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On Bounding the Excitation of A Terminated Wire Behind an Aperture in a Shield

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ABSTRACT

A formulation is developed for determining the voltage and current induced on an impedance-terminated wire behind an aperture in a shield. Loading effects on the aperture are included by using receiving antenna theory. Upper bounds for the current and voltage are obtained and numerical results are presented for a thin slot antenna.

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INTRODUCTION

An electromagnetic shield simply provides a barrier to the propagation of electromagnetic fields from one region to another. The diffusion of the fields through the walls of the shield has been well studied [1-5]. For shields that are perforated, either deliberately or inadvertently, a substantial degradation in shielding effectiveness may occur. Normally the quantitation of the degradation can be accomplished by the consideration of three processes (assumed independent): external interaction , aperture penetration, and internal coupling [6]. Utilizing this approach, techniques have been developed for establishing upper bounds on signals induced in multiconductor lines behind apertures [7,8]. These formulations require the aperture to be electrically small and the multiconductor line to be separated from aperture a distance sufficient for the dipole field approximation to be valid.

In determining internal coupling within a shielded region, a standard approach has been to neglect loading effects on the aperture from internal wiring (and termination impedances) in proximity to the aperture. The aperture-wiring interaction can be appreciable as was found by Butler and Umashankar [9], who obtained a numerical solution to an integral equation for the current induced on an unterminated wire behind an aperture. Also Umashankar and Wait [10] have used a numerical solution technique to determine the current induced on an infinite wire behind a slot-perforated screen. And more recently Naiheng and Harrington [11] have determined not only the current on an infinite wire but also considered the impedance terminated wire, as well as the non-TEM mode excitation.

In this paper an analysis is developed for obtaining the excitation of terminated wiring behind aperture perforated shields. Coupling between the aperture and the wire are included by considering the aperture to be a receiving antenna with the wire configuration providing the load impedance presented to the antenna. This load impedance is determined by using conventional transmission line theory and therefore only the TEM mode current is obtained. However, if the wire spacing is small in terms of the wavelength and the wire length, only the TEM mode is significant.

Using the analysis, expressions are derived for the upper bounds on the termination current and voltage for a wire behind a slot-perforated screen.

Simple formulas are also derived for low frequency applications. And numerical results are presented for a thin slot with a few wire/termination configurations where the slot-aperture is allowed to go through a few resonances.

ANALYSIS

In order to make the analysis tractable a simplified geometrical configuration is considered. A convenient shield-aperture-wire configuration is shown in Figure 1. It is further considered that the wire is run parallel to the shield surface and is separated from it by an electrically short distance. Accordingly the aperture acts as a receiving antenna with impedance loading $Z_{i1} + Z_{i2}$, where Z_{i1} and Z_{i2} are the impedances seen looking into the transmission lines formed on each side of the aperture and terminated in impedances Z_{L1} and Z_{L2}, respectively. See the Appendix for further discussion of this model.

The equivalent circuit for the receiving antenna being considered is shown in Figure 2. Following standard receiving antenna theory

$$V_{oc}^{(s)} = \vec{h}_e^{(s)} \cdot \vec{E}^{inc} = I_{sc}^{(s)} Z_{in}^{(s)} \tag{1}$$

is the open-circuit voltage of the receiving antenna in terms of the effective height and incident electric field. Here $Z_{in}^{(s)}$ is the input impedance of the aperture antenna, and $I_{sc}^{(s)}$ is the short circuit current. Often it is more convenient to consider the complementary antenna with effective height $h_e^{(d)}$ and input impedance $Z_{in}^{(d)}$ where [3]

$$Z_{in}^{(s)} Z_{in}^{(d)} = \frac{1}{4} \eta_o^2 \tag{2}$$

and

$$\vec{k} \times \vec{h}_e^{(s)} = \frac{1}{2} \frac{\eta_o}{Z_{in}^{(d)}} k \vec{h}_e^{(d)} \tag{3}$$

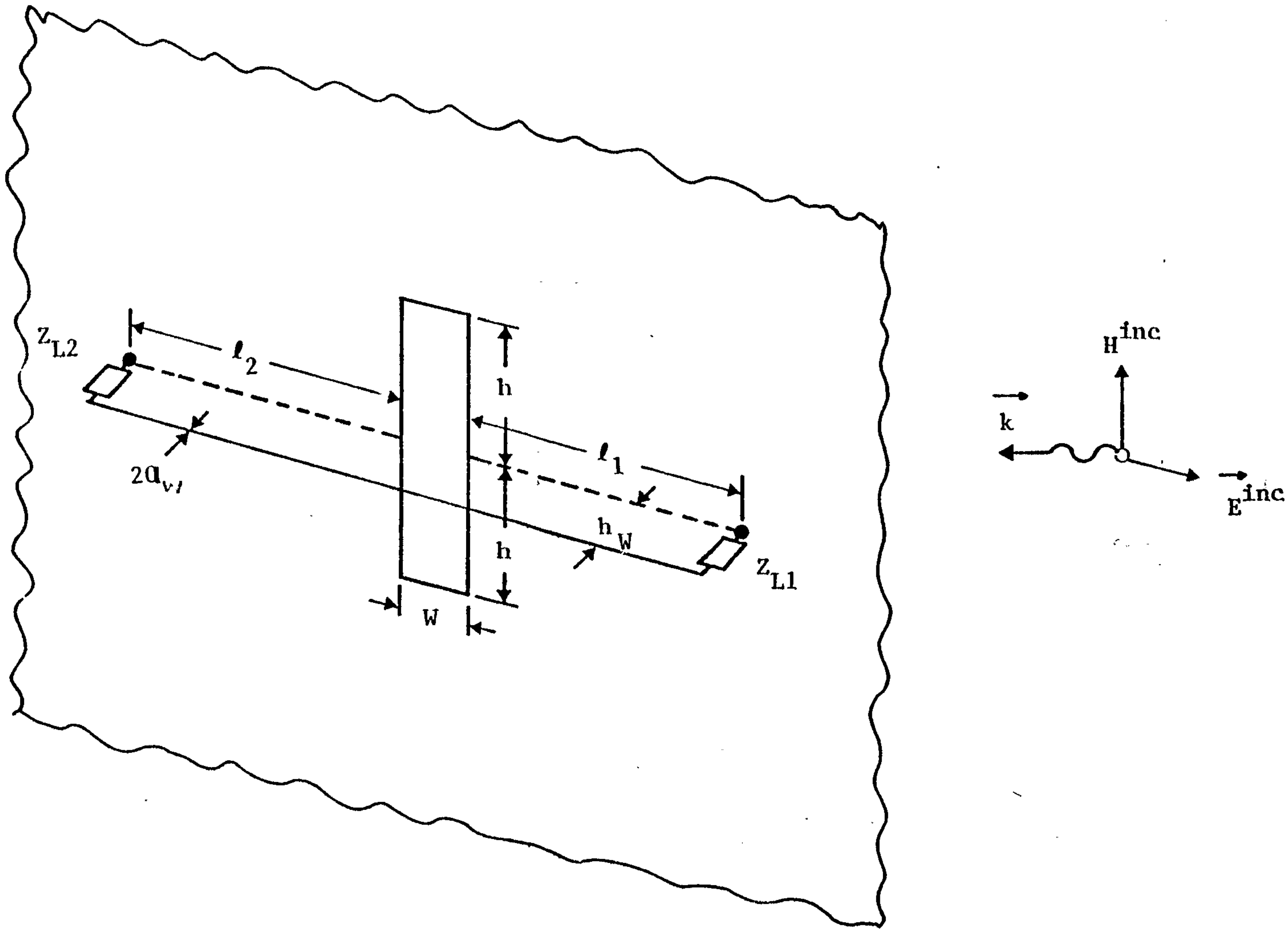

FIGURE 1. TERMINATED WIRE CONFIGURATION BEHIND A RECTANGLAR APERTURE IN A METAL PLATE SHIELD.

Here η_o is the intrinsic wave impedance of free space and $\vec{k}$ is the propagation vector of the incident plane wave [12,13]. Accordingly (1)-(3) yields

$$V_{oc}^{(s)} = (2\vec{H}^{inc} \cdot h_e^{(d)}) \eta_o^2 / 4Z_{in}^{(d)} \qquad (4)$$

consequently

$$I_{sc}^{(s)} = 2\vec{H}^{inc} \cdot \vec{h}_e^{(d)} = \hat{n} \cdot (\vec{h}_e^{(d)} \times \vec{J}_s) \qquad (5)$$

is the short-circuit current of the aperture antenna, $\hat{n}$ is the unit vector outward normal to the aperture surface and $\vec{J}_s$ is the surface current density at the aperture that would exist if the aperture were completely sealed.

From Figure 2., the solution for I_s, the wire current at the aperture, is

$$I_s = \frac{V_{oc}}{Z_{in}^{(s)} + Z_{i1} + Z_{i2}} \qquad (6)$$

It is readily shown that the maximum wire current occurs when

$$Z_{i1} + Z_{i2} = - j\,\mathrm{Im}[Z_{in}^{(s)}] \qquad (7)$$

Consequently the upper bound on the induced wire current is

$$|I_s|_{max} = \frac{|I_{sc}^{(s)}|}{\cos\phi_p} \qquad (8)$$

where ϕ_p is the argument of the complex input impedance of the aperture antenna.

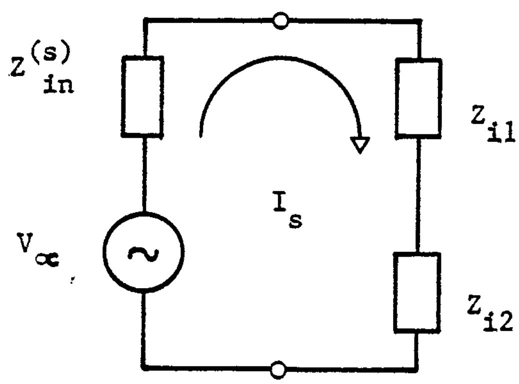

FIGURE 2. EQUIVALENT CIRCUIT FOR THE SLOT RECEIVING ANTENNA WITH THE LOAD IMPEDANCE THAT IS A SERIES COMBINATION OF Z_{i1} AND Z_{i2}

It is instructive to consider the current induced in the terminations of the transmission line behind the slot aperture. Generally it is this load current that is of most interest. First standard transmission line theory is used to relate the load current I_L to I_s. Accordingly

$$I_L = \frac{Z_c}{Z_c \cos k\ell + jZ_L \sin k\ell} I_s \tag{9}$$

where Z_L is the load impedance located a distance ℓ from the aperture and Z_c is the characteristic impedance of the transmission line,

$$Z_c = \frac{\eta_o}{2\pi} \operatorname{arccosh} \left(\frac{h_w}{a_w}\right) \tag{10}$$

Here h_w is the wire height and a_w is the radius of the wire behind the aperture (see Figure 1). Considering, for convenience, that the transmission line configuration is symmetric about the slot, i.e.

$$Z_{i2} = Z_{i1} = Z_c \frac{Z_L \cos k\ell + jZ_c \sin k\ell}{Z_c \cos k\ell + jZ_L \sin k\ell} \tag{11}$$

then combining (6) and (9) yields

$$I_L = \frac{Z_{in}^{(s)} Z_c \hat{n} \cdot (h_e^{(d)} \times \vec{J}_s)}{(Z_{in}^{(s)} + 2Z_L) Z_c \cos(k\ell) + j(Z_{in}^{(s)} Z_L + 2Z_c^2) \sin k\ell} \tag{12}$$

and the corresponding load voltage is given by $V_L = I_L Z_L$. Note that (12) yields finite currents at all frequencies and that there are no zeros of the denominator of (12) along the jw axis of the complex frequency plane.

The maximum voltage developed across the transmission line at the aperture is the maximum voltage across the antenna, i.e.

$$\left|V_S\right|_{max} = \left|V_{oc} - I_s Z_{in}^{(s)}\right|_{max} \tag{13}$$

However, if the transmission line is oriented symmetrically about the aperture, then only one-half the voltage of (13) is developed on each of the symmetrical segments of transmission line.

From the foregoing considerations the maximum transmission line current at the aperture occurs for a transmission line with either an open or a shorted termination. However, it is the maximum termination current that is of most interest. Accordingly (12) is manipulated to obtain the following expression for the termination current:

$$\left|I_L\right| = \frac{\left|1 - \Gamma_L\right|\left|1 - \Gamma_{in}\right|}{\{1 + \left|\Gamma_L \Gamma_{in}\right|^2 + 2\mathrm{Re}\,[\Gamma_{in}\Gamma_L e^{-j2k\ell}]\}^{\frac{1}{2}}} \frac{\left|I_{sc}^{(s)}\right|}{2} \tag{14}$$

where

$$\Gamma_{in} = \frac{2Z_c - Z_{in}^{(s)}}{2Z_c + Z_{in}^{(s)}} \tag{15}$$

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} \tag{16}$$

The termination current considering all terminations and all line lengths in (14) is a maximum when

$$\Gamma_L = -1 \quad \text{and} \quad 2k\ell = \angle\Gamma_{in} \tag{17}$$

Therefore the maximum termination current occurs for a line with shorted terminations. Using (17) in (14) yields

$$\left|I_L\right|_{max} = \frac{2\left|Z_{in}^{(s)}\right|}{\left|2Z_c + Z_{in}^{(s)}\right| - \left|2Z_c - Z_{in}^{(s)}\right|}\left|I_{sc}^{(s)}\right| , \tag{18}$$

that occurs when the line length ℓ satisfies

$$\tan(2k\ell) = -\frac{4Z_c \operatorname{Im}[Z_{in}^{(s)}]}{4Z_c^2 - \left|Z_{in}^{(s)}\right|^2} \tag{19}$$

Consequently (18) provides an upper bound on the termination current. Moreover it is the least upper bound.

Similarly the maximum termination voltage can also be obtained. Using $V_L = I_L Z_L$ and (14) yields after some mathematical manipulation,

$$\left|V_L\right| = \frac{\left|1 + \Gamma_L\right|\left|1 - \Gamma_{in}\right|}{\{1 + \left|\Gamma_{in}\Gamma_L\right|^2 + 2\operatorname{Re}[\Gamma_{in}\Gamma_L\, e^{-j2k\ell}]\}^{\frac{1}{2}}}\,\frac{Z_c\left|I_{sc}^{(s)}\right|}{2} \tag{20}$$

By inspection the maximum voltage occurs when

$$\Gamma_L = +1 \text{ and } 2k\ell = \angle\Gamma_{in} + \pi \tag{21}$$

In contrast to the conditions for maximum current, the maximum termination voltage occurs for lines with open-circuit terminations. Using (21) in (20) yields,

$$\left|V_L\right|_{max} = Z_c\left|I_L\right|_{max} \tag{22}$$

where $\left|I_L\right|_{max}$ is given by (18). However the maximum termination voltage occurs when line length ℓ satisfies

$$\tan(2k\ell-\pi) = \frac{-4Z_c \operatorname{Im}\left\{Z_{in}^{(s)}\right\}}{4Z_c^2 - \left|Z_{in}^{(s)}\right|^2} \tag{23}$$

Note that the voltage and current maxima appearing in (22) occur for different line lengths. Also note that the upper bound on the termination voltage is a least upper bound that occurs for lines symmetrically oriented with respect to the aperture.

Low Frequency Approximation

If the dimensions of the aperture are small in terms of the wavelength, then certain simple approximations are valid. The complement of a thin slot is the narrow strip dipole and at low frequency ($k_o h \lesssim 0.5$) the following approximations can be used [14]:

$$\vec{h}_e^{(d)} \simeq \frac{h(\Omega-1)}{\Omega-.61}\hat{z}$$

$$Z_{in}^{(d)} \simeq \frac{\eta_o}{6\pi}(kh)^2 - j\frac{\eta_o}{2\pi}\frac{\Omega-3.39}{k\,h} \tag{24}$$

where $\eta_o = \sqrt{\mu_o/\varepsilon_o} \simeq 120\pi$ Ohms and the shape factor for the strip dipole is $\Omega = 2\ln(\frac{8h}{w})$. Here w is the width of the strip dipole complement of the narrow slot.

Applying the foregoing approximations yields

$$\left|I_s\right|_{max} \simeq \frac{3(\Omega-3.39)}{(k\ h)^3} \left|I_{sc}^{(s)}\right| \tag{25}$$

or

$$\left|I_s\right|_{max} \simeq \frac{3h(\Omega-3.39)(\Omega-1)}{(\Omega-.61)} \frac{\left|\hat{z}x\vec{J}_s\right|}{(kh)^3} \tag{26}$$

as the upper bound of the wire current behind the aperture. It will be demonstrated subsequently, that the wire current in general will be substantially less than the upper bound and equal to it only at discrete frequencies, or over narrow frequency bands. Also if $Z_L = 0$, then using (24) in (2) and (12) yields

$$I_L = \frac{h\left[\frac{\Omega-1}{\Omega-.61}\right]\left|\hat{z}\ x\vec{J}_s\right|}{\cos(k\ell) + \frac{\Omega-3.39}{\eta_o\pi kh} 4Z_c\sin(k\ell) + j\,\frac{4Z_c(kh)^2}{3\pi\eta_o}\sin(k\ell)} \tag{27}$$

The low frequency approximation for the maximum load current and voltage can be obtained by using (24) in (18) and (22), respectively. Consequently, after some mathematical manipulation,

$$\left|I_L\right|_{max} \simeq \left[1 + \frac{\eta_o^2(\Omega-3.39)^2}{16\pi^2 Z_c^2(kh)^2}\right]^{\frac{1}{2}} \left|I_s\right|_{max} \tag{28}$$

and

$$\left|V_L\right|_{max} \simeq \left[1 + \frac{\eta_o^2(\Omega-3.39)^2}{16\pi^2 Z_c^2 (kh)^2} \right]^{\frac{1}{2}} Z_c \left|I_s\right|_{max} \tag{29}$$

NUMERICAL RESULTS

The upper bound on the current induced in a wire behind an aperture in a shield depends upon the impedance and effective length of the aperture when used as an antenna. Considering a thin slot aperture and using transmission line thoery, expressions are derived for the least upper bounds on the current both at the aperture and at the terminations. Sample results are obtained for the maximum wire current at the aperture and presented in Figure 3 for a few slot dimensions. In order to obtain these data, cylindrical dipole results from King and Harrison [15] are used for the impedance and effective lengths of the narrow strip electric dipole complement of the narrow slot.

For the maximum induced source current or voltage to occur, the wire with its terminations must present a purely reactive load to the aperture antenna. And for the maximum wire current at the aperture this load reactance must equal the negative of the antenna reactance. Consequently this maximum current can occur only at specific frequencies or effectively over narrow frequency bands where the total reactance is small compared to the antenna resistance. This behavior is exhibited in Figure 4 where the wire current at the aperture is presented for a few load configurations. Note that below 100 MHz the unterminated wire suffers the largest current and the wire with the shorted terminations has the smallest induced current.

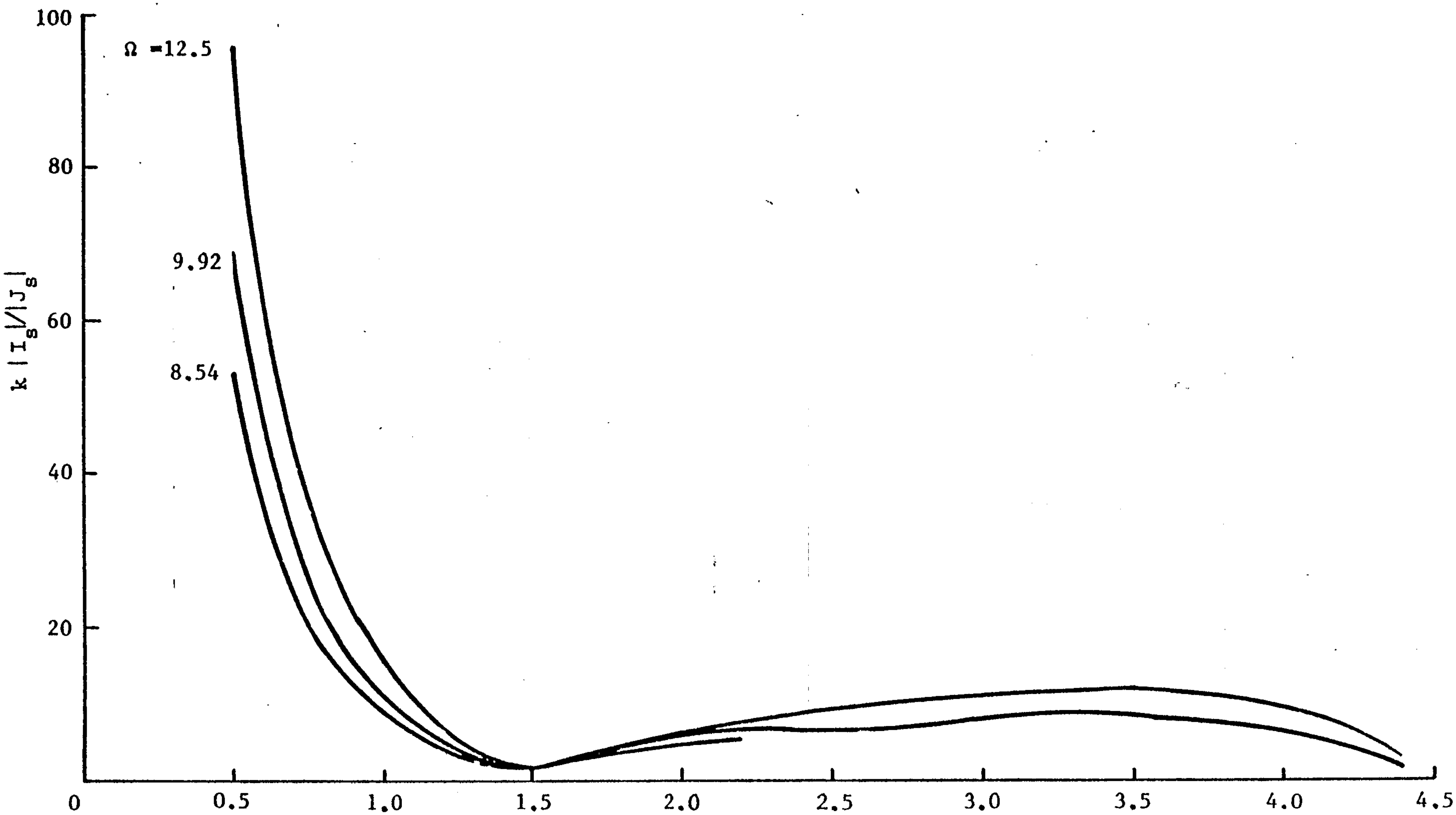

FIGURE 3. MAXIMUM WIRE CURRENT FOR A WIRE CONFIGURATION BEHIND A THIN SLOT IN A CONDUCTING PLATE.

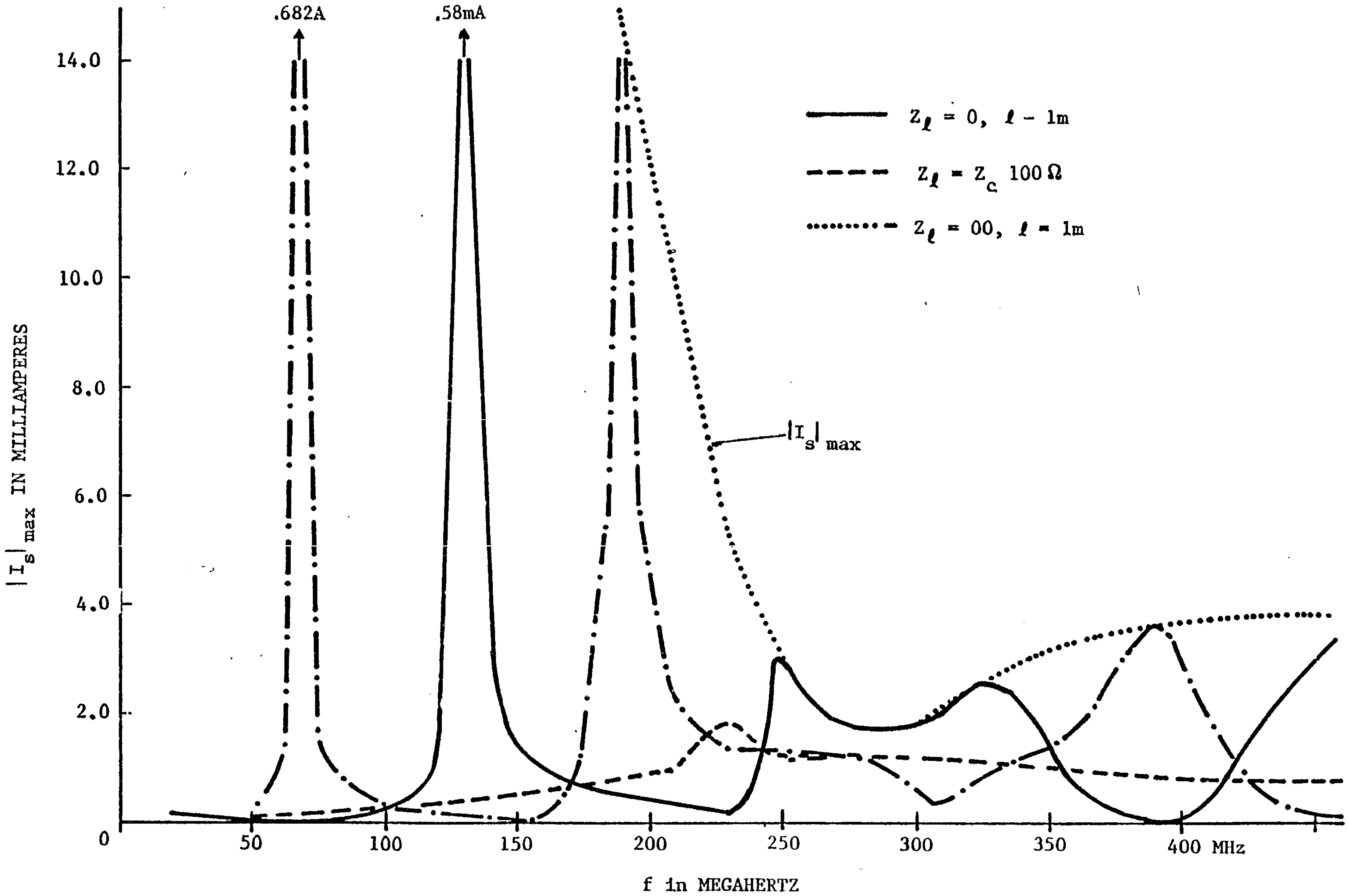

FIGURE 4. WIRE CURRENT AT THE RECTANGULAR APERTURE WITH h = 0.25 and W = 0.014m, AND FOR DIFFERENT LOAD CONFIGURATIONS. HERE E^{inc} = 1 V/M AND Z_c = 100 Ω

It is also shown that the maximum current at the termination occurs for short-circuit terminations and that the maximum termination voltage occurs for open-circuit terminations. However these occur for different wire lengths. Moreover they occur for different wire lengths from those that yield maximum wire current at the aperture.

CONCLUSION

By using receiving antenna theory, the maximum load current and voltage induced in a terminated wire configuration behind an aperture perforated shield has been determined. The analysis fully included loading effects of the wire configuration on the aperture. Approximations valid at low frequency are also presented.

Results of the formulation are presented for a few wire/aperture configurations.

APPENDIX

When conventional receiving antenna theory is used to determine the excitation of a wire behind an aperture, the Thevenin equivalent circuit for the aperture is used, where $V_{oc}^{(s)}$ is the source voltage and $Z_{in}^{(s)}$ is the source impedance. This of course assumes well-defined terminals exist for the aperture and that their separation is a small fraction of a wavelength. Moreover linearity must be assumed as well as negligible interaction between the aperture fields and the source producing the field incident on the aperture.

In the treatment of the excitation of a wire behind an electrically small aperture generally two sources are introduced - a series voltage source and a shunt current source [16,17]. The use of two sources arises from the separate consideration of the electric and magnetic dipole moments of the aperture and is a convenient procedure. In addition to the sources, Lee and Yang [17] have obtained the equivalent impedance loading of the wire resulting from the aperture. This impedance involves a series inductance and a shunt (negative) capacitance.

The formulation presented in this paper is not completely equivalent to the small aperture formulation of Lee and Yang even for electrically-short thin slots. Specifically the electric dipole effects (the shunt current source and negative shunt capacitance) are not included. However these effects are negligible for thin-slot apertures especially when conditions for maximum coupling to the wire are considered.

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