Interaction Notes

Note 406

March 1981

Effect of Environment Conductivity on the Electromagnetic Penetration of Apertures

John Lam

Dikewood Corporation
Santa Monica, California

Abstract

The nuclear electromagnetic environment of a missile underground shelter is a conducting environment. The environment conductivity affects the penetration of environment electromagnetic energy through apertures on the shelter shield into the shelter's interior. To study this effect an analytical model is formulated and solved. It is concluded that the electric field penetration is enhanced by the environment conductivity, while the magnetic field penetration is unaffected.

CONTENTS

ILLUSTRATIONS

I. INTRODUCTION

The electromagnetic shield of a missile shelter is perforated by an assortment of apertures. These apertures are essential to the operation of the shelter system, and serve a variety of functions such as communication, power supply, access and ventilation. They are, however, imperfections of the shelter shield, and, in the event of a nuclear attack, become inadvertent points of entry for electromagnetic energy in the nuclear environment into the shelter's interior. To ensure the survival of the shelter system in the nuclear environment, the shelter shield must be adequately hardened to reduce the aperture penetration to a safe level.

The acquisition of quantitative data on the phenomenon of aperture penetration is essential to the hardness design and validation of the shelter shield to a specified level. There has already been a considerable amount of studies on aperture penetration performed under various programs of hardening military aircraft against the nuclear electromagnetic pulse (EMP). The results of these studies, however, are not immediately applicable to the missile shelter. The reason is that the type of nuclear environment the shelter shield is expected to be exposed to is the source region. The source region is a conducting environment whereas the EMP environment of military aircraft in flight is generally not. If the shelter shield aperture is above ground, the conducting environment is the air ionized by the nuclear radiation. If the aperture is underground, the conducting environment is the soil with or without ionization by the nuclear environment. In studying the source-region electromagnetic penetration of apertures on the missile shelter shield, one must therefore consider the problem of penetration from an external conducting environment into a non-conducting interior.

Despite the difference between the two types of penetration, there is reason to hope that, under certain circumstances, the existing data of aperture penetration from a non-conducting environment can be utilized, after appropriate modification, to describe aperture penetration from a conducting environment. The present study is performed to examine this possibility.

In this study one aims at deriving rigorous basic results pertaining to aperture penetration from a conducting medium. The approach is to formulate and solve an analytical model typifying the penetration. By comparing the analytical solution with that for penetration from a non-conducting medium, one can establish contact between the results for the two penetration situations.

II. ANALYTICAL MODEL

Figure 1 illustrates a typical configuration for the penetration of source-region electromagnetic energy into a missile shelter's interior by way of an aperture on the shelter shield. The shelter is depicted as a horizontal underground shelter buried entirely in the soil. The source region of a nuclear explosion nearby can be characterized by an air conductivity σ_{air}, a Compton current $\underline{J}^c$, and the electromagnetic fields $\underline{E}$ and $\underline{H}$ in the air above the ground. The electromagnetic fields can penetrate into the ground and interact with the conducting shelter shield. As a consequence of this interaction the fields furthermore penetrate the aperture on the shield and leak into the shelter's interior.

The dimensions of the aperture are in general much smaller than those of the shelter. In calculating the leakage electromagnetic fields inside the shelter, it becomes advantageous to decompose the calculation into two separate problems. The first problem is the external interaction problem of the shelter shield. In this problem the aperture is considered to be completely covered up so that the shelter shield can be regarded as a perfect shield. One then calculates the response of this perfect shield to the source-region electromagnetic environment. The response takes the form of induced charges and currents on the shield's outer surface. One can determine the total electromagnetic fields $\underline{E}_{sc}$ and $\underline{H}_{sc}$ at the site of the aperture. These are called the short-circuit fields at the aperture because they are determined with the aperture covered up or short-circuited.

The second problem is the aperture penetration problem. Here one considers the short-circuit fields and the ground conductivity as given, and one calculates the penetration of the short-circuit fields into the shelter's interior when the aperture is opened up. In the present study one will be concerned exclusively with the aperture penetration problem.

The aperture penetration problem will be studied with an analytical model shown in Figure 2. In this model the shelter shield is modeled by a conducting screen coinciding with the xy plane of a rectangular coordinate system. On the upper side of the screen ($z > 0$), one has a conducting medium

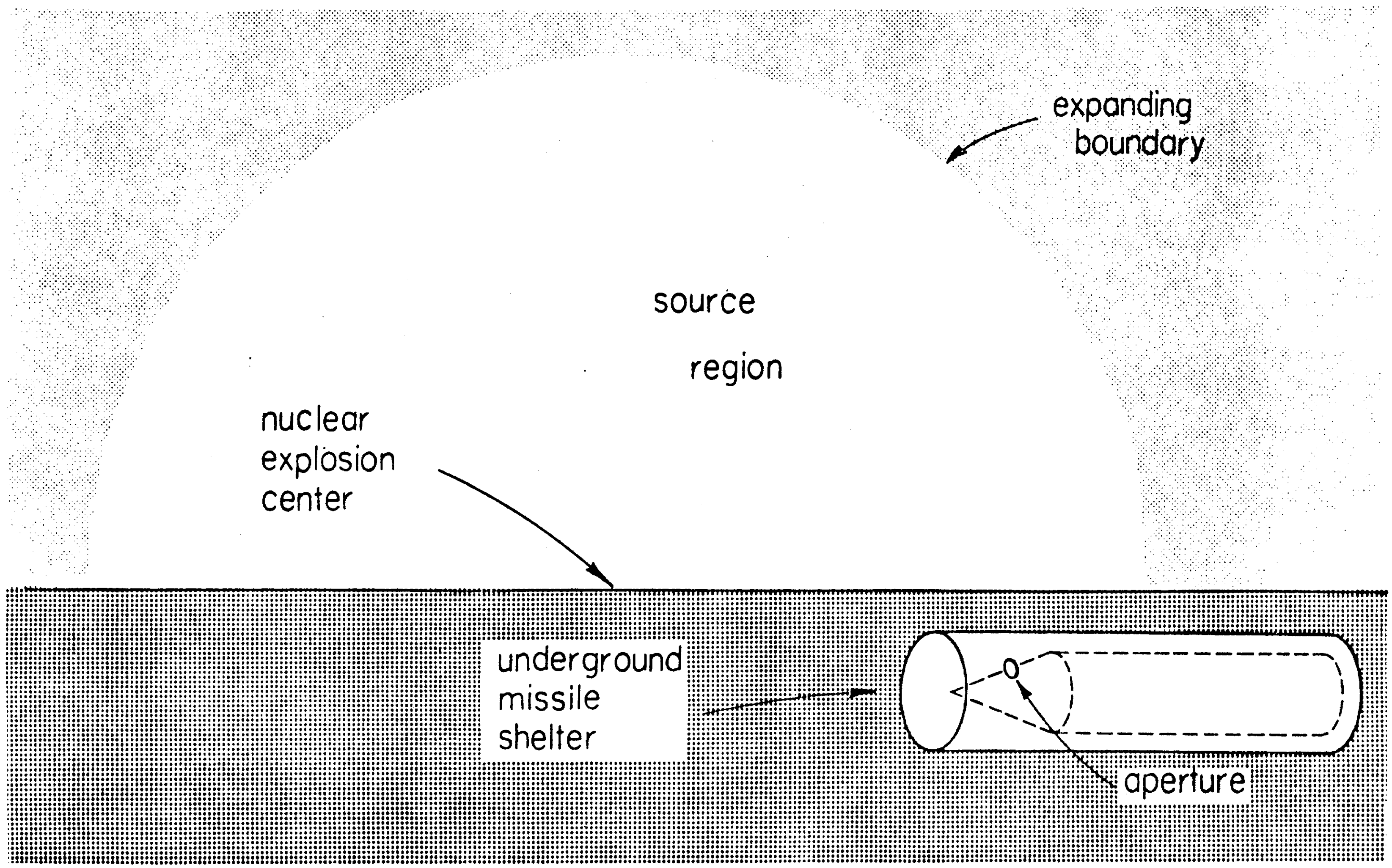

Figure 1. A missile underground shelter in the nuclear source-region environment. The electromagnetic energy in the source region can penetrate into the shelter's interior through any aperture on the shelter shield.

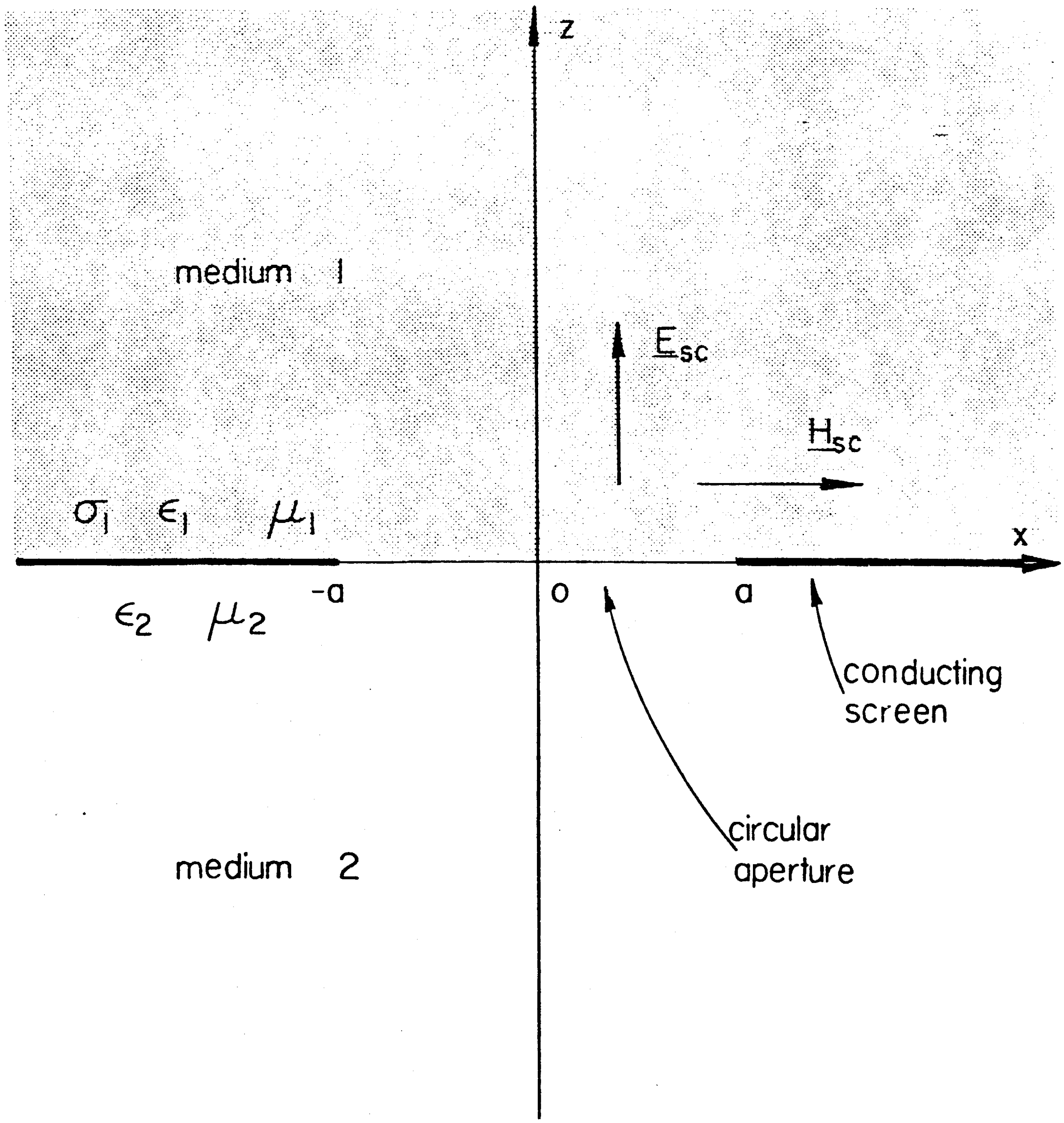

Figure 2. An analytical model for studying the effect of environment conductivity on the electromagnetic penetration of an aperture.

characterized by a conductivity σ_1, a dielectric constant ε_1 and a permeability μ_1. It will be referred to as medium 1. On the under side of the screen ($z < 0$), one has a non-conducting medium ($\sigma_2 = 0$) characterized by a dielectric constant ε_2 and a permeability μ_2. It will be referred to as medium 2.

Suppose there exists in medium 1 a given electromagnetic field of angular frequency ω and field vectors $\underline{E}_{sc}$ and $\underline{H}_{sc}$, satisfying the Maxwell equations in medium 1. At the surface of the screen ($z = 0$), $\underline{E}_{sc}$ is of necessity normal to the screen and $\underline{H}_{sc}$ is tangential to it. For the purpose of studying aperture penetration one can consider the fields to be uniform and put

$$\underline{E}_{sc} = E_{sc}\hat{e}_z$$

$$\underline{H}_{sc} = H_{sc}\hat{e}_x$$

The aperture penetration problem consists in calculating the electromagnetic fields leaking into medium 2 when a circular aperture of radius a and centered at the coordinate origin is opened up on the conducting screen.

The results of the calculation are summarized in Section III. The calculation is detailed in Appendices A, B and C.

III. SUMMARY OF RESULTS

Consider the aperture penetration problem depicted in Figure 2. The penetration is quasi-static if the radius a of the circular aperture is much smaller than the skin depth δ of the external conducting medium 1 defined by

$$\delta = \sqrt{\frac{2}{\omega \mu_1 \sigma_1}}$$

For σ_1 = 0.01 mho/m and a frequency of 1 MHz, δ is about 5 m. In the following the quasi-static criterion is assumed satisfied.

Results for $\sigma_1 = 0$

If medium 1 is non-conducting ($\sigma_1 = 0$), the electric field $\underline{E}_2$ and magnetic field $\underline{H}_2$ penetrating into medium 2 are given by

$$\underline{E}_2 = - \nabla V_2$$

$$\underline{H}_2 = - \nabla U_2$$

where

$$V_2 = - E_{sc} z \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2} \frac{2}{\pi} \left(\tan^{-1} \frac{1}{\zeta} - \frac{1}{\zeta} \right)$$

$$U_2 = - H_{sc} x \frac{\mu_1}{\mu_1 + \mu_2} \frac{2}{\pi} \left(\tan^{-1} \frac{1}{\zeta} - \frac{\zeta}{1 + \zeta^2} \right)$$

and ζ is defined by

$$\zeta = \left(\frac{r^2 - a^2 + \sqrt{(r^2 - a^2)^2 + 4a^2 z^2}}{2a^2} \right)^{\frac{1}{2}}$$

with

$$r^2 = x^2 + y^2 + z^2$$

At large distances from the aperture ($r >> a$) the fields are dipole fields:

$$V_2 \simeq \frac{1}{4\pi\varepsilon_2}\frac{\underline{p}\cdot\underline{r}}{r^3}$$

$$U_2 \simeq \frac{1}{4\pi}\frac{\underline{m}\cdot\underline{r}}{r^3}$$

where

$$\underline{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$$

The dipole moments are given by

$$\underline{p} = \frac{8}{3}\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2}a^3\underline{E}_{sc}$$

$$\underline{m} = -\frac{16}{3}\frac{\mu_1}{\mu_1 + \mu_2}a^3\underline{H}_{sc}$$

Results for $\sigma_1 \neq 0$

If medium 1 is conducting ($\sigma_1 \neq 0$), the electric field $\underline{E}_2$ and magnetic field $\underline{H}_2$ penetrating into medium 2 are given by

$$\underline{E}_2(\sigma_1 \neq 0) = \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1}\underline{E}_2(\sigma_1 = 0)$$

$$\underline{H}_2(\sigma_1 \neq 0) = \underline{H}_2(\sigma_1 = 0)$$

The electric and magnetic dipole moments of the far fields ($r >> a$) are given by

$$\underline{p}(\sigma_1 \neq 0) = \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1}\,\underline{p}(\sigma_1 = 0)$$

$$\underline{m}(\sigma_1 \neq 0) = \underline{m}(\sigma_1 = 0)$$

Conclusion

For the same external short-circuit electric and magnetic fields, the electric field penetration of an aperture is enhanced by a factor $(\varepsilon_1 + \varepsilon_2)/\varepsilon_1$ when the external medium is made conducting, while the magnetic field penetration remains unchanged.

This conclusion is reached through a quasi-static analysis of penetration through a circular aperture. It is believed that the same conclusion should hold true for apertures of other shapes.

APPENDIX A

OBLATE SPHEROIDAL COORDINATES

The aperture penetration is calculated by solving Maxwell's equations separately in media 1 and 2, and matching the solutions at the aperture. In medium 1 Maxwell's equations read

$$\nabla \cdot \underline{E} = 0$$

$$\nabla \times \underline{E} = - j\omega\mu_1 \underline{H}$$

$$\nabla \cdot \underline{H} = 0$$

$$\nabla \times \underline{H} = (\sigma_1 + j\omega\varepsilon_1)\underline{E} \tag{1}$$

In medium 2 they read

$$\nabla \cdot \underline{E} = 0$$

$$\nabla \times \underline{E} = - j\omega\mu_2 \underline{H}$$

$$\nabla \cdot \underline{H} = 0$$

$$\nabla \times \underline{H} = j\omega\varepsilon_2 \underline{E} \tag{2}$$

To solve Maxwell's equations for the circular aperture geometry, it is advantageous to introduce the dimensionless oblate spheriodal coordinates ξ,η and ϕ . They are related to the rectangular coordinates x,y and z through the equations

$$x = a\sqrt{(1+\xi^2)(1-\eta^2)}\ \cos\phi$$

$$y = a\sqrt{(1+\xi^2)(1-\eta^2)}\ \sin\phi$$

$$z = a\xi\eta \tag{3}$$

Their ranges are

$$-\infty < \xi < \infty$$

$$0 \leq \eta \leq 1$$

$$0 \leq \phi \leq 2\pi \tag{4}$$

As shown in Figure 3 the coordinate surfaces of constant ξ are confocal oblate spheroids. The coordinate surfaces of constant η are confocal hyperboloids of one sheet. In particular the circular aperture is the coordinate surface $\xi = 0$; the conducting screen is the coordinate surface $\eta = 0$. The z axis is given by $\eta = 1$. The upper half-space $z > 0$ corresponds to $\xi > 0$, and the lower half-space $z < 0$ corresponds to $\xi < 0$.

In the oblate spheroidal coordinates the Laplace operator is given by

$$\nabla^2 W = \frac{1}{a^2(\xi^2+\eta^2)}\left[\frac{\partial}{\partial\xi}(1+\xi^2)\frac{\partial W}{\partial\xi} + \frac{\partial}{\partial\eta}(1-\eta^2)\frac{\partial W}{\partial\eta} + \frac{\xi^2+\eta^2}{(1+\xi^2)(1-\eta^2)}\frac{\partial^2 W}{\partial\phi^2}\right] \tag{5}$$

The general solution of the Laplace equation

$$\nabla^2 W = 0 \tag{6}$$

is of the form

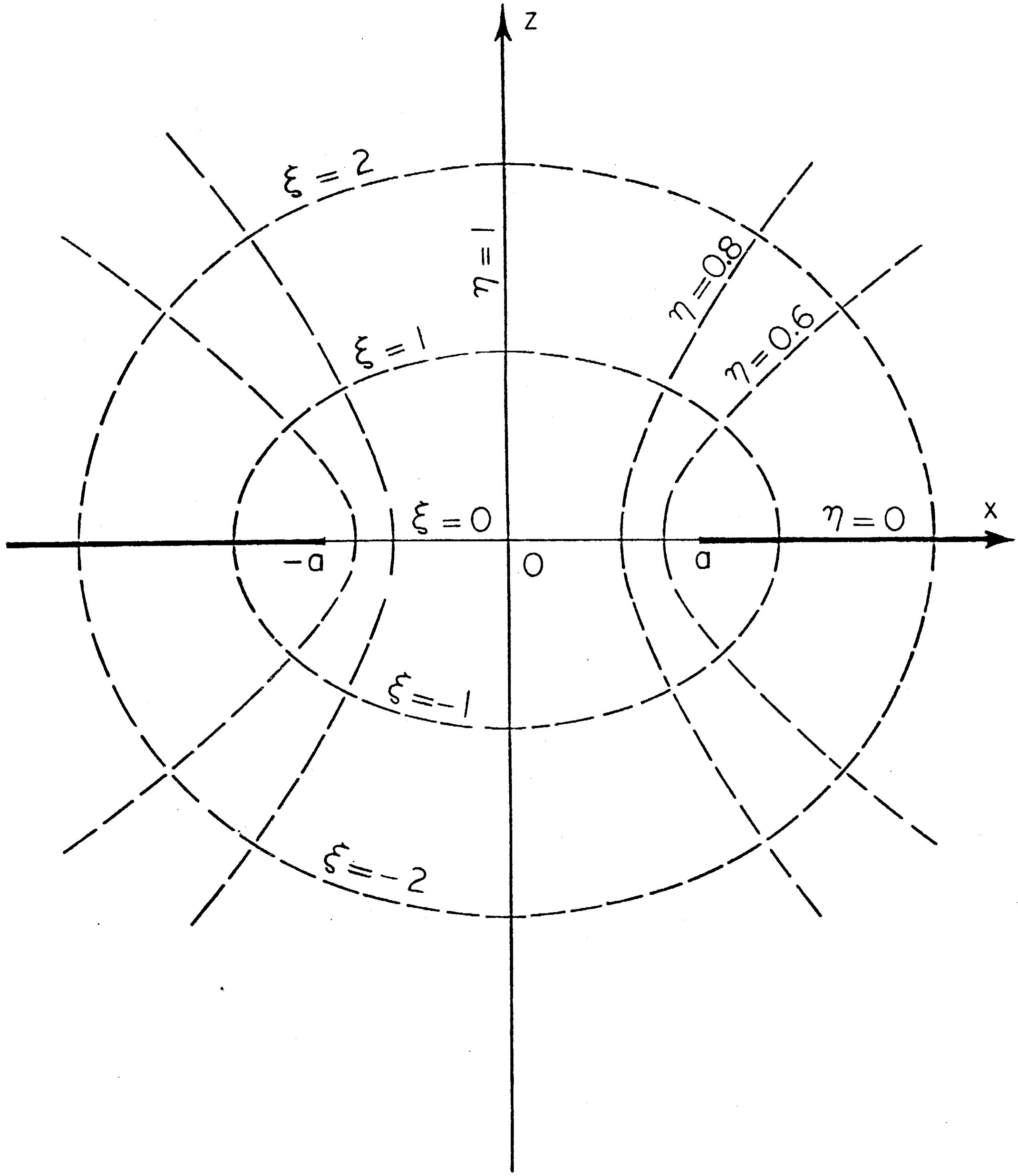

Figure 3. Oblate spheroidal coordinate system.

$$W = \sum_{n=o}^{\infty} \sum_{m=o}^{n} \left[A_{nm} P_n^m (j\xi) + B_{nm} Q_n^m (j\xi) \right] \cdot$$

$$\cdot P_n^m(\eta) \left[C_m \cos m\phi + D_m \sin m\phi \right] \tag{7}$$

where P_n^m and Q_n^m are the associated Legendre functions. In particular

$$P_1(j\xi) = j\xi$$

$$Q_1(j\xi) = \xi \tan^{-1} \frac{1}{\xi} - 1$$

$$P_1(\eta) = \eta \tag{8}$$

and

$$P_1^1(j\xi) = j\sqrt{1+\xi^2}$$

$$Q_1^1(j\xi) = \sqrt{1+\xi^2} \left(\tan^{-1} \frac{1}{\xi} - \frac{\xi}{1+\xi^2} \right)$$

$$P_1^1(\eta) = \sqrt{1-\eta^2} \tag{9}$$

The branch of the arc tangent is to be chosen to make

$$0 \leq \tan^{-1} \frac{1}{\xi} \leq \pi \tag{10}$$

APPENDIX B

ELECTRIC FIELD PENETRATION

Apertures are as a rule small. The smallness of an aperture is measured by the ratio of the linear dimension of the aperture to the skin depth of the external conducting medium 1 at the frequencies of the nuclear electromagnetic fields. The skin depth δ is given by the expression

$$\delta = \sqrt{\frac{2}{\omega\mu_1\sigma_1}} \tag{11}$$

For σ = 0.01 mho/m and a high frequency of 1 MHz, δ comes out to be about 5m, which is much greater than the size of apertures. The aperture penetration can be considered quasi-static.

In the quasi-static penetration of an aperture the penetration by the electric and the magnetic field can be analyzed independently. The electric field penetration is described by the pair of quasi-static Maxwell's equations

$$\nabla \cdot \underline{E} = 0 \qquad \nabla \times \underline{E} = 0 \tag{12}$$

in both media 1 and 2. The electric field can therefore be derived from an electrostatic potential V:

$$\underline{E} = - \nabla V \tag{13}$$

which satisfies the Laplace equation:

$$\nabla^2 V = 0 \tag{14}$$

Let the total electric fields and potentials in media 1 and 2 be denoted by $\underline{E}_1$, V_1 and $\underline{E}_2$, V_2, respectively. On the conducting screen the boundary conditions are

$$V_1 = V_2 = 0 \tag{15}$$

At the aperture the boundary conditions are

$$V_1 = V_2 \tag{16}$$

and

$$\left(\epsilon_1 + \frac{\sigma_1}{j\omega}\right) E_{1z} = \epsilon_2 E_{2z} \tag{17}$$

In the low-frequency limit, the condition (17) becomes

$$E_{1z} = 0 \tag{18}$$

at the aperture. This boundary condition differs from that for a non-conducting medium 1. It implies that the conduction current $\sigma_1 \underline{E}_1$ in medium 1 cannot flow through the aperture into the non-conducting medium 2.

The short-circuit electric field $\underline{E}_{sc}$ can be derived from a potential V_{sc}:

$$\begin{aligned} V_{sc} &= - \underline{E}_{sc} z \\ &= - E_{sc}\, a\, \xi\, \eta \\ &= j\, E_{sc}\, a\, P_1(j\xi) P_1(\eta) \end{aligned} \tag{19}$$

in terms of the oblate spheriodal coordinates. The total potentials V_1 and V_2 can be expanded as follows:

$$\begin{aligned} V_1 &= \left[AP_1(j\xi) + BQ_1(j\xi) \right] P_1(\eta) \\ V_2 &= \left[CP_1(j\xi) + DQ_1(j\xi) \right] P_1(\eta) \end{aligned} \tag{20}$$

These expansions automatically satisfy the boundary conditions (15) on the screen (z=0). The expansion coefficients A,B,C and D are determined by imposing the aperture boundary conditions (16) and (18), and the asymptotic boundary conditions:

$$\begin{aligned} V_1 &\to V_{sc} \qquad \text{as } \xi \to \infty \\ V_2 &\to 0 \qquad \text{as } \xi \to -\infty \end{aligned} \tag{21}$$

The results are

$$V_1 = E_{sc}\, a \left[jP_1(j\xi) + \frac{2}{\pi} Q_1(j\xi) \right] P_1(\eta)$$

$$V_2 = 2E_{sc}\, a \left[jP_1(j\xi) + \frac{1}{\pi} Q_1(j\xi) \right] P_1(\eta) \tag{22}$$

One can rewrite V_2 in the form

$$V_2 = - E_{sc} z \frac{2}{\pi} \left(\tan^{-1} \frac{1}{|\xi|} - \frac{1}{|\xi|} \right) \tag{23}$$

where $|\xi|$ is related to the rectangular coordinates through the equation

$$|\xi| = \left(\frac{r^2 - a^2 + \sqrt{(r^2 - a^2)^2 + 4a^2 z^2}}{2a^2} \right)^{\frac{1}{2}} \tag{24}$$

with

$$r^2 = x^2 + y^2 + z^2 \tag{25}$$

At large distances from the aperture ($r \gg a$), the field $\underline{E}_2$ penetrating into medium 2 has the form of a dipole field:

$$V_2 \simeq \frac{1}{4\pi\varepsilon_2} \frac{\underline{p} \cdot \underline{r}}{r^3} \tag{26}$$

The dipole moment $\underline{p}$ is given by

$$\underline{p} = \frac{8}{3} \varepsilon_2 a^3 \underline{E}_{sc} \tag{27}$$

If medium 1 had been non-conducting ($\sigma_1 = 0$), the electrostatic potential V_2 in medium 2 would have been equal to

$$V_2 = - E_{sc} z \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2} \frac{2}{\pi} \left(\tan^{-1} \frac{1}{|\xi|} - \frac{1}{|\xi|} \right) \tag{28}$$

Comparing equation (28) with equation (23), one concludes that the effect of the conductivity of medium 1 on the quasi-static electric field penetrating into medium 2 is to modify the field amplitude in medium 2 by a factor of $(\varepsilon_1 + \varepsilon_2)/\varepsilon_1$, the field pattern in medium 2 remaining the same.

Note that the conductivity σ_1 does not appear explicitly in the expression for V_2. The effect of environment conductivity enters indirectly through the new aperture boundary condition (18).

APPENDIX C

MAGNETIC FIELD PENETRATION

In the quasi-static limit the Maxwell equations for the magnetic field in medium 1 are

$$\nabla \cdot \underline{H}_1 = 0$$

$$\nabla \times \underline{H}_1 = \sigma_1 \underline{E}_1 \tag{29}$$

They differ from the equations for a non-conducting medium in the appearance of the conduction current density $\sigma_1 \underline{E}_1$. In medium 2 the Maxwell equations are simply

$$\nabla \cdot \underline{H}_2 = 0$$

$$\nabla \times \underline{H}_2 = 0 \tag{30}$$

The electric field $\underline{E}_1$ in medium 1 has already been calculated in Appendix B. It can be written as the sum of the short-circuit electric field $\underline{E}_{sc}$ and a term $\underline{E}'$ due to the aperture:

$$\underline{E}_1 = \underline{E}_{sc} + \underline{E}' \tag{31}$$

where, by equation (22), $\underline{E}'$ is given by

$$\underline{E}' = - \nabla \left[\frac{2}{\pi} E_{sc} \; aQ_1(j\xi) \right] P_1(\eta) \tag{32}$$

Then the conduction current density in medium 1 has two parts $\sigma_1 \underline{E}_{sc}$ and $\sigma_1 \underline{E}'$. The magnetic field penetration problem can be split up into two sub-problems involving each of these two current densities.

In the first sub-problem one considers the solution of the field equations

$$\nabla \cdot \underline{H}_1 = 0$$

$$\nabla \times \underline{H}_1 = \sigma_1 \underline{E}_{sc} \tag{33}$$

Since $\sigma_1 \underline{E}_{sc}$ is the source of the short-circuit magnetic field $\underline{H}_{sc}$, this problem is identical to that of the penetration of $\underline{H}_{sc}$ through the aperture. It is solved as follows.

Around the aperture, $\underline{H}_{sc}$ can be derived from a magnetostatic potential U_{sc} :

$$\begin{aligned} U_{sc} &= - H_{sc}\, x \\ &= - H_{sc}\, a \sqrt{(1+\xi^2)(1-\eta^2)} \cos\phi \\ &= jH_{sc}\, a\, P_1^1(j\xi) P_1^1(\eta) \cos\phi \end{aligned} \tag{34}$$

in terms of the oblate spheroidal coordinates. One can similarly derive the total magnetic fields $\underline{H}_1$ and $\underline{H}_2$ in media 1 and 2 from magnetostatic potentials U_1 and U_2 :

$$\underline{H}_1 = - \nabla U_1$$

$$\underline{H}_2 = - \nabla U_2 \tag{35}$$

The potentials satisfy the Laplace equation. They can be expanded as follows:

$$U_1 = \left[A\, P_1^1(j\xi) + B\, Q_1^1(j\xi) \right] P_1^1(\eta) \cos\phi$$

$$U_2 = \left[C\, P_1^1(j\xi) + D\, Q_1^1(j\xi) \right] P_1^1(\eta) \cos\phi \tag{36}$$

These expansions automatically satisfy the boundary condition for the magnetic field at the screen (η=0):

$$\frac{\partial}{\partial \eta} U_1 = \frac{\partial}{\partial \eta} U_2 = 0 \tag{37}$$

The expansion coefficients A,B,C and D are determined by imposing the boundary conditions at the aperture ($\xi = 0$) :

$$U_1 = U_2$$

$$\mu_1 \frac{\partial}{\partial \xi} U_1 = \mu_2 \frac{\partial}{\partial \xi} U_2 \tag{38}$$

and the asyumptotic boundary conditions:

$$U_1 \rightarrow U_{sc} \qquad \xi \rightarrow \infty$$

$$U_2 \rightarrow 0 \qquad \xi \rightarrow -\infty \tag{39}$$

The results of the determination are:

$$U_1 = H_{sc}\, a \left[j\, P_1^1 (j\xi) + \frac{\mu_2}{\mu_1 + \mu_2} \frac{2}{\pi} Q_1^1 (j\xi) \right] P_1^1(\eta) \cos \phi$$

$$U_2 = 2H_{sc}\, a \frac{\mu_1}{\mu_1 + \mu_2} \left[j\, P_1^1 (j\xi) + \frac{1}{\pi} Q_1^1 (j\xi) \right] P_1^1(\eta) \cos \phi \tag{40}$$

U_2 can be rewritten as

$$U_2 = - H_{sc}\, x \frac{\mu_1}{\mu_1 + \mu_2} \frac{2}{\pi} \left[\tan^{-1} \frac{1}{|\xi|} - \frac{|\xi|}{1 + |\xi|^2} \right] \tag{41}$$

where, as before, $|\xi|$ is a function of x,y and z as given in equation (24). At large distances from the aperture ($r >> a$), U_2 assumes the form of a dipole potential :

$$U_2 \simeq \frac{1}{4\pi} \frac{\underline{m} \cdot \underline{r}}{r^3} \tag{42}$$

The dipole moment $\underline{m}$ is given by

$$\underline{m} = - \frac{16}{3} \frac{\mu_1}{\mu_1 + \mu_2} a^3 \underline{H}_{sc} \tag{43}$$

Consider now the second sub-problem dealing with the penetration of the aperture by the magnetic field generated by the conduction current density $\sigma_1 \underline{E}'$. It can be seen that this portion of the magnetic field is a higher-order contribution, and should be dropped in a quasi-static calculation. The magnitude of this magnetic field is of order $\sigma_1 E_{sc} a$. But E_{sc} is related to H_{sc} by the characteristic impedance of medium 1:

$$E_{sc} \approx \sqrt{\frac{\omega\mu_1}{\sigma_1}} H_{sc} \tag{44}$$

Consequently the magnetic field generated by $\sigma_1 \underline{E}'$ is of order $(a/\delta)H_{sc}$ which is small in the quasi-static regime.

One concludes that, in the quasi-static regime, the magnetic field penetration of the aperture is not affected by the conductivity of medium 1.