Interaction Note

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On the Further Considerations of Infinitely Long Cable and Aperture Coupled Regions

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Abstract

The electromagnetic coupling to an infinitely long cable placed behind an arbitrarily shaped aperture is discussed as a boundary value problem. Extension to multiple parallel cables behind the aperture is also treated. In the quasi-static limit, an equivalent circuit consisting of lumped elements and sources is obtained for the coupled regions.

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I. INTRODUCTION

Electromagnetic energy penetration through apertures in conducting screens and its subsequent coupling interaction with bodies behind apertures is studied recently [1] from the transient analysis point of view. Such detailed analysis based on an integral equation formulation, in principle, leads one to characterize effectively and accurately interaction between complex aperture shapes and the coupled bodies behind it.

Certain aperture coupled geometries have been analyzed in the past [2-6], specifically to mention a few, modal analysis of braided shield cables [3], excitation of a terminated TEM transmission line through small apertures [6] and excitation of an infinite length cable behind narrow slot in a conducting screen [5]. Also a wire passing by a circular aperture in an infinite ground plane is discussed in [7] which follows the formulation based on the integral equation approach similar to the one derived in [5]. A lumped equivalent circuit is also obtained in [7] for a wire transmission line in the presence of an electrically small circular hole.

This note considers basically extensions of the work in [5, 7] dealing with thin wire structures placed behind protective screens. The excitation of an infinitely long loaded cable behind a narrow slot in a conduction screen [5] is extended to the case of arbitrary shaped apertures. A procedure is given to account for multiple parallel loaded cables behind a given aperture shape. A general formulation is discussed for obtaining the field distribution in an aperture with interaction with an infinite cable taken into account. The transfer admittance function obtained, allows one to further calculate the induced current on the infinitely long cable and also the portion of the power leaked along the cable from the aperture excitation [9].

Using quasi-static approximations [7], the lumped equivalent circuit (with appropriate source terms to account for the excitation), is given for the transmission-line cable in the presence of a narrow rectangular aperture. The aperture polarizabilities considered in the equivalent circuit do take into account the presence of the cable on the aperture distribution. Minor discrepancies found in the previous derivation of the equivalent circuit element values in [7] are corrected, and the appropriate equivalent circuit element values and the source terms are indicated.

II. FORMULATION

An expression for the electric current induced on an infinitely long loaded thin cable placed behind a general shaped aperture-perforated conducting screen is derived systematically by taking into account fully the interaction with the aperture. Also derived is a set of coupled integro-differential equations for the determination of the magnetic current or the tangential-electric-field distribution in the aperture for an arbitrarily polarized external incident field on the aperture by fully taking into account the presence of the infinitely long cable. The expressions obtained are in a form suitable for circuit modeling of the aperture-cable coupled region. When multiple parallel infinitely long cables are present behind the aperture perforated screen, a method is given to take into account their corresponding mutual coupling with the aperture.

A. Expression for the Electric Current $\tilde{I}(z,s)$ on the Infinitely Long Loaded Cable

Referring to Figure 2.1, the electric vector potential $\vec{\tilde{A}}_m(\vec{r},s)$ in the shadow side of $y > o$ of the aperture is given by [1],

$$\vec{\tilde{A}}_m(r,s) = \frac{(\sigma+s\varepsilon)}{2\pi s} \iint_{S_a} \vec{\tilde{J}}_{s_m}(\vec{r}_a',s)\, \tilde{G}(\vec{r},\vec{r}_a';s)\, dx_a' dz_a' \qquad y > o, \tag{2.1a}$$

where $\vec{\tilde{J}}_{s_m}(r_a',s)$ is the magnetic current distribution and

$$\tilde{G}(r,r_a';s) = \frac{e^{-\gamma|\vec{r}-\vec{r}_a'|}}{|\vec{r}-\vec{r}_a'|} \tag{2.1b}$$

$$|\vec{r}-\vec{r}_a'| = [(x-x_a')^2 + y^2 + (z-z_a')^2]^{\frac{1}{2}} \tag{2.1c}$$

and the propagation constant

$$\gamma = [s\mu\,(\sigma+s\varepsilon)]^{\frac{1}{2}} \tag{2,1d}$$

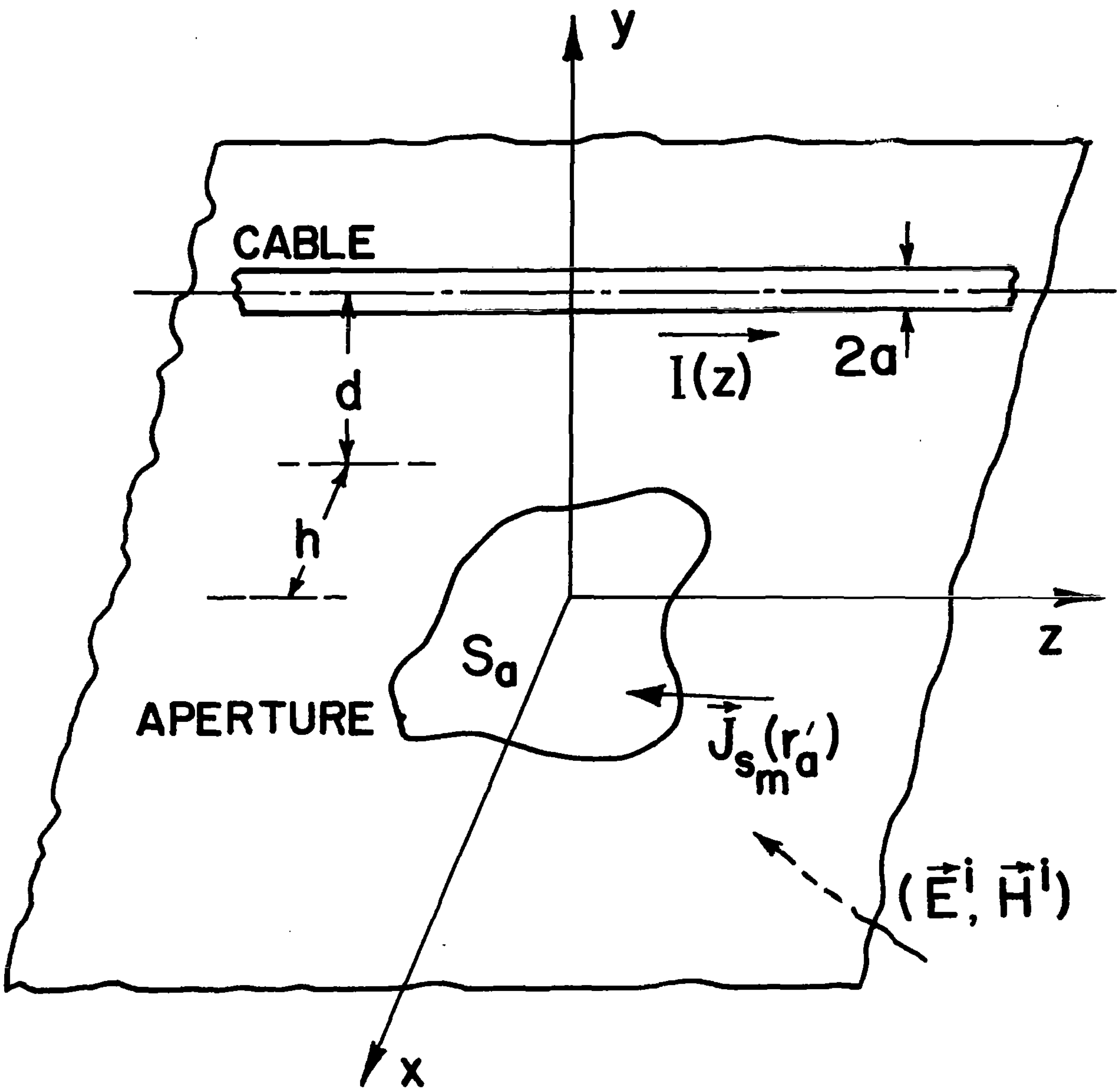

Figure 2.1 Infinitely Long Cable Behind an Aperture

which is applicable to both upper $y > o$ and lower $y < o$ half spaces having the same medium characteristics of permeability μ, permittivity ε and conductivity σ. Using the identity [8],

$$\frac{e^{-\gamma|\vec{r}-\vec{r}_a^{\,\prime}|}}{|\vec{r}-\vec{r}_a^{\,\prime}|} = \frac{1}{j\pi}\int_{C_\zeta} K_o(u\Psi)e^{\zeta(z-z_a^{\prime})}\, d\zeta \tag{2.2a}$$

$$u = [\gamma^2-\zeta^2]^{\frac{1}{2}} \tag{2.2b}$$

$$\Psi = [(x-x_a^{\prime})^2 + y^2]^{\frac{1}{2}} \tag{2.2c}$$

where K_o is the modified Bessel function of second kind, zero order and C_ζ is the Bromwich contour of integration in the complex ζ-plane as shown in Figure 2.2. Substituting (2.2a) into the expression (2.1a) we have,

$$\tilde{\vec{A}}_m(\vec{r},s) = \frac{(\sigma+s\varepsilon)}{j2\pi^2 s}\iint_{S_a} \tilde{\vec{J}}_{s_m}(\vec{r}_a^{\,\prime},s)\left[\int_{C_\zeta} K_o(u\Psi)e^{\zeta(z-z_a^{\prime})}\, d\zeta\right] dx_a^{\prime}\, dz_a^{\prime} \qquad y > o \tag{2.3}$$

In obtaining the above potential expression an e^{st} variation is considered for field quantities, s being the two sided Laplace transform variable,

$$s = \Omega + j\omega \tag{2.4}$$

In the above integral representation and in the analysis to follow, the Laplace transform definition given below is followed for the z coordinate variable,

$$\tilde{\tilde{T}}(\zeta,s) = \int_{-\infty}^{\infty} \tilde{T}(z,s)\, e^{-\zeta z}\, dz \tag{2.5a}$$

which has the inverse transform, Fig. 2.2,

$$\tilde{T}(z,s) = \frac{1}{2\pi j}\int_{C_\zeta} \tilde{\tilde{T}}(\zeta,s)\, e^{\zeta z}\, d\zeta \tag{2.5b}$$

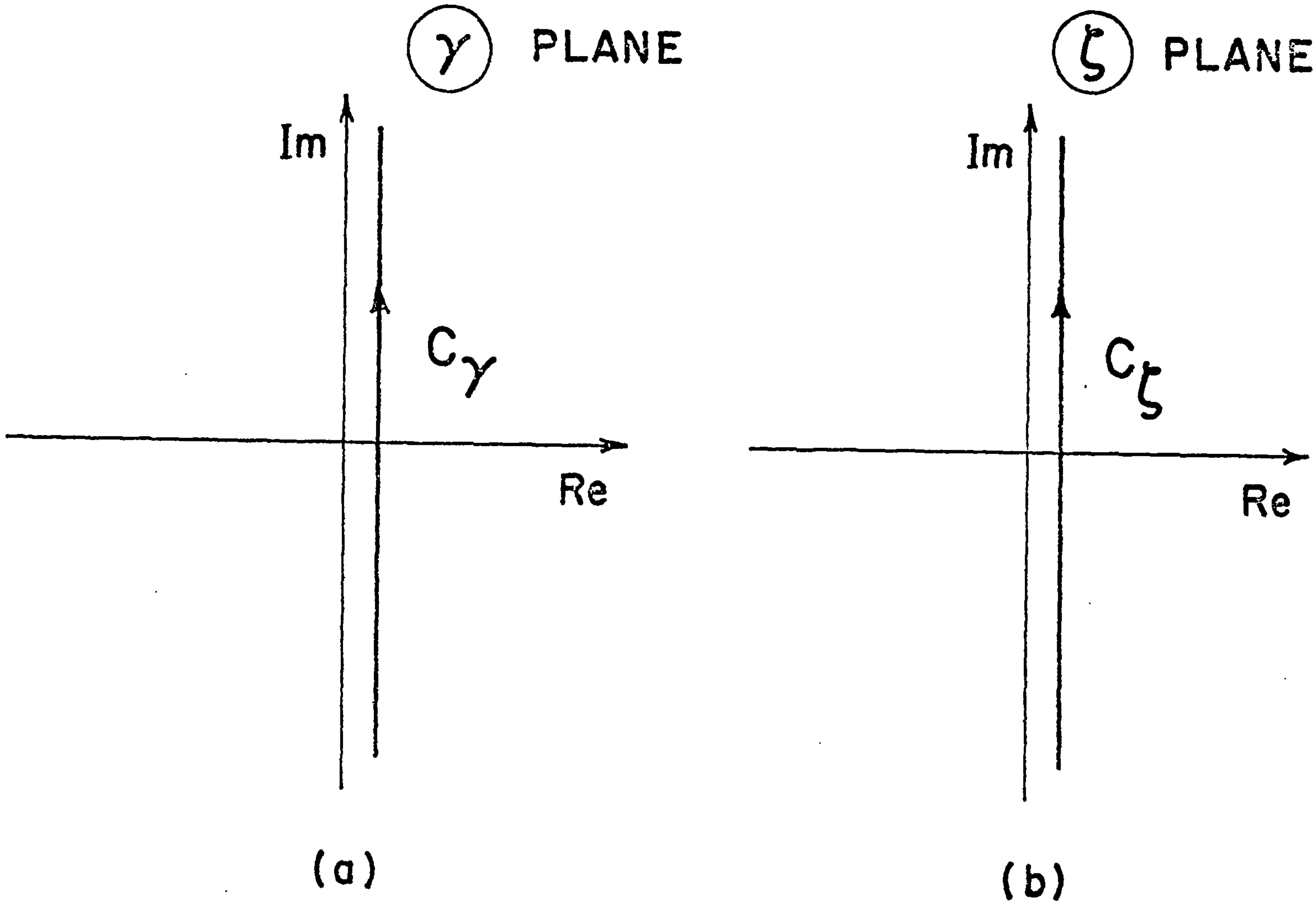

Figure 2.2
Contours of Integration in γ- and ζ-planes

The same definitions hold good for all other field quantities. The z component of the primary electric field due to the aperture magnetic current distribution is given by [1],

$$\tilde{E}_z^{(p)}(x,y,z;s) = \frac{s}{(\sigma+s\varepsilon)} \left[\nabla\times\tilde{\vec{A}}_m(\vec{r},s)\right]\cdot\vec{1}_z \tag{2.6a}$$

According to the expressions (2.3), (2.5b) and (2.6a),

$$\tilde{E}_z^{(p)}(x,y,\zeta;s) = \frac{1}{\pi}\iint_{S_a} \tilde{J}_{s_{m_x}}(\vec{r}_a{}',s)\left[\frac{uy}{\Psi}K_1(u\Psi)e^{-\zeta z_a'}\right]dx_a'dz_a' \qquad y>0, \tag{2.6b}$$

where K_1 is the modified Bessel function of the second kind, first order. Similarly the z component of the secondary magnetic vector potential due to the induced electric current on the infinitely long cable and its image is given by [9],

$$\tilde{A}_z(x,y,z;s) = \frac{1}{2\pi j}\int_{C_\zeta}\tilde{\xi}(\zeta,s)\left[K_0(u\Psi_1)-K_0(u\Psi_2)\right]e^{\zeta z}\,d\zeta \qquad y>0, \tag{2.7a}$$

$$\Psi_1 = [(x-h)^2 + (y-d)^2]^{1/2} \tag{2.7b}$$

$$\Psi_2 = [(x-h)^2 + (y+d)^2]^{1/2} \tag{2.7c}$$

It can be easily verified that the secondary magnetic vector potential as given by the expression (2.7a) has the right boundary condition $\tilde{A}_z(x,y,z;s)|_{y=0} = 0$. We also note that the spectral term $\tilde{\xi}(\zeta,s)$ in (2.7a) is a quantity proportional to the transform of the electric current $\tilde{I}(\zeta,s)$ on the infinitely long cable. The total electric current $\tilde{I}(z,s)$ at any point along the cable can be determined from the line integral of the magnetic field evaluated on the surface of the cable

$$\tilde{I}(z,s) = -\frac{1}{\mu}\int_{\phi=0}^{2\pi}\Psi'\frac{\partial\tilde{A}_z(\vec{r}',s)}{\partial\Psi'}\,d\phi'\Big|_{\Psi'=a} \tag{2.8}$$

where Ψ' and ϕ' are the local cylindrical coordinates about the cable axis and a is the radius of the cable (wire).

Hence from the expressions (2.5b), (2.7a) and (2.8), the transform of the electric current is given by,

$$\tilde{I}(\zeta,s) = \frac{2\pi a}{\mu} u \left[K_1(u\,a) - K_1(2ud)\right] \tilde{F}(\zeta,s) \tag{2.9}$$

If $\tilde{E}_z^{(s)}(x,y,z;s)$ is the z component of the secondary electric field due to currents on the infinitely long cable [1]

$$\tilde{E}_z^{(s)}(x,y,z;s) = \frac{s}{\gamma^2}\left[\nabla(\nabla\cdot\tilde{\vec{A}}(\vec{r},s)) - \gamma^2\tilde{\vec{A}}(\vec{r},s)\right]\cdot\vec{1}_z \tag{2.10}$$

On substituting the expression (2.7a) and using (2.5b), its transform is obtained as,

$$\tilde{E}_z^{(s)}(x,y,\zeta;s) = -\frac{u^2 s}{\gamma^2}\left[K_0(u\Psi_1) - K_0(u\Psi_2)\right]\tilde{F}(\zeta,s)$$

$$y>0, \tag{2.11}$$

We can now determine the spectral distribution $\tilde{F}(\zeta,s)$ by enforcing the cable impedance boundary condition that on the surface of the cable

$$\left[\tilde{E}_z^{(p)}(\zeta,s) + \tilde{E}_z^{(s)}(\zeta,s)\right]_{\Psi'=a} = \tilde{Z}_w'(s)\,\tilde{I}(\zeta,s) \tag{2.12}$$

where $\tilde{Z}_w'(s)$ is the impedance per unit length of the infinitely long cable which depends on the cross sectional geometry of the cable [10]. By substituting the expressions (2.6b), (2.9) and (2.11) into the impedance boundary condition (2.12), the spectral function $\tilde{F}(\zeta,s)$ proportional to the transform of the electric current on the cable is given by,

$$\tilde{\mathcal{E}}(\zeta,s) = \frac{\frac{\gamma^2}{\pi s}\iint_{S_a} \tilde{J}_{s_{m_x}}(x_a',z_a';s)\left[\frac{d}{\Psi_c}K_1(u\Psi_c)\,e^{-\zeta z_a'}\right]dx_a'\,dz_a'}{\tilde{\mathcal{D}}(\zeta,s)} \quad (2.13a)$$

$$\tilde{\mathcal{D}}(\zeta,s) = u\left[K_0(ua) - K_0(2ud)\right] + 2\pi a\gamma\left[K_1(ua) - K_1(2ud)\right]\frac{\tilde{Z}_w'(s)}{Z_o} \quad (2.13b)$$

where

$$\Psi_c = [(h - x_a')^2 + d^2]^{1/2} \quad (2.13c)$$

$$Z_o = [\frac{\mu s}{\sigma+s\varepsilon}]^{1/2} \quad (2.13d)$$

Only the x component of the magnetic current distribution which is perpendicular to the cable axis has the direct coupling to the infinitely long cable; even though $\tilde{J}_{s_{m_{x,z}}}(x_a',z_a';s)$ are intercoupled. Hence, the electric current distribution induced on the infinitely long cable can be obtained by the expressions (2.5b), (2.9) and (2.13a),

$$\tilde{I}(z,s) = \iint_{S_a} \tilde{J}_{s_{m_x}}(x_a',\ z_a';s)\ \tilde{T}(z,\ x_a',\ z_a';s)\ dx_a'\ dz_a' \quad (2.14a)$$

where

$$\tilde{T}(z,\ x_a',z_a';s) = \frac{1}{2\pi j}\int_{C_\zeta} \tilde{\mathcal{T}}(\zeta,x_a',z_a';s)\ e^{\zeta(z-z_a')}\ d\zeta \quad (2.14b)$$

$$\tilde{\mathcal{T}}(\zeta,x_a',z_a';s) = \frac{\tilde{\mathcal{N}}(\zeta,s)}{\tilde{\mathcal{D}}(\zeta,s)} \quad (2.14c)$$

$$\tilde{\mathcal{N}}(\zeta,s) = 2a\ (\sigma+s\varepsilon)\ u\ (\frac{d}{\Psi_c})\ K_1(u\Psi_c)\left[K_1(ua) - K_1(2ud)\right] \quad (2.14d)$$

The expression (2.14a) gives the electric current $\tilde{I}(z,s)$ on the infinitely long cable in terms of the aperture magnetic current distribution $\tilde{J}_{s_{m_x}}(x_a',z_a';s)$ and reduces to the form derived in [5] if the special narrow slot case is introduced for the aperture. $\tilde{I}(z,s)$ can be evaluated only after first determining the aperture magnetic current distribution $\tilde{\vec{J}}_{s_m}(\vec{r}_a',s)$ for a given shaped aperture. The expression (2.14b) for

$\tilde{T}(z,x_a',z_a';s)$ is well known as the transfer admittance function and clearly determines the amount of electromagnetic coupling between the aperture and the infinitely long cable placed behind it.

B. Coupled Integral Equations for the Magnetic Current Distribution in the Aperture

In the following a set of coupled integro-differential equations are derived by treating the magnetic current distribution $\tilde{\vec{J}}_{s_m}(\vec{r}_a',s)$ as an unknown distribution to be determined, taking into account the complete coupling of the infinite cable, for an external excitation incident on the aperture in the lower half space $y<0$. This is accomplished by writing down the corresponding expressions [1] for the total magnetic field $\tilde{\vec{H}}^+(\vec{r},s)$ and $\tilde{\vec{H}}^-(\vec{r},s)$ in the half space regions $y>0$ and $y<0$ respectively, and enforcing the remaining aperture boundary condition [1] that the transverse-to-y-direction components of the total magnetic field should be continuous in the region containing the aperture perforated conducting screen, i.e.,

$$\lim_{y \to 0_+} \tilde{\vec{H}}_+ \times \vec{1}_y = \lim_{y \to 0_-} \tilde{\vec{H}}_- \times \vec{1}_y \qquad (x,z) \in S_a \tag{2.15}$$

The total magnetic field $\tilde{\vec{H}}_+(\vec{r},s)$ valid for the region $y>0$ and $\tilde{\vec{H}}_-(\vec{r},s)$ for the region $y<0$ are given by [1],

$$\tilde{\vec{H}}_+(\vec{r},s) \begin{cases} = \tilde{\vec{H}}_+^{(s)}(\vec{r},s) + \tilde{\vec{H}}^{(w)}(\vec{r},s) \\ = -\dfrac{s}{\gamma^2}\left[\nabla(\nabla\cdot\tilde{\vec{A}}_{m_+}(\vec{r},s)) - \gamma^2\tilde{\vec{A}}_{m_+}(\vec{r},s)\right] \\ \quad + \dfrac{1}{\mu}\nabla \times \vec{1}_z\,\tilde{A}_z(\vec{r},s) \end{cases} \qquad y>0 \tag{2.16a}$$

$$\tilde{\vec{H}}_-(\vec{r},s) = \frac{s}{\gamma^2}\left[\nabla(\nabla.\tilde{\vec{A}}_{m_-}(\vec{r},s)) - \gamma^2\tilde{\vec{A}}_{m_-}(\vec{r},s)\right] + \tilde{\vec{H}}^{(sc)}(\vec{r},s) \qquad y<0 \tag{2.16b}$$

where the electric vector potential $\tilde{\vec{A}}_{m_\pm}(\vec{r},s)$ is exactly given by the expression (2.1a), since both half spaces have the same media characteristics (μ,ε,σ) and in (2.16b) the term $\tilde{\vec{H}}^{(sc)}(\vec{r},s)$ is the short-circuit magnetic field. On applying the boundary condition (2.15) to the expressions (2.16a and b), we obtain the following coupled set of integro-differential equations, valid in the aperture S_a,

$$\frac{2s}{\gamma^2}\left\{\left[\frac{\partial^2}{\partial x^2} - \gamma^2\right] \tilde{A}_{m_x}(\vec{r},s) + \frac{\partial^2}{\partial x \partial z} \tilde{A}_{m_z}(\vec{r},s)\right\}$$

$$- \frac{1}{\mu} \frac{\partial}{\partial y} \tilde{A}_z(\vec{r},s) = - 2 \tilde{H}_x^i (\vec{r},s) \tag{2.17a}$$

$$\vec{r} \in S_a$$

$$\frac{2s}{\gamma^2}\left\{\left[\frac{\partial^2}{\partial z^2} - \gamma^2\right] \tilde{A}_{m_z}(\vec{r},s) + \frac{\partial^2}{\partial x \partial z} \tilde{A}_{m_x}(\vec{r},s)\right\} = - 2 \tilde{H}_z^i (\vec{r},s) \tag{2.17b}$$

$$\vec{r} \in S_a$$

where the potentials $\tilde{A}_{m_{x,z}}(\vec{r},s)$ and $\tilde{A}_z(\vec{r},s)$ are given by (2.1a) and (2.7a).

$\tilde{H}_x^i(\vec{r},s)$ and $\tilde{H}_z^i(\vec{r},s)$ are the x and z components of the incident magnetic field excitation on the aperture. If a plane wave is assumed to excite the aperture with its direction of propagation making an angle ϕ_i and θ_i with the x and z axes respectively, the incident magnetic field can be written as,

$$\tilde{\vec{H}}^i(x,z;s) = \left[\vec{1}_x \tilde{H}_{xo}^i(s) + \vec{1}_z \tilde{H}_{zo}^i\right] e^{-\gamma f(x,z,\theta_i,\phi_i)} \tag{2.18a}$$

$$f(x,z,\theta_i,\phi_i) = x \sin\theta_i \cos\phi_i + z \cos\theta_i \tag{2.18b}$$

$$+ y \sin\theta_i \sin\phi_i$$

where $\tilde{H}^i_{xo}(s)$ and $\tilde{H}^i_{zo}(s)$ are the amplitude factors in the x and z directions and are determined by the plane wave polarization. Further substituting the potential expressions defined in (2.1a) and (2.7a) the coupled equations (2.17a and b) take the form,

$$\left[\frac{\partial^2}{\partial x^2} - \gamma^2\right] \iint_{S_a} \tilde{J}_{s_{m_x}}(x_a',z_a';s)\ \tilde{K}(x,x_a',z,z_a';s)\ dx_a',dz_a'$$

$$+ \iint_{S_a} \tilde{J}_{s_{m_x}}(x_a',z_a';s)\ \tilde{G}_m(x,x_a',\ z,\ z_a';s)\ dx_a'dz_a'$$

$$+ \frac{\partial^2}{\partial x \partial z} \iint_{S_a} \tilde{J}_{s_{m_z}}(x_a',z_a';s)\ \tilde{K}(x,x_a',z,z_a';s)\ dx_a',dz_a'$$

$$= -\ 2\pi\gamma Z_o \tilde{H}^i_x(x,z;s)$$

$$(x,z) \in S_a \qquad (2.19a)$$

$$\left[\frac{\partial^2}{\partial z^2} - \gamma^2\right] \iint_{S_a} \tilde{J}_{s_{m_z}}(x_a',z_a';s)\ \tilde{K}(x,x_a',z,z_a';s)\ dx_a'dz_a'$$

$$+ \frac{\partial^2}{\partial x \partial z} \iint_{S_a} \tilde{J}_{s_{m_x}}(x_a',z_a';s)\ \tilde{K}(x,x_a',z,z_a';s)\ dx_a'dz_a'$$

$$= -\ 2\pi\gamma Z_o \tilde{H}^i_z(x,z;s)$$

$$(x,z) \in S_a \qquad (2.19b)$$

where

$$\tilde{K}(x,x_a',z,z_a';s) = \frac{e^{-\gamma R_s}}{R_s} \qquad (2.19c)$$

$$R_s = [(x-x_a')^2 + (z-z_a')^2]^{1/2} \quad (2.19d)$$

$$\tilde{G}_m(x,x_a',z,z_a';s) =$$

$$-\frac{1}{j\pi}\int_{C_\zeta} \frac{u\gamma^2 \left(\frac{d}{\Psi_c}\right)\left(\frac{d}{\Psi_s}\right) K_1(u\Psi_c)\, K_1(u\Psi_s)\, e^{\zeta(z-z_a')}\, d\zeta}{\tilde{D}(\zeta,s)} \quad (2.19e)$$

$$\Psi_s = [(x-h)^2 + d^2]^{1/2} \quad (2.19f)$$

The coupled integro-differential equations (2.19a and b) for the unknown magnetic current distribution $\tilde{J}_{s_{m_x}}(\vec{r},s)$ and $\tilde{J}_{s_{m_z}}(\vec{r},s)$ are in a more convenient form for further analysis using numerical methods [4,11]. The second integral term in (2.19a) has inherently built in the transform of the electric current on the infinitely long cable and thus completely accounts for the interaction between the aperture and the infinitely long cable. With cable removed this particular integral term with (2.19e) as kernel vanishes, resulting in the coupled integral equations for the general shaped aperture [1]. The equations (2.19a,b) further reduce to a simple form for the case of excitation of the infinitely long cable through a narrow rectangular aperture [5]. Suppose the narrow slot, $w \ll \ell$, is oriented along x axis in the screen, Fig. 2.3, then the magnetic current $\tilde{J}_{s_{m_x}}(x,s)$ is predominant; and as far as a narrow slot and infinite cable is concerned, the integral equation (2.19b) is ignored and the integral term involving the distribution $\tilde{J}_{s_{m_z}}$ is deleted in the integral equation (2.19a). In fact the resulting one dimensional integral equation is identical to the one obtained in [5]. If the narrow slot is oriented along the z axis, then the axis of the cable is parallel to the slot axis and there is negligible coupling between the narrow slot and the infinitely long cable [1].

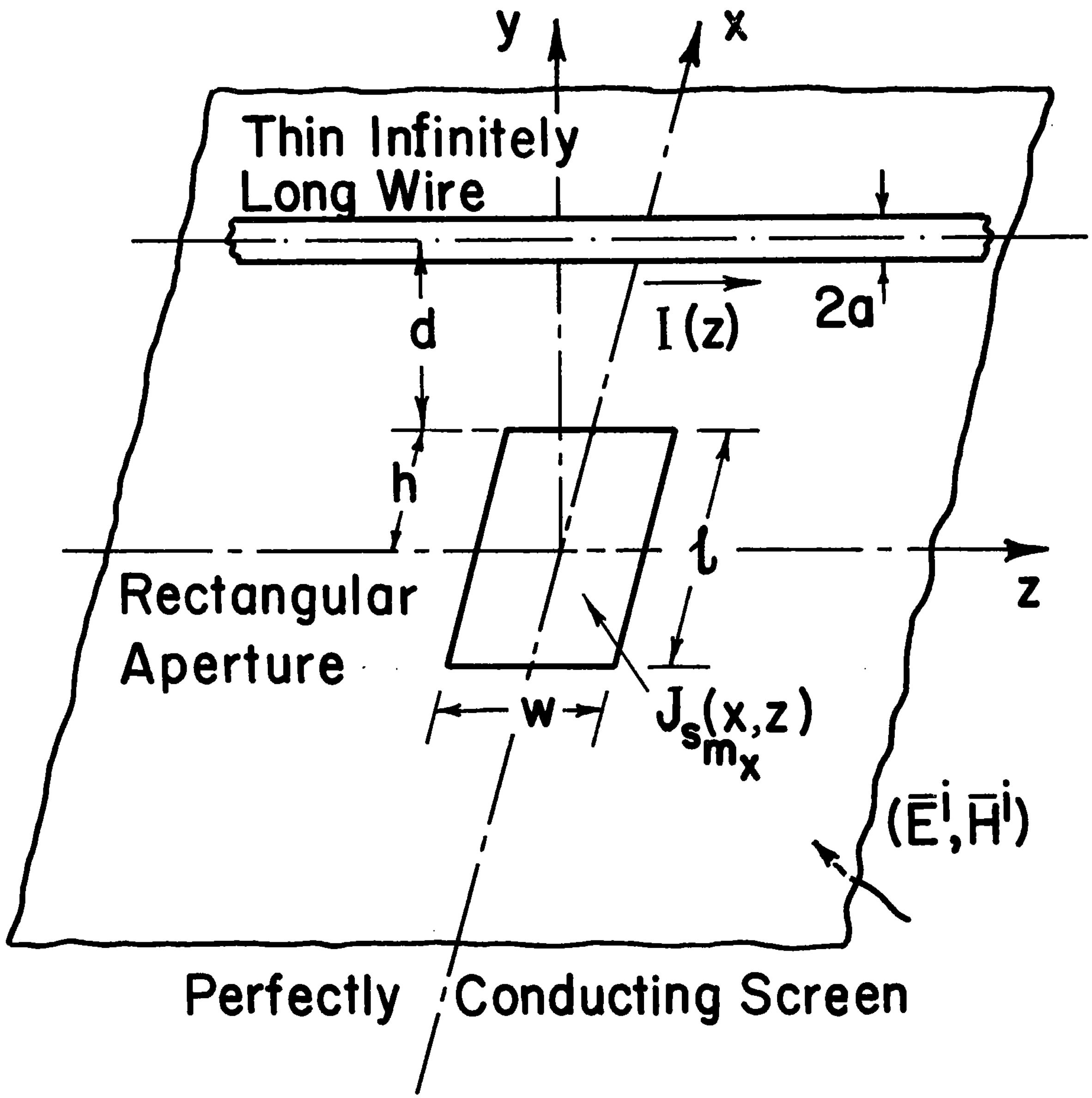

Figure 2.3 Infinitely Long Wire Behind a Narrow Slot

C. Extension to Multiple Cables

In Section A an expression for the total electric current on the infinitely long cable placed behind an aperture perforated conducting screen is obtained in terms of the magnetic current distribution in the aperture, and further in Section B, a set of coupled integro-differential equations is obtained for the magnetic current distribution in the aperture with full account of the interaction between the infinitely long cable and the aperture. Further extensions of the problem when multiple cables are present behind the aperture perforated conducting screen are now considered. In Fig. 2.4 is shown the configuration discussed in this section. A set of thin infinitely long N parallel cables of radius a_n are placed in the region $y > 0$, and are located at $x = h_n$, $y = d_n$, $n = 1, 2 \ldots N$ with the distance between any two cables at least a few radii apart. Without loss of generality, it is assumed that the axes of the parallel infinitely long thin cables are oriented along the z axis. The total z component of the secondary magnetic vector potential is obtained by the superposition of the individual cable contributions.

$$\tilde{A}_z^{(t)}(x,y,z\,;s) = \sum_{n=1}^{N} \frac{1}{2\pi j} \int_{C_\zeta} \tilde{F}_n(\zeta,s)\,[K_0(u\Psi_{n_1}) - K_0(u\Psi_{n_2})]\, e^{\zeta z}\, d\zeta \qquad (2.20a)$$

where

$$\Psi_{n_1} = [(x-h_n)^2 + (y-d_n)^2]^{1/2} \qquad (2.20b)$$

$$\Psi_{n_2} = [(x-h_n)^2 + (y+d_n)^2]^{1/2} \qquad (2.20c)$$

$\tilde{F}_n(\zeta,s)$ are the spectral distributions proportional to the respective induced electric currents on the corresponding cables [10]. According to the expressions (2.5b) and (2.8)

$$\tilde{I}_n(\zeta,s) = \frac{2\pi a_n}{\mu}\, u\, [K_1(ua_n) - K_1(2ud_n)]\, \tilde{F}_n(\zeta,s) \qquad (2.21)$$

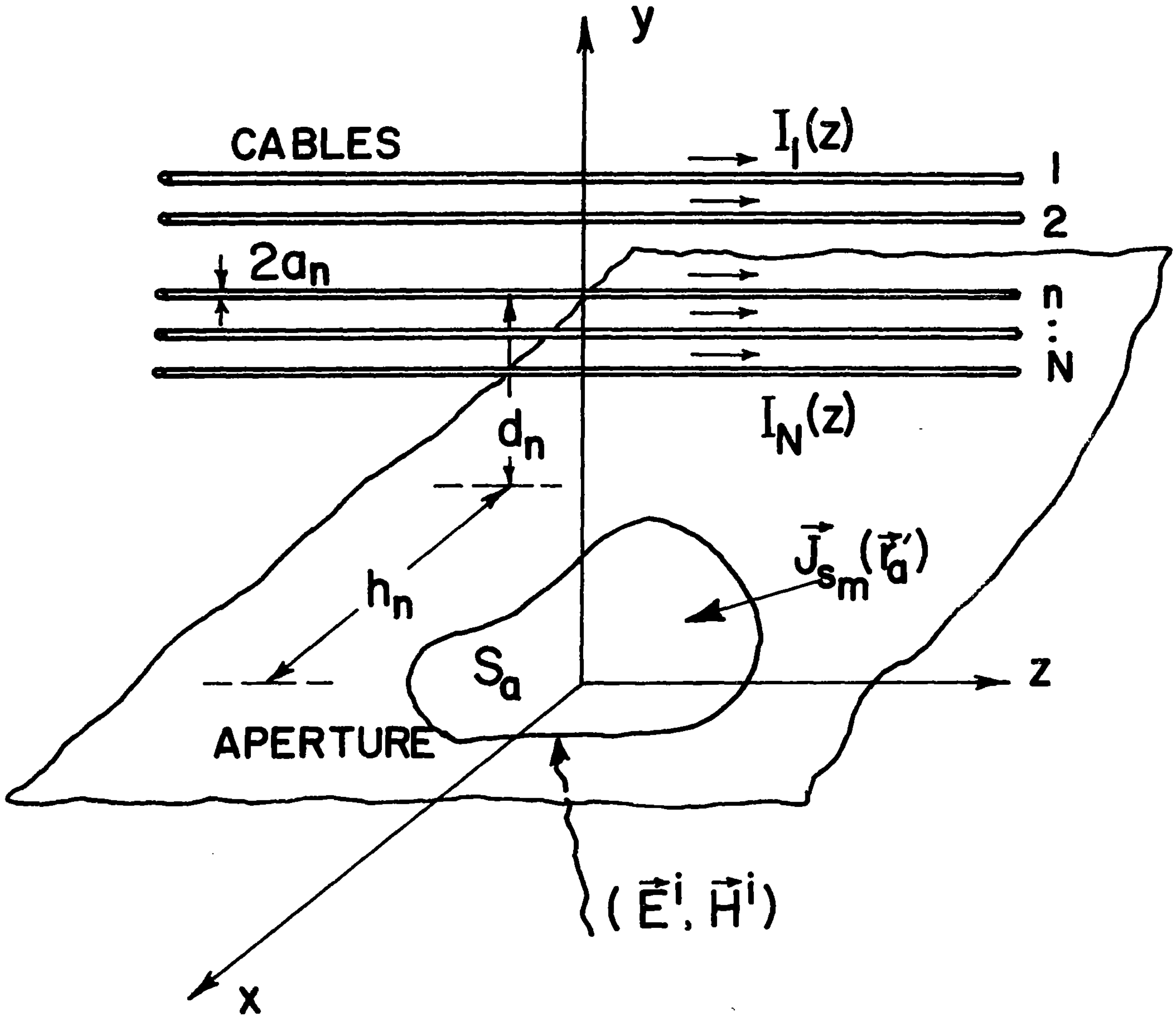

Figure 2.4
Infinitely Long Parallel Cables Behind an Aperture

The corresponding total secondary electric field due to the currents on the various cables is obtained with the substitution of the expression (2.20a) into (2.10). Hence using the definition (2.5b), the transform of the secondary electric field is

$$\tilde{E}_z^{s(t)}(x,y,z;s) = \sum_{n=1}^{N} - \frac{u^2 s}{\gamma^2} [K_o(u\Psi_{n_1}) - K_o(u\Psi_{n_2})] \tilde{F}_n(\zeta,s) \quad (2.22)$$

The spectral distrubtions $\tilde{F}_n(\zeta,s)$, n = 1,2, ...N are determined by enforcing the impedance boundary condition on each of the cables

$$\tilde{E}_z^{(p)}(\zeta,s) + E_z^{s(t)}(\zeta,s)\Big|_{\Psi' = a_m} = \tilde{Z}'_{w,m}(s)\, \tilde{I}_m(\zeta,s) \quad (2.23)$$

for m = 1, 2 .....N

where $\tilde{Z}'_{w,m}(s)$ is again the impedance of the m^{th} infinitely long cable per unit length. On substituting (2.6b), (2.21) and (2.22) into the impedance boundary condition, the following matrix equation is obtained for the spectral distribution $\tilde{F}_n(\zeta,s)$,

$$\left[\tilde{S}_{mn}\right]\left[\tilde{F}_n\right] = \left[\tilde{V}_m\right] \quad (2.24a)$$

where

$$\tilde{S}_{mn} = u\,[K_o(ua_n) - K_o(2ud_n)] + 2\pi a_n \gamma\,[K_1(ua_n) - K_1(2ud_n)]\,\frac{\tilde{Z}'_{w,n}(s)}{Z_o} \quad (2.24b)$$

m = n

$$= u\,[K_o(u\Psi_{mn_1}) - K_o(2u\Psi_{mn_2})] \quad (2.24c)$$

m ≠ n

$$\Psi_{mn_1} = [(h_m-h_n)^2 + (d_m-d_n)^2]^{1/2} \tag{2.24d}$$

$$\Psi_{mn_2} = [(h_m-h_n)^2 + (d_m+d_n)]^{1/2} \tag{2.24e}$$

$$V_m = \frac{\gamma^2}{s} \frac{1}{\pi} \iint_{S_a} J_{s_{m_x}} (\vec{r}\,'_a, s) \left[\frac{d_m}{\Psi_{m_c}} K_1 (u\Psi_{m_c}) e^{-\zeta z'_a}\right] dx'_a\, dz'_a \tag{2.24f}$$

$$\Psi_{m_c} = [(h_m - x'_a)^2 + d_m{}^2]^{1/2} \tag{2.24g}$$

Hence the spectral functions $\tilde{\underset{\sim}{F}}_n (\zeta,s)$, n = 1, 2 ....N are obtained as the solution of,

$$\left[\tilde{\underset{\sim}{F}}_n\right] = \left[\tilde{\underset{\sim}{S}}_{mn}\right]^{-1} \cdot \left[\tilde{\underset{\sim}{V}}_m\right] \tag{2.25}$$

The electric current induced on the infinitely long loaded cables is given by substituting (2.21) and (2.25) into the expression (2.5b),

$$\tilde{I}_n(z,s) = \iint_{S_a} \tilde{J}_{s_{m_x}} (x'_a, z'_a; s)\, \tilde{T}_n (z, x'_a, z'_a; s)\, dx'_a\, dz'_a \tag{2.26a}$$

where

$$\tilde{T}_n (z,x'_a,z'_a;s) = \frac{1}{2\pi j} \int_{C_\zeta} \tilde{\underset{\sim}{T}}_n (\zeta, x'_a, z'_a; s)\, e^{\zeta(z-z'_a)}\, d\zeta \tag{2.26b}$$

$$\left[\tilde{\underset{\sim}{T}}_n (\zeta,x'_a,z'_a;s)\right] = $$

$$2a_n u\, (\sigma+s\varepsilon) \left[K_1(ua_n) - K_1\, (2ud_n)\right] \left\{[S_{mn}]^{-1} \cdot \left[\frac{d_m}{\Psi_{m_c}} K_1\, (u\Psi_{m_c})\right]\right\} \tag{2.26c}$$

Again $\tilde{T}_n$ is the transfer admittance function which gives the amount of coupling between the aperture and the n^{th} infinitely long cable. The induced currents can be evaluated only if the aperture distribution is known. One can use the integro-differential equations (2.17a and b) derived in Section B for determining the aperture distribution with the expression (2.20a) used for $\tilde{A}_z(\vec{r},s)$.

D. Alternate Form of Integral Equation for the Aperture Distribution

The coupled set of integro-differential equations (2.19a and b) derived in Section B are in a proper form (1) convenient for numerical applications [4,11] to determine the aperture magnetic current distribution. These integral equations are formulated based on the aperture boundary condition (2.15) that the transverse-to-y-direction components of the magnetic field should be continuous in the aperture region. In this section an alternative integral equation is obtained by enforcing the obvious aperture boundary condition that the normal component of the total electric field in the aperture should be continuous in the region containing the aperture perforated conducting screen,

$$\lim_{y\to 0_+} \tilde{\vec{E}}_+ \cdot \vec{1}_y = \lim_{y\to 0_-} \tilde{\vec{E}}_- \cdot \vec{1}_y \tag{2.27}$$

The total electric field $\tilde{\vec{E}}_+(\vec{r},s)$ valid in the region y>0 and $\tilde{\vec{E}}_-(\vec{r},s)$ for the region y<0 are given by [1],

$$\tilde{\vec{E}}_+(\vec{r},s) \begin{cases} = \tilde{\vec{E}}^{(w)}(\vec{r},s) + \tilde{\vec{E}}_+^{(s)}(\vec{r},s) \\ = \frac{s}{\gamma^2}\left[\nabla(\nabla\cdot\tilde{\vec{A}}(\vec{r},s)) - \gamma^2\tilde{\vec{A}}(\vec{r},s)\right] \\ + \frac{s}{(\sigma+s\varepsilon)} \nabla \times \tilde{\vec{A}}_m(\vec{r},s) \end{cases} \quad y>0 \tag{2.28a}$$

$$\tilde{\vec{E}}_{-}(\vec{r},s) = -\frac{s}{(\sigma+s\varepsilon)} \nabla \times \tilde{\vec{A}}_m(\vec{r},s) + \tilde{\vec{E}}^{sc}(\vec{r},s) \qquad (2.28b)$$

$$y<0$$

where $\tilde{\vec{E}}^{sc}(\vec{r},s)$ is the short circuit electric field. On substituting the expressions (2.28 a and b) into the aperture boundary condition (2.27), the following integral equation is obtained,

$$\frac{s}{\gamma^2}\left[\frac{\partial^2}{\partial y \partial z} \tilde{A}_z(\vec{r},s)\right] + \frac{2s}{(\sigma+s\varepsilon)}\left[\frac{\partial}{\partial z}\tilde{A}_{m_x}(\vec{r},s) - \frac{\partial}{\partial x}\tilde{A}_{m_z}(\vec{r},s)\right]$$

$$= 2\,\tilde{E}^i_y(\vec{r},s) \qquad (2.29)$$

$$\vec{r} \in S_a$$

where the potentials $\tilde{A}_{m_{x,z}}(\vec{r},s)$ and $\tilde{A}_z(\vec{r},s)$ are given by the expressions (2.1a) and (2.7a) respectively, and $\tilde{E}^i_y(\vec{r},s)$ is the y component of the incident electric field. On substituting (2.1a) and (2.7a) into the expression (2.29), we have

$$\frac{\partial}{\partial z}\iint \tilde{J}_{s_{m_x}}(x'_a,z'_a;s)\, \tilde{G}_n(x,x'_a,z,z'_a;s)\, dx'_a dz'_a$$

$$+ \frac{\partial}{\partial z}\iint \tilde{J}_{s_{m_x}}(x'_a,z'_a;s)\, \tilde{K}(x,x'_a,z,z'_a;s)\, dx'_a dz'_a$$

$$- \frac{\partial}{\partial x}\iint \tilde{J}_{s_{m_x}}(x'_a,z'_a;s)\, \tilde{K}(x,x'_a,z,z'_a;s)\, dx'_a dz'_a$$

$$= 2\pi\, \tilde{E}^i_y(x,z;s)$$

$$(x,z) \in S_a \qquad (2.30a)$$

where

$$\tilde{G}_n(x,x_a',z,z_a';s) = -\frac{1}{\gamma^2}\tilde{G}_m(x,x_a',z,z_a';s) \tag{2.30b}$$

The two components $\tilde{J}_{s_{m_x}}$ and $\tilde{J}_{s_{m_z}}$ are to be solved from the integral equation (2.30a). Since there are two components, the solution is obtained in conjunction with the integral equations, either (2.19a) or (2.19b).

III. NUMERICAL RESULTS AND APERTURE CHARACTERIZATION IN THE PRESENCE OF INFINITE CABLE

In the previous section, the interaction with an infinitely long cable placed behind an aperture perforated conducting screen is discussed. A set of coupled integral equations are obtained for determining the tangential electric field distribution in the aperture in the presence of the infinitely long cable. For a given specific aperture shape, the numerical solution of the integral equations (2.19a and b) even though elaborate, is straight forward [12-14]. The Green's function (2.19e) should be carefully evaluated [15] noting the proper branches of u as discussed in Appendix A. As an example, for a narrow slot in a conducting screen Fig. 2.3, the integral equations (2.19a and b) simplifiy to one dimensional form [5]. The distribution $\tilde{J}_{s_{m_x}}(x,z)$ for a narrow slot can be written as

$$\tilde{J}_{s_{m_x}}(x,z) = \tilde{m}(x)\ \xi(z) \tag{3.1a}$$

$$\xi(z) = [(\frac{w}{2})^2 - z^2]^{-1/2}\ (1/\pi) \tag{3.1b}$$

The magnetic current distribution $\tilde{m}(x)$ along the center line of the narrow slot is plotted in Fig. 2.5 for a few locations of the perfectly conducting $(\tilde{Z}_w'(s) = 0)$ infinitely long cable of radius a = 0.005m. Both half spaces are assumed to have free space characteristics with incident plane wave exciting the narrow slot normally θ_i=90°, ϕ_i=90°, and $\tilde{H}^i_{xo}$=1, $\tilde{H}^i_{zo}$=0. Since it is a resonant slot, there is not marked asymmetry in the distribution, but the interaction of the infinitely long cable alters the magnitude of the field distribution; the closer is the location of the cable, then the stronger the interaction with the narrow slot.

The current induced on the infinitely long cable $\tilde{I}(z)$ can now be evaluated using the expression (2.14) and the appropriate magnetic current distribution obtained in Fig. 2.5. The results of the induced current distribution are shown in Fig. 2.6. Away from the axis $|z|$ = 0, the current on the infinitely long cable does exhibit a small damping behavior and oscillates approximately at the wave length of the incident plane wave field. For large values of $|z|$, the current on the wire has a TEM distribution (which is discussed in the next sections) and all the higher order TM modes in the localized infinitely-long-cable narrow-slot region are evanescent in nature.

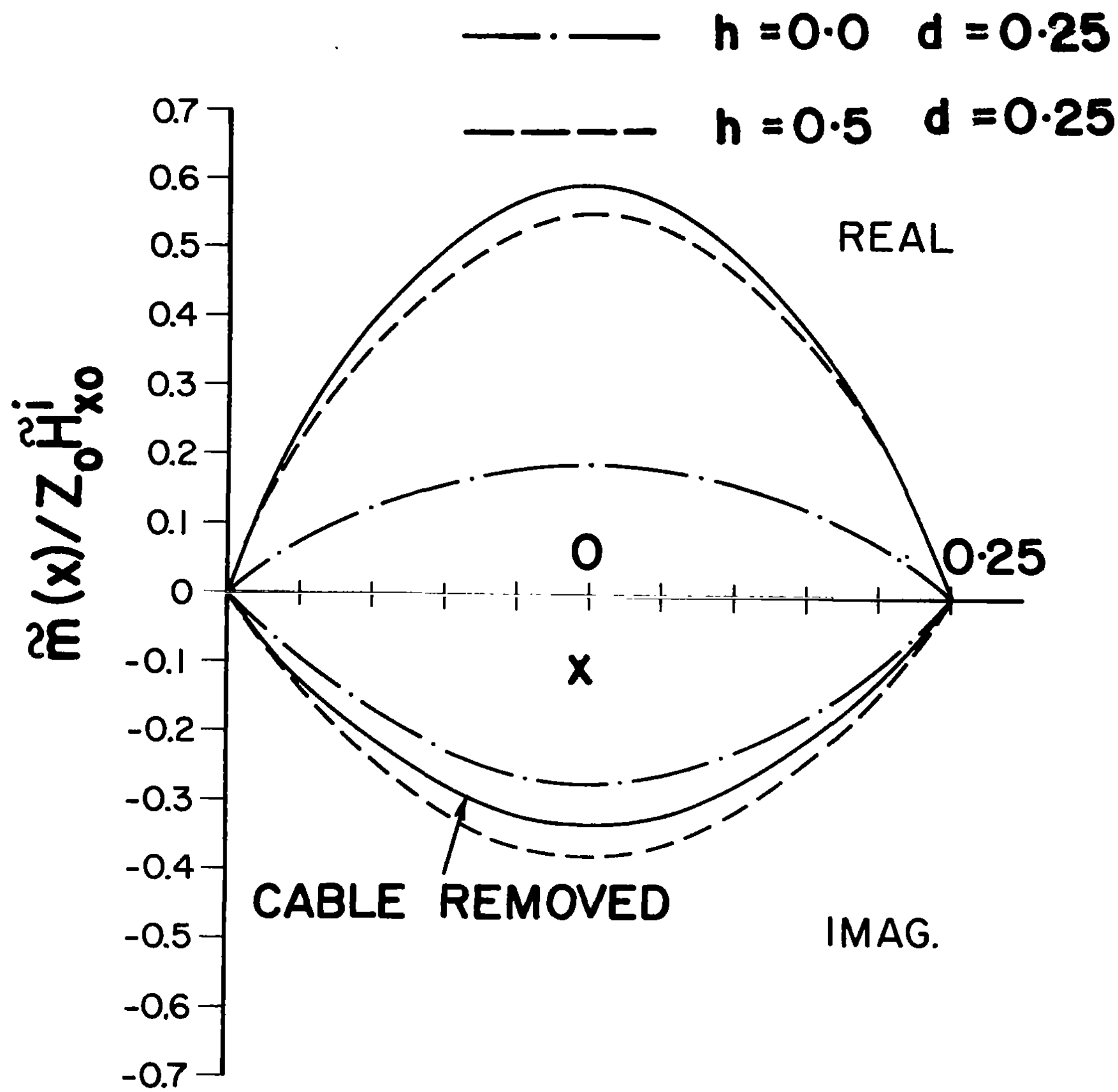

Figure 2.5

Distribution of the Magnetic Current in the Narrow Resonant Slot in the Presence of Infinitely Long Cable at Frequency 300 MHz, ℓ=0.5m, ℓ/w=10, a=0.005m, $\tilde{Z}_w'(s)=0$, ($\sigma=0$, $\mu=\mu_o$, $\varepsilon=\varepsilon_o$)

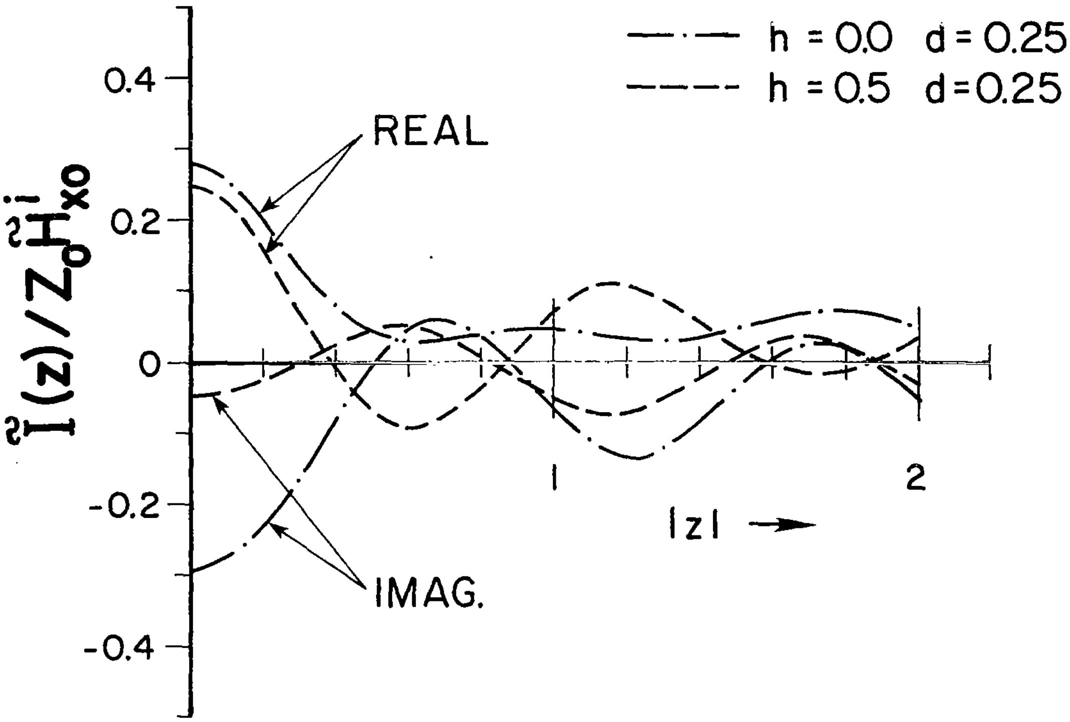

Figure 2.6

Induced Electric Current on the Perfectly Conducting Infinitely Long Cable Behind Narrow Slot at Frequency 300 MHz, ℓ=0.5m, ℓ/w=10, a=0.005m,

$\tilde{Z}_w'(s)=0$, ($\sigma=0$, $\mu=\mu_o$, $\varepsilon=\varepsilon_o$)

Once the integral equations (2.19a and b) are solved for the aperture magnetic current distribution, the fields scattered from the aperture into the two half spaces can be calculated using the expressions (2.16a and b) and (2.28a and b). For electrically small apertures [13,16] the fields scattered can be obtained in terms of the imaged aperture magnetic $m_{a_{im}}$ and electric $p_{a_{im}}$ dipole moments [1,6] (with $\tilde{\vec{H}}^{sc}$ and $\tilde{\vec{E}}^{sc}$ short circuit fields in the y<0 illuminated side),

$$\tilde{\vec{m}}_{a_{im}} = \frac{1}{s\mu} \iint_{S_a} \tilde{\vec{J}}_{s_m}(\vec{r}_a') \, dx_a' dz_a' \tag{3.2a}$$

$$= - \tilde{\vec{\vec{\alpha}}}_m \cdot \tilde{\vec{H}}^{sc} (\vec{r}_a') \tag{3.2b}$$

$$\tilde{\vec{p}}_{a_{im}} = \frac{(\sigma+s\varepsilon)}{2s} \iint_{S_a} [\tilde{\vec{J}}_{s_m}(\vec{r}_a') \times \vec{r}_a'] \, dx_a' dz_a' \tag{3.3a}$$

$$= \frac{(\sigma+s\varepsilon)}{s} \alpha_e \tilde{\vec{E}}^{sc}(\vec{r}_a') \tag{3.3b}$$

and

$$\vec{r}_a' = \vec{1}_x \, x_a' + \vec{1}_z \, z_a' \tag{3.3c}$$

where α_e and $\tilde{\vec{\vec{\alpha}}}_m$ are the imaged electric and the magnetic polarizabilities. The characterization of apertures in terms of the aperture dipole moments introduces errors [1] in the localized near field region and holds good only for large distances away from the aperture.

In a conventional method of moment technique [5,11] for a given shape of aperture, the coupled integral equations (2.19a and b) are solved by dividing the aperture surface into a number of small rectangular patches ($\Delta x_n'$, $\Delta z_n'$) and expanding the two unknown aperture magnetic current distributions in terms of suitable piecewise functions,

$$\tilde{J}_{s_{m_x}}(x_a', z_a'; s) = \sum_{n=1}^{N_x} \tilde{J}_{s_{m_x}}^{(n)}(s)\, P_n(|x_a' - x_{a_n}'|)\, Q_n(|z_a' - z_{a_n}'|) \tag{3.4a}$$

$$J_{s_{m_z}}(x_a', z_a'; s) = \sum_{\ell=1}^{N_y} J_{s_{m_z}}^{(\ell)}(s)\, P_\ell(|x_a' - x_{a_\ell}'|)\, Q_\ell(|z_a' - z_{a_\ell}'|) \tag{3.4b}$$

where P_n, P_ℓ and Q_n, Q_ℓ are piecewise functions defined within patches $(\Delta x_n', \Delta z_n')$ and $(\Delta x_\ell', \Delta z_\ell')$. $\tilde{J}_{s_{m_x}}^{(n)}$ and $\tilde{J}_{s_{m_z}}^{(\ell)}$ are the numerical amplitude factors at the center of patches which are determined in the solution of the integral equations. Once the field distributions are known in the aperture, the fields radiated can be calculated using (2.16 and (2.28). Also for small apertures, the aperture dipole moments can be calculated using the expressions (3.2) and (3.3).

IV. EQUIVALENT CIRCUIT FOR THE APERTURE INFINITELY-LONG-CABLE COUPLED REGION

This section is mainly devoted to representing the coupled region of the arbitrary shaped aperture and the infinitely long cable in terms of simple lumped circuit elements, and the excitation of the aperture in terms of simple equivalent sources. If the aperture is removed by short circuiting it in the conducting ground plane, we are left with a simple transmission line consisting of the infinitely long cable and the infinitely long ground plane. Hence the presence of the aperture acts like a transition and in the localized coupled region perturbs the otherwise TEM mode by exciting the higher order TM modes. The higher order TM modes are evanescent in nature. For large values of $|z|$ away from the aperture, there exists only the TEM mode between the infinitely long cable and the conducting ground plane.

A detailed follow-up procedure to arrive at the equivalent circuits from the field theory as applied to wave guide transitions is discussed in [17]. Based on this technique the braided-shield coaxial cable [3] was analyzed and the appropriate equivalent circuit for the coaxial line was obtained. As such the same technique can be extended to arrive at the equivalent circuit for the transmission line over a ground plane in the presence of an aperture, or one can use the procedure of [7] where in an infinitely long cable passing by a small circular hole is discussed.

But, as far as the lossless coupled region of the arbitrarily shaped aperture and the infinitely long cable is concerned, we arrive at the appropriate equivalent circuit from a straightforward and simple evaluation of the expression (2.14) derived in Section II for the current induced on the infinitely long cable. Relevant equivalent-circuit element values and sources are obtained for the case of a rectangular aperture and further specialized to the case of a narrow slot in the conducting ground plane. For the case of an infinitely long cable located close to the ground plane, appropriate correction terms affecting the distribution are also obtained which do not appear in the derivation of [7] due to nature of approximations introduced.

Referring to the expression (2.14) the current on the infinitely long cable is given by

$$\tilde{I}(z,s) = \iint_{S_a} \tilde{J}_{s_{m_x}}(x_a';s)\,\tilde{T}(z,x_a',z_a';s)\,dx_a'dz_a' \tag{4.1a}$$

where the transfer admittance function is

$$\tilde{T}(z,x_a',z_a';s) = \frac{1}{2\pi j}\int_{C_\zeta} \frac{\tilde{N}(\zeta,s)}{\tilde{D}(\zeta,s)}\, e^{\zeta(z-z_a')}\,d\zeta \tag{4.1b}$$

In the above integral C_ζ is the Bromwich contour in the complex ζ-plane, Fig. A-1b. For $(z-z_a') > 0$, the contour C_ζ is closed in the left half of ζ-plane and applying the residue theorem, the complex admittance function simplifies [9] to,

$$\tilde{T}(z,x_a',z_a';s) = \sum_n \frac{\tilde{N}(\zeta_n,s)}{\tilde{D}'(\zeta_n,s)}\, e^{\zeta_n(z-z_a')} + \tilde{T}_{bc}(z,x_a',z_a';s) \tag{4.2a}$$

in which ζ_n are the poles of the admittance function integrand and are also zeros of the modal equation

$$\tilde{\mathcal{D}}(\zeta_n,s) = 0 \tag{4.2b}$$

and in the expression (4.2a),

$$\tilde{\mathcal{D}}'(\zeta_n,s) = \frac{d}{d\zeta}\tilde{\mathcal{D}}(\zeta,s)\Big|_{\zeta=\zeta_n} \tag{4.2c}$$

and the term $\tilde{T}_{bc}$ is the branch cut contribution (continuous spectrum of higher order evanescent modes) associated with the branch point $\zeta=-\gamma$ shown in the Fig. A-1b.

For the case of perfectly conducting, infinitely long cable, the impedance loading function $\tilde{Z}_w'(s)$ is zero, and the TEM part of the solution which exists for large values of $|z|$ is obtained by evaluating the integral expression (4.1b) at the branch point $\zeta=-\gamma$. Hence, for $(z-z_a') > 0$, around the branch point $\zeta=-\gamma$ we have

$$\tilde{T}(z,x_a',z_a';s) = \frac{C_f}{2Z_o \ln(\frac{2d}{a})}\left[\frac{d}{\Psi_c^2}\right] e^{-\gamma|z-z_a'|} \qquad (4.3a)$$

$$|z| \rightarrow \infty$$

$$C_f = (1 - \frac{a}{2d}) \qquad (4.3b)$$

where the term C_f is the correction factor for the admittance function which accounts for the closer location of the infinitely long cable relative to the ground plane. The correction factor C_f can be taken equal to unity if the vertical distance d of the cable is large compared to the radius of the cable. Hence, the TEM excited current on the infinitely long cable is obtained by substituting (4.3a) into the expression (4.1a),

$$\tilde{I}(z,s) = \frac{C_f}{2\pi Z_c}\iint_{S_a} (\frac{d}{\Psi_c^2})\ e^{-\gamma|z-z_a'|}\ \tilde{J}_{s_{m_x}}(x_a',z_a',s)\ dx_a'dz_a' \qquad (4.4a)$$

and

$$Z_c = \frac{Z_o}{2\pi} \ln(\frac{2d}{a}) \qquad (4.4b)$$

is the characteristic impedance of the infinitely long lossless cable transmission line over the ground plane.

Similarly, the TEM voltage $\tilde{V}(z,s)$ across the infinitely long cable and the ground plane can be calculated [3,17] using $\tilde{\vec{E}}^{(w)}(\vec{r},s)$ in the expression (2.28a) or the well known TEM relationship between the voltage and the current [6,18]

$$\tilde{V}(z,s) = Z_c\tilde{I}(z,s) \qquad (4.5a)$$

and thus we have the voltage expression

$$\tilde{V}(z,s) = \frac{C_f}{2\pi}\iint_{S_a} (\frac{d}{\Psi_c^2})\ e^{-\gamma|z-z_a'|}\ \tilde{J}_{s_{m_x}}(x_a',z_a';s)\ dx_a'dz_a' \qquad (4.5b)$$

It is interesting to note that, to arrive at the solutions of $\tilde{V}(z,s)$ and $\tilde{I}(z,s)$ for the infinitely long cable, no approximations have been made (except for thin wire model wherein uniform current density is assumed around the cable). For a given aperture shape the distribution $\tilde{J}_{s_{m_x}}(x_a', z_a', s)$ is to be determined as a solution to the coupled integral equations (2.19a and b).

A. Quasi-Static Circuit Representation (Lossless Coupled Region)

In the quasi-static approximation $|\gamma A| << 1$, where A is the largest dimension of the aperture, the exponential term in (4.4a) and (4.5b) can be written as

$$e^{-\gamma(z-z_a')} \simeq e^{-\gamma z} (1 + \gamma z_a') \tag{4.6a}$$

and if

$$A << [\Psi_0^2 = (h^2 + d^2)] \tag{4.6b}$$

then the expressions (4.4a) and (4.5b) for the TEM line current and voltage can be written in terms of the imaged magnetic and the electric dipole moments,

$$\tilde{I}(z,s) = \frac{e^{-\gamma z}}{2} \left[\frac{s\mu}{\pi Z_c} (\tilde{m}_{a_{im}}^{(x)}) - \frac{2sZ_0}{\pi Z_c} (\tilde{p}_{a_{im}}^{(x)})\right] \left(\frac{d}{\Psi_0^2}\right) C_f \tag{4.7a}$$

$$\tilde{V}(z,s) = \frac{e^{-\gamma z}}{2} \left[\frac{s\mu}{\pi} (\tilde{m}_{a_{im}}^{(x)}) - \frac{2sZ_0}{\pi} (\tilde{p}_{a_{im}}^{(x)})\right] \left(\frac{d}{\Psi_0^2}\right) C_f \tag{4.7b}$$

where referring to the dipole-moment definitions (3.2) and (3.3),

$$\tilde{m}_{a_{im}}^{(x)} = \frac{1}{s\mu} \iint_{S_a} \tilde{J}_{s_{m_x}}(x_a', z_a'; s)\, dx_a' dz_a' \tag{4.7c}$$

$$\tilde{p}_{a_{im}}^{(x)} = \frac{\varepsilon}{2} \iint_{S_a} [-z_a' \tilde{J}_{s_{m_x}}(x_a', z_a'; s)]\, dx_a' dz_a' \tag{4.7d}$$

which are assumed to exist at the "geometric center" of the aperture. Since the lossless coupled region formed by the infinitely long wire and the aperture is linear [18], referring to the Appendix B for appropriate Green functions, the TEM current and voltage solutions (4.7a and b) satisfy the linear transmission line equations with sources,

$$\frac{d\tilde{V}(z,s)}{dz} = -sL'\tilde{I}(z,s) + \tilde{B}_1(s)\,\delta(z) \tag{4.8a}$$

$$\frac{d\tilde{I}(z,s)}{dz} = -sC'\tilde{V}(z,s) + \tilde{B}_2(s)\,\delta(z) \tag{4.8b}$$

where L' and C' are the inductance and capacitance per unit length (Appendix C) of the single wire transmission line over the conducting ground plane. The terms $\tilde{B}_1(s)$ and $\tilde{B}_2(s)$ are the coupled region coefficients from which the equivalent sources and the transmission line-aperture parameters can be determined,

$$\tilde{B}_1(s) = \frac{s\mu}{\pi}\,(m_{a_{im}}^{(x)})\,\left(\frac{d}{\Psi_o^2}\right)\,C_f \tag{4.9a}$$

$$\tilde{B}_2(s) = -\,\frac{2sZ_o}{\pi Z_c}\,(\tilde{p}_{a_{im}}^{(x)})\,\left(\frac{d}{\Psi_o^2}\right)\,C_f \tag{4.9b}$$

In the above expressions, the imaged dipole moments $\tilde{m}_{a_{im}}^{(x)}$ and $\tilde{p}_{a_{im}}^{(x)}$ account for the short circuit fields due to the external excitation as well as the fields radiated by the infinitely long wire. For very small apertures, the dipole moments can be written in terms of aperture polarizabilities. We again note that the imaged dipole moments $\tilde{m}_{a_{im}}$ defined in (4.7c) and $\tilde{p}_{a_{im}}^{(x)}$ defined in (4.7d) are in fact obtained solely by the magnetic current distribution $\tilde{J}_{s_{m_x}}^{(s)}$. A word of caution is to note that the electric dipole moment defined in the expression (3.3a) is actually obtained both from the x and z components of the magnetic current distribution.

The imaged dipole moments can now be written (for small apertures) in terms of aperture polarizabilities,

$$\tilde{m}^{(x)}_{a_{im}} = \tilde{\alpha}^{(xx)}_{m} \left[\tilde{H}^{sc}_{x}(o_+) + \tilde{H}^{(w)}_{x}(o_+)\right] \tag{4.10a}$$

$$\tilde{p}^{(x)}_{a_{im}} = -\varepsilon\tilde{\alpha}^{(x)}_{e}\left[\tilde{E}^{sc}_{y}(0_+) + \tilde{E}^{(w)}_{y}(0_+)\right] \tag{4.10b}$$

The $\tilde{m}^{(x)}_{a_{im}}$ and $\tilde{p}^{(x)}_{a_{im}}$ are in fact referred to the y>0 shadow region in the above expressions. $\tilde{\alpha}^{(xx)}_{m}$ is the x component of the imaged magnetic polarizability and $\tilde{\alpha}^{(x)}_{e}$ is the imaged partial electric polarizability in the y- direction. Further, $\tilde{H}^{sc}_{x}(o_+)$ and $\tilde{E}^{sc}_{y}(0_+)$ are the short circuit magnetic and electric field components. Similarly $\tilde{H}^{(w)}_{x}(0_+)$ and $\tilde{E}^{(w)}_{y}(0_+)$ are the scattered magnetic and electric field components set up due to the induced currents on the infinitely long wire. Since we are interested with TEM circuit characterization, referring to Appendix C,

$$\tilde{E}^{(w)}_{y}(0_+) = \tilde{V}_{o}(z)\ \tilde{e}_{o}(x,y) = \left[\frac{Z_o}{Z_c}\right]\left[\frac{d}{\pi\Psi_o^2}\right]\tilde{V}(z) \tag{4.11a}$$

and

$$\tilde{H}^{(w)}_{x}(0_+) = \tilde{I}_{o}(z)\ \tilde{h}_{o}(x,y) = -\left[\frac{d}{\pi\Psi_o^2}\right]\tilde{I}(z) \tag{4.11b}$$

Hence substituting the expressions (4.10) and (4.11) into the coefficients (4.9), the linear transmission line equations take the form,

$$\frac{d\tilde{V}(z,s)}{dz} = -sL'\ \tilde{I}(z,s) - sL_a\ \tilde{I}(z,s)\ \delta(z)$$

$$+ \tilde{V}_{eq}(s)\ \delta(z) \tag{4.12a}$$

$$\frac{d\tilde{I}(z,s)}{dz} = -sC'\tilde{V}(z,s) + sC_a\tilde{V}(z,s)\,\delta(z)$$

$$+ \tilde{I}_{eq}(s)\,\delta(z) \qquad (4.12b)$$

where

$$L_a = \mu\left[\frac{d}{\pi\Psi_o^2}\right]^2 \tilde{\alpha}_m^{(xx)}\, C_f \qquad (4.12c)$$

$$C_a = 2\varepsilon\left[\frac{Z_o}{Z_c}\,\frac{d}{\pi\Psi_o^2}\right]^2 \tilde{\alpha}_e^{(x)}\, C_f \qquad (4.12d)$$

$$\tilde{V}_{eq}(s) = \frac{s\mu}{\pi}\,\frac{d}{\Psi_o^2}\left[\tilde{\alpha}_m^{(xx)}\tilde{H}_x^{sc}(0_+)\right] C_f \qquad (4.12e)$$

$$\tilde{I}_{eq}(s) = \frac{2s\varepsilon Z_o}{\pi Z_c}\,\frac{d}{\Psi_o^2}\left[\tilde{\alpha}_e^{(x)}\tilde{E}_y^{sc}(0_+)\right] C_f \qquad (4.12f)$$

The above equations give the complete TEM circuit representation, Fig. 4.1, of the coupled region formed by the electrically small aperture and the infinitely long wire for the TEM mode on the wire transmission line. The element values L_a and C_a account for interaction between the TEM transmission line and the aperture. Further the equivalent voltage source $\tilde{V}_{eq}(s)$ and the current source $\tilde{I}_{eq}(s)$ corresponds to the excitation of the aperture in the $y<0$ region.

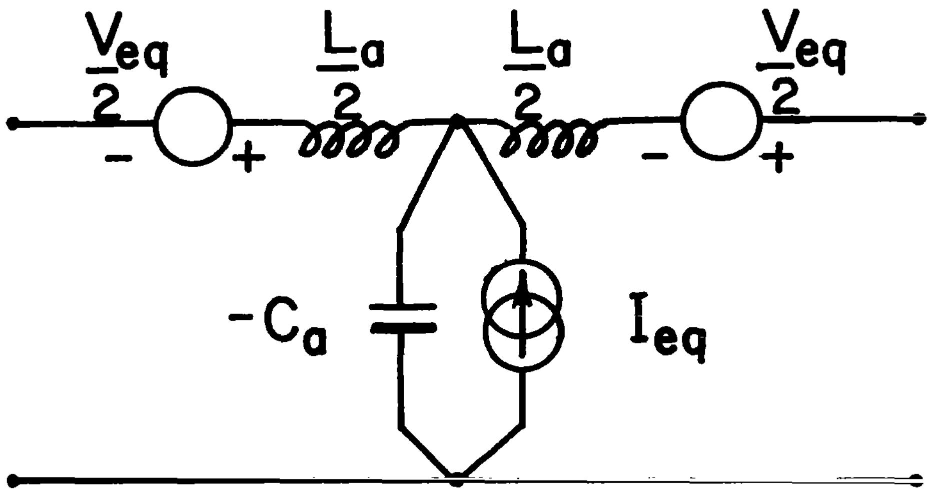

Figure 4.1

Equivalent Circuit for the Infinitely Long Wire-Aperture Coupled Region, T-Form

V. REMARKS

A systematic treatment of the excitation of an infinitely long loaded cable behind an arbitrarily shaped aperture is obtained in this note. Extension of the analysis to multiple parallel cables is also considered. In the quasi-static limit, an equivalent circuit consisting of lumped elements and sources is obtained for the coupled regions.

APPENDIX A

Branch of Parameter "u" in ζ-Plane

Consider the following integral (expression (2.7a)) for two dimensional field representation in ζ-plane

$$\tilde{F}(z,s) = \frac{1}{2\pi j}\int_{C_\zeta} \tilde{F}(\zeta,s)\, K_0(u\Psi)\, e^{\zeta z}\, d\zeta \tag{A-1}$$

where

$$u = [\gamma^2 - \zeta^2]^{1/2} \tag{A-2}$$

and C_ζ is the Bromwich contour, Fig. 2.2. The radial wave number u is a double valued function and hence its branch should be properly defined in the complex ζ-plane in order that the integral (A-1) uniquely represents the function $\tilde{F}(z,s)$. Assuming the spectral term $\tilde{F}(\zeta,s)$ can be an artibrary distribution, we have

$$K_0(u\Psi)\, e^{\zeta z} \sim e^{-u\Psi}\, e^{\zeta z} \tag{A-3}$$

For z>0, the contour C_ζ is closed in the LHS of the ζ-plane and for radiation condition to hold,

$$\text{Re}\,[u] > 0 \tag{A-5}$$

$$\text{Im}\,[u] > 0 \tag{A-6}$$

To uniquely define the double valued function u, the complex ζ-plane is viewed as two Riemann sheets in which the top or proper sheet has the value of Re u>0 and in the bottom or improper sheet has the value of Re u<0. The branch points are located at $\zeta = \pm\gamma$ as shown in Fig. A-1. Further

$$u^2 = (\gamma_1^2 - \gamma_2^2) - (\zeta_1^2 - \zeta_2^2) + 2j\,(\gamma_1\gamma_2 - \zeta_1\zeta_2) \tag{A-7}$$

$$= |u|^2\, e^{j\phi} \tag{A-8}$$

The plots of $\text{Re}[u^2]$ and $\text{Im}[u^2]$ separating the various regions in the ζ-plane are shown in Fig. A-1a. The horizontally shaded region corresponds to $\text{Re}[u^2]<0$ and the vertically shaded region corresponds to $\text{Im}[u^2]<0$, and in the remaining region of the ζ-plane $\text{Re}[u^2]>0$, $\text{Im}[u^2]>0$. For Re u>0 on the top proper Riemann sheet, the angle $|\phi|<\pi$ which dictates the choice of the branch to be $\phi=\pi$ or $\text{Im}[u^2]=0$ and $\text{Re}[u^2]<0$.

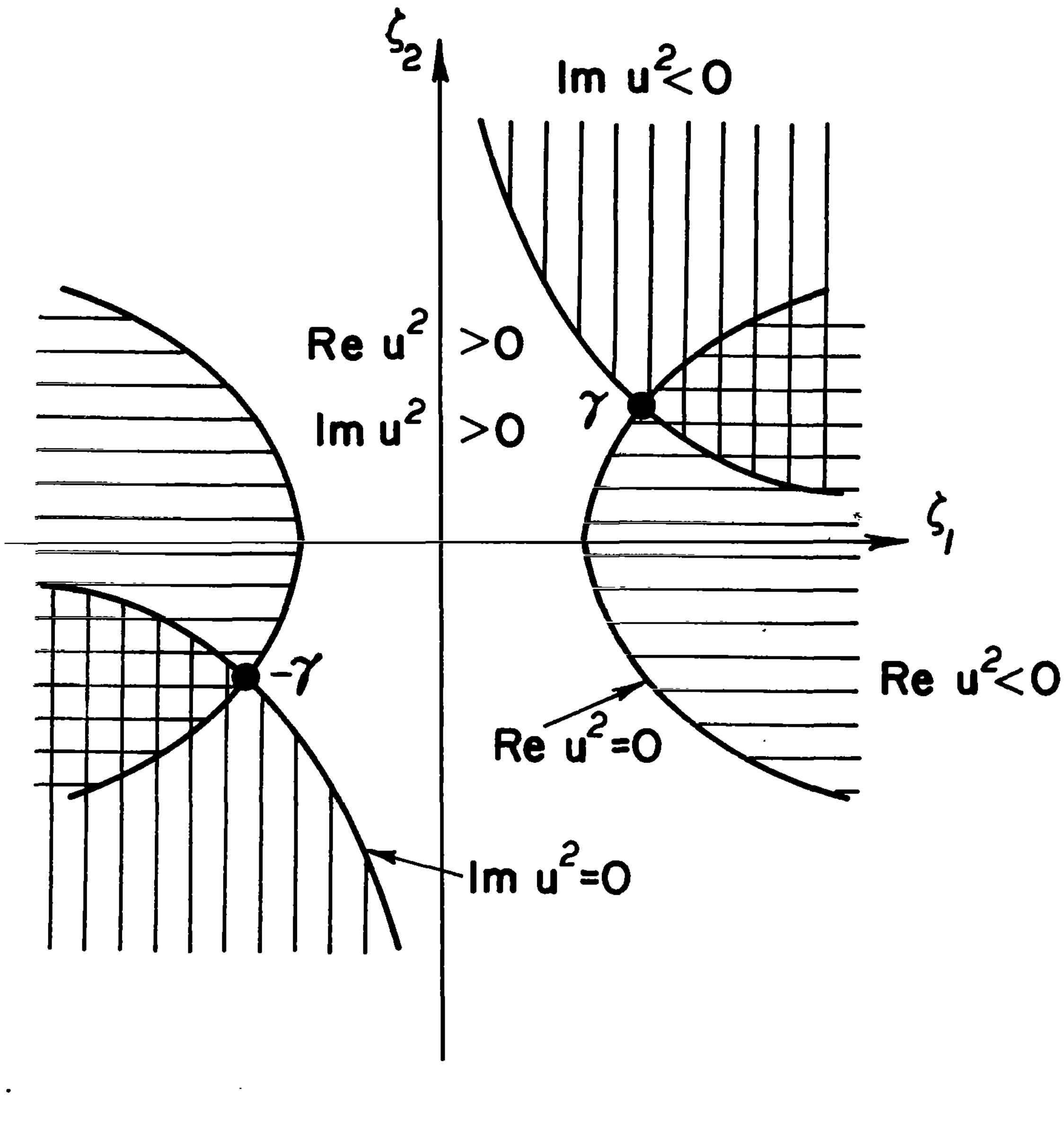

Figure A-1a

Plot of u^2 in Complex ζ-plane

$\gamma=\gamma_1+j\gamma_2$, $\zeta=\zeta_1+j\zeta_2$

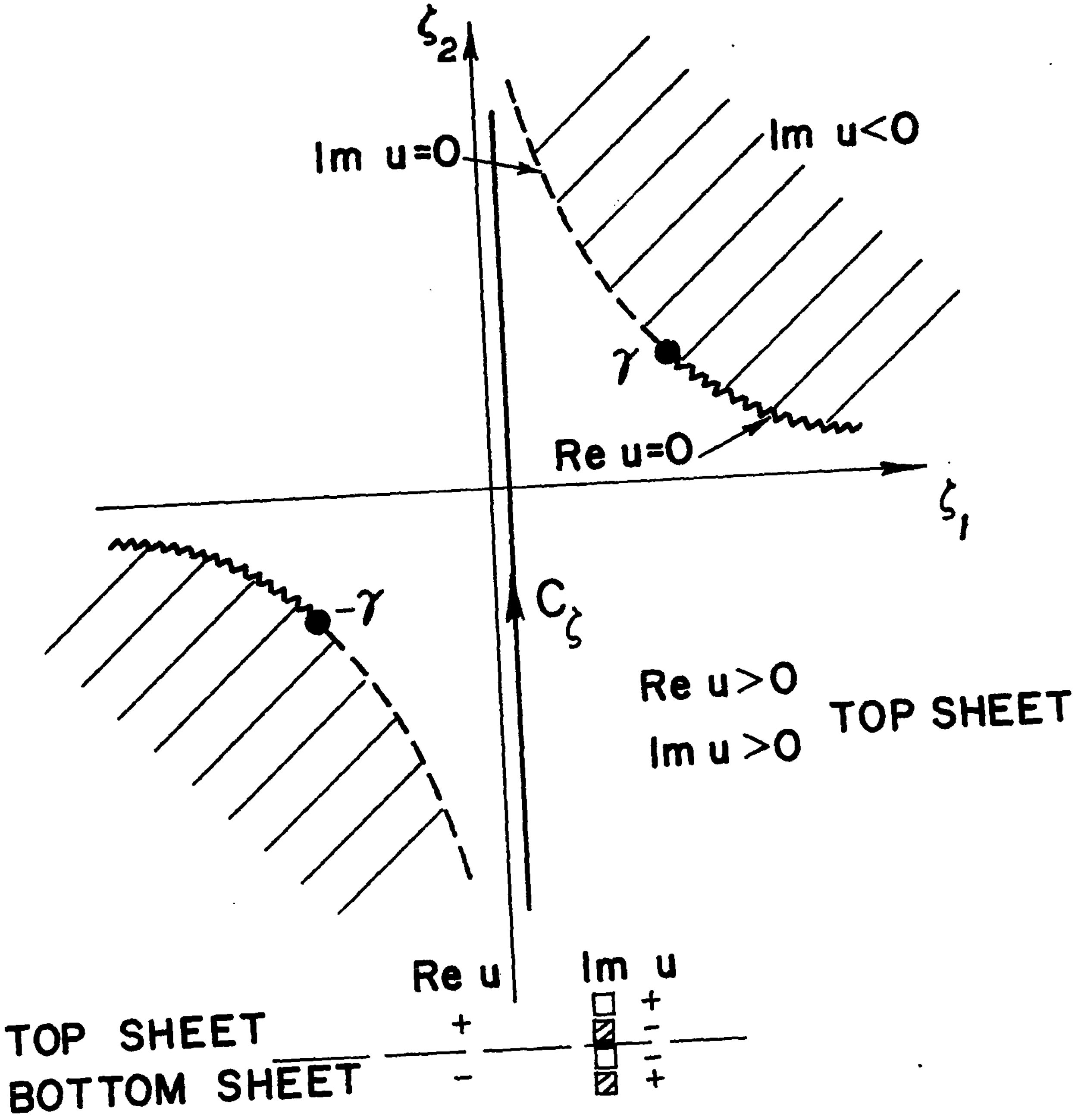

Figure A-1b

Branch Cuts of $u=(\gamma^2-\zeta^2)^{1/2}$ in Complex γ Plane; $\gamma=\gamma_1+j\gamma_2$, $\zeta=\zeta_1+j\zeta_2$

In Fig. A-1b is shown the appropriate branch cuts for the function u in the complex ζ-plane.

APPENDIX B

Green Functions for Transmission Line with Sources

A detailed analysis of a TEM transmission line with sources based on a Green's function approach can be found in [18] with linearity and superposition assumed. With general current and voltage sources, one has to solve the first order linear differential equations,

$$\frac{d\tilde{V}(z)}{dz} = -Z\tilde{I}(z) + \tilde{V}^{(s)}(z) \qquad \text{(B-1)}$$

$$\frac{d\tilde{I}(z)}{dz} = -Y\tilde{V}(z) + \tilde{I}^{(s)}(z) \qquad \text{(B-2)}$$

where

$Z' = R' + sL'$	impedance of transmission line per unit length
$Y' = G' + sC'$	admittance of transmission line per unit length
$\tilde{V}^{(s)}(z)$	voltage source distribution
$\tilde{I}^{(s)}(z)$	current source distribution

$\tilde{V}(z)$ and $\tilde{I}(z)$ are the TEM voltage and current responses. The general problem can be split into two by invoking superposition and linearity. The voltage and current can be written as

$$\tilde{V}(z) = V^{(\nu)}(z) + V^{(i)}(z) \qquad \text{(B-3)}$$

$$\tilde{I}(z) = I^{(\nu)}(z) + I^{(i)}(z) \qquad \text{(B-4)}$$

where $V^{(\nu)}$ and $I^{(\nu)}$ are solutions with voltage sources acting alone and are solutions of, Fig. B-2,

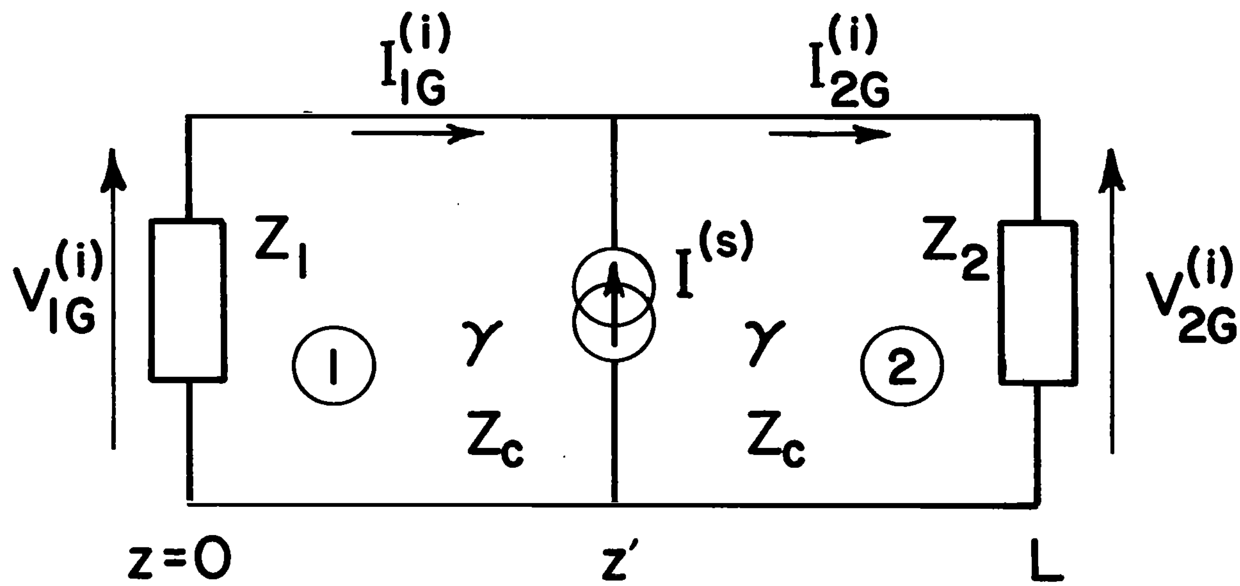

Figure B-1

Matched Transmission Line With Current Source

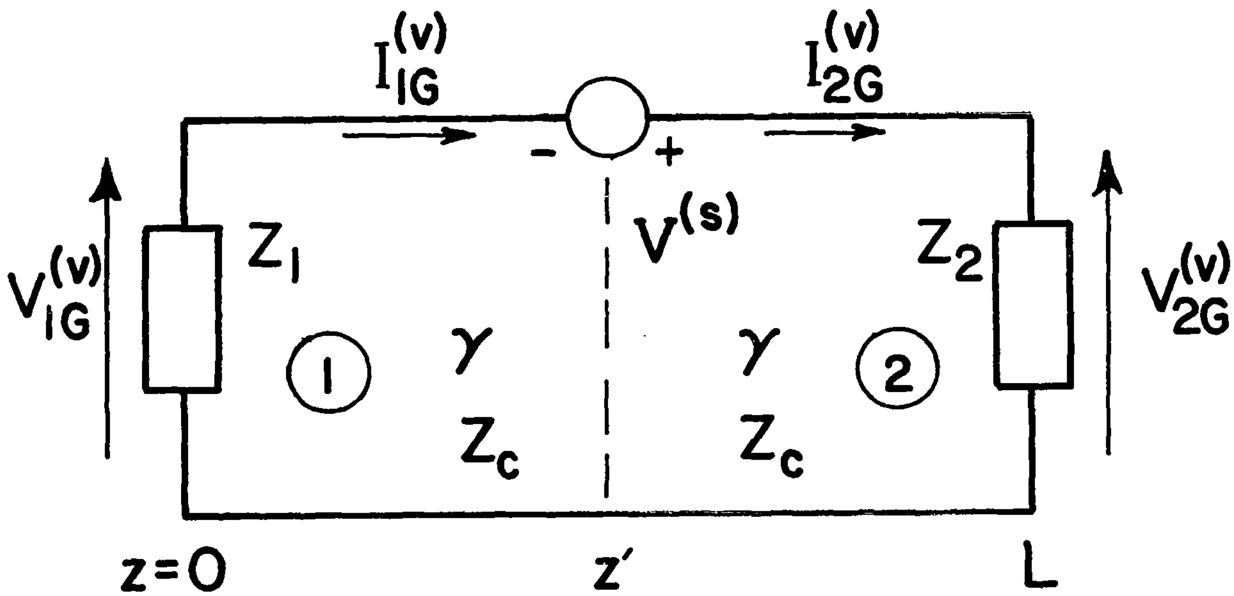

Figure B-2

Matched Transmission Line with Voltage Source

$$\frac{d\tilde{V}^{(\nu)}(z)}{dz} = -Z'\tilde{I}^{(\nu)}(z) + \tilde{V}^{(s)}(z) \qquad \text{(B-5)}$$

$$\frac{d\tilde{I}^{(\nu)}(z)}{dz} = -Y'\tilde{V}^{(\nu)}(z) \qquad \text{(B-6)}$$

Similarly $\tilde{V}^{(i)}$ and $\tilde{I}^{(i)}$ are solutions with current sources acting alone and are solutions of, Fig. B-1

$$\frac{d\tilde{V}^{(i)}(z)}{dz} = -Z'\tilde{I}^{(i)}(z) \qquad \text{(B-7)}$$

$$\frac{d\tilde{I}^{(i)}(z)}{dz} = -Y'\tilde{V}^{(i)}(z) + \tilde{I}^{(s)}(z) \qquad \text{(B-8)}$$

The two sets of equations (B-5), (B-6) and (B-7), (B-8) form simple linear differential equations and the solutions can be obtained by a straightforward method once the appropriate boundary conditions are specified.

The specific transmission line problems with current and voltage sources treated separately are shown in Figs. B-1 and B-2. To obtain appropriate Green's functions point or delta function sources are assumed and the Green's functions are given in the following.

Referring to the Fig. B-1, wherein a transmission line of length L is shown with both ends matched, and a unit delta function current source located at $z=z'$, the equations (B-7) and (B-8) yield the Green functions,

$$\tilde{V}_{1G}^{(i)}(z) = \frac{Z_c}{2}\ e^{\gamma(z-z')} \qquad \text{(B-9)}$$

$$V_{2G}^{(i)}(z) = \frac{Z_c}{2}\ e^{-\gamma(z-z')} \qquad \text{(B-10)}$$

$$\tilde{I}_{1G}^{(i)}(z) = -\frac{1}{2}\, e^{\gamma(z-z')} \qquad \text{(B-11)}$$

$$\tilde{I}_{2G}^{(i)}(z) = \frac{1}{2}\, e^{-\gamma(z-z')} \qquad \text{(B-12)}$$

with

$$\int_0^L \tilde{I}^{(s)}(z') \, \delta(z-z') \, dz' = 1 \qquad \text{(B-13)}$$

Similarly, referring to the Fig. B-2, with a unit delta function voltage source located at $z=z'$, the equations (B-5) and (B-6) yield the Green functions,

$$\tilde{V}_{1G}^{(v)}(z) = -\frac{1}{2} e^{\gamma(z-z')} \qquad \text{(B-14)}$$

$$\tilde{V}_{2G}^{(v)}(z) = \frac{1}{2} e^{-\gamma(z-z')} \qquad \text{(B-15)}$$

$$\tilde{I}_{1G}^{(v)}(z) = \frac{1}{2Z_c} e^{\gamma(z-z')} \qquad \text{(B-16)}$$

$$\tilde{I}_{2G}^{(v)}(z) = \frac{1}{2Z_c} e^{-\gamma(z-z')} \qquad \text{(B-17)}$$

with

$$\int_0^L \tilde{V}^{(s)}(z') \, \delta(z-z') \, dz' = 1 \qquad \text{(B-18)}$$

In fact the above Green functions are applicable even for an infinitely long transmission line with appropriate sources. For any other source distribution along the transmission line, the solution is obtained by constructing the superposition integral since the transmission line is linear [18]; with general current source distribution,

$$\tilde{V}^{(i)}(z) = \int \tilde{I}^{(s)}(z') \, \tilde{V}_G^{(i)}(z,z') \, dz' \qquad \text{(B-19)}$$

$$\tilde{I}^{(i)}(z) = \int \tilde{I}^{(s)}(z') \, \tilde{I}_G^{(i)}(z,z') \, dz' \qquad \text{(B-20)}$$

and for general voltage source distribution,

$$\tilde{V}^{(\nu)}(z) = \int \tilde{V}^{(s)}(z') \, V_G^{(\nu)}(z,z') \, dz' \tag{B-21}$$

$$\tilde{I}^{(\nu)}(z) = \int \tilde{V}^{(s)}(z') \, \tilde{I}_G^{(\nu)}(z,z') \, dz' \tag{B-22}$$

The above results are utilized in Section IV for deriving the equivalent circuit of aperture-infinitely long wire coupled region. The above solutions can also be written in the combined voltage and current form discussed in [19].

APPENDIX C

Normalization Considerations From Field to Circuit Quantities

A systematic reduction of the field equations to the corresponding circuit equations is rigorously discussed in [17]. As applied to TEM representative quantities, the transverse field vectors are written as

$$\tilde{\vec{E}}_{to}(\vec{r}) = V_o(z)\ \tilde{\vec{e}}_o(x.y) \tag{C-1}$$

$$\tilde{\vec{H}}_{to}(\vec{r}) = I_o(z)\ \vec{h}_o(x.y) \tag{C-2}$$

where $\tilde{V}_o(z)$ and $\tilde{I}_o(z)$ are the modal voltage and current quantities; $\tilde{\vec{e}}_o(x,y)$ and $\tilde{\vec{h}}_o(x,y)$ are the TEM distributions in the transverse plane and are chosen in such a way as to satisfy the power orthognality condition

$$\iint (\tilde{\vec{e}}_o \times \tilde{\vec{h}}_o) \cdot \vec{1}_z\, dz = 1 \tag{C-3}$$

Also we have

$$\tilde{\vec{h}}_o = \vec{1}_z \times \tilde{\vec{e}}_o \tag{C-4}$$

The Equation (C-3) helps us to choose proper normalization constants in reducing the field equations to the circuit form. The terminal voltage $\tilde{V}(z)$ and current $\tilde{I}(z)$ in a TEM line satisfy the relationship

$$\frac{\tilde{V}(z)}{\tilde{I}(z)} = Z_c \text{ (line characteristic impedance)} \tag{C-5}$$

$$= NZ_o \tag{C-6}$$

and the field modal voltage and current satisfy

$$\frac{\tilde{V}_o(z)}{\tilde{I}_o(z)} = Z_o \text{ (medium characteristic impedance)} \tag{C-7}$$

where N is the normalization constant. Also the condition (C-3) gives the power flow expression

$$P = \frac{1}{2} \text{Re} (\tilde{V}\tilde{I}^*) = \frac{1}{2} \text{Re} (\tilde{V}_o \tilde{I}_o^*) \qquad \text{(C-8)}$$

which yields the basic relationships between line and modal voltage and current quantities,

$$\tilde{V}_o(z) = \frac{\tilde{V}(z)}{\sqrt{N}} \qquad \text{(C-9)}$$

$$\tilde{I}_o(z) = \tilde{I}(z)\sqrt{N} \qquad \text{(C-10)}$$

For the case of an infinitely long wire over a ground plane, we refer to Fig. 2.1 with the aperture short circuited,

$$\tilde{\vec{E}}_{to}(\vec{r}) = \frac{V_o(z)}{\pi\sqrt{N}} (\tilde{e}_{xo} \vec{1}_x + \tilde{e}_{yo} \vec{1}_y) \qquad \text{(C-11)}$$

$$N = \frac{1}{2\pi} \ln \left(\frac{2d}{a}\right) \qquad \text{(C-12)}$$

$$\tilde{e}_{xo} = \frac{2(x-h)yd}{d_{xy}} \qquad \text{(C-13)}$$

$$\tilde{e}_{yo} = \frac{-d\,[(x-h)^2 - y^2 + d^2]}{d_{xy}} \qquad \text{(C-14)}$$

$$d_{xy} = [(x-h)^2 + (y-d)^2]\,[(x-h)^2 + (y+d)^2] \qquad \text{(C-15)}$$

Hence for the infinitely long transmission line over a ground plane, we have

$$\frac{d\tilde{V}(z)}{dz} = -sL' \, \tilde{I}(z) \qquad \text{(C-16)}$$

$$\frac{d\tilde{I}(z)}{dz} = -sC'\tilde{V}(z) \qquad \text{(C-17)}$$

with the circuit constants

$$L' = \frac{\mu}{2\pi} \ln \left(\frac{2d}{a}\right) \tag{C-18}$$

$$C' = \frac{2\pi\varepsilon}{\ln\left(\frac{2d}{a}\right)} \tag{C-19}$$

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