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Interaction Notes

Note 416

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Sublayer Sets and Relative Shielding Order in Electromagnetic Topology

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Abstract

This note explores some aspects of qualitative electromagnetic topology for system design. Sublayers are partitioned into sets for separate treatment in the design process. The concept of relative shielding order between pairs of subsets is introduced and constraints on this non-negative-integer parameter are explored. Special cases are considered, including that of uniform relative shielding order between pairs of a set of sublayers designated primary sublayers.

I. Introduction

Electromagnetic topology considers certain aspects of the interaction of electromagnetic fields with various objects, especially complex ones. It is an abstraction which looks at the connectivity or continuity of the object (especially its conductors) so as to order the electromagnetic interaction process according to important electromagnetic properties of parts of the object [1,2].

Let us divide electromagnetic topology into two areas designated qualitative and quantitative. Quantitative electromagnetic topology is concerned with the ordering of the electromagnetic equations describing the transport of electromagnetic signals through a system according to the various topological entities defined, and using this ordering to express the properties of the whole in terms of the properties of the topological parts [3,5].

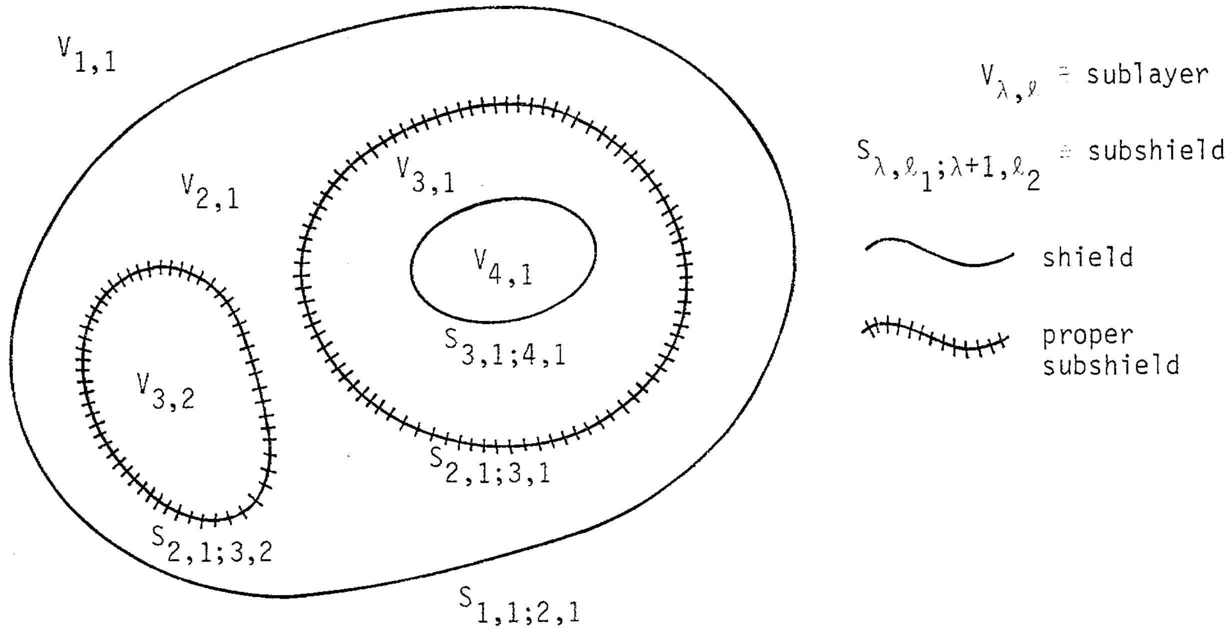
Qualitative electromagnetic topology, on the other hand, is concerned with the definition and use of appropriate topological entities to construct electromagnetic designs of systems from a macroscopic viewpoint. It is discrete in the sense that a set of entities (such as surfaces and volumes) are defined with certain connectivity relationships among these entities. Making these topological entities coincide with various physical features in a realized system design allows one to control the electromagnetic propagation through the system.

There are various possible electromagnetic topologies one can choose for a system design. One would like to know which were more efficient and effective for particular applications. This note begins an exploration of such questions by considering an electromagnetic topology defined to the level of sublayers and subshields. The concept of relative shielding order is introduced and applied to selected sublayers. Constraining the shielding order between various sublayers constrains certain aspects of the topology from which solutions for and optimization of the topology can be sought.

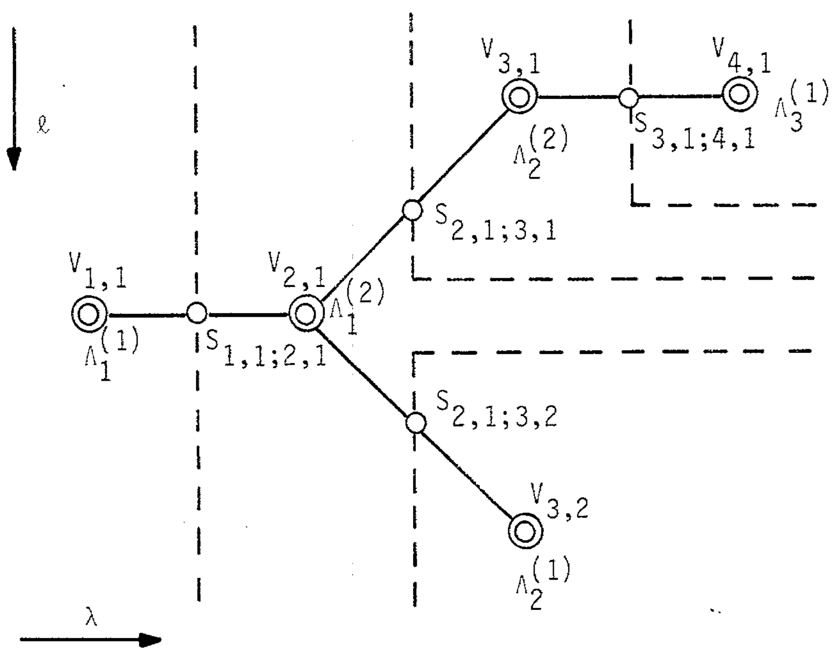
II. Hierarchical Topology at Sublayer Level

The basics of a hierarchical electromagnetic topology have been discussed previously [2,5]. In this formalism space is divided into a set of volumes with associated boundary surfaces. In a hierarchical topology euclidean space is divided into a set of nested volumes V_λ for $\lambda = 1,2,\ldots,\lambda_{max}$ which begin from the outside and work progressively inward (deeper into the system); the separating boundaries $S_{\lambda;\lambda+1}$ are closed surfaces referred to as shields (labelled by the outer V_λ and inner $V_{\lambda+1}$ layers). As illustrated in fig. 2.1A the individual layers need not be connected as a simple volume but can exist in more than one part; each part is called a sublayer $V_{\lambda,\ell}$ for $\ell = 1,2,\ldots,\ell_{max}(\lambda)$. In this example the layer V_3 is composed of two proper sublayers $V_{3,1}$ and $V_{3,2}$ while the other layers are also each sublayers. Similarly, the shield $S_{2;3}$ is composed of two proper subshields $S_{2,1;3,1}$ and $S_{2,1;3,2}$ while the other shields are also each subshields. Further division of these topological entities into elementary volumes and elementary surfaces is possible but is not pursued here. For more discussion of these points see [5].

Associated with a volume/surface topology there is a dual graph for which there is a one-to-one correspondence between volumes and vertices and between surfaces and edges. For our purposes there is a special vertex placed in each branch (dividing it in two), this vertex being identified with the corresponding surface; the resulting dual graph is then a bipartite graph. For a hierarchical topology defined down to sublayer level (and no further) this dual graph is a tree graph [4,5]. For the example electromagnetic topology in fig. 2.1A the dual bipartite graph is exhibited in fig. 2.1B. A useful feature of the dual graph is that it clearly exhibits which subshields signals must pass through in going from one sublayer to another because the path connecting two vertices in a tree graph is unique (assuming a restriction that any edges may be traversed at most once).



A. Volume/surface topology



B. Dual bipartite graph (a tree)

Fig. 2.1. Sublayers and Subshields in Hierarchical Topology

III. Relative Shielding Order

As a simplification, assign a weight $a_{\lambda,\ell_1';\lambda+1,\ell_2'}$ to each subshield $S_{\lambda,\ell_1';\lambda+1,\ell_2'}$. This weight can be used to represent the effectiveness of the shield in appropriate units. For our purposes this may be the (natural) logarithm of the attenuation, or negative logarithm of the transmission, appropriately defined. One interesting way to define transmission is in a norm sense of an appropriate block of the interaction matrix belonging to the subshield [5]. Neglecting attenuation through the sublayers then one can define the weight between any two sublayers as

$$A_{\lambda_1,\ell_1;\lambda_2,\ell_2} \equiv \sum_{P_{\lambda_1,\ell_1;\lambda_2,\ell_2}} a_{\lambda,\ell_1';\lambda+1,\ell_2'} \tag{3.1}$$

where $P_{\lambda_1,\ell_1;\lambda_2,\ell_2}$ is the unique path in the dual tree graph going from V_{λ_1,ℓ_1} to V_{λ_2,ℓ_2} (or the reverse) and $(\lambda,\ell_1';\lambda+1,\ell_2')$ is chosen to correspond to each subshield along this path. Note that if one assumes reciprocity as well as linearity we have

$$\begin{aligned} a_{\lambda,\ell_1';\lambda+1,\ell_2'} &= a_{\lambda+1,\ell_2';\lambda,\ell_1'} && \text{subshield weight reciprocity} \\ A_{\lambda_1,\ell_1;\lambda_2,\ell_2} &= A_{\lambda_2,\ell_2;\lambda_1,\ell_1} && \text{relative shielding reciprocity} \end{aligned} \tag{3.2}$$

Now $A_{\lambda_1,\ell_1;\lambda_2,\ell_2}$ can be thought of as the relative shielding between sublayers (or perhaps a lower bound on the shielding).

Going a step further in simplification, suppose that all subshields have about the same attenuation and hence about the same "weights." Normalizing these one can set all the $a_{\lambda,\ell_1';\lambda+1,\ell_2'}$ to 1.0. This gives what can be termed the relative shielding order which we designate as

$$R_{\lambda_1,\ell_1;\lambda_2,\ell_2} \equiv \sum_{P_{\lambda_1,\ell_1;\lambda_2,\ell_2}} 1$$

$\equiv$ relative shielding order between V_{λ_1,ℓ_1} and V_{λ_2,ℓ_2}

$\equiv$ number of subshields crossed (only once) in going from V_{λ_1,ℓ_1} to V_{λ_2,ℓ_2} along the unique path $P_{\lambda_1,\ell_1;\lambda_2,\ell_2}$

$$\equiv R_{\lambda_2,\ell_2;\lambda_1,\ell_1} \quad \text{(path reversal symmetry)} \tag{3.3}$$

Note that this is a non-negative integer.

$$R_{\lambda_1,\ell_1;\lambda_2,\ell_2} \begin{cases} = 0 \text{ for } (\lambda_1,\ell_1) = (\lambda_2,\ell_2) \\ \geq 1 \text{ for } (\lambda_1,\ell_1) \neq (\lambda_2,\ell_2) \end{cases} \tag{3.4}$$

As an example of relative shielding order one can construct a matrix corresponding to the example topology in fig. 2.1 in the following table.

$V_{\lambda,\ell}$ \ $V_{\lambda,\ell}$	$V_{1,1}$	$V_{2,1}$	$V_{3,1}$	$V_{3,2}$	$V_{4,1}$
$V_{1,1}$	0	1	2	2	3
$V_{2,1}$	1	0	1	1	2
$V_{3,1}$	2	1	0	2	1
$V_{3,2}$	2	1	2	0	3
$V_{4,1}$	3	2	1	3	0

(matrix entries: $R_{\lambda_1,\ell_1;\lambda_2,\ell_2}$)

Table 3.1. Relative Shielding Order Among Sublayers (Corresponding to Fig. 2.1)

IV. Partitioning Sublayers into Sets

In constructing a topological design for a system one may not wish all the sublayers to have the same role. Some sublayers may contain sensitive equipment, others may contain strong sources of electromagnetic interference, and yet others may have neither of the above but are included so as to provide additional subshields as desired. In this context one may wish to separate the system topology into different sets of sublayers, and impose different constraints on each of these sets so as to construct an efficient electromagnetic topology for a given system.

Define an index ν which is used with $\Lambda^{(\nu)}$ where

$$\begin{aligned}
\Lambda_n^{(\nu)} &\equiv \text{nth member of } \nu\text{th set of sublayers} \\
\{\Lambda_n^{(1)}\} &\equiv \text{set of primary sublayers} \\
\{\Lambda_n^{(2)}\} &\equiv \text{set of secondary sublayers} \\
\{\Lambda_n^{(3)}\} &\equiv \text{set of tertiary sublayers} \\
&\text{etc.} \\
\{\Lambda_n^{(\nu)}\} &\equiv \text{set of all sublayers} \\
\nu &= 1,2,\ldots,\nu_{max} \\
\nu_{max} &\equiv \text{partition order of the system (topology)} \\
n &= 1,2,\ldots,n_{max}(\nu) \\
n_{max}(\nu) &\equiv \text{partition order of the } \nu\text{th set of sublayers}
\end{aligned} \tag{4.1}$$

These symbols apply to both proper sublayers and improper sublayers (layers consisting of a single sublayer).

Like the sublayers and layers, the above sets of volumes completely partition the (euclidean) space in which our system topology is defined. We might define

$$\Lambda^{(\nu)+} \equiv \bigcup_{n=1}^{n_{max}(\nu)} \Lambda_n^{(\nu)+} \equiv \nu\text{th space} \tag{4.2}$$

where a superscript + indicates that the volume (sublayer in this case) is augmented by its adjoining boundaries (as in [5]). Then we have

$$\Lambda^{+} \equiv \bigcup_{\nu=1}^{\nu_{max}} \Lambda^{(\nu)+} \equiv \text{all space (euclidean)} \equiv \text{universe} \tag{4.3}$$

For this partitioning of the sublayers one can write the relative shielding order as

$$R_{n;m}^{(\nu_1;\nu_2)} \equiv \text{relative shielding order between } \Lambda_n^{(\nu_1)} \text{ and } \Lambda_m^{(\nu_2)} \tag{4.4}$$

which for sublayers in the same subset can be reduced to

$$R_{n,m}^{(\nu)} \equiv R_{n;m}^{(\nu;\nu)} \tag{4.5}$$

with the special cases

$$R_{n,n}^{(\nu)} = R_{n;n}^{(\nu;\nu)} = 0 \tag{4.6}$$

$$R_{n;m}^{(\nu_1;\nu_2)} \geq 1 \quad \text{for} \quad \nu_1 \neq \nu_2 \quad \text{or } n \neq m$$

A motivation for defining primary, secondary, etc. sublayers is that one may wish to select certain numbers of these and constrain the shielding that isolates them. Using the concept of relative shielding order let us consider the primary sublayers. For present purposes let us think of primary sublayers as the following:

> primary sublayers--sublayers containing sensitive equipment to be shielded (by keeping undesirable electromagnetic signals out), or intense electromagnetic sources to be shielded (by keeping undesirable electromagnetic signals in).

Similarly, let us think of secondary sublayers as:

> secondary sublayers--sublayers that are not primary sublayers.

Secondary sublayers in this context are included in the topology to provide additional subshields separating primary sublayers. It should be noted that this is not the only way to partition the sublayers into sets. Future notes may alter and expand these partitions.

Using this concept of primary sublayers, as an example consider the electromagnetic topology in fig. 2.1 and partition the sublayers as in the following:

$$\begin{aligned}
V_{1,1} &\equiv \Lambda_1^{(1)} && \text{(primary sublayer)} \\
V_{2,1} &\equiv \Lambda_1^{(2)} && \text{(secondary sublayer)} \\
V_{3,1} &\equiv \Lambda_2^{(2)} && \text{(secondary sublayer)} \\
V_{3,2} &\equiv \Lambda_2^{(1)} && \text{(primary sublayer)} \\
V_{4,1} &\equiv \Lambda_3^{(1)} && \text{(primary sublayer)} \\
\nu_{max} &= 2 && \text{(partition order of the system)} \\
n_{max}(1) &= 3 && \text{(number of primary sublayers)} \\
n_{max}(2) &= 2 && \text{(number of secondary sublayers)}
\end{aligned} \tag{4.7}$$

In this example the vertices at the "ends" of the tree-graph interaction sequence diagram have been chosen as the primary sublayers so as to give in a certain sense maximum shielding between pairs of primary sublayers. This gives relative shielding orders for pairs of primary sublayers in the following table.

$\Lambda_n^{(1)}$ \ $\Lambda_n^{(1)}$	$\Lambda_1^{(1)}$	$\Lambda_2^{(1)}$	$\Lambda_3^{(1)}$
$\Lambda_1^{(1)}$	0	2	3
$\Lambda_2^{(1)}$	2	0	3
$\Lambda_3^{(1)}$	3	3	0

(values: $R_{n,m}^{(1)}$)

Table 4.1. Relative Shielding Order Among Primary Sublayers (Corresponding to Fig. 2.1)

V. Constraints on Relative Shielding Order for Sets of Sublayers

For the general case let us define some minimum relative shielding orders.

$$R_{min} \equiv \min_{\substack{\nu_1 \neq \nu_2 \\ \text{or} \\ n \neq m}} R_{n;m}^{(\nu_1;\nu_2)}$$

$\equiv$ minimum relative shielding order between any two distinct sublayers

$$\begin{cases} =1 & \text{for at least two distinct sublayers} \\ \text{undefined} & \text{if only one distinct sublayer} \end{cases}$$

(5.1)

$$R_{min}^{(\nu)} \equiv \min_{n \neq m} R_{n,m}^{(\nu)}$$

$\equiv$ minimum relative shielding order between any two distinct sublayers in the νth set of sublayers

$$\begin{cases} \geq 1 & \text{for at least two distinct sublayers in the } \nu\text{th set of sublayers} \\ \text{undefined} & \text{if only one distinct sublayer in the } \nu\text{th set of sublayers} \end{cases}$$

$$R_{min}^{(\nu)} \geq R_{min} \quad \text{for all } \nu \text{ if defined}$$

For our example in fig. 2.1 we have

$$R_{min} = 1$$

$$R_{min}^{(1)} = 2 \qquad (5.2)$$

$$R_{min}^{(2)} = 1$$

For the general case let us define some maximum shielding orders

$$R_{max} \equiv \max_{\substack{\nu_1 \neq \nu_2 \\ \text{or} \\ n \neq m}} R_{n;m}^{(\nu_1;\nu_2)}$$

$\equiv$ maximum relative shielding order between any two distinct sublayers

$$\begin{cases} \geq 1 & \text{for at least two distinct sublayers} \\ \text{undefined} & \text{if only one distinct sublayer} \end{cases}$$

(5.3)

$$R_{max}^{(\nu)} \equiv \max_{n \neq m} R_{n,m}^{(\nu)}$$

$\equiv$ maximum relative shielding order between any two distinct sublayers in the νth set of sublayers

$$\begin{cases} \geq 1 & \text{for at least two distinct sublayers in the } \nu\text{th set of sublayers} \\ \text{undefined} & \text{if only one distinct sublayer in the } \nu\text{th set of sublayers} \end{cases}$$

$$R_{max} \geq R_{max}^{(\nu)} \quad \text{for all } \nu \text{ if defined}$$

In traversing from $V_{1,1}$ (the outside of the system) into the system one can go as "deep" as $V_{\lambda_{max},\ell}$ crossing $\lambda_{max} - 1$ subshields giving

$\lambda_{max} - 1 \equiv$ shielding order of system (topology)

$\equiv$ maximum relative shielding order from "outside" sublayer to any other sublayer

(5.4)

Hence we have

$$R_{max} \geq \lambda_{max} - 1 \tag{5.5}$$

since the "outside" sublayer is only one of the sublayers; some maximum path from another sublayer may be "longer" and at least one path (from the "outside" sublayer) is this "long." For our example we have

$$
\begin{aligned}
R_{max} &= 3 \\
R^{(1)}_{max} &= 3 \\
R^{(2)}_{max} &= 1 \\
\lambda_{max} - 1 &= 3
\end{aligned}
\qquad (5.6)
$$

Combining these results we have both lower and upper bounds for relative shielding order as

$$
\begin{aligned}
&1 = R_{min} \leq R^{(\nu_1;\nu_2)}_{n;m} \leq R_{max} \qquad \text{for } \Lambda^{(\nu_1)}_{n} \neq \Lambda^{(\nu_2)}_{m} \\
&1 \leq R^{(\nu)}_{min} \leq R^{(\nu)}_{n,m} \leq R^{(\nu)}_{max} \leq R_{max} \qquad \text{for } \Lambda^{(\nu)}_{n} \neq \Lambda^{(\nu)}_{m}
\end{aligned}
\qquad (5.7)
$$

Note that $R^{(\nu)}_{n,m}$, since it pertains to the νth subset of the sublayers, is in general more constrained than $R^{(\nu_1;\nu_2)}_{n;m}$ which applies to the entire set of sublayers. This property of the $R^{(\nu)}_{n,m}$ can be used to constrain the minimum and/or maximum relative shielding orders within one or more subsets of sublayers in synthesizing appropriate topological designs for systems.

VI. Constraints on Combinations of Relative Shielding Orders for Sets of Sublayers

Now in choosing some subset of the sublayers, and assigning relative shielding orders between pairs of these sublayers, there is the requirement of assigning these relative shielding orders in a consistent manner, i.e., in a topologically possible manner. The previous section discussed constraints on individual relative shielding orders. This section considers combinations of relative shielding orders.

Suppose in the equivalent dual graph we have two sublayers $\Lambda_n^{(\nu_1)}$ and $\Lambda_m^{(\nu_2)}$ denoted by the notation for elements of sublayer sets. Denote the path connecting these two by

$$P_{n;m}^{(\nu_1;\nu_2)} = P_{m;n}^{(\nu_2;\nu_1)} \tag{6.1}$$

This path is unique because the dual graph is a tree graph. The relative shielding order $R_{n;m}^{(\nu_1;\nu_2)}$ represents the minimum number of subshields traversed in going from $\Lambda_n^{(\nu_1)}$ to $\Lambda_m^{(\nu_2)}$ (or reverse); this is precisely the number of subshields traversed on the above path with no subshields being traversed more than once.

Consider a set of sublayers denoted by $\Lambda_{p_q}^{(\eta_q)}$ for $q = 1,2,\ldots,q_{max}$ with

$$\begin{aligned} p_1 &= n \quad , \quad \eta_1 = \nu_1 \\ p_{q_{max}} &= m \quad , \quad \eta_{q_{max}} = \nu_2 \end{aligned} \tag{6.2}$$

so that

$$\begin{aligned} \Lambda_{p_1}^{(\eta_1)} &= \Lambda_n^{(\nu_1)} \\ \Lambda_{p_{q_{max}}}^{(\eta_{q_{max}})} &= \Lambda_m^{(\nu_2)} \end{aligned} \tag{6.3}$$

are the ends of the previously defined path in (6.1). Now let us consider a set of paths $P_{p_q;p_{q+1}}^{(\eta_q;\eta_{q+1})}$ connecting $\Lambda_{p_q}^{(\eta_q)}$ and $\Lambda_{p_{q+1}}^{(\eta_{q+1})}$ for $q = 1,2,\ldots,q_{max}-1$ with associated relative shielding orders $R_{p_q;p_{q+1}}^{(\eta_q;\eta_{q+1})}$. Traversing these paths (q_{max} - 1 in number) in order ($q = 1,2,\ldots,q_{max}-1$) traces a path from $\Lambda_{p_1}^{(\eta_1)}$ to $\Lambda_{p_{q_{max}}}^{(\eta_{q_{max}})}$ (i.e., $\Lambda_n^{(\nu_1)}$ to $\Lambda_m^{(\nu_2)}$) with perhaps various edges in the graph traversed more than once.

Now since the path in (6.1) is the minimum path we have

Theorem 6.1: Minimum path relative shielding order

$$R_{p_1;p_{q_{max}}}^{(\eta_1;\eta_{q_{max}})} \leq \sum_{q=1}^{q_{max}-1} R_{p_q;p_{q+1}}^{(\eta_q;\eta_{q+1})} \tag{6.4}$$

An interesting case has $q_{max} = 3$ (three sublayers) for which we set

$$\begin{aligned} p_1 &= n \quad , \quad \eta_1 = \nu_1 \\ p_2 &= u \quad , \quad \eta_2 = \nu_u \\ p_3 &= m \quad , \quad \eta_3 = \nu_2 \end{aligned} \tag{6.5}$$

giving

Corollary: Topological triangle inequality

$$R_{n;m}^{(\nu_1;\nu_2)} \leq R_{n;u}^{(\nu_1;\nu_u)} + R_{u;m}^{(\nu_u;\nu_m)} \tag{6.6}$$

In the topological sense this says that the shortest path between two sublayers is sometimes increased by an intermediate detour to some other sublayer.

Confining our attention to the νth set of sublayers (which might be, say, the primary sublayers) theorem 6.1 becomes

Theorem 6.2: Minimum path relative shielding order for sublayers in the νth set

$$R^{(\nu)}_{p_1,p_{q_{max}}} \le \sum_{q=1}^{q_{max}-1} R^{(\nu)}_{p_q,p_{q+1}} \tag{6.7}$$

In this form all the q_{max} - 1 individual paths have both ends at $\Lambda^{(\nu)}_n$ sublayers but may connect to sublayers not in the νth set at intermediate points. Again for q_{max} = 3 we now set

$$p_1 = n \quad , \quad p_2 = u \quad , \quad p_3 = m$$
$$\eta_1 = \eta_2 = \eta_3 = \nu \tag{6.8}$$

giving

Corollary: Topological triangle inequality for sublayers in the νth set

$$R^{(\nu)}_{n,m} \le R^{(\nu)}_{n,u} + R^{(\nu)}_{u,m} \tag{6.9}$$

Combining the results of (5.7) with the results of this section gives lower and upper bounds for the sum of the relative shielding orders along various paths connected in sequence. Substituting from (5.7) into (6.4) gives

Theorem 6.3: Bounds on relative shielding order along a sequence of connected paths

$$1 = R_{min} \le R^{(\eta_1;\eta_{q_{max}})}_{p_1;p_{q_{max}}} \le \sum_{q=1}^{q_{max}-1} R^{(\eta_q;\eta_{q+1})}_{p_q;p_{q+1}} \le (q_{max} - 1)R_{max}$$

$$\text{for } \Lambda^{(\eta_1)}_{p_1} \ne \Lambda^{(\eta_{q_{max}})}_{p_{q_{max}}} \tag{6.10}$$

Stated in words, the sum of the relative shielding orders of the sequence of connected paths is no less than the relative shielding order of the minimum path connecting the end sublayers, and is no greater than the number of these sequential paths times the maximum relative shielding order in the entire system topology. This inequality holds for every sequence of such paths in the system topology. Again selecting the case $q_{max} = 3$ we have the indices in (6.5) giving

Corollary: Bounds on relative shielding order among three sublayers

$$1 = R_{min} \leq R_{n;m}^{(\nu_1;\nu_2)} \leq R_{n;u}^{(\nu_1;\nu_u)} + R_{u;m}^{(\nu_u;\nu_m)} \leq 2R_{max}$$

$$\text{for } \Lambda_n^{(\nu_1)} \neq \Lambda_m^{(\nu_2)} \qquad (6.11)$$

If the individual paths are non-trivial so that each relative shielding order is at least 1 we have

Theorem 6.4: Bounds on relative shielding order along a sequence of non-trivial connected paths

$$\text{greater of} \left[R_{p_1,p_{q_{max}}}^{(n_1;n_{q_{max}})} , q_{max} - 1 \right]$$

$$\leq \sum_{q=1}^{q_{max}-1} R_{p_q;p_{q+1}}^{(n_q;n_{q+1})} \leq (q_{max} - 1)R_{max} \qquad (6.12)$$

Corollary: Bounds on relative shielding order among three distinct sublayers

$$\text{greater of} \left[R_{n;m}^{(\nu_1;\nu_2)} , 2 \right]$$

$$\leq R_{n;u}^{(\nu_1;\nu_u)} + R_{u;m}^{(\nu_u;\nu_2)} \leq 2R_{max} \qquad (6.13)$$

Consider now the above results specialized to the case that all the sublayers are elements of the νth set which does not in general include all of the sublayers. An important new feature is the fact that $R_{min}^{(\nu)}$ need not be 1 but may be greater (as in (5.1)). In this case (6.10) becomes

Theorem 6.5: Bounds on relative shielding order along a sequence of connected paths with all end sublayers in the νth set

$$1 \leq R_{min}^{(\nu)} \leq R_{p_1,p_{q_{max}}}^{(\nu)} \leq \sum_{q=1}^{q_{max}-1} R_{p_q,p_{q+1}}^{(\nu)} \leq (q_{max} - 1)R_{max}^{(\nu)}$$

$$\text{for } \Lambda_{p_1}^{(\nu)} \neq \Lambda_{p_{q_{max}}}^{(\nu)} \qquad (6.14)$$

and (6.11) becomes

Corollary: Bounds on relative shielding order among three sublayers all in the νth set

$$1 \leq R_{min}^{(\nu)} \leq R_{n,m}^{(\nu)} \leq R_{n,u}^{(\nu)} + R_{u,m}^{(\nu)} \leq 2R_{max}^{(\nu)}$$

$$\text{for } \Lambda_{n}^{(\nu)} \neq \Lambda_{m}^{(\nu)} \qquad (6.15)$$

If the individual paths are non-trivial (each relative shielding order being at least 1) then we have for sublayers all in the νth set

Theorem 6.6: Bounds on relative shielding order along a sequence of non-trivial connected paths with all end sublayers in the νth set

$$\text{greater of } \left[R_{p_1,p_{q_{max}}}^{(\nu)} , (q_{max} - 1)R_{min}^{(\nu)} \right]$$

$$\leq \sum_{q=1}^{q_{max}-1} R_{p_q;p_{q+1}}^{(\nu)} \leq (q_{max} - 1)R_{max}^{(\nu)} \qquad (6.16)$$

$$R_{min}^{(\nu)} \geq 1$$

Note now that it is quite possible for $R_{min}^{(\nu)}$ to be assigned arbitrary positive integer values by appropriate definition of sublayers not in the νth subset to provide at least as many subshields as desired between each pair in the νth subset of sublayers. This inclusion of $R_{min}^{(\nu)}$ in the lower bound makes it possible to raise the lower bound. The special case of three sublayers now becomes

Corollary: Bounds on relative shielding order among three distinct sublayers in the νth set

$$\text{greater of } \left[R_{n,m}^{(\nu)} , 2R_{min}^{(\nu)}\right]$$

$$\leq R_{n,u}^{(\nu)} + R_{u,m}^{(\nu)} \leq 2R_{max}^{(\nu)} \qquad (6.17)$$

which again gives some design flexibility by appropriate choices in the lower bound.

Letting the νth set be the primary sublayers in our example (fig. 2.1 and table 4.1) (6.17) is satisfied with

$$R_{1,2}^{(1)} = 2 \quad , \quad R_{1,3}^{(1)} = 3 \quad , \quad R_{2,3}^{(1)} = 3$$

$$R_{min}^{(1)} = 2 \quad , \quad R_{max}^{(1)} = 3 \qquad (6.18)$$

Note that this example provides three separate instances of satisfying (6.17) based on the three possible choices of $R_{n,m}^{(\nu)}$ $(=R_{m,n}^{(\nu)})$.

Our example shows then that it is possible to choose $R_{min}^{(1)} > 1$, and by extension as large as we want. An easy way to do this, as in fig. 2.1, is to place the primary sublayers out at the "ends" of the tree graph by extending the paths from such primary sublayers to other primary sublayers to as large a relative shielding order as desired.

VII. Inversion of Electromagnetic Topology

An interesting aspect of electromagnetic topology is its properties under spatial inversion. Considering our example topology in fig. 2.1, let us choose a point within some sublayer, say $V_{3,1}^{(1)}$ as in fig. 7.1A, and perform an inversion to give the topology in fig. 7.1B. While this latter topology may look different from the former they share some important features so that they may be considered equivalent. In particular they both have the same dual bipartite graph (interaction sequence diagram). The two topologies are thus in one-to-one correspondence with the graph and with each other. Of course, one could have taken the inversion point in any sublayer to make that sublayer become the outside sublayer, giving five possible equivalent topologies for our example. In general with N sublayers then there are N electromagnetic topological diagrams equivalent to each other in the above sense.

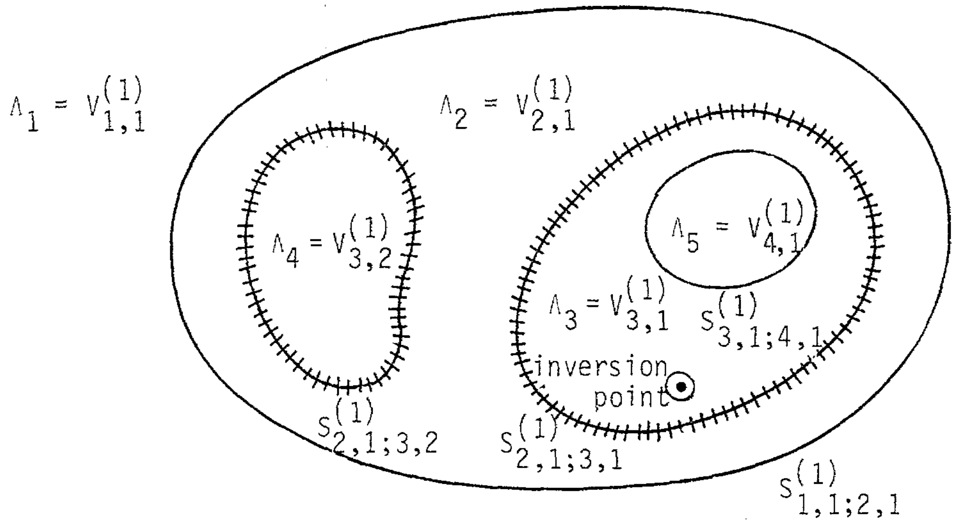
To illustrate some of the transformations under inversion consider table 7.1. In this table there are some notational features. A superscript is used to designate volumes and surfaces in the original topology (1) and after inversion (2). Note that, as exhibited by this example, the shielding order of the system, $\lambda_{max} - 1$, [2,5] is in general not conserved under inversion. Futhermore, layers (or principal volumes) are not necessarily layers after inversion. Layers consist of one or more sublayers (unconnected volumes in the same layer); in the case of two or more sublayers these sublayers are called proper sublayers. Similarly, shields (or principal surfaces) are not necessarily shields after inversion. Shields consist of one or more subshields (unconnected closed surfaces in the same shield); in the case of two or more subshields these are called proper subshields.

Note, however, that we have:

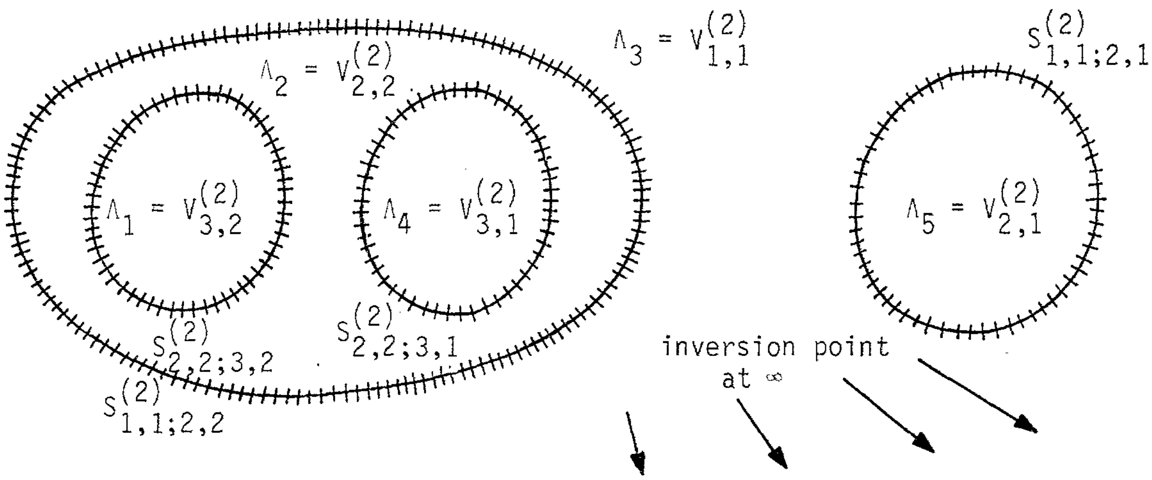
<u>Theorem 7.1</u>: Conservation of relative shielding order

> Relative shielding order is conserved under inversion of the volume/surface topology defined to sublayer level.

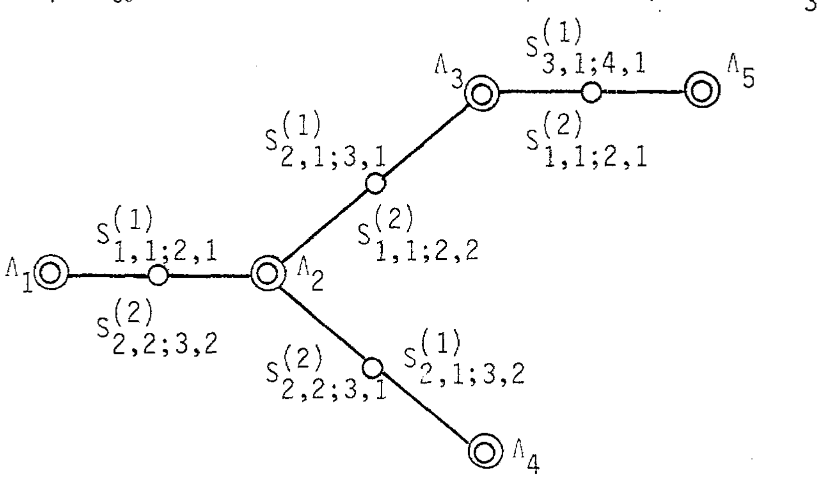
This result follows from the fact that the inversion of the electromagnetic topology does not alter the dual graph (interaction sequence diagram). Since the vertices in this graph are the sublayers and the sublayers and their connections (subshields) to other sublayers are mapped one to one under inversion, then the number of subshields on a path (unique) from one sublayer to another is conserved. This is the relative shielding order.



A. Original electromagnetic topology



B. Topology after inversion with respect to point in Λ_3



C. Dual bipartite graph (interaction sequence diagram)

Fig. 7.1. Inversion of Electromagnetic Topology

Sublayer	Original Topology		Topology After Inversion	
	Designation	Layer or Proper Sublayer	Designation	Layer or Proper Sublayer
Λ_1	$V^{(1)}_{1,1}$	Layer	$V^{(2)}_{3,2}$	Proper Sublayer
Λ_2	$V^{(1)}_{2,1}$	Layer	$V^{(2)}_{2,2}$	Proper Sublayer
Λ_3	$V^{(1)}_{3,1}$	Proper Sublayer	$V^{(2)}_{1,1}$	Layer
Λ_4	$V^{(1)}_{3,2}$	Proper Sublayer	$V^{(2)}_{3,1}$	Proper Sublayer
Λ_5	$V^{(1)}_{4,1}$	Layer	$V^{(2)}_{2,1}$	Proper Sublayer
Subshield	Designation	Shield or Proper Subshield	Designation	Shield or Proper Subshield
$\Lambda^+_1 \cap \Lambda^+_2$	$S^{(1)}_{1,1;2,1}$	Shield	$S^{(2)}_{2,2;3,2}$	Proper Subshield
$\Lambda^+_2 \cap \Lambda^+_3$	$S^{(1)}_{2,1;3,1}$	Proper Subshield	$S^{(2)}_{1,1;2,2}$	Proper Subshield
$\Lambda^+_2 \cap \Lambda^+_4$	$S^{(1)}_{2,1;3,2}$	Proper Subshield	$S^{(2)}_{2,2;3,1}$	Proper Subshield
$\Lambda^+_3 \cap \Lambda^+_5$	$S^{(1)}_{3,1;4,1}$	Shield	$S^{(2)}_{1,1;2,1}$	Proper Subshield
Shielding Order of System $\lambda_{max} - 1$	3		2	

Table 7.1 Transformation of Electromagnetic Topology Under Inversion

A special case of interest occurs if the inversion point is located on one of the subshields. In this case the associated subshield under inversion extends to infinity, dividing euclidean space in this special sense. This can be considered a generalization of the inversion procedure for which the dual graph and conservation of relative shielding order still apply.

VIII. Uniform Relative Shielding Order for Primary Sublayers

Now we come to a very interesting application of the relative-shielding-order concept. In designing the electromagnetic topology for a system one could try to specify the relative shielding orders between some or all the pairs of some set of sublayers of particular interest. Denote this set, without loss of generality, as the primary sublayers $\{\Lambda_n^{(1)}\}$.

Let us consider the case that the shielding between one primary sublayer and another is in some sense uniform [4]. While one can in principal realize subshields with somewhat arbitrary attenuations, this case illustrates some of the power of topological concepts while presenting an example of some practical interest. A basic result is:

Theorem 8.1: Uniform Relative Shielding Order for Primary Sublayers

If we have:

1) Three or more distinct primary sublayers $\Lambda_n^{(1)}$, i.e.,

$$n = 1,2,\ldots,n_{max}(1)$$

with $n_{max}(1) \geq 3$

2) Uniform relative shielding order among all pairs of the $\{\Lambda_n^{(1)}\}$, i.e.,

$$R_{n,m}^{(1)} = R \quad \text{independent of } n,m$$

with $n,m = 1,2,\ldots,n_{max}(1)$, but $n \neq m$

Then:

$$\begin{aligned} &R = \text{even} \\ &\quad \text{and} \\ &\text{the smallest non-trivial } R = 2. \end{aligned} \tag{8.1}$$

To prove this, note that $R \geq 0$ and hence R is a non-negative even or odd integer. Consider both cases.

Case 1: R = even

A solution exists for this case by exhibition as in fig. 8.1. This shows a tree graph (the dual graph for electromagnetic topology defined to the sublayer level). In this example we take one secondary volume $\Lambda_1^{(2)}$ as the

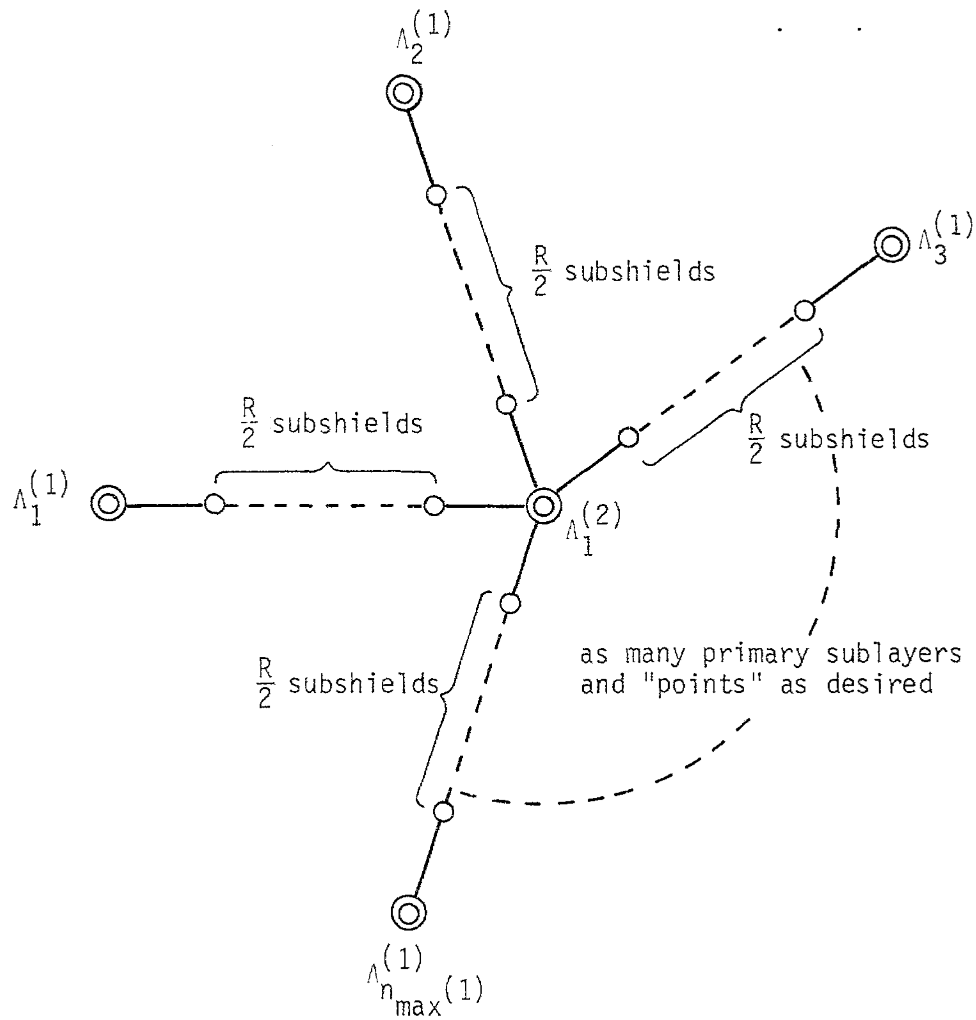


Fig. 8.1. Uniform Relative Shielding Order for R = Even: Star Graph

"center" (a particular vertex) in this special form of tree graph which we might call a star graph. This graph has $n_{max}(1)$ "points" corresponding to paths from the "center" to each primary sublayer. Each such path has an integer number of subshields and an integer number of sublayers (secondary etc. sublayers). The relative shielding order from one primary sublayer to another is R and from any primary sublayer to the "center" is R/2. If R is even then R/2 is an integer and the conditions are satisfied.

Case 2: R = odd

No solution exists for this case as is illustrated in fig. 8.2. Begin with $\Lambda_1^{(1)}$ and $\Lambda_2^{(1)}$ and connect them by a path with relative shielding order R = odd. Next connect a path from another primary volume $\Lambda_3^{(1)}$ to some sublayer on the first path denoted Λ_a which may or may not be one of the end sublayers of the first path. Note only one such path from $\Lambda_3^{(1)}$ to the first path is allowed since the graph must be a tree graph and paths between any two sublayers must be unique. Now the relative shielding order from $\Lambda_1^{(1)}$ to Λ_a (say R_1') plus that from Λ_a to $\Lambda_2^{(1)}$ (say R_2') must give (by our construction of the first path)

$$R_{1,2}^{(1)} = R_1' + R_2' \equiv R = \text{odd} \tag{8.2}$$

By the introduction of the path from $\Lambda_3^{(1)}$ to Λ_a with relative shielding order R_3' we have

$$R_{1,3}^{(1)} = R_1' + R_3'$$
$$R_{2,3}^{(1)} = R_2' + R_3' \tag{8.3}$$

Subtracting

$$R_{1,3}^{(2)} - R_{2,3}^{(1)} = R_1' - R_2' \tag{8.4}$$

Now since

$$2R_2' = \text{even} \tag{8.5}$$

then

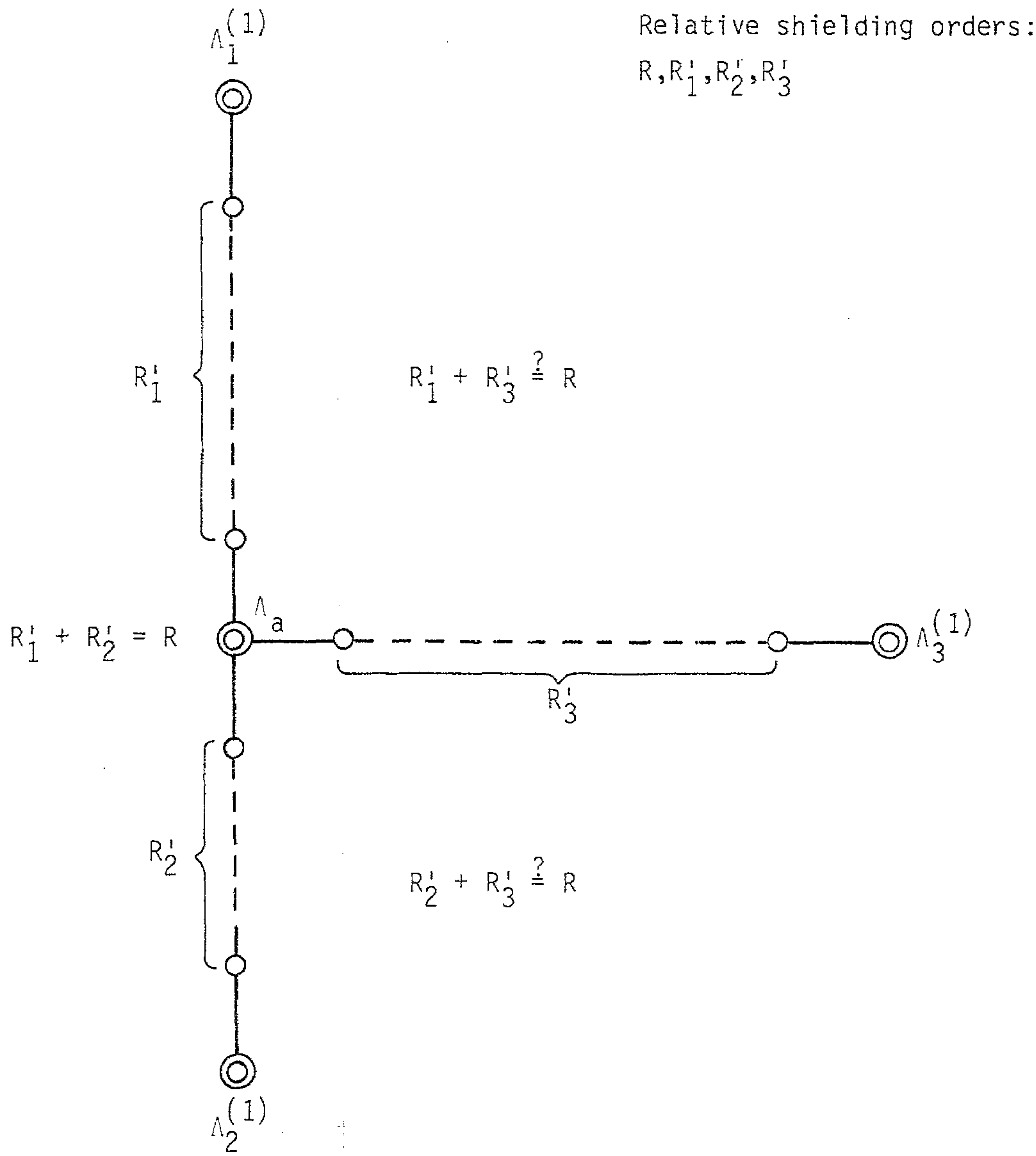


Fig. 8.2. Uniform Relative Shielding Order for R = Odd

$$R_1' + R_2' - 2R_2' = R_1' - R_2' = \text{odd} - \text{even} = \text{odd} \qquad (8.6)$$

Hence

$$R_1' - R_2' \neq 0 \qquad (8.7)$$

and

$$R_{1,3}^{(1)} - R_{2,3}^{(1)} \neq 0 \qquad (8.8)$$

or

$$R_{1,3}^{(1)} \neq R_{2,3}^{(1)} \qquad (8.9)$$

Thus

$$R_{1,3}^{(1)} \neq R \quad \text{or} \quad R_{2,3}^{(1)} \neq R \qquad (8.10)$$

and not all three relative shielding orders can be R. Therefore, three primary sublayers cannot satisfy all relative shielding orders equal to the same odd integer R and the case of $n_{max}(1) = 3$ is impossible.

Can we have $n_{max}(1) > 3$ with uniform relative shielding order? Add sequentially additional primary sublayers $\Lambda_4^{(1)}$ etc. with connecting paths to the tree graph which do not destroy the tree property. Since the paths among $\Lambda_1^{(1)}$, $\Lambda_2^{(1)}$, and $\Lambda_3^{(1)}$ are then not changed (the paths being unique in a tree graph), then the addition of these additional primary sublayers does not allow $R_{1,2}^{(1)}$, $R_{1,3}^{(1)}$, and $R_{2,3}^{(1)}$ to be all equal, and hence the relative shielding orders for four or more primary sublayers cannot all be equal.

Hence, odd R for $n_{max}(1) \geq 3$ is impossible. Non-negative even R are all possible. The smallest non-trivial possible R is 2.

Q.E.D.

IX. Examples of Uniform Relative Shielding Order

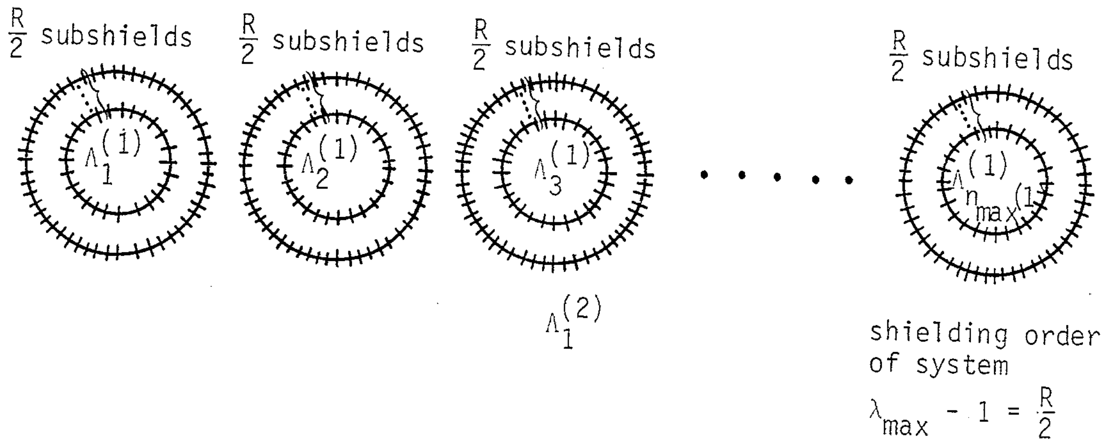
Corresponding to this case of uniform relative shielding order there are various volume/surface topological diagrams. Figure 9.1A shows the symmetric case corresponding to the graph in fig. 8.1. Each primary sublayer is individually surrounded by R/2 subshields. The exterior sublayer $\Lambda_1^{(2)}$ is a secondary sublayer as are the other sublayers separating the subshields surrounding each primary volume. The shielding order of this system topology is $\lambda_{max} - 1 = R/2$.

Now perform an inversion about a point in $\Lambda_1^{(1)}$. This gives the diagram in fig. 9.1B in which one of the primary sublayers is now the exterior. Primary sublayers $\Lambda_2^{(1)}$ through $\Lambda_{n_{max}(1)}^{(1)}$ have R/2 subshields surrounding each separately and one subset of R/2 subshields surrounding all of these together, these latter R/2 subshields being shields as well. The secondary sublayer $\Lambda_1^{(2)}$ now assumes an intermediate role in the hierarchical shielding topology. The shielding order of this system topology is $\lambda_{max} - 1 = R$.

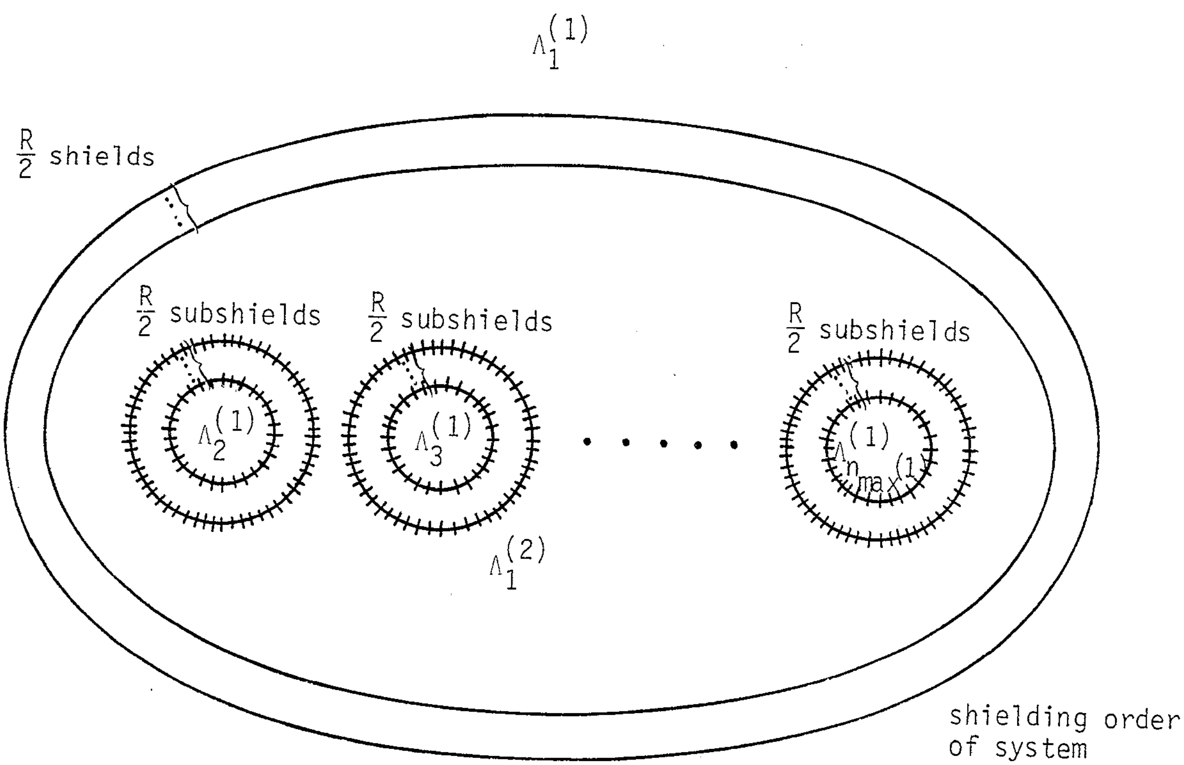
Note that the latter example corresponds to some practical conditions. The exterior sublayer (also layer) $\Lambda_1^{(1)}$ corresponds to the location of various important electromagnetic interference sources such as EMP, lightning, radar, etc. The interior primary sublayers can be used to separately contain strong interference sources (transmitting equipment, etc.), equipment susceptible to interference (computers, etc.), and equipment carrying signals that one does not wish to be detected in $\Lambda_1^{(1)}$ (secure communications, etc.).

Since the smallest non-trivial uniform relative shielding order is 2, we have the interesting and practical case shown in fig. 9.2 with $\Lambda_1^{(1)}$ taken again as the exterior sublayer (also layer). Primary sublayers $\Lambda_2^{(1)}$ through $\Lambda_{n_{max}(1)}^{(1)}$ have individual subshields around each. These subshields might be realized by cable shields, box shields, perhaps buffer circuits, as well as filters and limiters at penetrations. Another common subshield (also shield) surrounds all of these. This outer shield might be realized by structural metal (as in an aircraft skin), bulkheads, conduits, and various penetration protective devices.

In this latter example the secondary sublayer (also layer) $\Lambda_1^{(2)}$ can assume a special role. It, of course, separates subshields so as to give a uniform relative shielding order for primary sublayers R=2. In addition, however, $\Lambda_1^{(2)}$ is shielded from the other sublayers (the primary sublayers) by a uniform relative shielding order R=1. So suppose one has some class of



A. Symmetric case



B. Case that the exterior is one of the primary sublayers

Fig. 9.1. Example Topological Diagrams for Uniform Relative Shielding Order: R = Even

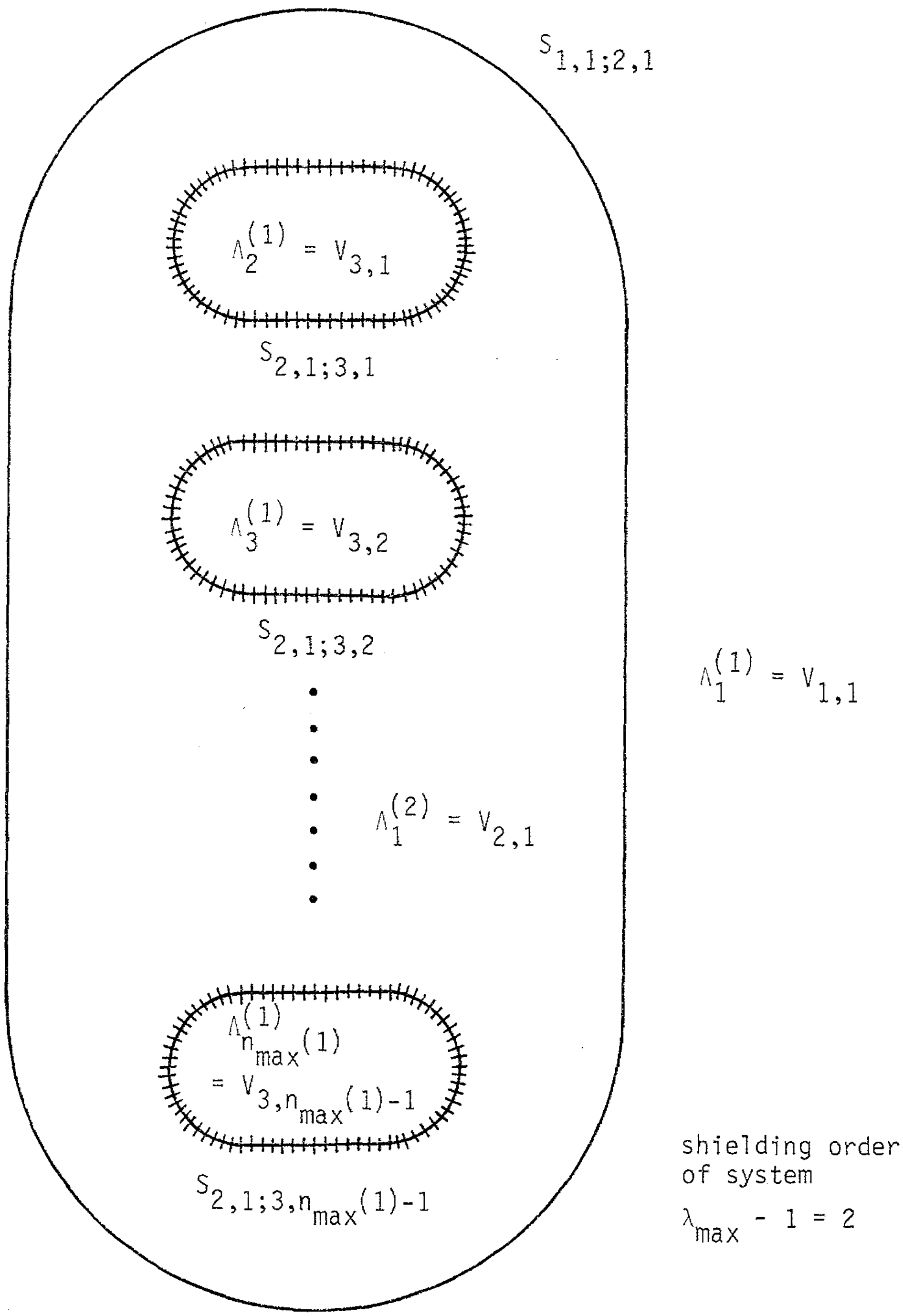


Fig. 9.2. Exterior Primary Sublayer for Smallest Non-Trivial Uniform Relative Shielding Order: R = 2

equipment which is fairly insensitive to electromagnetic interference but requires some shielding. Also this equipment may generate interference, but say only a modest amount (compared to some other sources). Then some such equipment might be placed in $\Lambda_1^{(2)}$ to achieve an efficient shielding design.

Consider the case of the electric power source and distribution in say an aircraft (not including any sensitive power control circuits). This set of robust conductors, transformers, etc. may be well suited for such a secondary sublayer. In transporting the power to any primary sublayer only one subshield (and hence one set of heavy duty filters, etc.) is needed (corresponding to R=1). If, however, this equipment is placed in another primary sublayer then two subshields (and hence two sets of filters, etc.) must be traversed by the power (corresponding to R=2). Thus the use of $\Lambda_1^{(2)}$ for power generation and distribution can reduce cost, weight, and complexity, at least in some circumstances.

X. Summary

This note has explored some aspects of qualitative electromagnetic topology. By partitioning sublayers into sets and imposing desired relative shielding orders, to the extent possible, between pairs of sublayers in various of these sets, one can synthesize various specific electromagnetic topologies appropriate to system design problems. Here some results have been presented but much extension appears possible.

Given the variety of possible assumptions concerning relative shielding orders between pairs of sublayers which may contain particular kinds of equipment with various electromagnetic interference sources and sensitivities, one may expect some interesting design concepts to be developed. The discrete nature of qualitative electromagetic topology (as in integer relative shielding order) may lead to combinatoric and group theoretic aspects to such designs. Perhaps this subject can develop into one of topological synthesis.

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