$\|g(t)\, o\|_p \leq \|g(t)\|_1 \quad \text{for } 1 < p < \infty$$

$$\|g(t)\, o\|_1 = \|g(t)\, o\|_\infty = \|g(t)\|_1 \tag{4.12}$$

so that the convolution operator g(t)o p-norm is simply related to the 1-norm of the associated time-domain function g(t). The 2-norm has the special result (Parseval theorem)

$$\|f(t)\|_2 = \frac{1}{\sqrt{2\pi}} \|\tilde{f}(j\omega)\|_2$$

$$\|g(t)o\|_2 = |\tilde{g}(j\omega)|_{\max} \equiv |\tilde{g}(j\omega_{\max})| \leq [g(t)]_1 \tag{4.13}$$

Here the 2-norm in frequency domain merely replaces t by ω in the integration. The convolution operator norm is just the maximum value of the frequency-domain function (on the jω axis of the s plane).

5. BOUNDS ASSOCIATED WITH SUBSHIELDS

Now consider an aggregate of signals passing through the various parts of a subshield such as characterized by a scattering matrix for $S_{\lambda,\, \ell_1;\, \lambda+1,\, \ell_2}$. In section 3 these appear in the good-shielding approximation in (3.10) through (3.12). The general form is

$$\left(\tilde{V}_n^{(out)}(s)\right) = \left(\tilde{T}_{n,m}(s)\right) \cdot \left(\tilde{V}_n^{(in)}(s)\right) \quad , \quad \left(V_n^{(out)}(t)\right) = \left(T_{n,m}(t)\right) \overset{o}{\cdot} \left(V_n^{(in)}(t)\right) \tag{5.1}$$

where we now have matrix convolution operators in time domain. Here "in" designates the input to the subshield and "out" designates the output from the subshield into the sublayer, the signals being in the form of combined voltages (waves). In this linear form with transfer-function matrices we have in time domain

$$\left\| \left(\tilde{V}_n^{(out)}(t)\right) \right\| \leq \left\| \left(T_{n,m}(t)\right) o \right\| \; \left\| \left(V_n^{(in)}(t)\right) \right\| \tag{5.2}$$

Taking the norms in p-norm sense we have the results for vectors [25]

$$\left\| \left(V_n(t)\right) \right\|_p = \left\| \, \left\| \left(V_n(t)\right) \right\|_{pv} \right\|_{pf} = \left\| \left(\left\| V_n(t) \right\|_{pf} \right) \right\|_{pv} \tag{5.3}$$

where subscript v denotes vector (4.4) or associated matrix sense and subscript f denotes function (4.9) or associated convolution operator sense. So here the norm can be taken in the sense of a norm of norms. For the associated convolution operator this is

$$\begin{aligned} \left\| \left(T_{n,m}(t)\right) o \right\|_p &\leq \left\| \left(\left\| T_{n,m}(t) \right\|_{pf} \right) \right\|_{pv} \\ \left\| \left(T_{n,m}(t)\right) \right\|_{pf} &\leq \left\| T_{n,m}(t) \right\|_{1f} \end{aligned} \tag{5.4}$$

For the special case that the transfer-function matrix is diagonal (and hence square N × N) we have

$$\begin{aligned} \left\| \left(T_{n,m}(t)\right) o \right\| &= \max_n \left\| T_{n,n}(t) \, o \right\|_{pf} \\ &\leq \max_n \left\| T_{n,n}(t) \right\|_{1f} \end{aligned} \tag{5.5}$$

This linear case corresponds to a set of penetrations (localized) through an otherwise good shield (an important case). Diffusion through the shielding material (often less important) can be included as a distributed term, but this complicates matters considerably. For the discrete penetrations the most important are conductors (wires) with protective filters at the shield. The transfer-function matrix elements can be appropriately defined and measured as in [13]. Similarly apertures (electrically small) can be characterized by appropriate experimental configurations involving transmission and reception wires and/or bowls as in [13]. Specifying a shielding requirement in frequency or time domains corresponding to the transfer-function matrix, then one can measure the individual transfer functions of the various penetrations and combine them in the norm sense as above to determine if the required shielding is obtained.

Extending the norm concept to include nonlinear protection devices (clamps, fuses, spark gaps, etc.) at the subshield one can introduce the concept of a norm limiter [22, 27]. For this consider the nth signal reaching the inside of the shield and require

$$\left\| V_n^{(out)}(t) \right\| \leq X_n \tag{5.6}$$

no matter what the incoming signal $V_n^{(in)}$ is. This is the simple case corresponding to a diagonal transfer-function matrix. An ideal norm limiter would not affect the $V^{(out)}$

waveform until such time as X_n is reached, at least in a p-norm sense as defined by a time integral. The X_n might correspond to the peak signal allowed through (∞ - norm) or the energy allowed through (related to 2-norm). As discussed in [22, 27] it is desirable that there be terminating filters on both sides of the nonlinear element to minimize the effect of reflections from this element. For the p-norm of a vector of signals (5.6) becomes

$$\left\| \left(V_n^{(out)}(t) \right) \right\| = \left\| \left(\left\| V_n^{(out)}(t) \right\|_{pf} \right) \right\|_{pv} \leq \left\| (X_n) \right\|_{pv} = \left\{ \sum_{n=1}^{N} X_n^p \right\}^{\frac{1}{p}} \tag{5.7}$$

The norm limiters can be combined with filters on both sides. Consider the simple (but common) case of a diagonal filter/limiter combination so that $V_n^{(out)}$ is only influenced by $V_n^{(in)}$. Let there be a first filter $T_{n,n}^{(1)}$, the norm limiter X_n, and a second filter $T_{n,n}^{(2)}$. Then as in [25] we have the combined result

$$\left\| V_n^{(out)}(t) \right\|_{pf} \leq \left\| T_{n,n}^{(2)}(t) \circ \right\|_{pf} \left\{ \text{lesser of} \left[X_n, \left\| T_{n,n}^{(1)}(t) \circ \right\|_{pf} \left\| V_n^{(in)}(t) \right\|_{pf} \right] \right\}$$

$$\leq \left\| T_{n,n}^{(2)}(t) \right\|_{1f} \left\{ \text{lesser of} \left[X_n, \left\| T_{n,n}^{(1)}(t) \right\|_{1f} \left\| V_n^{(in)}(t) \right\|_{pf} \right] \right\} \tag{5.8}$$

$$\left\| \left(V_n^{(out)}(t) \right) \right\|_p = \left\| \left(\left\| V_n^{(out)}(t) \right\|_{pf} \right) \right\|_{pv}$$

This gives a bound in terms of any p-norm (with X_n defined for that norm). This allows for both linear attenuation (filters) and nonlinear attenuation (norm limiters).

6. CONCLUDING REMARKS

This has been a brief summary of the basic aspects of quantitative and qualitative EM topology, this forming a general theory of electromagnetic interference control by controlling unwanted signal propagation in complex electronic systems. For more details one can consult various items in the bibliography, the present paper only touching the highlights. Nevertheless, one can now see a thread running through the whole scheme. Beginning with the qualitative or descriptive aspects one organizes the problem. In turn this orders the interaction equations (BLT equation) from which one obtains the good-shielding approximation with a product formula in which the transfer-function matrices for the subshields can control unwanted electromagnetic signals. Introducing norms, bounds are obtained on such signals in both frequency and time domains including the effects of special limiting nonlinear devices.

The basic pieces are now in place, but various aspects of the theory will likely be expanded. Perhaps more important there is much to be done to implement various aspects of this in test procedures and associated specifications.

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