C.1-NC

INTERACTION NOTES

Note 393

April 1980

EXPERIMENTAL, TIME-DOMAIN CHARACTERIZATION OF PARTIALLY DEGENERATE THREE-CONDUCTOR TRANSMISSION LINES

Ashok K. Agrawal and Harold J. Price
Mission Research Corporation, Albuquerque, N.M. 87108

ABSTRACT

In a recent paper [1], a method for the time-domain characterization of lossless multiconductor transmission lines with cross-sectionally inhomogeneous dielectrics was presented. This method is limited to lines with completely nondegenerate propagation; that is, all the modes have distinct propagation velocities. In this paper, a method is presented for the characterization of lossless, partially degenerate three-conductor lines, together with experimental data. The results are in good agreement with independent frequency-domain measurements.

CONTENTS

ILLUSTRATIONS

I. INTRODUCTION

In general, a multiconductor transmission line (N conductors plus a reference) with cross-sectionally inhomogeneous dielectric will have N propagation modes, each having a different velocity. A knowledge of the modal velocities and the modal amplitudes is needed in order to obtain per-unit-length inductance and capacitance matrices from the characteristic-impedance matrix[1]. The characteristic-impedance matrix of the line can be measured by the method described in [2]. In a recent paper [1], a method for the time-domain characterization of multiconductor transmission lines in a cross-sectionally inhomogeneous medium was presented. This method assumes completely nondegenerate propagation on the line; that is, all the modes have distinct propagation velocities. Degeneracy among some modes may occur due to symmetry. For partially degenerate propagation, where some groups of modes have the same propagation velocity in each group, the degenerate modes cannot be resolved in time. Since modes with the same velocity travel together, only the sum of the modal voltage or current amplitudes for these modes can be measured.

Consider a lossless transmission line formed by N conductors, plus a reference conductor (ground or shield). The line is assumed to be uniform along its length, but with arbitrary cross section. The dielectric surrounding the conductors is inhomogeneous (e.g., cable made of insulated conductors partially separated by air). The transmission line equations are*:

$$\frac{\partial}{\partial z} [V_n(z,t)] = -[L_{nm}] \frac{\partial}{\partial t} [I_m(z,t)] \tag{1}$$

$$\frac{\partial}{\partial z} [I_n(z,t)] = -[C_{nm}] \frac{\partial}{\partial t} [V_m(z,t)] \tag{2}$$

with $n = 1, 2, \cdots, N$; $m = 1, 2, \cdots, N$

where V_m and I_m represent the voltage and current on the m th conductor, respectively. $[L_{nm}]$ and $[C_{nm}]$ are, respectively, per-unit-length inductance and capacitance matrices.

*Note on the use of variables: the variables with single subscript in a bracket represent vectors and the variable with double subscript in a bracket represent matrices.

The components of the forward traveling voltage or current vectors are components of the eigenvectors of the matrix product $[L_{nm}][C_{nm}]$ or $[C_{nm}][L_{nm}]$, respectively, [1], [3]. A velocity matrix, which is similar to $[L_{nm}][C_{nm}]$ or $[C_{nm}][L_{nm}]$, was introduced in [1]. The velocity matrix can be constructed from the voltage or current eigenvectors and the eigenvalues determined from Time-Domain-Reflectometer (TDR) measurements, using the relations:

$$[v_{nm}] = [V_{nm}][v_i][V_{nm}]^{-1} \tag{3}$$

$$[v'_{nm}] = [I_{nm}][v_i][I_{nm}]^{-1} \tag{4}$$

where $[v_{nm}]$ is the velocity matrix whose eigenvalues are v_i's and the eigenvectors are $[V_n]_i$; the matrix $[V_{nm}]$ is formed by vectors $[V_n]_i$ as its columns. $[v'_{nm}]$ is the velocity matrix whose eigenvalues are v_i's and the eigenvectors are $[I_n]_i$; the matrix $[I_{nm}]$ is formed by the vectors $[I_n]_i$ as its columns. v_i's are the inverse of the square root of the eigenvalues of either $[L_{nm}][C_{nm}]$ or $[C_{nm}][L_{nm}]$.

The per-unit-length inductance and capacitance matrices are obtained from the following relations[1]:

$$[L_{nm}] = [v_{nm}]^{-1}[Y_{c_{nm}}]^{-1} \tag{5}$$

$$[C_{nm}] = [Y_{c_{nm}}][v_{nm}]^{-1} \tag{6}$$

or

$$[L_{nm}] = [Y_{c_{nm}}]^{-1}[v'_{nm}]^{-1} \tag{7}$$

$$[C_{nm}] = [v'_{nm}]^{-1}[Y_{c_{nm}}] \tag{8}$$

where $[Y_{c_{nm}}]$ is the characteristic-admittance matrix of the multiconductor line, and can be obtained from the TDR measurements[2].

In the case of nondegenerate propagation, the eigenvalues are all distinct, a set of linearly independent eigenvectors exists, and equations (3) or (4) are applicable. In the case of partial degeneracy, the existence of linearly independent eigenvectors, for a real nonsymmetric matrix, is not guaranteed in general. It is shown in this paper that since the matrices $[L_{nm}]$ and $[C_{nm}]$ are real, symmetric, and positive definite, they can be simultaneously diagonalized, and thus, the matrix product $[L_{nm}][C_{nm}]$ or $[C_{nm}][L_{nm}]$ can also be diagnoalized. For this situation, it will be shown that a set of linearly independent eigenvectors can be found for multiple eigenvalues.

In order to use equations (3) or (4), the modal amplitudes for the individual modes must be known. For the particular case of a three-conductor line, the modal amplitudes, for the degenerate modes, can be obtained from their sum. Here, the orthogonality property of the modes is used. Finally, experimental results for a three-conductor transmission line over a ground plane are presented. These results are found to be in good agreement with the independent frequency-domain measurements.

It should be noted that for a multiconductor transmission line with homogeneous dielectric, all modes are degenerate. In this case, the inductance and capacitance parameters can be obtained from the knowledge of the characteristic-impedance matrix and the velocity of propagation, which is identical for all modes, using the relations:

$$[L_{nm}] = \frac{1}{v}[Z_{c_{nm}}] \tag{9}$$

$$[C_{nm}] = \frac{1}{v}[Z_{c_{nm}}]^{-1} \tag{10}$$

where $[Z_{c_{mm}}]$ is the characteristic-impedance matrix of the multiconductor line.

II. OROTHOGONAL PROPERTIES OF THE MODES

Consider the transmission-line equations (1) and (2) in the matrix from:

$$\frac{\partial}{\partial z}\begin{bmatrix} V_n(z,t) \\ I_n(z,t) \end{bmatrix} = -\begin{bmatrix} [0_{nm}] & [L_{nm}] \\ [C_{nm}] & [0_{nm}] \end{bmatrix} \frac{\partial}{\partial t}\begin{bmatrix} V_n(z,t) \\ I_n(z,t) \end{bmatrix} \tag{11}$$

where $[0_{nm}]$ is a null matrix of N x N size. Suppose $[T_{nm}]$ is a nonsingular matrix. Then making the transformation

$$\begin{bmatrix} V_n(z,t) \\ I_n(z,t) \end{bmatrix} = \begin{bmatrix} [T_{nm}] & [0_{nm}] \\ [0_{nm}] & [T^t_{nm}]^{-1} \end{bmatrix} \begin{bmatrix} V'_n(z,t) \\ I'_n(z,t) \end{bmatrix} \tag{12}$$

in equation (11) yields:

$$\begin{bmatrix} [T_{nm}] & [0_{nm}] \\ [0_{nm}] & [T^t_{nm}]^{-1} \end{bmatrix} \frac{\partial}{\partial z}\begin{bmatrix} V'_n(z,t) \\ I'_n(z,t) \end{bmatrix} = \begin{bmatrix} [0_{nm}] & [L_{nm}] \\ [C_{nm}] & [0_{nm}] \end{bmatrix} \begin{bmatrix} [T_{nm}] & [0_{nm}] \\ [0_{nm}] & [T^t_{nm}]^{-1} \end{bmatrix} \frac{\partial}{\partial t}\begin{bmatrix} V'_n(z,t) \\ I'_n(z,t) \end{bmatrix} \tag{13}$$

where the supercript 't' represents the transpose. Premultiplying both sides by the inverse of the transformation matrix leads to:

$$\frac{\partial}{\partial z}\begin{bmatrix} V'_n(z,t) \\ I'_n(z,t) \end{bmatrix} = -\begin{bmatrix} [0_{nm}] & [T_{nm}]^{-1}[L_{nm}][T^t_{nm}]^{-1} \\ [T^t_{nm}][C_{nm}][T_{nm}] & [0_{nm}] \end{bmatrix} \frac{\partial}{\partial t}\begin{bmatrix} V'_n(z,t) \\ I'_n(z,t) \end{bmatrix} \tag{14}$$

The matrix $[L_{nm}]$ is symmetric, as is its inverse $[L_{nm}]^{-1}$. Thus, consider the term:

$$\left[[T_{nm}]^{-1}[L_{nm}][T_{nm}^t]^{-1}\right]^{-1} = [T_{nm}^t][L_{nm}]^{-1}[T_{nm}] \qquad (15)$$

Now consider the matrices $[T_{nm}^t][L_{nm}]^{-1}[T_{nm}]$ and $[T_{nm}^t][C_{nm}][T_{nm}]$. Since the matrices $[L_{nm}]^{-1}$ and $[C_{nm}]$ are symmetric and $[C_{nm}]$ is positive definite, there exists a nonsingular matrix $[S_{nm}]$ such that [4]:

$$[C_{nm}] = [S_{nm}][S_{nm}^t] \qquad (16a)$$

$$[L_{nm}]^{-1} = [S_{nm}][\mathrm{diag}\ \lambda_{kk}][S_{nm}^t] \qquad (16b)$$

substituting $[S_{nm}]^{-1} = [T_{nm}^t]$ in equation (16), we obtain:

$$[T_{nm}^t][L_{nm}]^{-1}[T_{nm}] = [\mathrm{diag}\ \lambda_{kk}] \qquad (17a)$$

and

$$[T_{nm}^t][C_{nm}][T_{nm}] = [U] \qquad (17b)$$

where [U] is an identity matrix, and is defined as

$$[U] = [\delta_{nm}]; \quad \delta_{nm} = 1 \text{ if } n = m$$
$$\delta_{nm} = 0 \text{ if } n \neq m$$

The inverse of both sides of equation (17a) yields:

$$[T_{nm}]^{-1}[L_{nm}][T_{nm}^t]^{-1} = [\mathrm{diag}\ \lambda_{kk}]^{-1} = [\mathrm{diag}\ L_{kk}] \qquad (18)$$

Therefore, the matrix $[T_{nm}]$ will simultaneously diagonalize matrices $[L_{nm}]$ and $[C_{nm}]$ and so will decouple equation (11). Note that no assumption is been made about the eigenvalues of the inductance and capacitance matrices.

Now multiply equations (17b) and (18) to obtain:

$$[T_{nm}]^{-1}[L_{nm}][T^t_{nm}]^{-1}[T^t_{nm}][C_{nm}][T_{nm}] = [\text{diag } L_{kk}][U] = [\text{diag } L_{kk}] \quad (19)$$

and

$$[T^t_{nm}][C_{nm}][T_{nm}][T_{nm}]^{-1}[L_{nm}][T^t_{nm}]^{-1} = [U][\text{diag } L_{kk}] = [\text{diag } L_{kk}] \quad (20)$$

Simplification of equations (19) and (20) yields:

$$[T_{nm}]^{-1}[L_{nm}][C_{nm}][T_{nm}] = [\text{diag } L_{kk}] \quad (21)$$

and

$$[T^t_{nm}][C_{nm}][L_{nm}][T^t_{nm}]^{-1} = [\text{diag } L_{kk}] \quad (22)$$

Thus, the matrix $[T_{nm}]$ and $[T^t_{nm}]^{-1}$ diagonalize the matrix products $[L_{nm}][C_{nm}]$ and $[C_{nm}][L_{nm}]$. Note that in equations (21) and (22), the right-hand-side of both equations is the same, so that the eigenvalues of the matrix products $[L_{nm}][C_{nm}]$ and $[C_{nm}][L_{nm}]$ are identical. Since the transformations in equations (21) and (22) are similarity transformations, $[T_{nm}]$ and $[T^t_{nm}]^{-1}$ are the eigenvector matrices of the matrix products $[L_{nm}][C_{nm}]$ and $[C_{nm}][L_{nm}]$, respectively. They will be referred to as the voltage and current eigenvector matrices. Thus:

$$[T_{nm}] = [V_{nm}]$$

and

$$[T^t_{nm}]^{-1} = [I_{nm}].$$

Consider $[V_{nm}][I^t_{nm}]$ and $[V^t_{nm}][I_{nm}]$

$$[V_{nm}][I^t_{nm}] = [T_{nm}] \left\{[T^t_{nm}]^{-1}\right\}^t = [T_{nm}][T_{nm}]^{-1} = [U] \quad (23)$$

and

$$[I_{nm}][V^t_{nm}] = [T^t_{nm}]^{-1}[T^t_{nm}] = [U] \quad (24)$$

From equatio^-^ (23) and (24),

$$[V_n]_i \cdot [I_n]_j = 0 \text{ if } i \neq j \quad (25)$$

where $[\ \ ]_i$ and $[I_n]_j$ are the i th and j th columns of the matrices $[V_{nm}]$ and $[I_{nm}]$, 'espectively. $[V_n]_i$ and $[I_n]_j$ are the i th and j th voltage and current vectors, respectively, corresponding to the eigenvalues v_i and v_j. Equation (25) is valid in the case of degeneracy, i.e., $v_i = v_j$. Thus the voltage and current eigenvectors are orthogonal to each other, including the ones corresponding to degenerate eigenvalues.

In the preceding discussion we have established that 1) the transmission-line equations (1) and (2) can be decoupled, 2) there exists a matrix $[T_{nm}]$ which will simultaneously diagonalize the inductance and capacitance matrices, 3) the columns of the matrix $[T_{nm}]$ are linearly independent, 4) the voltage and current eigenvectors are mutually orthogonal.

One way to find the matrix $[T_{nm}]$ is as follows:

Since $[C_{nm}]$ is a symmetric positive-definite matrix, there exists an orthogonal matrix $[A_{nm}]$ which will diagonalize $[C_{nm}]$, so that:

$$[A^t_{nm}][C_{nm}][A_{nm}] = [\text{diag } C_{ii}] \quad (26)$$

and

$$[A_{nm}^t] = [A_{nm}]^{-1} \tag{27}$$

Furthermore, there is another diagonal matrix $[Y_{ii}]$ which reduces the matrix $[\text{diag } C_{ii}]$ to the identity matrix; i.e.,

$$[Y_{ii}][A_{nm}^t][C_{nm}][A_{nm}][Y_{ii}] = [U] \tag{28}$$

Making these same transformations on the matrix $[L_{nm}]^{-1}$ we obtain:

$$[Y_{ii}][A_{nm}^t][L_{nm}]^{-1}[A_{nm}][Y_{ii}] = [L'_{nm}]^{-1} \tag{29}$$

Since the matrix $[L_{nm}]^{-1}$ is symmetric, then from equation (29), $[L'_{nm}]^{-1}$ is also symmetric. Now, since $[L'_{nm}]$ is symmetric, there exists an orthogonal matrix $[P_{nm}]$ which will diagonalize $[L'_{nm}]^{-1}$, so that:

$$[P_{nm}^t][L'_{nm}]^{-1}[P_{nm}] = [\text{diag } L'_{kk}] \tag{30}$$

from equations (29) and (30), we obtain:

$$[P_{nm}^t][Y_{ii}][A_{nm}^t][L_{nm}]^{-1}[A_{nm}][Y_{ii}][P_{nm}] = [\text{diag } L'_{kk}] \tag{31}$$

Since the matrix $[P_{nm}]$ is orthogonal, we can write equation (28) in the following form:

$$[P_{nm}^t][Y_{ii}][A_{nm}^t][C_{nm}][A_{nm}][Y_{ii}][P_{nm}] = [P_{nm}^t][U][P_{nm}] = [U] \tag{32}$$

From equations (16), (17), (31), and (32)

$$[T_{nm}] = [A_{nm}][Y_{ii}][P_{nm}] \tag{33}$$

A congruence transformation to decouple equation (11) was discussed in [5]. The method described above is general, and the orthogonal properties of the modes for the partially-degenerate case have been established.

III. DETERMINATION OF THE VELOCITY MATRIX FOR THREE-CONDUCTOR LINE (PARTIALLY DEGENERATE CASE)

Consider a lossless line formed by three-conductors, plus a reference conductor (ground). The dielectric surrounding the line is inhomogenous (e.g., the dielectric insulation on the conductors and air). In general, for this inhomogeneous case, there will be three propagation modes traveling with different velocities. Degeneracy among modes may occur due to symmetry. Consider the three-wire line shown in figure 1. The line is made of identical conductors with identical insulations. In this case degeneracy occurs among the two differential modes. The common mode can always be separated from the differential modes, since it has the largest propagation velocity. This is due to the fact that for the common mode, a substantial amount of the electric field resides outside the dielectric sheaths of the conductros, thereby lowering the dielectric constant seen by this mode. The differential modes, on the otherhand, have large electric fields between the individual wires and thus see a larger effective dielectric constant.

The voltage and current eigenvectors of the line can be measured using the method described in [1]. Let us denote the common mode as mode 1, and the differential modes as modes 2 and 3. Since the propagation velocities of modes 2 and 3 are assumed equal, i.e., $v_2 = v_3$, these modes cannot be resolved in time and we can only measure their sum. Thus, we can measure $[V_n]_1$, $[V_n]_2 + [V_n]_3$, $[I_n]_1$, and $[I_n]_2 + [I_n]_3$. From the knowledge of the above measured qualities, we need to determine all the elements of the voltage and current eigenvector matrices. The voltage and current eigenvector matrices can be normalized in the following form:

$$\begin{bmatrix} 1 & 1 & 1 \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 1 & 1 \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \tag{34}$$

From the orthogonal properties of the modes, described in Section II, we have:

$$[V_n]_1 \cdot [I_n]_2 = 0; \; [V_n]_1 \cdot [I_n]_3 = 0 \tag{35}$$

$$[V_n]_2 \cdot [I_n]_1 = 0; \; [V_n]_2 \cdot [I_n]_3 = 0 \tag{36}$$

$$[V_n]_3 \cdot [I_n]_1 = 0; \; [V_n]_3 \cdot [I_n]_2 = 0 \tag{37}$$

These result in the following relations:

$$V_{21}I_{22} + V_{31}I_{32} = -1 \tag{38}$$

$$V_{21}I_{23} + V_{31}I_{33} = -1 \tag{39}$$

$$V_{22}I_{21} + V_{32}I_{31} = -1 \tag{40}$$

$$V_{22}I_{23} + V_{32}I_{33} = -1 \tag{41}$$

$$V_{23}I_{21} + V_{33}I_{31} = -1 \tag{42}$$

$$V_{23}I_{22} + V_{33}I_{32} = -1 \tag{43}$$

In addition to the above we have following relationships from the measured eigenvectors 2 and 3:

$$V_{22} + V_{23} = A \text{ (Measured)} \tag{44}$$

$$V_{32} + V_{33} = B \text{ (Measured)} \tag{45}$$

$$I_{22} + I_{23} = C \text{ (Measured)} \tag{46}$$

$$I_{32} + I_{33} = D \text{ (Measured)} \tag{47}$$

In equations (38) thru (47), there are eight unknowns, namely, V_{22}, V_{23}, V_{32}, V_{33}, I_{22}, I_{23}, I_{32}, and I_{33}. The nonlinear equations (38) thru (47) can be solved for the above unknowns, to obtain the voltage and current eigenvectors.

IV. EXPERIMENTAL RESULTS

For the purpose of demonstrating the validity of the methods described, a three-wire cable (over a ground plane) 10 meter in length was constructed using three identical wires insulated with polyvinylchloride. The inner and outer diameters of the wires are 0.42926 cm and 0.6096 cm, respectively. The cable was supported with styrofoam blocks above an aluminum ground plane in the configuration shown in Fig. 1.

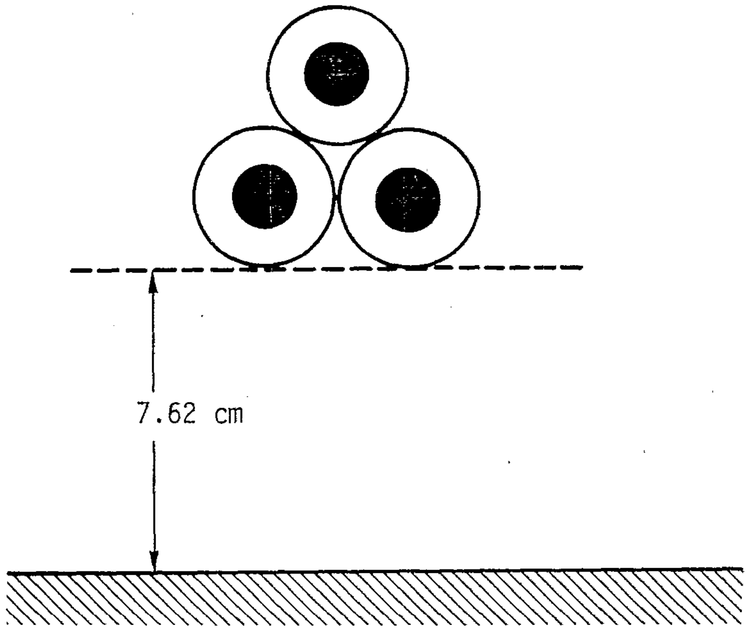


Figure 1. Cross section of a three-wire line over a ground plane.

The Time-Domain-Reflectometer (TDR) method described in [2] was used to determine the characteristic-admittance matrix of the multiconductor line. TDR recordings obtained by driving each wire in turn with the others grounded at the input end and with the load end shorted, are presented in Fig. 2. TDR recordings obtained by driving wires 1 and 2, 2 and 3, and 1 and 3 in parallel with the other wire grounded at the input end and with the load end shorted, are presented in Fig. 3. The diagonal and off-diagonal terms of the characteristic-admittance matrix are given by the following relations [2]:

$$Y_{ii} = 1/Z^m_{ii}$$

$$Y_{ij} = Y_{ji} = 1/2\ (1/Z^m_{ij} - 1/Z^m_{ii} - 1/Z^m_{jj}) \qquad (48)$$

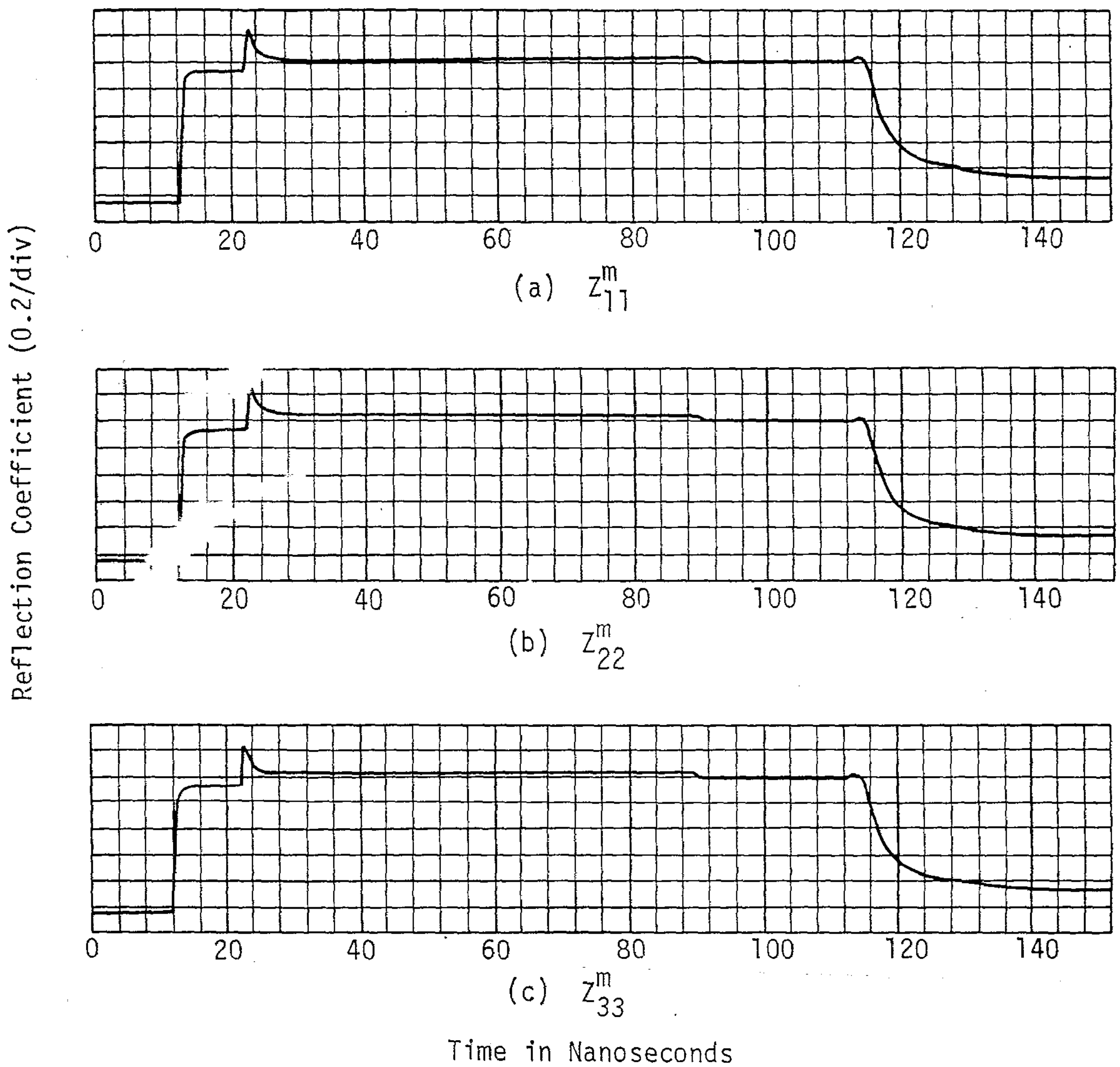


Figure 2. Waveforms measured with a time-domain reflectometer to determine the impedance Z_{ii}^m and the modal velocities.

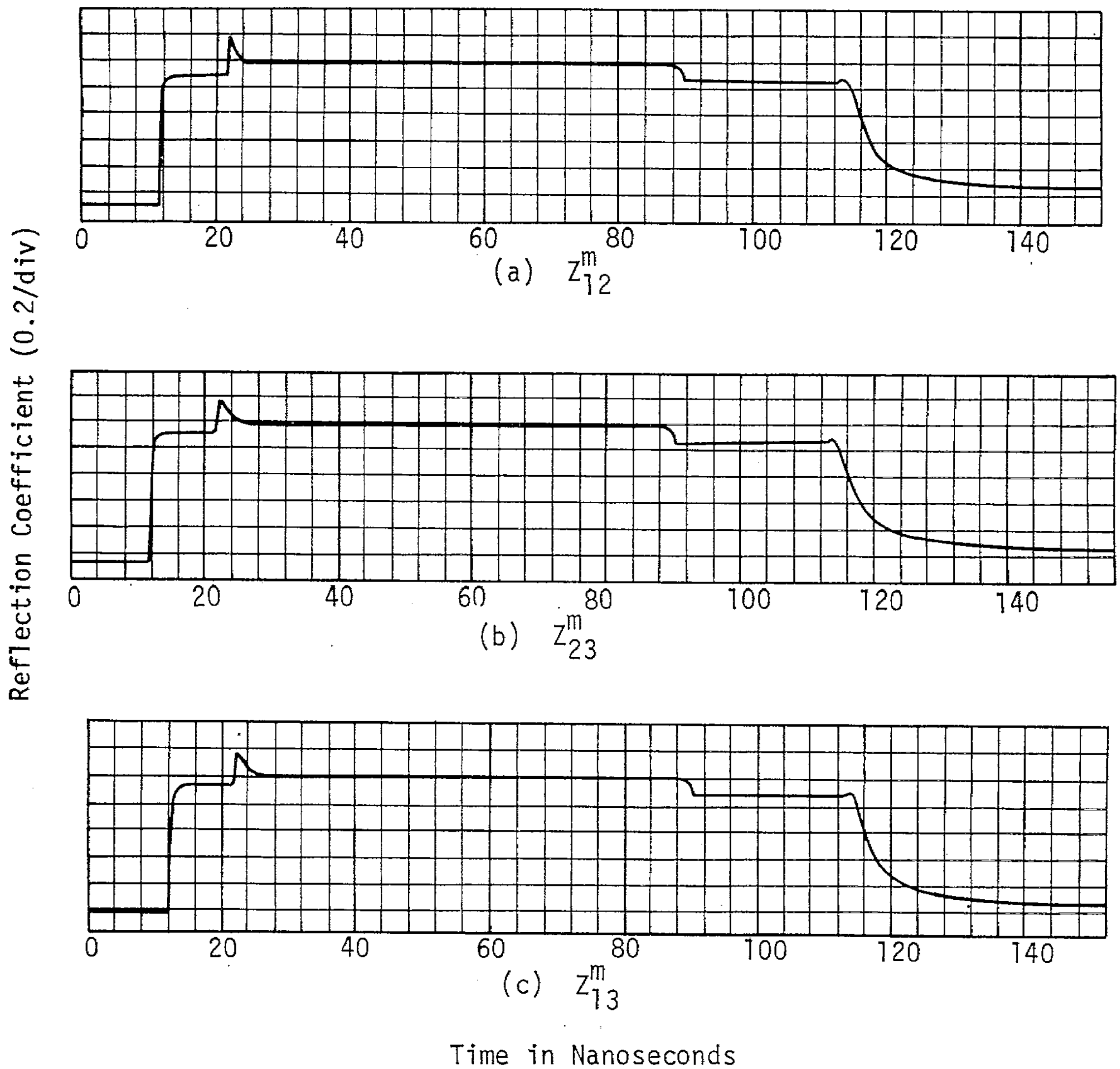
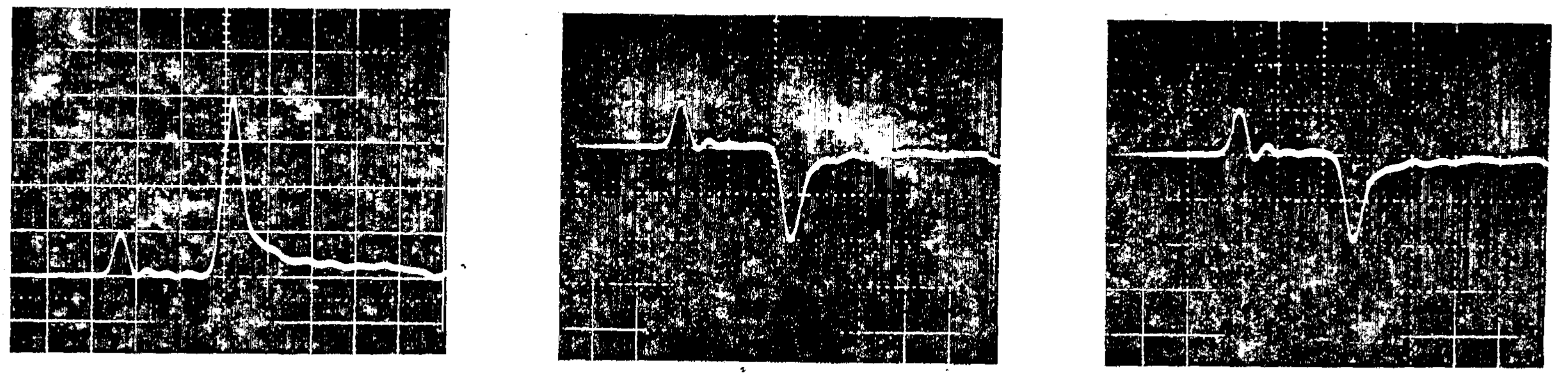
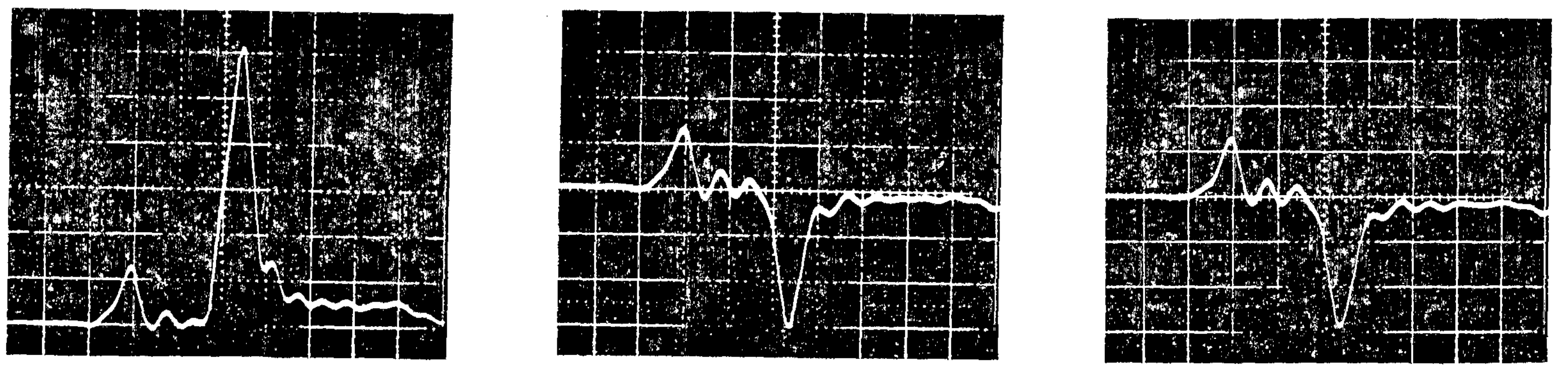


Figure 3. Waveforms measured with a time-domain reflectometer to determine the impedance Z^m_{ij} and the modal velocities.



(a) voltage on wire 1 (b) voltage on wire 2 (c) voltage on wire 3

Figure 4. Voltage waveforms at the load end when wire 1 is driven at the input end. Vertical scale 0.5v/div; horizontal scale 5 ns/div.



(a) current on wire 1 (b) current on wire 2 (c) current on wire 3

Figure 5. Current waveforms at the load end when wire 1 is driven at the input end. Vertical scale 5 ma/div; horizontal scale 5 ns/div.

where Z^m_{ii} is the measured impedance of wire i with all other wires grounded at the input, and Z^m_{ij} is the measured impedance of wires i and j connected in parallel at the input end with all other wires grounded at the input.

The reflected pulses shown in Figs. 2 and 3 exhibit three time-delayed step functions corresponding to the two discrete propagation modes on the line. The first mode is the common mode and the second mode is a combination of the two differential modes.

The speed of propagation of a mode is determined from the ratio of the length of the line to one-half of the round-trip travel time for that mode. The measured round-trip travel time for the two distinct modes corresponds to propagation speeds of 2.88×10^8 and 2.13×10^8 meters/sec.

Measurement of the eigenvectors was accomplished by driving one of the wires with a short-duration pulse from a 50-Ω source and terminating the ends of each wire in 50-Ω resistive loads. The output voltage and current pulses on each wire were recorded using a high-impedance voltage probe and a low-impedance current probe, respectively, with a 200-MHz oscilloscope. The recorded voltage and current pulse data are shown in Fig. 3.

The modal voltage amplitudes were computed from the measured load voltages using equation 9 in [1]. The modal current amplitudes were computed using a similar relation for current pulses.

The characteristic-admittance matrix calculated from the TDR data is:

$$[Y_{c_{nm}}] = \begin{bmatrix} 1.60 & -0.724 & -0.716 \\ -0.724 & 1.587 & -0.709 \\ -0.716 & -0.709 & 1.578 \end{bmatrix} \times 10^{-2} \text{ mho} \qquad (49)$$

The voltage and current eigenvectors calculated from the load voltages and currents are:

$$[V_n]_1 = \begin{bmatrix} 6.428 \\ 6.491 \\ 6.518 \end{bmatrix} ; \quad [V_n]_2 + [V_n]_3 = \begin{bmatrix} 3.567 \\ -1.882 \\ -1.786 \end{bmatrix} \tag{50}$$

$$[I_n]_1 = \begin{bmatrix} 1.346 \\ 1.359 \\ 1.401 \end{bmatrix} ; \quad [I_n]_2 + [I_n]_3 = \begin{bmatrix} 12.96 \\ -6.536 \\ -6.13 \end{bmatrix} \tag{51}$$

The voltage and current eigenvector matrices computed from these eigenvectors, using the method described in Section III, are:

$$[V_{nm}] = \begin{bmatrix} 1 & 1 & 1 \\ 1.009 & 0.828 & -1.8495 \\ 1.014 & -1.828 & 0.8495 \end{bmatrix}$$

$$[I_{nm}] = \begin{bmatrix} 1 & 1 & 1 \\ 1.009 & 0.1078 & -1.1168 \\ 1.04 & -1.1078 & 0.1168 \end{bmatrix}$$

The inductance and capacitance matrices obtained from the voltage and current eigenvectors are:

(a) From voltage eigenvectors

$$[L_{nm}] = \begin{bmatrix} 0.8792 & 0.6791 & 0.6787 \\ 0.6775 & 0.8827 & 0.6785 \\ 0.6760 & 0.6774 & 0.8829 \end{bmatrix} \mu H/m$$

$$[C_{nm}] = \begin{bmatrix} 74.54 & -34.57 & -34.2 \\ -34.63 & 73.87 & -33.96 \\ -34.29 & -34.0 & 73.41 \end{bmatrix} pF/m$$

(b) From current eigenvectors

$$[L_{nm}] = \begin{bmatrix} 0.8775 & 0.6782 & 0.6779 \\ 0.6773 & 0.8831 & 0.6791 \\ 0.6763 & 0.6785 & 0.8841 \end{bmatrix} \mu H/m$$

$$[C_{nm}] = \begin{bmatrix} 74.48 & -34.61 & -34.23 \\ -34.64 & 73.89 & -33.91 \\ -34.28 & -33.96 & 73.45 \end{bmatrix} pF/M$$

These results show that the voltage and current eigenvectors give almost identical results. Further, the inductance and capacitance matrices are known to be symmetric (e.g. $A_{ij} = A_{ji}$, $i \neq j$). The asymmetry present in the inductance and capacitance matrices and difference in results obtained from the voltage and current eigenvectors must be attributed to the measurement and data-reduction errors.

Independent measurements of the per-unit-length parameters were also carried out in the frequency domain using the technique described in [6]. The line parameters determined from the time-and-frequency-domain techniques are compared in Table I. The time-domain results are averages of the parameters obtained from voltage and current eigenvectors, and symmetry has been imposed by replacing the off-diagonal matrix elements A_{ij} with average values $(A_{ij} + A_{ji})/2$. In the calculation of the propagation velocities from the experimental data using the frequency-domain method [6], the velocities do not turn out to be nearly equal, not exactly, due to numerical approximations in the computations.

TABLE I

COMPARISON OF PER-UNIT-LENGTH PARAMETERS FROM THE TIME-DOMAIN AND FREQUENCY-DOMAIN MEASUREMENTS

Parameters	Time Domain	Frequency Domain
$[L_{nm}]$ (μH/m	$\begin{bmatrix} 0.8783 & 0.6779 & 0.6772 \\ 0.6779 & 0.8829 & 0.6784 \\ 0.6772 & 0.6784 & 0.8835 \end{bmatrix}$	$\begin{bmatrix} 0.8797 & 0.6554 & 0.6547 \\ 0.6554 & 0.8622 & 0.6470 \\ 0.6547 & 0.6470 & 0.8655 \end{bmatrix}$
$[C_{nm}]$ (pF/m)	$\begin{bmatrix} 74.51 & -34.61 & -34.25 \\ -34.61 & 73.88 & -33.96 \\ -34.25 & -33.96 & 73.43 \end{bmatrix}$	$\begin{bmatrix} 77.15 & -35.53 & -35.73 \\ -35.53 & 78.59 & -35.82 \\ -35.73 & -35.82 & 77.44 \end{bmatrix}$
$[Y_{c_{nm}}]$ (mho)	$\begin{bmatrix} 1.60 & -0.724 & -0.716 \\ -0.724 & 1.587 & -0.709 \\ -0.716 & -0.709 & 1.578 \end{bmatrix} \times 10^{-2}$	$\begin{bmatrix} 1.571 & -0.7089 & -0.7039 \\ -0.7089 & 1.598 & -0.7041 \\ -0.7039 & -0.7041 & 1.579 \end{bmatrix} \times 10^{-2}$

CONCLUDING REMARKS

A measurement technique for the characterization of three-conductor lossless line in a cross-sectionally inhomogeneous medium has been presented for the case of partially-degenerate propagation on the line. The orthogonality properties of the propagation modes have been established for the degenerate case.

The results obtained from this method are found to be in good agreement with frequency-domain results. The method described in this paper can be applied to the case where two modes have nearly equal propagation velocities. In practice, due to symmetry in the configuration, the propagation velocities of the modes will be nearly equal, rather than being exactly equal.

The method described here can be extended to N conductor lines, where several groups of modes, but not all, have the same velocities.

REFERENCES

1. A. K. Agrawal, H. M. Fowles, and L. D. Scott, "Experimental Characterization of Multiconductor Transmission Lines in Inhomogeneous Media Using Time-Domain Techniques," AFWL Interaction Notes, Note 332, Feb 1978; also published as IEEE Trans., EMC, Vol. EMC-21, No. 1, pp. 28-32, February 1979.

2. V. L. Carey, T. R. Scott, and W. T. Weeks, "Characterization of Multiple Parallel Transmission Lines Using Time-Domain Reflectometry", IEEE Trans. Instrum. Meas., Vol. IM-18, No. 3, September 1969.

3. K. D. Marx, "Propagation Modes, Equivalent Circuits, and Characteristic Terminations for Multiconductor Transmission Lines with Inhomogeneous Dielectrics", IEEE Trans. Microwave Theory Tech., Vol. MTT-21, No. 7, July 1973.

4. R. Bellman, Introduction to Matrix Analysis, McGraw Hill, New York, 1960.

5. F. Y. Chang, "Transient Analysis of Lossless Coupled Transmission Lines in a Nonhomogeneous Dielectric Medium", IEEE Trans. Microwave Theory Tech., Vol. MTT-18, No. 9, September 1970.

6. A. K. Agrawal, et al, "Experimental Characterization of Multiconductor Transmission Lines in the Frequency Domain," AFWL Interaction Notes, Note 311, June 1977; also published as IEEE Trans. EMC, Vol. EMC-21, No. 1, pp. 20-27, February 1979.