REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER RADC-TR-76-101, Vol II (of seven)	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) APPLICATIONS OF MULTICONDUCTOR TRANSMISSION LINE THEORY TO THE PREDICTION OF CABLE COUPLING Vol II - Computation of the Capacitance Matrices for Ribbon Cables		5. TYPE OF REPORT & PERIOD COVERED Final Technical Report
		6. PERFORMING ORG. REPORT NUMBER N/A
7. AUTHOR(s) Arthur E. Feather Clayton R. Paul		8. CONTRACT OR GRANT NUMBER(s) F30602-72-C-0418
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Kentucky Department of Electrical Engineering Lexington KY 40506		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62702F 45400130
11. CONTROLLING OFFICE NAME AND ADDRESS Rome Air Development Center (RBCT) Griffiss AFB NY 13441		12. REPORT DATE April 1976
		13. NUMBER OF PAGES 146
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Same		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Same

18. SUPPLEMENTARY NOTES

RADC Project Engineer: James Brodock (RBCT)

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Electromagnetic Compatibility	Wire-to-wire Coupling
Cable Coupling	Ribbon Cable
Transmission Lines	Flat Pack Cable
Multiconductor Transmission Lines	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Multi-conductor ribbon cables utilizing closely coupled dielectric-coated conductors are now used extensively to interconnect electronic subsystems. General techniques involving solution of the multiconductor transmission line equations can be used to predict unwanted interference or crosstalk in the cable. Inherent in these techniques is knowledge of the per-unit-length transmission line capacitance and inductance matrices of the ribbon cable. A computer implemented numerical technique for determining these matrices for

(Cont'd)

DD FORM 1 JAN 73 1473   EDITION OF 1 NOV 65 IS OBSOLETE

ribbon cables is described in this report.

The per-unit-length charge distributions on the conductor and dielectric surfaces are described by Fourier series with the potential and displacement vector equations at these surfaces enforced as boundary conditions. A matrix inversion techniques is used to develop elements of the generalized and transmission line capacitance matrices.

Computer program GETCAP was written in a user-oriented format. The user simply describes the physical characteristics and GETCAP computes the per-unit-length capacitance matrices for the ribbon cable described.

The Post-Doctoral Program at Rome Air Development Center is pursued via
Project 9567 under the direction of Mr. Jacob Scherer. The Post-Doctoral
Program is a cooperative venture between RADC and the participating
universities: Syracuse University (Department of Electrical and Computer
Engineering), the U.S. Air Force Academy (Department of Electrical Engineer-
ing), Cornell University (School of Electrical Engineering), Purdue University
(School of Electrical Engineering), University of Kentucky (Department of
Electrical Engineering). Georgia Institute of Technology (School of Electrical
Engineering), Clarkson College of Technology (Department of Electrical
Engineering), State University of New York at Buffalo (Department of Electri-
cal Engineering), North Carolina State University (Department of Electrical
Engineering), Florida Technological University (Department of Electrical
Engineering), Florida Institute of Technology (College of Engineering), Air
Force Institute of Technology (Department of Electrical Engineering), and the
University of Adelaide (Department of Electrical Engineering) in South
Australia. The Post-Doctoral Program provides, via contract, the opportunity
for faculty and visiting faculty at the participating universities to spend a
year full-time on exploratory development and operational problem-solving
efforts with the post-doctoral splitting their time between RADC (or the
ultimate customer) and the educational institutions.

The Post-Doctoral Program is totally customer-funded with current projects
being undertaken for Air Defense Command (NORAD), Air Force Communications
Service, Federal Aviation Administration, Defense Communications Agency,
Aeronautical Systems Division (AFSC), Aero-Propulsion Laboratory (AFSC), and

ACKNOWLEDGEMENTS

iii

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

INTRODUCTION

Ribbon, or "flat-pack" cables, as described in this report, are a particular type of multiconductor cable consisting of n dielectric-coated conductors arranged in a linear array as shown in Fig. 1.1. Each conductor is cylindrical in shape, with all conductors in the cable of identical radii, r_c. Surrounding each conductor are identical, cylindrically-shaped homogeneous dielectrics, or insulations, of equal radii, r_d, and relative dielectric constant, ε_r. The conductors are uniformly separated by a distance, d, in the horizontal plane. In many types of ribbon cables, the conductor orientations are maintained in this linear array by bonding the adjacent dielectric insulations together. When the separation between con- ductors does not allow the dielectric surfaces to touch, a
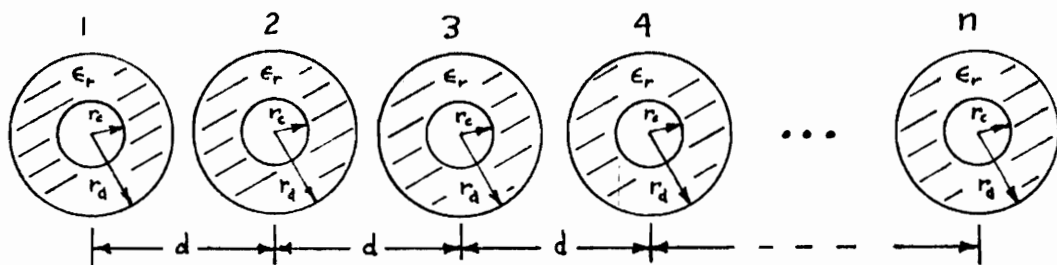

Figure 1.1 Orientation of ribbon or "flat-pack" cable.

1

thin film of the insulation connects the adjacent conductor insulations.

Ribbon cables have been used for many years in computer bus connections as in the Digital Equipment Corporation PDP-8 minicomputer [1]. They are recently finding greater usage in other types of systems such as aircraft and missiles. In these applications, the various conductors connect electronic devices at either end of the cable. Of considerable importance when using these cables is the ability to predict interference or crosstalk. Crosstalk is the unintentional coupling of signals from one wire onto another, resulting in possible bit errors in computer signals and the mixing of signals in analog systems.

Paul [2,13,19] has developed general techniques for characterizing a system of wires as a multiconductor transmission line, which can be used to predict crosstalk in ribbon cables. These techniques require that the per-unit-length transmission line capacitance and inductance matrices of the system, $\underset{\sim}{C}$ and $\underset{\sim}{L}$, be obtained. The purpose of this report is to develop a numerical technique yielding these matrices for ribbon cables.

Approximations to the elements of the transmission line capacitance matrix can be obtained for cases with no dielectric insulation, providing the separation between the conductors is at least ten times the conductor radius. One such approximation will be derived in Chapter V, and will be used to develop an approximate expression for the trans-

mission line inductance matrix. Boast [3] describes an approximate method of determining the transmission line capacitance matrix for bare conductors above an infinite ground plane. The smallest ratio of conductor separation to wire radius must be greater than ten. In this case, one can assume that the per-unit-length charge on each conductor surface is uniformly distributed around the conductor periphery.

Ribbon cables, however, have a much smaller conductor separation than is required for these approximations to be valid, and in addition have dielectric insulations. Clements [4] showed that, in fact, the approximate formulas based on constant charge distributions are no longer sufficiently valid for close spacing and dielectric material surrounding the conductors.

Clements [4,9] has developed a general computer numerical technique for characterizing systems of circular conductors with circular dielectric insulations. This general technique is specialized in this report to the particular case of ribbon cables. As a result, many sim- plifications in the technique are made possible. This specialization of the general technique and the simpli- fications which result are described in detail in Chapter II. Certain special considerations in defining the boundary conditions and in choosing the matchpoints when applying the technique to ribbon cables are described in Chapter III. Chapter IV describes two alternate methods which were

investigated to reduce computer solution time; these methods did not yield valid results.

Chapter V describes the determination of the per-unit-length transmission line capacitance and inductance matrices, $\underset{\sim}{C}$ and $\underset{\sim}{L}$, needed to analyze multiconductor systems. An approximate expression for the transmission line inductance matrix is also derived.

A computer program was written to implement the results of this report to provide a means of finding the transmission line capacitance and inductance matrices of ribbon cables. Chapter VI describes the operation of this computer program and the separate subroutines used. Results of the program are compared to both known results and to the approximate technique of Chapter V. Chapter VII is basically a users manual describing input formats and displaying typical outputs from the program. Finally, Chapter VIII summarizes the main points of the report, and Appendix B gives a full listing of the computer program.

CHAPTER II

DESCRIPTION OF THE METHOD

II.1 Capacitance and the Capacitance Matrix

To understand the concepts involved in determining the capacitance matrix for a system of wires, it is necessary to have a knowledge of the meaning of the term capacitance. This can be accomplished by considering the case of two arbitrary conducting bodies, as shown in Fig. 2.1. Q_1 and Q_2 are the total free charges on the bodies, and $\emptyset_1$ and $\emptyset_2$ are the potentials of the bodies with respect to some arbitrary reference point. The charges Q_1 and Q_2 can be related to the potentials of the bodies as shown in Eq. (2.1).

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \emptyset_1 \\ \emptyset_2 \end{bmatrix} \tag{2.1}$$

Figure 2.1.   Two arbitrary conducting bodies.

5

The matrix

$$\underset{\sim}{C} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \tag{2.2}$$

is known as the "generalized" capacitance matrix [7].
Hereafter, ~ will be used to denote matrix quantities, and
an underbar, _, will denote nx1 column vectors. A matrix
with m rows and n columns is of order mxn.

When the system of Fig. 2.1 is considered as an elec-
trical network problem, it is customarily assumed that the
bodies have been excited in such a way that charges of equal
magnitude and opposite sign are present on the surfaces of
the bodies. Enforcing the condition $Q=Q_1=-Q_2$ appropriate
to network problems, Eq. (2.1) becomes

$$\begin{aligned} Q &= C_{11}\phi_1 + C_{12}\phi_2 \\ -Q &= C_{21}\phi_2 + C_{22}\phi_2 \end{aligned} \tag{2.3}$$

In most texts on electromagnetic theory, a network
consisting of only two bodies is said to have a capacitance,
C, between the bodies defined as [5,6,7]

$$C = \frac{Q}{V_o} \tag{2.4}$$

where V_o is the difference in potential of the bodies; i.e.,
$V_o = \phi_1 - \phi_2$.

Equation (2.3) can then be solved for the ratio
$Q/(\phi_1 - \phi_2)$ yielding an expression for the capacitance

6

between the two bodies of Fig. 2.1 as [7]

$$C = \frac{Q}{\phi_1 - \phi_2} = \frac{c_{11}c_{22} - c_{12}c_{21}}{c_{11} + c_{22} + c_{12} + c_{21}} \quad . \qquad (2.5)$$

The concept illustrated by Eq. (2.1) can be extended to the case of n charged bodies. Then, the nxn generalized capacitance matrix is defined by the equation

$$
\begin{bmatrix} Q_1 \\ Q_2 \\ \cdot \\ \cdot \\ \cdot \\ Q_n \end{bmatrix}
=
\begin{bmatrix}
c_{11} & c_{12} & \cdots & c_{1n} \\
c_{21} & c_{22} & \cdots & c_{2n} \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
c_{n1} & c_{n2} & \cdots & c_{nn}
\end{bmatrix}
\begin{bmatrix} \phi_1 \\ \phi_2 \\ \cdot \\ \cdot \\ \cdot \\ \phi_n \end{bmatrix}
\quad . \qquad (2.6)
$$

In matrix notation, Eq. (2.6) is written as

$$\underline{Q} = \underset{\sim}{C}\,\underline{\phi} \qquad (2.7)$$

where $\underset{\sim}{C}$ is the generalized capacitance matrix of the system.

When considering infinitely long parallel conductors, total free charge is no longer finite. The charge on each conductor is customarily expressed as a function of its length. This per-unit-length free charge, q_f, can be related to the conductor potentials by the same relationship as in Eq. (2.6). This yields the matrix equation

7

$$\begin{bmatrix} q_{1f} \\ q_{2f} \\ \vdots \\ q_{nf} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{bmatrix} \begin{bmatrix} \emptyset_1 \\ \emptyset_2 \\ \vdots \\ \emptyset_n \end{bmatrix} \qquad \text{(2.8a)}$$

or

$$\underline{q}_f = \underset{\sim}{C} \, \underline{\emptyset} \qquad \text{(2.8b)}$$

where q_{if} is the per-unit-length free charge on conductor i, and $\underset{\sim}{C}$ is the per-unit-length generalized capacitance matrix of the system.

II.2 Dielectric-Coated Conductors

Shown in Fig. 2.2 is a cross-section of two infinitely long circular conductors covered with a cylindrical diel-ectric. These dielectric-coated conductors will be referred to as wires. The dielectric material is assumed to be linear, homogeneous and isotropic. The quantities r_{c1} and r_{c2} in Fig. 2.2 designate the radii of the conductors, r_{d1} and r_{d2} designate the radii of the dielectrics, and ε_{r1} and ε_{r2} specify the relative dielectric constants of the diel-
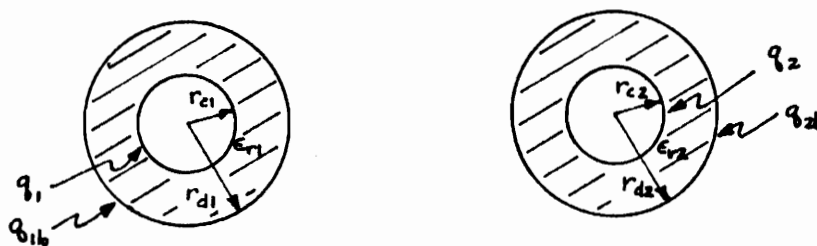
Fig. 2.2. Two dielectric-coated conductors.

8

ectric materials relative to free space. The term "conductor surface" will designate the conductor-dielectric boundary, and the term "dielectric surface" will designate the dielectric-free space interface.

The potentials on each conductor cause quantities of "free" charge to build up on the surfaces of the conductors. These "free" charges produce an electric field in the region surrounding the conductors. The introduction of dielectric material into this field will cause the dipoles in the dielectric to separate their positive and negative charges to align themselves with the electric field. This separation results in a net drift of "bound" charge to the dielectric surface, q_{1b} and q_{2b} in Fig. 2.2. An equivalent amount of charge equal in magnitude but of opposite sign will be induced on the conductor surface. The total (bound plus free) charges q_1 and q_2 on the conductor surfaces are

$$q_1 = q_{1f} - q_{1b} \text{ and } q_2 = q_{2f} - q_{2b} \quad . \quad (2.9)$$

Thus, the "free" charge on the conductor is found by adding the values for the bound charge at the dielectric surface and the total charge at the conductor surface;

$$q_{1f} = q_1 + q_{1b} \text{ and } q_{2f} = q_2 + q_{2b} \quad . \quad (2.10)$$

Since the generalized capacitance matrix relates "free" charge to conductor potentials, the per-unit-length charges on both the conductor and dielectric surfaces must be found

9

in order to develop this matrix for dielectric-coated con-
ductors.

II.3 Analytical Formulation of the Method

To find the generalized capacitance matrix of ribbon
cables, a general computer numerical technique developed by
Clements [4,9] is used, which utilizes the matrix methods
described by Adams [7,8]. The technique is associated with
the general "method of moments" [10] for solution of fields
problems.
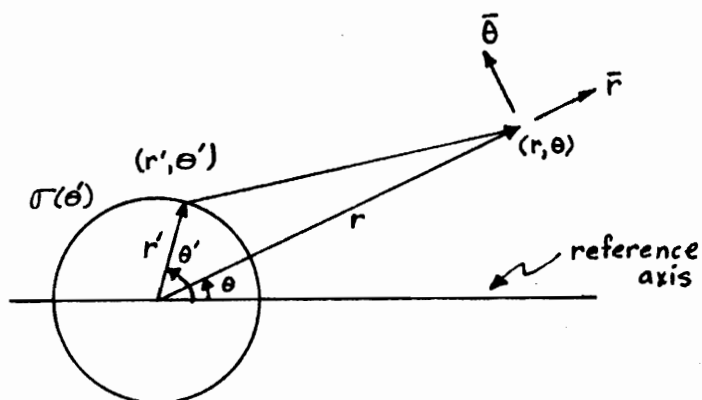
Figure 2.3.   Fourier series charge distribution around
a cylindrical surface.

In examining the method, the per-unit-length charge
distribution, σ, around an infinitely long cylindrical
surface as shown in Fig. 2.3, will be represented as a
Fourier series,

$$\sigma(\theta') = \sigma_0 + \sum_{m=1}^{k} (\sigma_m \cos m\theta' + \hat{\sigma}_m \sin m\theta') \quad . \qquad (2.11)$$

where k is the number of harmonics included in the repre-
sentation.   Then, the potential, with respect to an arbi-

trary reference point, at a point (r,θ) outside this surface (i.e., $r > r'$) is [4] (ε_v is the permittivity of free space)

$$\emptyset(r,\theta) = -\sigma_0 \frac{r' \ln r}{\varepsilon_v} + \frac{1}{2\varepsilon_v} \sum_{m=1}^{k} \sigma_m \frac{(r')^{m+1} \cos m\theta}{mr^m}$$

$$+ \frac{1}{2\varepsilon_v} \sum_{m=1}^{k} \hat{\sigma}_m \frac{(r')^{m+1} \sin m\theta}{mr^m} \quad . \tag{2.12}$$

The reference potential term (defined in Appendix A) has been omitted in Eq. (2.12). It is shown in Appendix A that this reference term will vanish for a system with zero net charge and the reference point at infinity. Also, the electric field at the point (r,θ) can be found as [4]

$$\overline{E}(r,\theta) = \frac{\sigma_0 \left(\frac{r'}{r}\right)}{\varepsilon_v}\overline{r} + \frac{1}{2\varepsilon_v} \sum_{m=1}^{k} \sigma_m \left(\frac{r'}{r}\right)^{m+1} (\cos m\theta \overline{r} + \sin m\theta \overline{\theta})$$

$$+ \frac{1}{2\varepsilon_v} \sum_{m=1}^{k} \hat{\sigma}_m \left(\frac{r'}{r}\right)^{m+1} (\sin m\theta \overline{r} - \cos m\theta \overline{\theta}) \quad , \tag{2.13}$$

where $\overline{r}$ is a unit vector in the radial direction, and $\overline{\theta}$ is a unit vector in the tangential direction.

Similarly, if the point (r,θ) is inside the surface, that is, $r < r'$, [4]

$$\emptyset(r,\theta) = \frac{-\sigma_0 r' \ln r'}{\varepsilon_v} + \frac{1}{2\varepsilon_v} \sum_{m=1}^{k} \sigma_m \frac{r^m \cos m\theta}{m(r')^{m-1}} + \frac{1}{2\varepsilon_v} \sum_{m=1}^{k} \hat{\sigma}_m \frac{r^m \sin m\theta}{m(r')^{m-1}}$$

$$\tag{2.14}$$

$$\overline{E}(r,\theta) = \frac{-1}{2\epsilon_v} \sum_{m=1}^{k} \sigma_m \left(\frac{r}{r^\intercal}\right)^{m-1} (\cos m\theta \overline{r} - \sin m\theta \overline{\theta})$$

$$+ \frac{-1}{2\epsilon_v} \sum_{m=1}^{k} \hat{\sigma}_m \left(\frac{r}{r^\intercal}\right)^{m-1} (\sin m\theta \overline{r} + \cos m\theta \overline{\theta}) . \qquad (2.15)$$

Equations (2.12), (2.13), (2.14), and (2.15) can be used to find an approximation to the potential and electric field vector at any point due to a per-unit-length charge distribution on a cylindrical surface of the form of Eq. (2.11). A consideration of the geometry of the "flat-pack" cable results in a reduction of the number of terms in these equations.

Figure 2.4 shows the orientation of the wires in a "flat-pack" cable. All wires are identical and are oriented in a linear horizontal array.
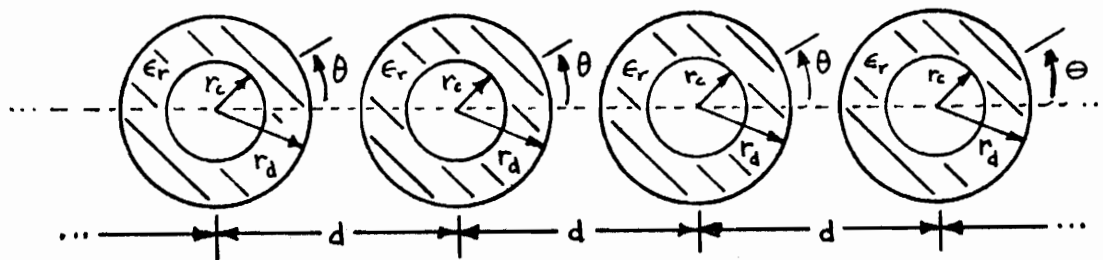
Figure 2.4   Physical geometry of "flat-pack" cable.

One would expect from the extreme symmetry of the "flat-pack" cable with respect to the line connecting centers of the wires, that the surface charges would be symmetric with respect to this line. This "even" function of the per-unit-length charge on the surfaces can be de-

scribed by constant and cosine terms only. In analytical form, the per-unit-length charge on a conductor surface is of the form

$$\sigma(\theta) = \sigma_0 + \sum_{m=1}^{k} \sigma_m \cos m\theta \quad . \qquad (2.16)$$

Thus for "flat-pack" wire orientations, only k+1 of the full 2k+1 Fourier coefficients need to be retained.

The potential and electric field equations for r>r', Eq. (2.12) and Eq. (2.13), can then be reduced to

$$\emptyset(r,\theta) = -\sigma_0 \frac{r'\ln r}{\varepsilon_v} + \frac{1}{2\varepsilon_v} \sum_{m=1}^{k} \sigma_m \frac{(r')^{m+1} \cos m\theta}{mr^m} \qquad (2.17)$$

and

$$\overline{E}(r,\theta) = \sigma_0 \frac{\left(\frac{r'}{r}\right)}{\varepsilon_v} \overline{r} + \frac{1}{2\varepsilon_v} \sum_{m=1}^{k} \sigma_m \left(\frac{r'}{r}\right)^{m+1} (\cos m\theta \overline{r} + \sin m\theta \overline{\theta}) \quad . \qquad (2.18)$$

Similarly, for r<r', Eq. (2.14) and Eq. (2.15) become

$$\emptyset(r,\theta) = -\sigma_0 \frac{r'\ln r'}{\varepsilon_v} + \frac{1}{2\varepsilon_v} \sum_{m=1}^{k} \sigma_m \frac{r^m \cos m\theta}{m(r')^{m-1}} \qquad (2.19)$$

and

$$\overline{E}(r,\theta) = \frac{-1}{2\varepsilon_v} \sum_{m=1}^{k} \sigma_m \left(\frac{r}{r'}\right)^{m-1} (\cos m\theta \overline{r} - \sin m\theta \overline{\theta}) \quad . \qquad (2.20)$$

Similarly, the per-unit-length bound charge on a dielectric surface is of the form

13

$$\sigma'(\theta) = \sigma'_0 + \sum_{m=1}^{l} \sigma'_m \cos m\theta \quad . \tag{2.21}$$

The potential and electric field vector due to the per-unit-length bound charge on the dielectric surface can be found from Eqns. (2.17) through (2.20) with σ replaced by σ'. The angle θ in the charge distributions in Eqns. (2.16) and (2.21) for each wire is with respect to the line connecting wire centers as shown in Fig. 2.4.

II.4 Matrix Organization in the Method

In the previous section, equations were developed for the potential and the electric field due to a charge distribution on a cylindrical surface, where the charge distribution is represented as a Fourier series. These equations are utilized in this section in a matrix formulation to solve for the per-unit-length generalized capacitance matrix.

Fourier series representations of k+1 terms are chosen for the charge distribution around each conductor surface, where k is the number of cosine terms in the series. Since the potential is equal at any point on the conductor surface, k+1 matchpoints on each conductor surface are used to define the potential resulting from the charge distributions on all the surfaces.

Similarly, Fourier series representations with l+1 terms are chosen to describe the bound charge distribution around the dielectric surface. The boundary conditions at

14

the dielectric surfaces require that the normal components of the displacement vector $\overline{D} = \varepsilon \overline{E}$ must be equal just inside and just outside the dielectric boundary [7]. Therefore, 1+1 matchpoints are chosen on each dielectric surface at which this condition will be enforced. The electric field will change by ρ_s / ε_v as a field point passes through the charged surface (where ρ_s is the surface charge in coulombs per square meter), but the normal component of the displacement vector will remain continuous across the boundary, i.e.,

$$\varepsilon_i E_n^i - \varepsilon_v E_n^o = 0$$

$$\varepsilon_v (\varepsilon_r E_n^i - E_n^o) = 0 \qquad (2.22)$$

$$\varepsilon_r E_n^i - E_n^o = 0$$

where E_n^i and E_n^o are the normal components of the electric field just inside and just outside the boundary, respectively, ε_i and ε_r are the dielectric constant and the relative dielectric constant of the insulation, and ε_v is the permittivity of free space.

The matrix equation which results will be of the form

$$
\underbrace{\begin{bmatrix}
\underset{\sim}{D}_{11} & \underset{\sim}{D}_{11'} & \underset{\sim}{D}_{12} & \underset{\sim}{D}_{12'} & & \cdots & & \underset{\sim}{D}_{1n} & \underset{\sim}{D}_{1n'} \\
\underset{\sim}{D}_{1'1} & \underset{\sim}{D}_{1'1'} & \underset{\sim}{D}_{1'2} & \underset{\sim}{D}_{1'2'} & & & & \underset{\sim}{D}_{1'n} & \underset{\sim}{D}_{1'n'} \\
\underset{\sim}{D}_{21} & \underset{\sim}{D}_{21'} & \underset{\sim}{D}_{22} & \underset{\sim}{D}_{22'} & & \cdots & & \underset{\sim}{D}_{2n} & \underset{\sim}{D}_{2n'} \\
\underset{\sim}{D}_{2'1} & \underset{\sim}{D}_{2'1'} & \underset{\sim}{D}_{2'2} & \underset{\sim}{D}_{2'2'} & & & & \underset{\sim}{D}_{2'n} & \underset{\sim}{D}_{2'n'} \\
& \vdots & & \vdots & & \ddots & & & \vdots \\
\underset{\sim}{D}_{n1} & \underset{\sim}{D}_{n1'} & \underset{\sim}{D}_{n2} & \underset{\sim}{D}_{n2'} & & & & \underset{\sim}{D}_{nn} & \underset{\sim}{D}_{nn'} \\
\underset{\sim}{D}_{n'1} & \underset{\sim}{D}_{n'1'} & \underset{\sim}{D}_{n'2} & \underset{\sim}{D}_{n'2'} & & & & \underset{\sim}{D}_{n'n} & \underset{\sim}{D}_{n'n'}
\end{bmatrix}}_{\underset{\sim}{D}}
\begin{bmatrix}
\underline{\sigma}_1 \\
\underline{\sigma}'_1 \\
\underline{\sigma}_2 \\
\underline{\sigma}'_2 \\
\vdots \\
\underline{\sigma}_n \\
\underline{\sigma}'_n
\end{bmatrix}
=
\begin{bmatrix}
\underline{\emptyset}_1 \\
\underline{0} \\
\underline{\emptyset}_2 \\
\underline{0} \\
\vdots \\
\underline{\emptyset}_n \\
\underline{0}
\end{bmatrix}
\tag{2.23}
$$

where unprimed quantities relate to the conductors, primed quantities relate to the dielectrics, and n equals the total number of wires in the cable. The σ_i terms are vectors of length k+1 containing the Fourier coefficients of free plus bound charge densities on the i-th conductor boundary, σ'_i are vectors of length l+1 containing the Fourier coefficients of the bound charge densities on the i-th dielectric boundary, and $\emptyset_i$ are column vectors of length k+1, defining the conductor potential of the i-th conductor for i=1,$\cdots$,n. These three vector quantities have an element organization as shown below:

$$
\underline{\sigma}_i =
\begin{bmatrix}
\sigma_{i0} \\
\sigma_{i1} \\
\vdots \\
\sigma_{ik}
\end{bmatrix}
\qquad
\underline{\sigma}'_i =
\begin{bmatrix}
\sigma'_{i0} \\
\sigma'_{i1} \\
\vdots \\
\sigma'_{il}
\end{bmatrix}
\qquad
\underline{\emptyset}_i =
\begin{bmatrix}
\emptyset_i \\
\emptyset_i \\
\vdots \\
\emptyset_i
\end{bmatrix}
\tag{2.24}
$$

16

where the per-unit-length charge distribution on the conductor surface of the i-th wire is represented by

$$\sigma_i = \sigma_{i0} + \sum_{m=1}^{k} \sigma_{im} \cos m\theta \qquad (2.25a)$$

and the per-unit-length bound charge distribution on the dielectric surface of the i-th wire is represented by

$$\sigma_i' = \sigma_{i0}' + \sum_{m=1}^{l} \sigma_{im}' \cos m\theta \qquad . \qquad (2.25b)$$

The submatrices of D have the significances noted below:

$\underset{\sim}{D}_{ij}$ contains the contributions to the potential at the matchpoints of conductor i due to unit charge distribution coefficients on conductor j. This submatrix is of dimension $(k+1) \times (k+1)$.

$\underset{\sim}{D}_{ij'}$ contains the contributions to the potential at the matchpoints of conductor i due to unit charge distribution coefficients on dielectric j'. This submatrix is of dimension $(k+1) \times (l+1)$.

$\underset{\sim}{D}_{i'j}$ contains the contributions to the difference in the normal components of the displacement vector at the matchpoints on the surface of dielectric i' due to unit charge distribution coefficients on conductor j. This submatrix is of dimension $(l+1) \times (k+1)$.

17

$D_{i'j'}$ contains the contributions to the difference in
the normal components of the displacement vector
at the matchpoints on the surface of dielectric i'
due to unit charge distribution coefficients on
dielectric j'. This submatrix is of dimension
$(1+1) \times (1+1)$.

For convenience, the matchpoints on each conductor
surface (dielectric surface) will be chosen according to the
same rules for all conductor surfaces (dielectric surfaces).
An example of this is shown in Fig. 2.5, where the match-
points are denoted by X's.
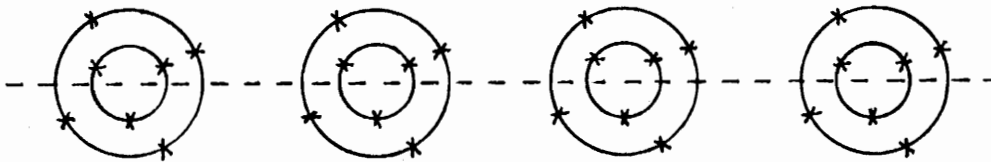
Figure 2.5.   Matchpoint orientations for the conductor
and dielectric surfaces.

Due to (1) the horizontal orientation of the flat-pack
cable, (2) the identical size of all the wires (see Fig.
1.1), and (3) this particular method of choosing the match-
points, many of the submatrices of D are identical. For
example, D_{11} contains the contribution to the potential at
the matchpoints of conductor 1 due to the components of its
own charge distribution. Obviously, then,

$$D_{11} = D_{22} = \cdots = D_{nn} \qquad .\qquad (2.26a)$$

18

Thus, the charge distribution on any conductor produces the same contribution to the potential at the matchpoints on its own surface. Also, the charge distribution on the surface of any dielectric produces the same contribution to the potential at the matchpoints on the conductor surface inside it:

$$\underset{\sim}{D}_{11'} = \underset{\sim}{D}_{22'} = \cdots = \underset{\sim}{D}_{nn'} \quad . \tag{2.26b}$$

Similarly, it can be seen that

$$\underset{\sim}{D}_{1'1} = \underset{\sim}{D}_{2'2} = \cdots = \underset{\sim}{D}_{n'n}$$

$$\underset{\sim}{D}_{1'1'} = \underset{\sim}{D}_{2'2'} = \cdots = \underset{\sim}{D}_{n'n'} \quad . \tag{2.26c}$$

This shows that all the submatrices on the main diagonal of $\underset{\sim}{D}$ are identical, i.e.,

$$\begin{bmatrix} \underset{\sim}{D}_{11} & \underset{\sim}{D}_{11'} \\ \underset{\sim}{D}_{1'1} & \underset{\sim}{D}_{1'1'} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{D}_{22} & \underset{\sim}{D}_{22'} \\ \underset{\sim}{D}_{2'2} & \underset{\sim}{D}_{2'2'} \end{bmatrix} = \cdots = \begin{bmatrix} \underset{\sim}{D}_{nn} & \underset{\sim}{D}_{nn'} \\ \underset{\sim}{D}_{n'n} & \underset{\sim}{D}_{n'n'} \end{bmatrix}$$

$$\tag{2.27}$$

The same is true for the other diagonal partitions: hence the matrix in Eq. (2.23) can be written as

$$\begin{bmatrix} \underset{\sim}{D}_0 & \underset{\sim}{D}_{-1} & \underset{\sim}{D}_{-2} & \cdots & \underset{\sim}{D}_{-(n-1)} \\ \underset{\sim}{D}_1 & \underset{\sim}{D}_0 & \underset{\sim}{D}_{-1} & \cdots & \underset{\sim}{D}_{-(n-2)} \\ \underset{\sim}{D}_2 & \underset{\sim}{D}_1 & \underset{\sim}{D}_0 & \cdots & \underset{\sim}{D}_{-(n-3)} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \underset{\sim}{D}_{(n-1)} & \underset{\sim}{D}_{(n-2)} & \underset{\sim}{D}_{(n-3)} & \cdots & \underset{\sim}{D}_0 \end{bmatrix} \begin{bmatrix} \underline{\sigma}^*_1 \\ \underline{\sigma}^*_2 \\ \underline{\sigma}^*_3 \\ \cdot \\ \cdot \\ \cdot \\ \underline{\sigma}^*_n \end{bmatrix} = \begin{bmatrix} \underline{X}_1 \\ \underline{X}_2 \\ \underline{X}_3 \\ \cdot \\ \cdot \\ \cdot \\ \underline{X}_n \end{bmatrix}$$

$$(2.28)$$

where

$$\underline{\sigma}^*_i = \begin{bmatrix} \underline{\sigma}_i \\ \underline{\sigma}'_i \end{bmatrix} \quad , \quad \underline{X}_i = \begin{bmatrix} \underline{\emptyset}_i \\ \underline{0} \end{bmatrix} \quad ,$$

and the subscripts on the $\underset{\sim}{D}_i$ submatrices in Eq. (2.28) are found from the subscripts of the corresponding submatrices of $\underset{\sim}{D}$ in Eq. (2.23) by subtracting the second subscript from the first. For example,

$$\begin{bmatrix} \underset{\sim}{D}_{1n} & \underset{\sim}{D}_{1n'} \\ \underset{\sim}{D}_{1'n} & \underset{\sim}{D}_{1'n'} \end{bmatrix} \equiv \underset{\sim}{D}_{-(n-1)} \quad . \qquad (2.29)$$

The matrix $\underset{\sim}{D}$ in Eq. (2.28) is in the block Toeplitz form allowing fast generation of the complete matrix, since only $2n-1$ of the original n^2 submatrices need be computed.

The matrix $\underset{\sim}{D}$ of equations (2.23) and (2.28) is inverted to obtain the matrix $\underset{\sim}{T} = \underset{\sim}{D}^{-1}$, and the equation becomes

$$
\begin{bmatrix}
\underline{\sigma}_1 \\
\underline{\sigma}_1' \\
\underline{\sigma}_2 \\
\underline{\sigma}_2' \\
\vdots \\
\underline{\sigma}_n \\
\underline{\sigma}_n'
\end{bmatrix}
=
\begin{bmatrix}
\underset{\sim}{T}_{11} & \underset{\sim}{T}_{11'} & \underset{\sim}{T}_{12} & \underset{\sim}{T}_{12'} & \ldots & \underset{\sim}{T}_{1n} & \underset{\sim}{T}_{1n'} \\
\underset{\sim}{T}_{1'1} & \underset{\sim}{T}_{1'1'} & \underset{\sim}{T}_{1'2} & \underset{\sim}{T}_{1'2'} & & \underset{\sim}{T}_{1'n} & \underset{\sim}{T}_{1'n'} \\
\cdot & & \cdot & & \cdot & & \cdot \\
\cdot & & \cdot & & \cdot & & \cdot \\
\cdot & & \cdot & & \cdot & & \cdot \\
\underset{\sim}{T}_{n1} & \underset{\sim}{T}_{n1'} & \underset{\sim}{T}_{n2} & \underset{\sim}{T}_{n2'} & \ldots & \underset{\sim}{T}_{nn} & \underset{\sim}{T}_{nn'} \\
\underset{\sim}{T}_{n'1} & \underset{\sim}{T}_{n'1'} & \underset{\sim}{T}_{n'2} & \underset{\sim}{T}_{n'2'} & & \underset{\sim}{T}_{n'n} & \underset{\sim}{T}_{n'n'}
\end{bmatrix}
\begin{bmatrix}
\underline{\emptyset}_1 \\
0 \\
\underline{\emptyset}_2 \\
0 \\
\vdots \\
\underline{\emptyset}_n \\
0
\end{bmatrix}
$$

$$(2.30)$$

The total (bound plus free) per-unit-length charge, q_i, on the i-th conductor surface is the integral of the per-unit-length surface charge density, σ_i, over that surface. Thus,

$$q_i = \int_0^{2\pi} \left(\sigma_{i0} + \sum_{m=1}^{k} \sigma_{im} \cos m\theta \right) r_c \, d\theta \qquad (2.31)$$

which reduces to

$$q_i = \int_0^{2\pi} \sigma_{i0} r_c \, d\theta = 2\pi r_c \sigma_{i0} \qquad (2.32)$$

since the integration of a cosine function over a multiple of 2π is zero. The term r_c is the conductor radius, and σ_{i0} is the average value of the total per-unit-length charge density (the constant term of the Fourier expansion) on conductor i.

Similarly, the per-unit-length bound charge on the dielectric surface is

$$q_i' = 2\pi r_d \sigma_{i0}' \quad , \qquad (2.33)$$

21

where r_d is the dielectric radius, and σ'_{i0} is the average value of the per-unit-length bound charge density on the i-th dielectric surface. As a consequence of Eq. (2.10), the per-unit-length free charge on the surface of conductor i is

$$q_{if} = q_i + q'_i \quad . \tag{2.34}$$

Thus, combining equations (2.32) and (2.33) yields the per-unit-length free charge on the i-th conductor,

$$q_{if} = 2\pi(r_c \sigma_{i0} + r_d \sigma'_{i0}) \quad . \tag{2.35}$$

From Eq. (2.30), the average per-unit-length charge density on the i-th conductor, σ_{i0}, (the first term in the $\underline{\sigma}_i$ vector) is

$$\sigma_{i0} = \underline{T}^1_{i1}\underline{\emptyset}_1 + \underline{T}^1_{i2}\underline{\emptyset}_2 + \cdots + \underline{T}^1_{in}\underline{\emptyset}_n \tag{2.36}$$

where $\underline{T}^1_{ij}$ indicates a 1xn vector consisting of the first row of $\underset{\sim}{T}_{ij}$. Similarly, the average per-unit-length charge density on the i-th dielectric, σ'_{i0}, is

$$\sigma'_{i0} = \underline{T}^1_{i'1}\underline{\emptyset}_1 + \underline{T}^1_{i'2}\underline{\emptyset}_2 + \cdots + \underline{T}^1_{i'n}\underline{\emptyset}_n \quad . \tag{2.37}$$

Adding equations (2.36) and (2.37), according to equation (2.35), yields

22

$$q_{if} = 2\pi \left[r_c (\underline{T}^1_{i1}\underline{\emptyset}_1 + \underline{T}^1_{i2}\underline{\emptyset}_2 + \cdots + \underline{T}^1_{in}\underline{\emptyset}_n) \right.$$

$$\left. + r_d (\underline{T}^1_{i'1}\underline{\emptyset}_1 + \underline{T}^1_{i'2}\underline{\emptyset}_2 + \cdots + \underline{T}^1_{i'n}\underline{\emptyset}_n) \right]$$

$$= 2\pi \sum_{j=1}^{n} \left[(r_c\underline{T}^1_{ij} + r_d\underline{T}^1_{i'j})\underline{\emptyset}_j \right] \quad . \tag{2.38}$$

Since $\underline{\emptyset}_i$ is a vector of equal conductor potentials, that is,

$$\underline{\emptyset}_j = \begin{bmatrix} \emptyset_i \\ \emptyset_i \\ \vdots \\ \emptyset_i \end{bmatrix} , \tag{2.39}$$

then Eq. (2.38) can be written as

$$q_{if} = 2\pi \sum_{j=1}^{n} \left\{ r_c \sum_{m=1}^{n} T^{1m}_{ij} + r_d \sum_{m=1}^{n} T^{1m}_{i'j} \right\} \emptyset_j \tag{2.40}$$

where T^{pq}_{ij} is the element of $\underset{\sim}{T}_{ij}$ in the p-th row and q-th column. The desired generalized capacitance matrix is

$$\begin{bmatrix} q_{1f} \\ q_{2f} \\ \vdots \\ q_{nf} \end{bmatrix} = \begin{bmatrix} \mathcal{C}_{11} & \mathcal{C}_{12} & \cdots & \mathcal{C}_{1n} \\ \mathcal{C}_{21} & \mathcal{C}_{22} & \cdots & \mathcal{C}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{C}_{n1} & \mathcal{C}_{n2} & \cdots & \mathcal{C}_{nn} \end{bmatrix} \begin{bmatrix} \emptyset_1 \\ \emptyset_2 \\ \vdots \\ \emptyset_n \end{bmatrix} \tag{2.41}$$

and the per-unit-length free charge on conductor i is

$$q_{if} = \sum_{j=1}^{n} (\mathcal{C}_{ij}) \, \emptyset_j \quad . \tag{2.42}$$

23

Thus comparing equations (2.40) and (2.42), the elements of the per-unit-length generalized capacitance matrix can be written as

$$\mathcal{C}_{ij} = 2\pi\left\{ r_c \sum_{m=1}^{n} T_{ij}^{1m} + r_d \sum_{m=1}^{n} T_{i'j}^{1m} \right\} \quad . \quad (2.43)$$

CHAPTER III

MATCHPOINT SELECTION

III.1 Introduction

In the previous chapter, a method for computing the
generalized capacitance matrix of ribbon cables was de-
veloped. Several major considerations concerning choice of
matchpoints will be discussed in this chapter. In the first
part of this chapter, necessary criteria for choosing the
position of the matchpoints are discussed. A matchpoint
selection technique applicable to the ribbon cable problem
was developed and is shown to meet these criteria. The last
portion of the chapter provides some guidelines on the
number of Fourier series coefficients and matchpoints
required for the surface charges to yield accurate capa-
citance matrices.

III.2 Matchpoints and Nonsingularity of the $\underset{\sim}{D}$ Matrix

A key step in the method outlined in Chapter II is the
inversion of the computed $\underset{\sim}{D}$ matrix. The necessary criterion
to insure solution is that $\underset{\sim}{D}$ be nonsingular thus assuring
that $\underset{\sim}{D}^{-1}$ exists. Therefore, the matchpoints should be
chosen such that singular $\underset{\sim}{D}$ matrices are not generated.

A matrix is singular when any of its rows (or columns)
can be expressed as a linear combination of the other rows

(columns). The most obvious case of this condition results when two rows (columns) are equal. Criteria for matchpoint selection which will guarantee that no two potential or displacement vector continuity equations are equal will be developed in this section. It should be pointed out, however, that these will constitute only necessary conditions for the nonsingularity of $\underset{\sim}{D}$.

As described in Chapter II, the matrix equation defining $\underset{\sim}{D}$ is of the following form:

$$
\begin{bmatrix}
\underset{\sim}{D}_{11} & \underset{\sim}{D}_{11'} & \underset{\sim}{D}_{12} & \underset{\sim}{D}_{12'} & \cdots & \underset{\sim}{D}_{1n} & \underset{\sim}{D}_{1n'} \\
\underset{\sim}{D}_{1'1} & \underset{\sim}{D}_{1'1'} & \underset{\sim}{D}_{1'2} & \underset{\sim}{D}_{1'2'} & \cdots & \underset{\sim}{D}_{1'n} & \underset{\sim}{D}_{1'n'} \\
\underset{\sim}{D}_{21} & \underset{\sim}{D}_{21'} & \underset{\sim}{D}_{22} & \underset{\sim}{D}_{22'} & \cdots & \underset{\sim}{D}_{2n} & \underset{\sim}{D}_{2n'} \\
\underset{\sim}{D}_{2'1} & \underset{\sim}{D}_{2'1'} & \underset{\sim}{D}_{2'2} & \underset{\sim}{D}_{2'2'} & \cdots & \underset{\sim}{D}_{2'n} & \underset{\sim}{D}_{2'n'} \\
\vdots & & \vdots & & \ddots & \vdots & \\
\underset{\sim}{D}_{n1} & \underset{\sim}{D}_{n1'} & \underset{\sim}{D}_{n2} & \underset{\sim}{D}_{n2'} & \cdots & \underset{\sim}{D}_{nn} & \underset{\sim}{D}_{nn'} \\
\underset{\sim}{D}_{n'1} & \underset{\sim}{D}_{n'1'} & \underset{\sim}{D}_{n'2} & \underset{\sim}{D}_{n'2'} & \cdots & \underset{\sim}{D}_{n'n} & \underset{\sim}{D}_{n'n'}
\end{bmatrix}
\begin{bmatrix}
\underline{\sigma}_1 \\ \underline{\sigma}'_1 \\ \underline{\sigma}_2 \\ \underline{\sigma}'_2 \\ \vdots \\ \underline{\sigma}'_2 \\ \underline{\sigma}'_n
\end{bmatrix}
=
\begin{bmatrix}
\underline{\phi}_1 \\ \underline{0} \\ \underline{\phi}_2 \\ \underline{0} \\ \vdots \\ \underline{\phi}_n \\ \underline{0}
\end{bmatrix}
. \tag{3.1}
$$

It will be assumed that there are k+1 matchpoints around each of the conductor surfaces and l+1 matchpoints around each of the dielectric surfaces. The first block row of $\underset{\sim}{D}_{ij}$ and $\underset{\sim}{D}_{i'j'}$ submatrices in a partitioned row of Eq. (3.1) consists of k+1 equations for the potential at the conductor matchpoints of wire i. The next block row of $\underset{\sim}{D}_{i'j}$ and $\underset{\sim}{D}_{i'j'}$ submatrices consist of l+1 equations containing

26

the differences in the normal components of the displacement vector at the matchpoints on the dielectric surface of wire i.

Examining the conditions for identical potential equations, the case where the identical equations are in different partitioned rows of the matrix in Eq. (3.1) will be considered first. The matchpoints corresponding to the two equations are located on different wires since the partitioning in Eq. (3.1) combines only those equations for a particular wire in each block row. Figure 3.1 illustrates two matchpoints, A and B, on the surfaces of the conductors of two different wires, i and j.
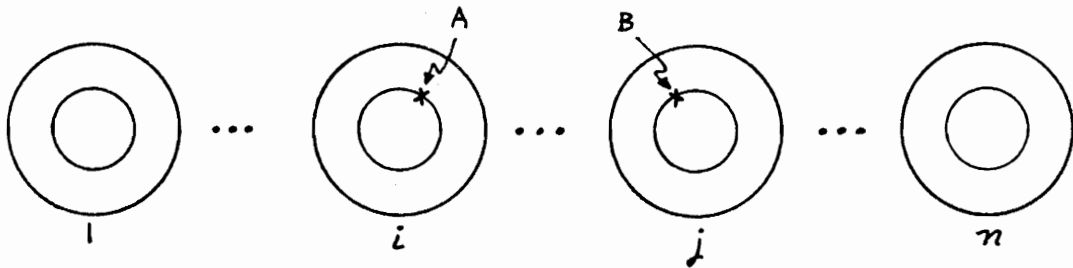
Figure 3.1 Matchpoints on two different wires in the cable.

Referring to the matrix equation (3.1), the rows associated with matchpoints A and B will be equal if the respective rows of the submatrices in each column partition are the same. That is,

$$\underline{D}_{im}^{A} = \underline{D}_{jm}^{B} \qquad \text{for all } m=1, 1', \ldots, n, n' \qquad , \qquad (3.2)$$

27

where D_{pq}^R indicates the row associated with matchpoint R in the submatrix D_{pq}. This implies that the contributions to the potentials at matchpoints A and B due to the components of the charge distributions on the m-th surface must be the same. It is obvious that this will be true only when matchpoints A and B are at the same location. For wires having dielectric insulations, however, matchpoints on two different conductors will never be coincident.

A similar analysis for the displacement vector continuity equations shows the same result which is illustrated in Fig. 3.2. For ribbon cables with separation greater than twice the dielectric radius, however, this will never occur, since the dielectric surfaces will not be touching.
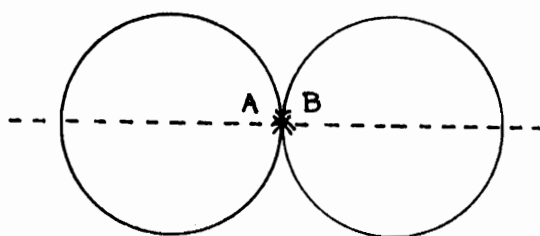
Figure 3.2 Coincident matchpoints from different surfaces.

When the adjacent dielectric surfaces in a ribbon cable are touching, this may occur and a general matchpoint selection method should avoid this condition.

Another instance in which the D matrix in Eq. (3.1) will be singular occurs when two identical equations are in the same row partition, implying that the matchpoints associated with these identical equations are associated

28

with the same surface. As in the previous case, the ele-
ments of the two equations will be equal when the effects at
the two matchpoints (on potential or continuity of the
displacement vector) due to components of the charge dis-
tributions on the m-th surface are the same. This will
result when choosing two matchpoints on the same surface of
some wire as images of one another, above and below the line
between the centers of the wires.   Fig. 3.3(a) illustrates
a typical occurance of this condition.
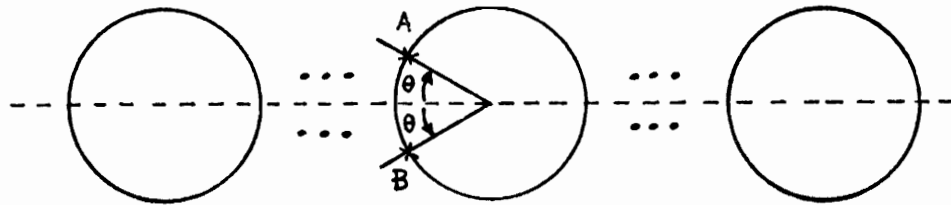
(a)   Matchpoints symmetrically disposed with respect
      to the center line on a charged surface
      (dielectric or conductor).
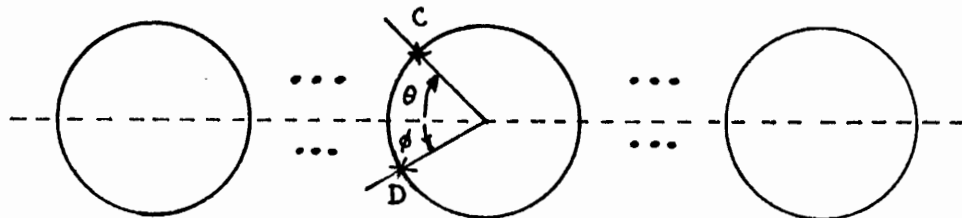
(b)   Alternate choice of matchpoints on the surface.

Figure 3.3.

Matchpoints A and B in   Fig. 3.3(a) are symmetrically
disposed with respect to the charge distributions on all the

29

wires. The potentials (or fields) at these points due to terms of the Fourier series representation of the charge distributions (assumed to be an even function with respect to the line between centers) on any surface will be the same, thus generating two identical rows. This can be easily avoided by choosing the matchpoints on any surface according to the method in Fig. 3.3(b). If the angles θ and ϕ are not equal, the rows in the D matrix associated with these matchpoints will not be equal.

A method for matchpoint selection must then meet two necessary criteria:

1) the matchpoints on two adjacent wires must not both be on the line between centers, and

2) the matchpoints on any surface must not be chosen symmetric with respect to the line between centers.

These criteria, of course, do not guarantee the nonsingularity of D. They do, however, avoid the obvious cases in which a singular D matrix will be generated.

III.3 Matchpoints and Symmetry of the C Matrix

Due to the extreme symmetry of the "flat-pack" problem (conductor radii identical, dielectric radii identical, separation of the wires identical, and the wires oriented in a horizontal plane), some of the elements of C must be identical. As an example, consider the case of three wires

30

shown in Fig. 3.4(a). The generalized capacitance matrix becomes

$$
\begin{bmatrix}
\hat{c}_{11} & \hat{c}_{12} & \hat{c}_{13} \\
\hat{c}_{21} & \hat{c}_{22} & \hat{c}_{21} \\
\hat{c}_{13} & \hat{c}_{12} & \hat{c}_{11}
\end{bmatrix}
\tag{3.3}
$$
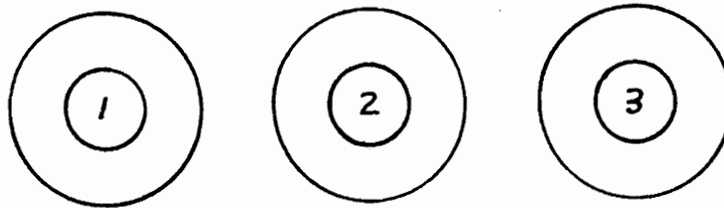
(a) A three-wire cable .
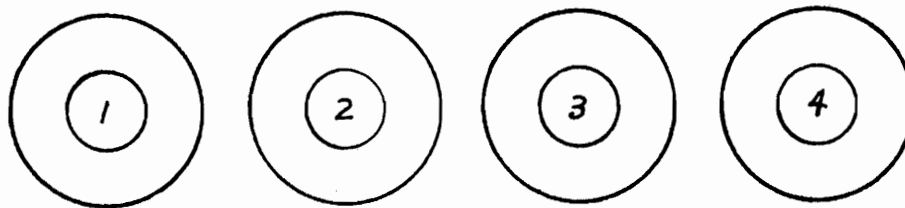
(b) A four-wire cable .

Figure 3.4 Three and four-wire cables illustrating the geometrical symmetry.

The resulting structure of $\underset{\sim}{\hat{C}}$ in Eq. (3.3) can be easily shown by recalling that $\hat{c}_{ij}$ is the ratio of the total per-unit-length free charge on the i-th conductor with a unit potential applied to the j-th conductor and all other conductor potentials set equal to zero; i.e.,

$$\mathcal{C}_{ij} = \left. \frac{q_{if}}{\phi_j} \right|_{\substack{\phi_m=0 \\ m \neq j}} \qquad \qquad . \qquad \qquad (3.4)$$

Clearly then from geometrical considerations

$$
\begin{aligned}
\mathcal{C}_{33} &= \mathcal{C}_{11} \\
\mathcal{C}_{23} &= \mathcal{C}_{21} \\
\mathcal{C}_{32} &= \mathcal{C}_{12} \\
\mathcal{C}_{31} &= \mathcal{C}_{13}
\end{aligned}
\qquad (3.5)
$$

as indicated in Eq. (3.3).

For four wires $\mathcal{C}$ becomes

$$
\mathcal{C} = \begin{bmatrix}
\mathcal{C}_{11} & \mathcal{C}_{12} & \mathcal{C}_{13} & \mathcal{C}_{14} \\
\mathcal{C}_{21} & \mathcal{C}_{22} & \mathcal{C}_{23} & \mathcal{C}_{24} \\
\mathcal{C}_{24} & \mathcal{C}_{23} & \mathcal{C}_{22} & \mathcal{C}_{21} \\
\mathcal{C}_{14} & \mathcal{C}_{13} & \mathcal{C}_{12} & \mathcal{C}_{11}
\end{bmatrix}
\qquad . \qquad (3.6)
$$

Thus it can be shown from Eq. (3.4) that for this four wire case (see Fig. 3.4(b)),

$$
\begin{aligned}
\mathcal{C}_{33} &= \mathcal{C}_{22} \\
\mathcal{C}_{44} &= \mathcal{C}_{11} \\
\mathcal{C}_{31} &= \mathcal{C}_{24} \\
\mathcal{C}_{32} &= \mathcal{C}_{23} \\
\mathcal{C}_{41} &= \mathcal{C}_{14} \\
\mathcal{C}_{42} &= \mathcal{C}_{13} \\
\mathcal{C}_{43} &= \mathcal{C}_{12}
\end{aligned}
\qquad (3.7)
$$

Therefore, the matchpoint selection technique should yield this "geometrical symmetry". Matrices with these properties are called centro-symmetric [18]. An nxn centro-symmetric matrix M has the property that $\underset{\sim}{M} = \underset{\sim}{S}\underset{\sim}{M}\underset{\sim}{S}$ where $\underset{\sim}{S}$ is an nxn matrix with ones on the cross-diagonal and zeros elsewhere; i.e., $S_{ij} = 1$ for $i+j = n+1$ and zeros elsewhere [18].

Reciprocity relations [11] also show that the generalized capacitance matrix is symmetric with respect to the main diagonal. Thus, the generalized capacitance matrices for three and four wire cables can be written as

$$
\underset{\sim}{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{12} \\ C_{13} & C_{12} & C_{11} \end{bmatrix}
\tag{3.8a}
$$

$$
\underset{\sim}{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{12} & C_{22} & C_{23} & C_{13} \\ C_{13} & C_{23} & C_{22} & C_{12} \\ C_{14} & C_{13} & C_{12} & C_{11} \end{bmatrix}
\tag{3.8b}
$$

Therefore, the generalized capacitance matrix should be symmetric with respect to both the main diagonal and the cross-diagonal and those elements above the dashed lines in Eq. (3.8) completely characterize the matrix.

Another necessary criterion, then, in choosing the matchpoints for the method described in Chapter II is that the computed capacitance matrix be symmetric with respect to

33

both the main diagonal and the cross-diagonal.

III.4 A Method for Matchpoint Selection

In the implementation of the general technique, many choices of the number of matchpoints and expansion terms for each surface are possible. However, one particular choice will be used in this report and the resulting computer program. The number of terms in the Fourier series distributions associated with each dielectric surface of each wire will be the same and will be denoted by NFD. Also, the number of expansion terms associated with each conductor surface of each wire will be the same and will be denoted by NFC. The number of matchpoints associated with each dielectric surface of each wire will be equal to the number of expansion coefficients associated with this surface; i.e., NFD = 1+1. Similarly, the number of matchpoints associated with each wire will be equal to the number of expansion coefficients associated with this surface; i.e., NFC = k+1.

During the course of this work, a recursive method for selection of the matchpoint positions was developed. This will be described and then shown to meet the necessary criteria. The method is dependent on the number of Fourier coefficents chosen for the distribution of charge around the surface. The relation assumes the matchpoints are evenly divided around the surface, so that the only independent variable is the rotation of the matchpoint set from the zero degree axis. This is illustrated in Fig. 3.5 for five

34

matchpoints on a particular surface. The angle θ used in the potential and electric field expressions in Chapter II will be chosen with respect to the horizontal line between centers, as shown in Fig. 2.4 and Fig. 3.5.
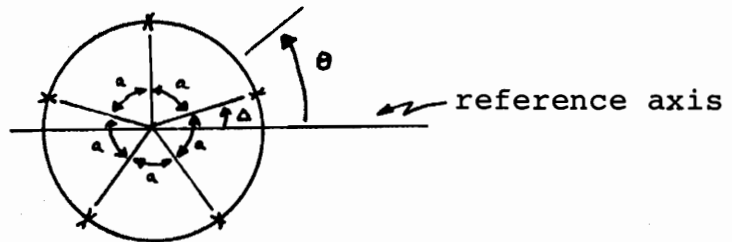
Figure 3.5   Example of matchpoint selection.

The angle, a, between any two adjacent matchpoints is determined by

$$a = \frac{2\pi}{NFS} \qquad , \qquad (3.9)$$

where NFS indicates the number of Fourier coefficients (matchpoints) associated with the surface (conductor or dielectric). For this particular case of five matchpoints,

$$a = \frac{2\pi}{5} = 72° \qquad .$$

The angle Δ, the rotation of the matchpoints from the θ=0 reference, is given by

$$\Delta = \frac{\pi}{(2\times NFS)} \qquad . \qquad (3.10)$$

Thus for the case of NFS=5, as above,

35
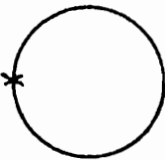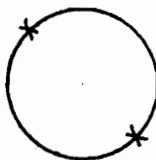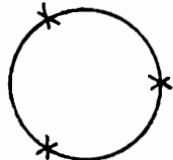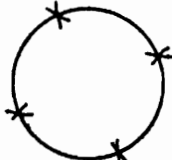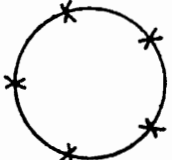
$$\Delta = \frac{\pi}{(2 \times 5)} = \frac{\pi}{10} \quad .$$

The matchpoints then lie at angles $\frac{\pi}{10}, \frac{5\pi}{10}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}$ from the reference.

Table 1 shows the typical matchpoint selections using the formulas in Eq. (3.9) and Eq. (3.10) for cases where the number of matchpoints is in the range of 1 to 10.

For all the cases shown in Table 1, the matchpoints on any particular surface (conductor or dielectric) are not symmetric with respect to the line connecting wire centers, nor do the matchpoints occur on the line connecting wire centers. This is true for all finite values of NFS. In general, then, this matchpoint selection technique will always meet the criteria which were necessary in avoiding $\underset{\sim}{D}$ matrices in which two rows of the potential equations or the displacement vector continuity equations are identical.

It was shown in Section III.3 that the generalized capacitance matrix should be symmetric about the main and cross-diagonals. This would be true if the exact charge distributions on the conductor and dielectric surfaces are obtained. In applying this technique, an approximation to the true charge distributions will be obtained. In the limit as the number of matchpoints on each surface increases without bound, these approximate charge distributions will approach the true charge distributions. In order to min-imize the computation time and required core storage, how-

36

NFS	a	Δ	Pictorial
6	$\frac{\pi}{3}$	$\frac{\pi}{12}$	
7	$\frac{2\pi}{7}$	$\frac{\pi}{14}$	
8	$\frac{\pi}{4}$	$\frac{\pi}{16}$	
9	$\frac{2\pi}{9}$	$\frac{\pi}{18}$	
10	$\frac{\pi}{5}$	$\frac{\pi}{20}$	

NFS	a	Δ	Pictorial
1	2π	$\frac{\pi}{2}$	
2	π	$\frac{\pi}{4}$	
3	$\frac{2\pi}{3}$	$\frac{\pi}{6}$	
4	$\frac{\pi}{2}$	$\frac{\pi}{8}$	
5	$\frac{2\pi}{5}$	$\frac{\pi}{10}$	

Table 1. Matchpoint positions resulting from formulae

$$a = \frac{2\pi}{\text{NFS}} \quad , \quad \Delta = \frac{\pi}{2\cdot\text{NFS}} \quad .$$

37

ever, the matchpoint selection technique should provide a rapid convergence of the generalized capacitance matrix to one which is symmetric about the main and cross diagonals.

As indicated above, the number of matchpoints on each conductor surface and each dielectric surface of each wire will be denoted by NFC and NFD, respectively. In addition, the number of matchpoints chosen on each surface will equal the number of Fourier series expansion terms associated with that surface. For the case of two wires, the matchpoint selection technique outlined previously can be shown to yield centro-symmetric generalized capacitance matrices for any value of NFC or NFD. To show this, consider the case of two wires with various matchpoint selections shown in Fig. 3.6. These cases should be sufficiently representative to illustrate the proof.

Note in Fig. 3.6 that when the number of matchpoints on a particular surface is odd, these matchpoints are mirror symmetric about a vertical line X-X' between wire centers. When the number of matchpoints on a particular surface is even, these matchpoints exhibit skew symmetry about the vertical line X-X'. These conclusions can be generalized for two wires and any value of NFC or NFD.

The generalized capacitance matrix for two wires becomes
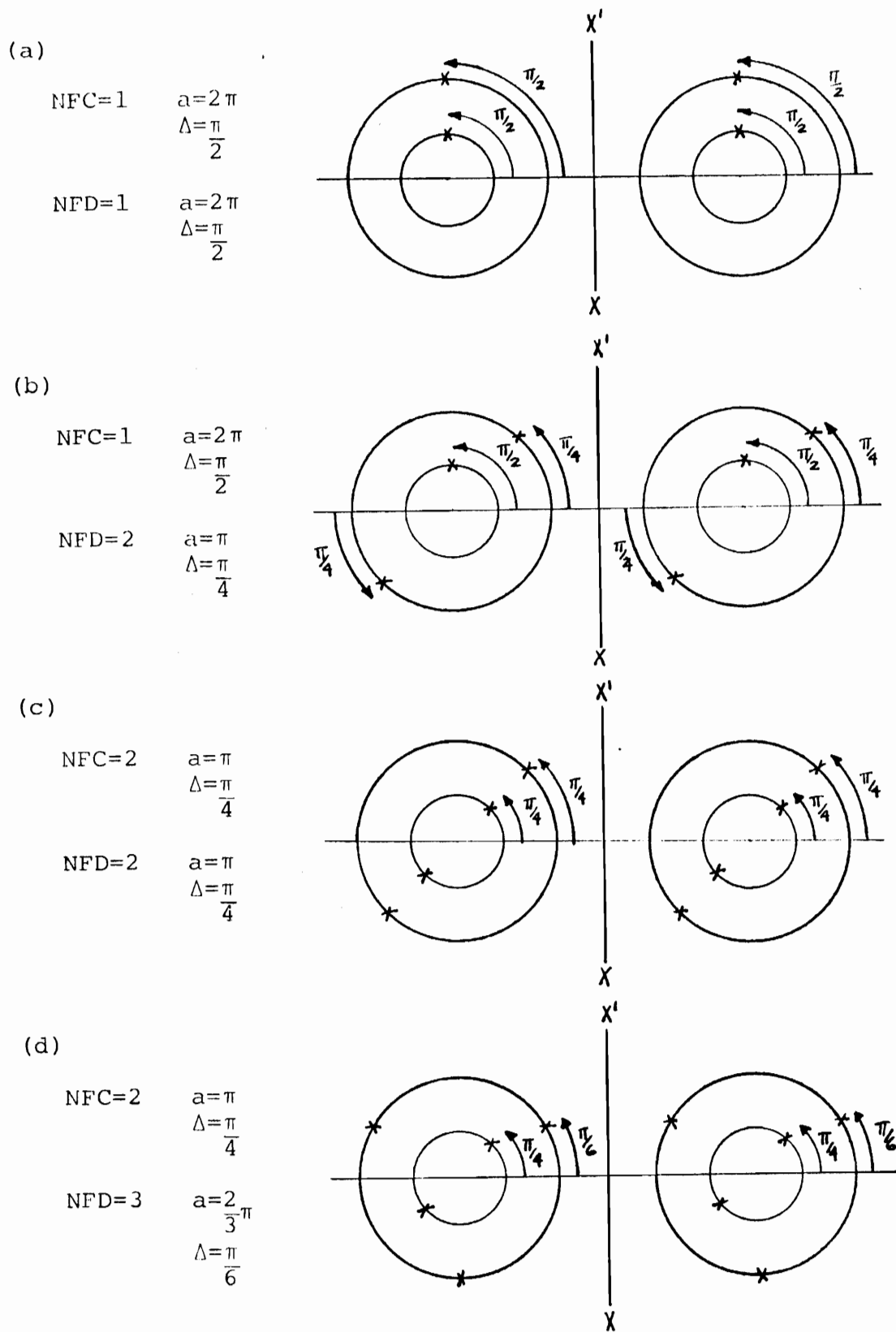
Figure 3.6. Matchpoint selections for two wires.

39

$$\underset{\sim}{c} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \qquad . \qquad (3.11)$$

Recall that c_{ij} is the free charge on the i-th conductor with a unit potential applied to the j-th conductor and the remaining conductors at zero potential. Since (1) the matchpoints on corresponding surfaces exhibit either mirror symmetry or skew symmetry with respect to the vertical line X-X', (2) the wires are identical, and (3) the assumed charge distributions are symmetric with respect to a line between centers, then clearly $\underset{\sim}{c}$ for two wires will exhibit centro-symmetry and

$$c_{11} = c_{22}$$
$$c_{12} = c_{21} \qquad . \qquad (3.12)$$

This is a direct result of the fact that due to the above observations, the computations performed when the cable is viewed from either end will be identical. Note that for two wires, centro-symmetry automatically guarantees symmetry of about the main diagonal. Computed results illustrating this are given in Table 2 for the cases of NFC=2, NFD=2 and NFC=2, NFD=3. Note that the matrices are symmetric with respect to the main and cross diagonals to sixteen digits. All computations were performed on an IBM 370/165 computer in double precision arithmetic.

For cables containing more than two wires, geometrical

40

Physical conditions:

r_c = 1.0

r_d = 2.0

sep = 4.0

ε_r = 4.0

Two wires:
NFC=2
NFD=2

$$\begin{bmatrix} 23.64630331172406D-12 & -50.61938353186358D-12 \\ -50.61938353186358D-12 & 23.64630331172406D-12 \end{bmatrix}$$

Two wires:
NFC=2
NFD=3

$$\begin{bmatrix} 28.13906061773703D-12 & -55.04463986326089D-12 \\ -55.04463986326089D-12 & 28.13906061773703D-12 \end{bmatrix}$$

Five Wires, NFC=3, NFD=7.   (elements $\times 10^{-12}$)

$$\begin{bmatrix} 48.001873820947 & -43.291394174935 & -4.6562119966386 & -3.3655186155275 & -6.4109370528478 \\ -43.291737014768 & 85.705646411657 & -40.156144007001 & -2.8839728747193 & -3.3654719979811 \\ -4.6562516882283 & -40.156144007001 & 86.008879896960 & -40.155930238945 & -4.6562516882283 \\ -3.3654719989811 & -2.8839728747193 & -40.155930238945 & 85.705646411656 & -43.291737014768 \\ -6.4109370528478 & -3.3655186155275 & -4.6562119966386 & -43.291394174935 & 48.001873820947 \end{bmatrix}$$

Table 2.   Computed generalized capacitance matrices for two and five wires.

41

symmetry (i.e., centro-symmetry) of $\c$ will again result. This has been verified computationally in numerous cases for cables involving up to twenty wires. However, for more than two wires, this geometrical symmetry does not automatically imply symmetry about the main diagonal. Again, the reason that $\c$ will not be symmetric with respect to the main diagonal is that only the approximate charge distributions are obtained. However, this matchpoint selection technique does provide rapid convergence to generalized capacitance matrices which are symmetric about the main and cross-diagonals. A typical result for a five wire cable with NFC=3, NFD=7 is shown in Table 2.

III.5 Number of Fourier Coefficients and Convergence of the Capacitance Matrix

One final topic to be discussed is the choice of the number of Fourier series coefficients around each boundary which are to yield results to a certain accuracy. This is most important in terms of computation time, since a single additional coefficient increases the size of the D matrix by an order of NW (the number of wires). This is significant since the number of operations (multiplications and divisions) required to invert an mxm matrix is approximately m^3. Therefore, the number of operations required to invert D is $(NFC + NFD)^3 (NW)^3$.

A number of sample cases were run to determine worst case needs. The problem considered is illustrated in Fig. 3.7,

42
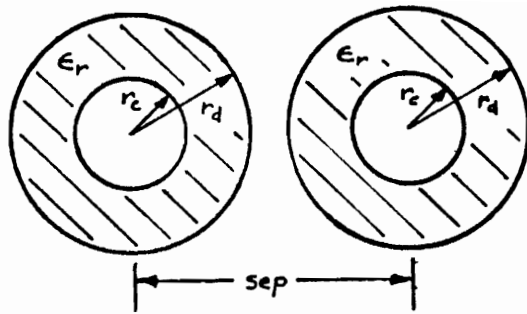
$r_c = 1.0$

$r_d = 2.0$

$\text{sep} = 4.0$

$\varepsilon_r = 4.0$

Figure 3.7 Two-wire ribbon cable.

where the dimensions of the cable indicate that the diel-
ectric surfaces are touching. This should correspond to a
worst case condition in terms of wire separation. Table 3
shows the calculated capacitance using Eq. (2.5), for
various numbers of Fourier coefficients associated with each
surface.

The circled items in Table 3 indicate the best result
for a particular number of total Fourier coefficients
assigned to each wire. The most interesting point is that
not as many terms are required around the conductor surface
as the dielectric surface. For example, with ten total
coefficients, the best organization would use three terms in
the representation of the charge distribution around the
conductor surface and seven for the dielectric surface.

Table 4(a) indicates similar results for five-wire
ribbon cable. Considering the convergence of the C_{33} term
given in Table 4(a), it is sufficient to assign approx-
imately one-third of the total number of expansion coef-
ficients alloted to the wire to the conductor surface expansion
and assign the remaining number to the dielectric surface

43

terms around conductor

# terms around dielectric	1	2	3	4	5	6	7	8	9	10
1	27.8									
2	33.2	37.1								
3	37.3	41.6	42.1							
4	39.7	44.4	45.0	45.0						
5	41.0	46.1	46.7	46.8	46.8					
6	41.7	47.1	47.7	47.8	47.8	47.8				
7	42.1	47.7	48.4	48.4	48.5		48.5			
8	42.4	48.1	48.7	48.8	48.8			48.8		
9	42.6	48.3	49.0	49.1	49.1				49.1	
10	42.7	48.4	49.1	49.2	49.2					49.2
11			49.2	49.3						
12			49.3							

Physical conditions: $r_c = 1.0$, $r_d = 2.0$, sep $= 4.0$, $\varepsilon_r = 4.0$.

Table 3.   Capacitance between two dielectric-coated conductors for various numbers of Fourier terms per boundary. (All entries $\times 10^{-12}$.)

44

# terms around dielectric	# terms around conductor						
	1	2	3	4	5	6	10
1	46.2						
2	56.3	63.5					
3	65.1	72.4	73.4				
4	70.5	78.1	79.4	79.5			
5	73.0	81.3	82.7	82.8	82.8		
6	74.7	83.5	84.8	85.0	85.0		
7	75.6	84.6	86.0	86.2	86.2		
8	76.2	85.3	86.8	87.0	87.0		
9	76.6	85.7	87.3	87.4	87.4		
10	76.8	86.0	87.6	87.7	87.8		87.8
11			87.8	87.9	87.9		
12			87.9	88.1	88.1		
13			88.0	88.1	88.2		
14			88.0	88.2	88.2		

Physical conditions:
$r_c=1.0$, $r_d=2.0$, sep=4.0, $\varepsilon_r=4.0$

Table 4(a).   Center element, $\mathcal{C}(3,3)$, in the generalized capacitance matrix of a five-wire ribbon cable for various numbers of Fourier terms per boundary. (All entries x10^{-12}.)

45

expansion. In applying these results to transmission line analysis, the transmission line capacitance matrix, C, is needed. The derivation of C from the generalized capacitance matrix is shown in Chapter V, and the C_{11} term for the five wire ribbon cable is shown in Table 4(b). Again, it is sufficient to proportion the total number of expansion coefficients allotted to a wire by assigning approximately one-third to the conductor surface. This allotment criterion has yielded the same convergence results for all computed cases which were examined and should serve as a general rule for users of this program.

A general rule for the total number of coefficients to be allotted to each wire cannot be as easily obtained since the convergence depends upon the relative separation of the wires; for larger wire separations, a smaller number of total expansion terms can be used. However, a worst case rule can be cited. Numerous examples have been run involving up to twenty wires in which the dielectric surfaces are touching (a worst case and generally typical situation for ribbon cables). The dielectric constant is assumed to be 4 (a representative upper bound on typically used insulation materials). For all of these cases, accuracy to within approximately five digits can be obtained by using a total of ten coefficients per each wire. With the above allotment rule, the recommendation is to use a total of ten terms for each wire and assign three to the conductor surface and seven to the dielectric surface.

# terms around dielectric	# terms around conductor						
	1	2	3	4	5	6	10
1	46.6						
2	56.6	63.7					
3	65.4	72.6	73.6				
4	70.8	78.3	79.6	79.7			
5	73.4	81.6	82.9	83.0	83.0		
6	75.1	83.7	85.1	85.2	85.2		
7	75.9	84.8	86.2	86.4	86.4		
8	76.6	85.5	87.0	87.2	87.2		
9	76.9	85.9	87.5	87.6	87.7		
10	77.1	86.2	87.8	88.0	88.0		88.0
11			88.0	88.1	88.2		
12			88.1	88.3	88.3		
13			88.2	88.3	88.4		
14			88.2	88.4	88.4		

Physical conditions:

$r_c = 1.0$, $r_d = 2.0$, sep$=4.0$, $\varepsilon_r = 4.0$

Table 4(b). C_{11} element in the transmission line capacitance matrix (to be described in Chapter V) of a five-wire ribbon cable for various numbers of Fourier terms per boundary. (All entries $\times 10^{-12}$.)

CHAPTER IV

ALTERNATE METHODS OF SOLUTION

IV.1 Introduction

A numerical method for determining the generalized capacitance matrix for ribbon cables was developed in Chapter II. Criteria for the selection of the matchpoints were given in Chapter III.

One difficulty with this technique is that for large numbers of wires, a reasonably accurate $\underset{\sim}{C}$ matrix requires that the $\underset{\sim}{D}$ matrix of Eq. (2.23) be of a very large order. As indicated previously, the number of operations required to obtain $\underset{\sim}{D}^{-1}$ is on the order of $(NFC+NFD)^3 (NW)^3$ where NW is the number of wires in the system and NFC and NFD are the number of matchpoints (expansion terms) associated with the conductor and dielectric boundaries, respectively.

The solution of the matrix equation (2.28) to yield the generalized capacitance matrix could be obtained in much less time if there were a large number of zero elements in the $\underset{\sim}{D}$ matrix. There exist many routines which take advantage of this "sparseness" of the coefficient matrix to optimize solution of the equations.

In this chapter, two methods which result in sparse $\underset{\sim}{D}$ matrices will be discussed. One is a "near-neighbor"

48

technique in which only close wires are assumed to affect one another. The other is a "term-dropping" technique based on the relative separation of the wires. Neither of these techniques proved to yield valid results.

IV.2 The Near-Neighbor Technique

In this technique, it is assumed that the potential and displacement vector at the matchpoints on a particular wire have contributions from the charge distributions on only the wires closest to it. A term will be defined to describe this closeness. Consider the multiwire ribbon cable shown in Fig. 4.1. The term "first near-neighbors" will refer to the wires directly adjacent to the match wire, "second near-neighbors" will refer to the wires separated by one wire from the match wire, etc., as noted in Fig. 4.1.

When calculating a particular row of the $\underset{\sim}{D}$ matrix, the terms corresponding to a particular number of near-neighbors will be included; the rest will be assumed to be small and
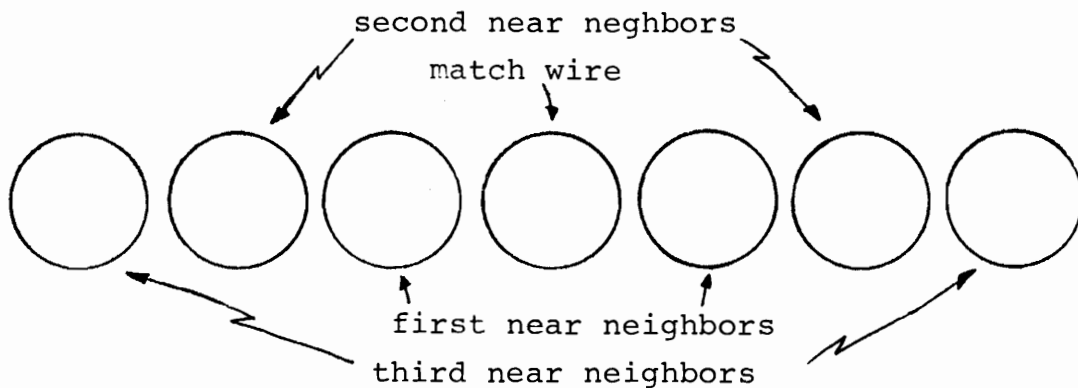
Figure 4.1. Near neighbors of a multiwire ribbon cable.
49

will be set equal to zero. The more terms that are ne-
glected, the more sparse the D matrix becomes.

The goal was to find an optimum number of near-neigh-
bors to include such that the results are valid, and yet the
D matrix is sparse enough to make solution worthwhile
through the sparse matrix techniques.

Results indicate, however, that a near-neighbor tech-
nique is not valid. Shown in Table 5 is a comparison of
several terms of the generalized capacitance matrix for the
exact (no assumptions) and the near-neighbor methods. Not
only are the magnitudes of the particular elements quite
different, but the required sign pattern of the generalized
capacitance matrix (positive diagonal elements, negative
off-diagonal elements) has been altered. Similar results
occcured for the case of ten wires with eight near-neighbors
included (only the effects of the end wires on one another
were neglected). It would seem from these results that this
method is unusable.

Careful investigation of the physical conditions of the
previous method shows that an assumption which was made is
invalid. In electromagnetics, it is valid to assume that
when two surface charge distributions are widely separated,
the distributions themselves are approximately constant
around the periphery of the charged surfaces.

Thus, assuming no effect on the potential or displace-
ment vector at a matchpoint on one wire due to the charge
distribution on another wire when the wires are widely

50

Elements of the Generalized Capacitance
Matrix for a Three-wire Ribbon Cable.

(elements $*10^{-12}$, pF/M)

Element	Control	First Near-neighbor
$C(1,1)$	36.44	-25.80
$C(1,2)$	-39.43	-22.96
$C(1,3)$	-13.73	40.20
$C(2,2)$	70.62	84.04
$C(2,3)$	-39.43	-22.96
$C(3,3)$	36.44	-25.80

Physical conditions: $r_c = 1.0$
$r_d = 2.0$
sep=4.0
$\varepsilon_r = 3.0$

Ten Fourier coefficients chosen
around each boundary.

Table 5.   Comparison of results using the
near-neighbor technique to
results from the control case
(no dropping of terms).

51

separated, as in the method above, is clearly not valid. However, assuming that this effect would be due to a constant distribution would seem more logical. This is the idea behind the second method which will be described in the next section.

IV.3 The Term-Dropping Technique

The results of the previous section indicate that the "near-neighbor" technique does not adequately describe the physical conditions of the ribbon cable. This method, then, will not be useful in reducing the computation time required to find the generalized capacitance matrix. The results did, however, point to another possible technique which will be described in this section.

The "term-dropping" technique is based on the assumption that the effects between two widely separated surface charge distributions are produced mainly by the average value of charge around the periphery of the charged surfaces. In this report, the surface charge distributions are represented by a Fourier series. Thus, the total contribution to the potential or displacement vector at a match point on the match wire should be adequately represented by a charge distribution on the source wire which has fewer number of Fourier series coefficients as the separation between the wires increases. An example of the "term-dropping" technique is illustrated in Fig. 4.2.

52
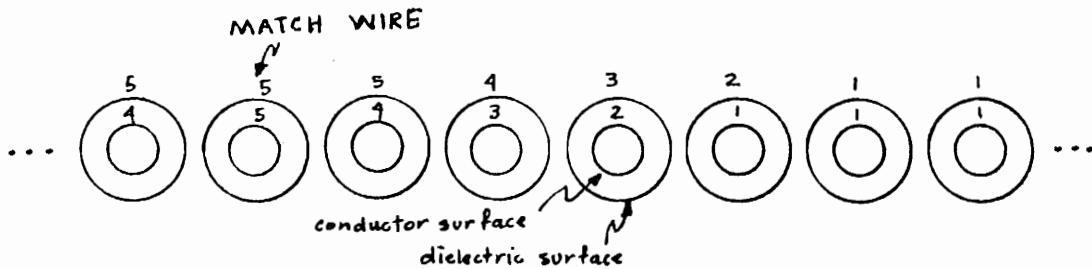
MATCH WIRE

Figure 4.2.   Multiwire flat-pack cable illustrating the
              "term-dropping" technique. (Five Fourier
              coefficients per boundary.)

The second wire from the left in Fig. 4.2 is considered
the match wire; that is, the contributions to the potential
and displacement vector at matchpoints on the surfaces of
the match wire due to all the wires are to be determined.
The numbers on the surfaces indicate the number of com-
ponents of the Fourier series representing the charge on
that surface which will be considered to effect the poten-
tial and electric field at the match wire. For example, the
number 3 will indicate that only the expansion terms 1,
$\cos\theta$, and $\cos2\theta$ are used to represent the distribution on
the particular surface; the number 6 indicates that the
terms 1, $\cos\theta$, $\cos2\theta$, $\cos3\theta$, $\cos4\theta$, and $\cos5\theta$ are retained.

A three wire flat-pack cable with conductor radius of
1.0, dielectric radius of 2.0, separation of 4.0, and
relative dielectric constant of 3.0, was analyzed using this
technique. Results from the term-dropping method are com-
pared to the control case (no dropping of terms) in Table 6.

The data in Table 6 indicate that accuracy is fairly
well maintained for all cases. In the case yielding 39.44%
sparsity (the percentage of the total elements in D which

53

Type	Distribution	Typical terms $*10^{-12}$	Sparcity & Accuracy
Control		$\ell(1,1)= 35.2894400$ $\ell(1,2)=-38.2562409$ $\ell(1,3)=-13.7449299$ $\ell(2,2)= 68.2786647$	900 elements 15 zeros 1.67% sparse
Terms less than 10^{-4} dropped		$\ell(1,1)= 35.2894800$ $\ell(1,2)=-38.2588920$ $\ell(1,3)=-13.7422123$ $\ell(2,2)= 68.2849897$	900 elements 115 zeros 12.8% sparse 4 digit accuracy
Terms less than 10^{-3} dropped		$\ell(1,1)= 35.3077153$ $\ell(1,2)=-38.3264668$ $\ell(1,3)=-13.6914884$ $\ell(2,2)= 68.4380311$	900 elements 235 zeros 26.1% sparse 2.5 digit accuracy
Terms less than 10^{-2} dropped		$\ell(1,1)= 34.7760576$ $\ell(1,2)=-38.4286168$ $\ell(1,3)=-12.9417964$ $\ell(2,2)= 69.1768575$	900 elements 355 zeros 39.4% sparse 1.5 digit accuracy

Three-wire ribbon cable.
Physical conditions:
$r_c=1.0$, $r_d=2.0$, sep=4.0, $\epsilon_r=3.0$.

Table 6.   Comparison of results from the control case (no dropping)
to results from the term-dropping technique.

54

are zero), the error in the elements of the generalized
capacitance matrix is at most 5.84%. Since this term-
dropping technique appears to work so well for three wires,
extension to more than three wires is called for. The term-
dropping technique was applied to the cases of 4, 5, and 6
wire ribbon cables, with the same wire characteristics,
dimensions and separations, as the three wire cable. The
data in Table 7A is a comparison of representative elements
of the generalized capacitance matrix which were computed
using this technique to the same elements from the control
case (no terms dropped). These elements are chosen to
indicate the relative behavior of all the elements of the
matrix.

For the case of 4 wires, the elements of $\underset{\sim}{C}$ agree to the
control case to within approximately 5% when elements of $\underset{\sim}{D}$
less than 10^{-3} are set to zero. However, for the 5 and 6
wire cables, with elements in $\underset{\sim}{D}$ less than 10^{-4} set to zero,
the error has increased to approximately 10% and 33% respec-
tively.

The error can be corrected by adding more Fourier
coefficients to the surface charge representations, while
simultaneously changing the threshold for setting elements
to zero to those less than 10^{-5}. This new threshold can be
illustrated by showing the number of Fourier series coeff-
icients representing each surface charge distribution which
will contribute to the potential and field equations on the
match wire, as in Fig. 4.3.

55

Representative Elements of the Generalized Capacitance
Matrix with 5 Fourier Coefficients per Boundary

(Elements $*10^{-12}$.)

Number of Wires	Element	No terms dropped	Terms dropped:		
			10^{-4}	10^{-3}	10^{-2}
4	$\hat{C}$ (1,4)	-8.63927	-8.62931	-8.17036	-6.99345
5	$\hat{C}$ (1,4)	-3.37309	-3.71094	-4.26695	-2.77220
	$\hat{C}$ (1,5)	-6.30418	-5.97030	-4.92757	-4.93595
6	$\hat{C}$ (1,5)	-2.41456	-3.20696	-2.98949	-.454844
	$\hat{C}$ (1,6)	-4.94645	-3.82304	-3.70573	-5.32934

Table 7A. Comparison of results from term-
dropping technique for 4, 5, and
6-wire ribbon cables with 5 Fourier
coefficients per boundary.

Representative Elements of the Generalized Capacitance
Matrix with 6 Fourier Coefficients per Boundary

(Elements $*10^{-12}$.)

Element	No terms dropped	Terms dropped:			
		10^{-5}	10^{-4}	10^{-3}	10^{-2}
$\hat{C}$ (1,5)	-2.41453	-2.41808	-2.68969	-3.25143	-2.09194
$\hat{C}$ (1,6)	-4.94634	-4.94287	-4.68013	-3.76534	-3.64053

Table 7B. Comparison of results from term-
dropping technique for a 6-wire
ribbon cable with 6 Fourier
coefficients per boundary.

MATCH
WIRE
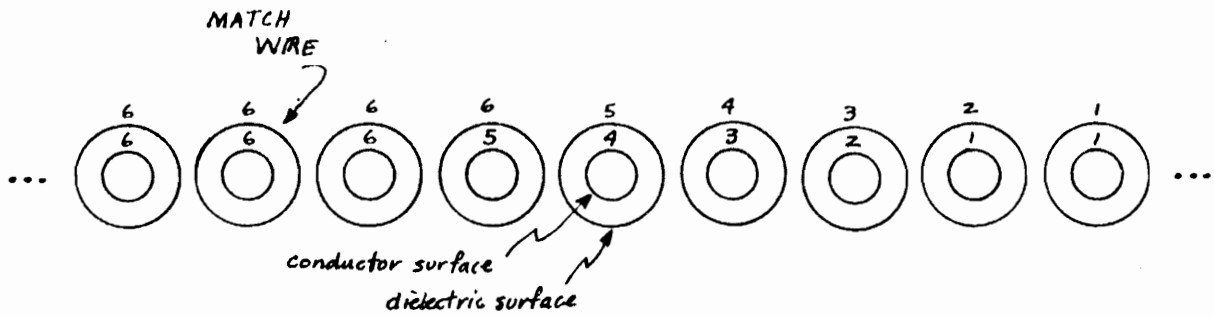
Fig. 4.3. Modified term-dropping technique with
           6 Fourier coefficients representing
           each surface charge.

These two changes were incorporated into the analysis
of the 6 wire ribbon cable. The results are shown in Table
7B. It is evident from these data that the combination of
increasing the number of Fourier coefficients used to
represent the surface charges and changing the dropping
threshold produces accurate results. But from a compu-
tational standpoint, this also increases the size of the $\underset{\sim}{D}$
matrix from a 60x60 matrix to a 72x72 matrix, while simul-
taneously reducing the sparsity of the $\underset{\sim}{D}$ matrix. Thus,
sparse matrix techniques would not be useful for these
problems.

The term-dropping technique, then, does not appear to
be particularly useful for two reasons:

(1) As the number of wires in the cable increases, the
     size of the $\underset{\sim}{D}$ matrix must be increased to allow
     the use of more Fourier coefficients in the
     representations for the surface charge distri-
     butions. This increase in size will increase

57

computation time significantly.

(2)   As the number of wires in the cable increases, the D matrix must be made less sparse to make the results accurate.

IV.4   Conclusions

The results of the previous two sections indicate that neither the near-neighbor technique nor the term-dropping technique yielded sufficiently valid results to be useful in determining the generalized capacitance matrix of ribbon cables.

An efficient method for finding the inverse of a block Toeplitz matrix would be useful, however, since the D matrix is in this form, as shown in Eq. (2.28) [14,17]. Methods for inversion of block Toeplitz matrices have been developed, but all require further constraints on the blocks. Sinnott's inversion algorithm [17] requires that the entire block Toeplitz matrix, as well as the block submatrices, be symmetric. Cramer's algorithm [14] requires that the block submatrices be scalar Toeplitz matrices. It does not appear that the requirements of either of these methods can be met with the structure of the D matrix for ribbon cables.

A method applicable to this problem yielding one-half of D^{-1} will be shown later in Chapter VI. This method will result in a time savings of approximately five-eighths over full inversion.

CHAPTER V

THE TRANSMISSION LINE CAPACITANCE AND INDUCTANCE MATRICES

V.1 Introduction

In Chapter II, a method for determining the generalized capacitance matrix of a ribbon cable with n wires was shown. Computation of crosstalk in this type of cable can be accomplished through a solution of the multiconductor transmission line equations [2]. This method utilizes the per-unit-length transmission line capacitance and inductance matrices. The transmission line capacitance matrix differs from the generalized capacitance matrix in that a particular conductor in the cable is chosen as a "reference conductor", and all cable voltages are referenced to this conductor.

In Section V.2, a technique for computing the transmission line capacitance matrix from the generalized capacitance matrix is given. In Section V.3, a formula yielding an approximate transmission line inductance matrix for ribbon cables is developed. Section V.4 deals with setting up the transmission line equations for cases involving multiple reference conductors in ribbon cables.

V.2   Determining the Transmission Line Capacitance Matrix From the Generalized Capacitance Matrix

The "generalized" capacitance matrix $\overset{\circ}{\underset{\sim}{C}}$ is defined by the matrix equation

$$
\begin{bmatrix} q_{1f} \\ q_{2f} \\ \vdots \\ q_{nf} \end{bmatrix} = \overset{\circ}{\underset{\sim}{C}} \begin{bmatrix} \emptyset_1 \\ \emptyset_2 \\ \vdots \\ \emptyset_n \end{bmatrix} \qquad (5.1)
$$

for an n-wire cable, with q_{if} indicating free charge on conductor i, and $\emptyset_i$ indicating the potential on conductor i with respect to some arbitrary reference point.

For the purpose of describing this method, the n-th conductor in the cable will be chosen as the reference for the transmission line voltages, defined by

$$
V_i = (\emptyset_i - \emptyset_n) \ , \ i = 1,2,\ldots,n-1 \ . \qquad (5.2)
$$

The transmission line capacitance matrix, $\underset{\sim}{C}$, is of order (n-1) and is defined by

$$
\begin{bmatrix} q_{1f} \\ \vdots \\ q_{(n-1)f} \end{bmatrix} = \begin{bmatrix} C_{11} & \cdots & C_{1(n-1)} \\ \vdots & & \vdots \\ C_{(n-1)1} & & C_{(n-1)(n-1)} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_{(n-1)} \end{bmatrix} \qquad (5.3)
$$

To determine the transmission line capacitance matrix, $\underset{\sim}{C}$,

60

from the generalized capacitance matrix, $\overset{c}{\underset{\sim}{}}$, we require the
per-unit-length free charge on the reference conductor to
satisfy the constraint

$$q_{nf} = - \sum_{i=1}^{n-1} q_{if} \quad .$$

(5.4)

Thus Eq. (5.1) can be written as

$$q_{1f} = c_{11}V_1 + c_{12}V_2 + \ldots + c_{1n}V_n + (\sum_{m=1}^{n} c_{1m})\emptyset_n$$

$$q_{2f} = c_{21}V_1 + c_{22}V_2 + \ldots + c_{2n}V_n + (\sum_{m=1}^{n} c_{2m})\emptyset_n$$

$$\vdots$$

$$- \sum_{i=1}^{n-1} q_{if} = c_{n1}V_1 + c_{n2}V_2 + \ldots + c_{nn}V_n + (\sum_{m=1}^{n} c_{nm})\emptyset_n$$

(5.5)

Adding all equations in Eq. (5.5) together results in

$$0 = (\sum_{m=1}^{n} c_{m1})V_1 + (\sum_{m=1}^{n} c_{m2})V_2 + \ldots + (\sum_{m=1}^{n} c_{mn})V_n$$

$$+ (\sum_{m=1}^{n} c_{1m} + \sum_{m=1}^{n} c_{2m} + \ldots + \sum_{m=1}^{n} c_{nm})\emptyset_n$$

(5.6)

Solving Eq. (5.6) for $\emptyset_n$ yields

$$\emptyset_n = - \frac{\sum_{k=1}^{n} \left[(\sum_{m=1}^{n} c_{mk})V_k \right]}{\sum_{l=1}^{n} \left[\sum_{m=1}^{n} c_{lm} \right]}$$

(5.7)

Substituting Eq. (5.7) into the first (n-1) equations in Eq. (5.5) yields the entries in the transmission line capacitance matrix as

$$[\underset{\sim}{C}]_{ij} = C_{ij} = \mathcal{C}_{ij} - \frac{\left[\sum\limits_{m=1}^{n} \mathcal{C}_{im}\right]\left[\sum\limits_{m=1}^{n} \mathcal{C}_{mj}\right]}{\sum\limits_{l=1}^{n}\left[\sum\limits_{m=1}^{n} \mathcal{C}_{lm}\right]} \tag{5.8}$$

for $i,j = 1,\ldots,(n-1)$. Eq. (5.8) can be rewritten as

$$C_{ij} = \frac{\mathcal{C}_{ij}\left\{\sum\limits_{l=1}^{n}\left[\sum\limits_{m=1}^{n} \mathcal{C}_{lm}\right]\right\} - \left\{\left[\sum\limits_{m=1}^{n} \mathcal{C}_{im}\right]\left[\sum\limits_{m=1}^{n} \mathcal{C}_{mj}\right]\right\}}{\sum\limits_{l=1}^{n}\left[\sum\limits_{m=1}^{n} \mathcal{C}_{lm}\right]} \tag{5.9}$$

Note that the denominator of Eq. (5.9) is simply the sum of all elements of the generalized capacitance matrix, $\underset{\sim}{\mathcal{C}}$. The second term of the numerator can be expanded by deleting the products with $\mathcal{C}_{ij}$ yielding for the numerator of Eq. (5.9)

$$\mathcal{C}_{ij}\left\{\sum\limits_{l=1}^{n}\left[\sum\limits_{m=1}^{n} \mathcal{C}_{lm}\right]\right\} - \mathcal{C}_{ij}\left(\sum\limits_{\substack{m=1 \\ m\neq i}}^{n} \mathcal{C}_{mj}\right) - \mathcal{C}_{ij}\left(\sum\limits_{\substack{m=1 \\ m\neq j}}^{n} \mathcal{C}_{im}\right) - \mathcal{C}^2_{ij}$$

$$-\left(\sum\limits_{\substack{m=1 \\ m\neq j}}^{n} \mathcal{C}_{im}\right)\left(\sum\limits_{\substack{m=1 \\ m\neq i}}^{n} \mathcal{C}_{mj}\right) \qquad . \tag{5.10}$$

Combining the first four terms of Eq. (5.10), the numerator of Eq. (5.9) becomes

$$c_{ij} \left\{ \sum_{l=1}^{n} \left[\sum_{m=1}^{n} c_{lm} \right] - \sum_{\substack{m=1 \\ m \neq i}}^{n} c_{mj} - \sum_{\substack{m=1 \\ m \neq j}}^{n} c_{im} - c_{ij} \right\} - \left(\sum_{\substack{m=1 \\ m \neq j}}^{n} c_{im} \right) \left(\sum_{\substack{m=1 \\ m \neq i}}^{n} c_{mj} \right)$$

(5.11)

or

$$c_{ij} \left\{ \sum_{\substack{l=1 \\ l \neq i}}^{n} \left[\sum_{\substack{m=1 \\ m \neq j}}^{n} c_{lm} \right] \right\} - \left(\sum_{\substack{m=1 \\ m \neq j}}^{n} c_{im} \right) \left(\sum_{\substack{m=1 \\ m \neq i}}^{n} c_{mj} \right) \quad . \tag{5.12}$$

The first term in Eq. (5.12) is the term c_{ij} multiplied by the sum of all the terms in $\underset{\sim}{c}$ except for those in the i-th row and j-th column. The second term in Eq. (5.12) is the product of the sum of the elements in the i-th row of $\underset{\sim}{c}$ (except c_{ij}) with the sum of the elements in the j-th column of $\underset{\sim}{c}$ (except c_{ij}).

The above results yield a simple procedure for determining the transmission line capacitance matrix from the generalized capacitance matrix with the n-th conductor chosen as the reference conductor for the transmission line voltages.

Rewriting Eq. (5.9) by inserting the numerator of Eq. (5.12) yields

63

$$C_{ij} = \frac{\mathfrak{c}_{ij}\left\{\sum\limits_{\substack{l=1\\l\neq i}}^{n}\left[\sum\limits_{\substack{m=1\\m\neq j}}^{n}\mathfrak{c}_{lm}\right]\right\} - \left(\sum\limits_{\substack{m=1\\m\neq j}}^{n}\mathfrak{c}_{im}\right)\left(\sum\limits_{\substack{m=1\\m\neq i}}^{n}\mathfrak{c}_{mj}\right)}{\sum\limits_{l=1}^{n}\left[\sum\limits_{m=1}^{n}\mathfrak{c}_{lm}\right]} \quad . \quad (5.13)$$

for $i,j = 1,\ldots,(n-1)$. Thus, given the generalized capacitance matrix $\underset{\sim}{\mathfrak{c}}$, as shown in Eq. (5.1), to find the term C_{ij} of the transmission line capacitance matrix $\underset{\sim}{C}$, in which the n-th conductor is chosen as reference for the transmission line voltages,

1) multiply $\mathfrak{c}_{ij}$ by the sum of the terms of the matrix $\underset{\sim}{\mathfrak{c}}$ with the i-th row and j-th column deleted,

2) subtract the product of the sum of all terms of the i-th row (except $\mathfrak{c}_{ij}$) with the sum of the terms of the j-th column (except $\mathfrak{c}_{ij}$), and

3) divide by the sum of all terms in the generalized capacitance matrix.

$$\overset{\displaystyle\text{j-th col.}}{\begin{bmatrix} \mathfrak{c}_{11} & \mathfrak{c}_{12} & \cdots & \mathfrak{c}_{ij} & \cdots & \mathfrak{c}_{1n} \\ \mathfrak{c}_{21} & \mathfrak{c}_{22} & \cdots & \mathfrak{c}_{2j} & \cdots & \mathfrak{c}_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathfrak{c}_{i1} & \mathfrak{c}_{i2} & & \mathfrak{c}_{ij} & & \mathfrak{c}_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathfrak{c}_{n1} & \mathfrak{c}_{n2} & \cdots & \mathfrak{c}_{nj} & \cdots & \mathfrak{c}_{nn} \end{bmatrix}} \quad (5.14)$$

i-th row $\longrightarrow$

The above method should be easy to do by hand especially for a small number of wires. For computer implementation, however, the form of Eq. (5.8) is used as shown below:

$$C_{ij} = \mathcal{C}_{ij} - \frac{\left(\sum\limits_{m=1}^{n} \mathcal{C}_{im}\right)\left(\sum\limits_{m=1}^{n} \mathcal{C}_{mj}\right)}{\sum\limits_{l=1}^{n}\left[\sum\limits_{m=1}^{n} \mathcal{C}_{lm}\right]} \qquad (5.15)$$

for $i,j = 1,\ldots,(n-1)$. The procedure is as follows:

1) Sum the elements in the n-th row of $\mathcal{C}$. This sum will be used to find the sum of the elements in $\tilde{\mathcal{C}}$.
2) For $i=1,\ldots,(n-1)$
   a) Sum the elements in the i-th row of $\mathcal{C}$. Add to the matrix sum. Insert this sum into the elements of the i-th row of the $\tilde{C}$ matrix.
   b) Sum the elements in the i-th column of $\mathcal{C}$. Store these sums.
3) For $i=1,\ldots,(n-1)$
   $j=1,\ldots,(n-1)$
   a) Multiply C_{ij} by the sum of column j, divide by the matrix sum, then subtract from $\mathcal{C}_{ij}$ and insert in C_{ij}.

This procedure was developed for obtaining the per-unit-length transmission line capacitance matrix for an n-wire cable with the n-th wire chosen as reference for the transmission line voltages. Clearly, this is readily adaptable to the case where any other wire in the cable is chosen as a reference for the transmission line voltages. To show this, it is sufficient to note that the rows and columns of the generalized capacitance matrix may be exchanged to obtain a representation with the per-unit-length free charge on the reference conductor, q_{rf}, as the last entry in the vector q_f and the potential of the reference conductor, ϕ_r,

65

as the last entry in the vector $\underline{\emptyset}$. The particular ordering
and structure of $\underset{\sim}{C}$ with respect to the conductor numbering
system used in the computer program will be discussed in
Section VII.4.

V.3 The Per-Unit-Length Transmission Line Inductance Matrix for Ribbon Cables

In order to solve the transmission line equations for a
cable system, the per-unit-length transmission line capaci-
tance and inductance matrices are needed. In the previous
section a method was developed yielding the transmission
line capacitance matrix from the generalized capacitance
matrix computed by the method described in Charter II. The
per-unit-length transmission line inductance matrix is a
function of the wire orientation and is independent of the
characteristics of the dielectric insulation materials.
This is true since the permeability of dielectrics is
typically that of free space. Thus, the inductance matrix
for a system of insulated wires is the same as that for a
system of uninsulated wires with an identical wire orien-
tation.

The per-unit-length transmission line inductance matrix
for a system of uninsulated conductors must obey the equa-
tion [15]

$$\underset{\sim}{L} = \mu_v \varepsilon_v \underset{\sim}{C}_0^{-1} \quad , \tag{5.16}$$

where μ_v is the permeability of free space, ε_v is the

permittivity of free space, and C_0^{-1} is the inverse of the per-unit-length transmission line capacitance matrix with the dielectrics removed.

Therefore, given the transmission line capacitance matrix for the ribbon cable with the dielectric insulation ignored, the transmission line inductance matrix can be easily found.

For most ribbon cables, the ratio of conductor radius to conductor separation usually is greater than 4. (If the insulation thickness is equal to the wire radius and the insulations were touching, the ratio would equal 4.) In many instances it may be sufficient to find an approximate form of the transmission line inductance matrix instead of recomputing the transmission line capacitance matrix with the dielectrics removed.

In developing this approximation, consider the case of n wires, in which the n-th wire is chosen as reference. Fig. 5.1 illustrates conditions on two typical conductors, i
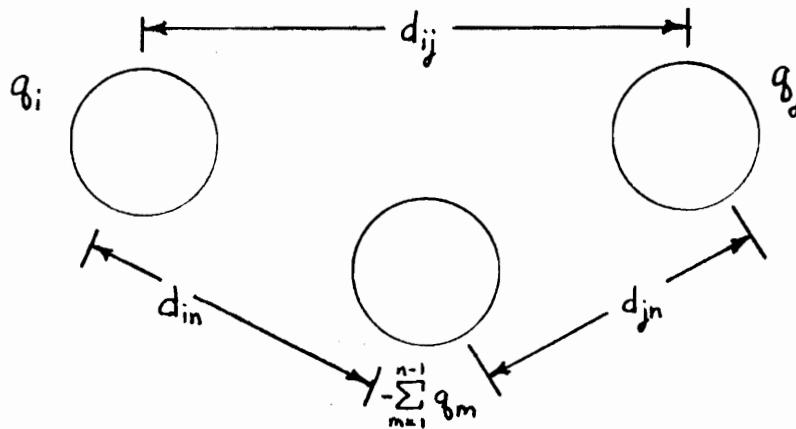
Figure 5.1. Two typical conductors and the reference conductor.

67

and j, as well as on the reference conductor, labeled n. The per-unit-length charge on conductor i (which for this uninsulated case will be entirely free charge) is q_i, d_{ij} is the distance between the centers of wires i and j, and V_i is the potential of conductor i with respect to the reference. The relationship between the conductor potentials, V, and the charge on the conductors, q, is represented by

$$
\begin{bmatrix} V_1 \\ \vdots \\ V_{(n-1)} \end{bmatrix} = \underbrace{\begin{bmatrix} C_{11} & \cdots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{(n-1)1} & \cdots & C_{(n-1)(n-1)} \end{bmatrix}^{-1}}_{\underset{\sim}{C}_0} \begin{bmatrix} q_1 \\ \vdots \\ q_{(n-1)} \end{bmatrix}
$$

(5.17)

Individual elements of $\underset{\sim}{C}_0^{-1}$ can be found from the conductor potentials and per-unit-length charges by the equation

$$
[\underset{\sim}{C}_0]_{ij}^{-1} = \frac{V_i}{q_j}\Big|_{q_m=0} \qquad \begin{matrix} m = 1, \ldots, (n-1) \\ m \neq j \end{matrix}
$$

(5.18)

The generalized capacitance matrix for this case can be found by using the results of Chapter II. Since the conductors are widely spaced, the match points can be chosen at the center of the conductors with constant charge distributions assumed on the conductors. However, when deter-

68

mining the potential of a wire due to its own charge dis-
tribution, the matchpoint will be chosen on the surface of
the conductor. Then, from equations (2.12) and (2.14),
there result n equations relating the per-unit-length charge
distributions, σ_{i0}, and potentials, $\emptyset_i$, in the form

$$
\begin{bmatrix} \emptyset_1 \\ \emptyset_2 \\ \vdots \\ \emptyset_n \end{bmatrix} = \frac{-1}{\varepsilon_v} \begin{bmatrix} r_{c1}\ln(r_{c1}) & r_{c2}\ln(d_{21}) & r_{c3}\ln(d_{31}) & \cdots \\ r_{c1}\ln(d_{21}) & r_{c2}\ln(r_{c2}) & r_{c3}\ln(d_{32}) & \cdots \\ \vdots & & & \\ r_{c1}\ln(d_{n1}) & & \cdots & \end{bmatrix} \begin{bmatrix} \sigma_{10} \\ \sigma_{20} \\ \vdots \\ \sigma_{n0} \end{bmatrix} .
$$

$$(5.19)$$

The potential of any conductor can then be written as a
function of the per-unit-length charges q_i. For example,
the first equation in (5.19) can be written as

$$
\emptyset_1 = \frac{-1}{2\pi\varepsilon_v} \left\{ 2\pi r_{c1}\sigma_{10}\ln(r_{c1}) + 2\pi r_{c2}\sigma_{20}\ln(d_{21}) + \cdots \right\}
$$

$$(5.20)$$

where $q_i = 2\pi r_{ci}\sigma_{i0}$ for $i = 1,2,\ldots,n$. Then, equation
(5.19) becomes:

$$
\begin{bmatrix} \emptyset_1 \\ \emptyset_2 \\ \vdots \\ \emptyset_n \end{bmatrix} = \frac{-1}{2\pi\varepsilon_v} \begin{bmatrix} \ln(r_{c1}) & \ln(d_{21}) & \cdots \\ \ln(d_{21}) & \ln(r_{c2}) & \cdots \\ \vdots & \vdots & \\ \ln(d_{n1}) & \ln(d_{n2}) & \cdots \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}
$$

$$(5.21)$$

Now, conductor n can be chosen as reference and the poten-
tial, V_i, between conductor i and the reference conductor
defined as

$$V_i = \emptyset_i - \emptyset_n \quad . \tag{5.22}$$

According to equation (5.21),

$$\emptyset_n = \frac{-1}{2\pi\varepsilon_v}\left\{ \ln(d_{n1})q_1 + \ldots + \ln(d_{ni})q_i + \ldots + \ln(d_{nj})q_j \right.$$
$$\left. + \ldots + \ln(r_{cn})q_n \right\} \tag{5.23a}$$

and

$$\emptyset_i = \frac{-1}{2\pi\varepsilon_v}\left\{ \ln(d_{i1})q_1 + \ldots + \ln(r_{ci})q_i + \ldots + \ln(d_{ij})q_j \right.$$
$$\left. + \ldots + \ln(d_{ni})q_n \right\} \quad . \tag{5.23b}$$

Then,

$$V_i = \emptyset_i - \emptyset_n$$
$$= \frac{-1}{2\pi\varepsilon_v}\left\{ \ln\left(\frac{d_{i1}}{d_{n1}}\right)q_1 + \ldots + \ln\left(\frac{r_{ci}}{d_{ni}}\right)q_i + \ldots + \ln\left(\frac{d_{ij}}{d_{nj}}\right)q_j + \right.$$
$$\left. \ldots + \ln\left(\frac{d_{ni}}{r_{cn}}\right)q_n \right\} \quad . \tag{5.24}$$

The requirement that the total charge on the reference conductor (conductor n) is minus the sum of the total charges on all the other conductors can again be invoked; that is,

$$q_n = -\sum_{i=1}^{n-1} q_i \quad . \tag{5.25}$$

Rewriting (5.24) by including (5.25), yields

$$V_i = \frac{-1}{2\pi\varepsilon_v}\left\{ \ldots + \ln\left(\frac{r_{cn}r_{ci}}{(d_{ni})^2}\right)q_i + \ldots + \ln\left(\frac{r_{cn}d_{ij}}{d_{ni}d_{nj}}\right)q_j + \ldots \right\}$$
$$\tag{5.26}$$

70

Then, from equations (5.18) and (5.26), individual elements of C_0^{-1} are

$$\left[C_0^{-1}\right]_{ij} = \begin{cases} \dfrac{1}{2\pi\epsilon_v}\ln\left[\dfrac{(d_{ni})^2}{r_{cn}r_{ci}}\right] & \text{for } i=j \\[2em] \dfrac{1}{2\pi\epsilon_v}\ln\left[\dfrac{d_{ni}d_{nj}}{r_{cn}d_{ij}}\right] & \text{for } i\neq j \end{cases} \qquad (5.27)$$

and $i,j = 1,2,\ldots,(n-1)$.

Finally, combining equations (5.16) and (5.27), the elements of an approximation to the per-unit-length transmission-line inductance matrix with conductor n chosen as reference are

$$[L]_{ij} = \begin{cases} \dfrac{\mu_v}{2\pi}\ln\left[\dfrac{(d_{ni})^2}{r_{cn}r_{ci}}\right] & \text{for } i=j \\[2em] \dfrac{\mu_v}{2\pi}\ln\left[\dfrac{d_{ni}d_{nj}}{r_{cn}d_{ij}}\right] & \text{for } i\neq j \end{cases} \qquad . \qquad (5.28)$$

V.4 Applications to the Analysis of Multiple Reference Conductor Systems

Computer and other digital system interconnections utilize a wire assignment in ribbon cables where many wires are reference conductors. The most common method is the ground-signal-ground technique, where the end wires and every other wire is a reference conductor, so that the signal wires are separated from each other by a reference conductor.

The transmission line equations can be used to solve

71

the multiple reference conductor problem by obtaining $\underset{\sim}{C}$ and
$\underset{\sim}{L}$ for the cable with one of the grounded conductors chosen
as reference. Then, the terminal impedance matrices, $\underset{\sim}{Z}$, can
be written by considering grounded wires as zero terminal
impedances [2, 13]. A typical example is illustrated in
Fig. 5.2.
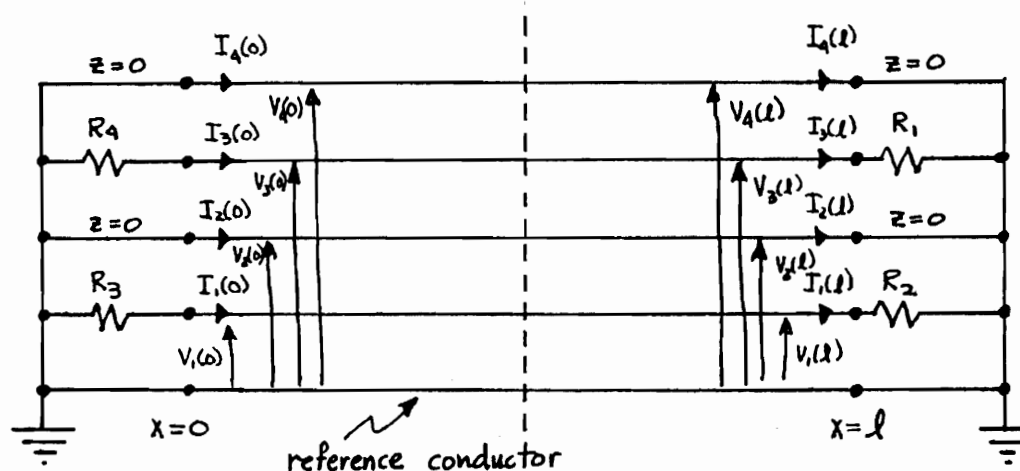
Figure 5.2 Five-wire, ground-signal-ground cable.

The equations defining the voltages at the ends of the
lines are [2]

$$\underline{V}(0) = \underline{E}_0 - \underset{\sim}{Z}(0)\underline{I}(0)$$

$$\underline{V}(1) = \underline{E}_1 + \underset{\sim}{Z}(1)\underline{I}(1) \qquad . \qquad (5.29)$$

where $\underline{E}_0$ and $\underline{E}_1$ are (n-1)x1 vectors of the equivalent open
circuit port excitations, and $\underline{I}(0)$ and $\underline{I}(1)$ are (n-1)x1
vectors of current in the lines directed in the direction of
increasing x. Writing the terminal impedance matrices, $\underset{\sim}{Z}(0)$
and $\underset{\sim}{Z}(1)$, at x=0 and x=1 respectively, for the 5 wire cable

72

in Fig. 5.2 yields [2, 13]:

$$Z(0) = \begin{bmatrix} R_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & R_4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Z(1) = \begin{bmatrix} R_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & R_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad . \tag{5.30}$$

The transmission line equations can then be solved in the normal fashion as described in [2, 13, 19].

CHAPTER VI

DESCRIPTION OF THE COMPUTER PROGRAM

VI.1   Introduction

The main purpose of this work was to implement on a
digital computer the techniques of Chapter II to solve for
the per-unit-length generalized capacitance matrix for a
multi-conductor ribbon cable.   Then, utilizing the results
of Chapter V, to solve for the per-unit-length transmission
line capacitance matrix needed for the analysis of crosstalk
in ribbon cable systems.   A listing of the program which
will be described is contained in Appendix B.

VI.2   Computer Program GETCAP

GETCAP (which is an acronym for GEneralized and Trans-
mission line CAPacitance matrices) is a FORTRAN computer
program which utilizes all the results of this report to
find the per-unit-length generalized and transmission line
capacitance matrices for ribbon cables.   The GETCAP main
computer program uses a subroutine which is also called
GETCAP.   This GETCAP subroutine may be used alone in cases
where it will be part of a larger program designed for
specific purposes, or with its controlling mainprogram which
provides a very user-oriented method of determining the
capacitance matrices.

The problem to be solved is described by the following input variables:

NW   —   The number of wires in the cable.

RC   —   The radii of the conductors.

RD   —   The radii of the outer dielectric insulation surfaces.

CCSEP —   The center-to-center separation of any two adjacent conductors.

ER   —   The relative dielectric constant of the insulation material (relative to free space).

NFC   —   The number of Fourier series terms to be used to represent the charge distributions around the conductor surfaces. (Note: The charge distribution is assumed to be of the form

$$\sigma_i(\theta) = \sum_{m=0}^{(NFC-1)} \sigma_{im} \cos m\theta$$

for i=1,...,NW.)

NFD   —   The number of Fourier series terms to be used to represent the charge distributions around the dielectric surfaces. (Note: The charge distribution is assumed to be of the form

$$\sigma_i'(\theta) = \sum_{m=0}^{(NFD-1)} \sigma_{im}' \cos m\theta$$

for i=1,...,NW.)

NF   —   The total number of Fourier coefficients per wire; (NF = NFC+NFD).

IREF   —   The reference conductor for the transmission

75

line voltages; $(1 \leq \text{IREF} \leq \text{NW})$.

IOPT - A program option selector.
The cable dimensions, RC, RD, and CCSEP can appear in any choice of units (mm., meters, mils., etc.), so long as all of the dimensions are in the same units.

Subroutine GETCAP recognizes three possible values for IOPT.

IOPT = 0 - Matrix partitioning is used to invert the $\underset{\sim}{D}$ matrix.

IOPT = 1 - Standard full inversion of the $\underset{\sim}{D}$ matrix.

IOPT = 2 - The dielectrics are removed. The program assumes a bare wire cable.

In addition, the GETCAP mainprogram recognizes options 10, 11, and 12. The second digit corresponds to options 0, 1, and 2 above. The first digit, 1, provides a copy of the upper triangle of the transmission line capacitance matrix in punched card format being generated for later use.

At this point, a discussion of the partitioning method which is selected with IOPT = 0 (or 10) is necessary. As was shown in Chapter III, the generalized capacitance matrix for ribbon cables is ideally symmetric with respect to both the main diagonal and the cross-diagonal. This means that only a few terms of the entire matrix must be found, as shown in Eq. (3.8). Further investigation shows that these terms can be developed from the elements of half of the inverse of the $\underset{\sim}{D}$ matrix.

The elements of $\underset{\sim}{C}$ are developed from the inverse of the

76

D matrix according to Eq. (2.43)

$$\dot{C}_{ij} = 2\pi \, r_c \left[\sum_{m=1}^{n} T_{ij}^{1m} + r_d \sum_{m=1}^{n} T_{i'j}^{1m} \right] \quad , \quad (6.1)$$

where $T=D^{-1}$. The symmetry of $\dot{C}$ with respect to both the main and cross-diagonals will allow those elements which completely specify $\dot{C}$ to be developed only once. Also, these elements will be chosen so that only the submatrices on the left half of T will be used.

Noble [12] describes a technique for finding the inverse of a square, nonsingular matrix D partitioned such that

$$D = \begin{bmatrix} P & Q \\ R & S \end{bmatrix} \qquad (6.2)$$

where P and S are square matrices. The inverse of D (if D and S are non-singular) is

$$D^{-1} = \begin{bmatrix} X & -XQS^{-1} \\ -S^{-1}RX & W \end{bmatrix}$$

where $X = (P-QS^{-1}R)^{-1}$ and $W = S^{-1} + S^{-1}RXQS^{-1}$ (6.3)

Partitioning the matrix D of Eq.(6.2) in halves results in the minimal amount of time required to solve for part of the inverse of D by the method in (6.3). This can be verified by counting the number of operations (multiplications and divisions) required to obtain the form of D^{-1} in Eq. (6.3). The number of operations required to multiply an nxk matrix

77

by a kxp matrix is npk.

Partitioning D in Eq. (6.2) in halves, the number of operations to solve for the required portions of D^{-1} as described above is approximately 5/8 of the number of operations required by a direct full inversion of D. To show this, consider D of order 2n such that

$$D = \begin{bmatrix} P & Q \\ R & S \end{bmatrix} \qquad , \qquad (6.4)$$

where P, Q, R, and S are n x n matrices. To solve for D^{-1} directly will result in approximately $8n^3$ operations. If the inverse of D is written as

$$D^{-1} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \qquad , \qquad (6.5)$$

then solving for α and γ will require $5n^3$ operations (remembering that n^3 operations are required to invert an nxn matrix and n^3 operations are also required to multiply two nxn matrices).

VI.3 Operational Details of the GETCAP Program

The entire GETCAP program consists of five program units:

MAIN   – The main program for inputting data and
          controlling output of results.

GETCAP – The subroutine which performs the actual
          computation of the capacitance matrices from
          the input data.

78

MINV - A matrix inversion subroutine from the IBM Scientific Subroutine Package (SSP) [16].

MPC   - A subroutine which multiplies two general matrices, then multiplies the resulting matrix by a constant.

MPRT - A subroutine which outputs a general matrix to the printer in matrix format with labeling.

Each of these program units will be described separately in this section, with the greatest emphasis on the GETCAP subroutine.

The main program was written so as to be used by persons who are not computer-specialists. Its operational characteristics are illustrated by the flowchart in Fig. 6.1. The main program is an executive over the GETCAP subroutine; it also checks the input data for obvious errors. The main program also provides the matrix and array storage areas used in the computations by subroutine GETCAP. The user must ensure that the array dimensions are large enough to deal with the problems to be considered. Directions for this dimensioning are given at the beginning of the listing of the main program. Assuming a maximum number of wires, NW, and a maximum number of total Fourier series expansion terms per wire, NF=(NFC+NFD), for all problems to be considered, the dimensions of the matrices required in subroutine GETCAP will be as follows:

$$C(NW^2)$$
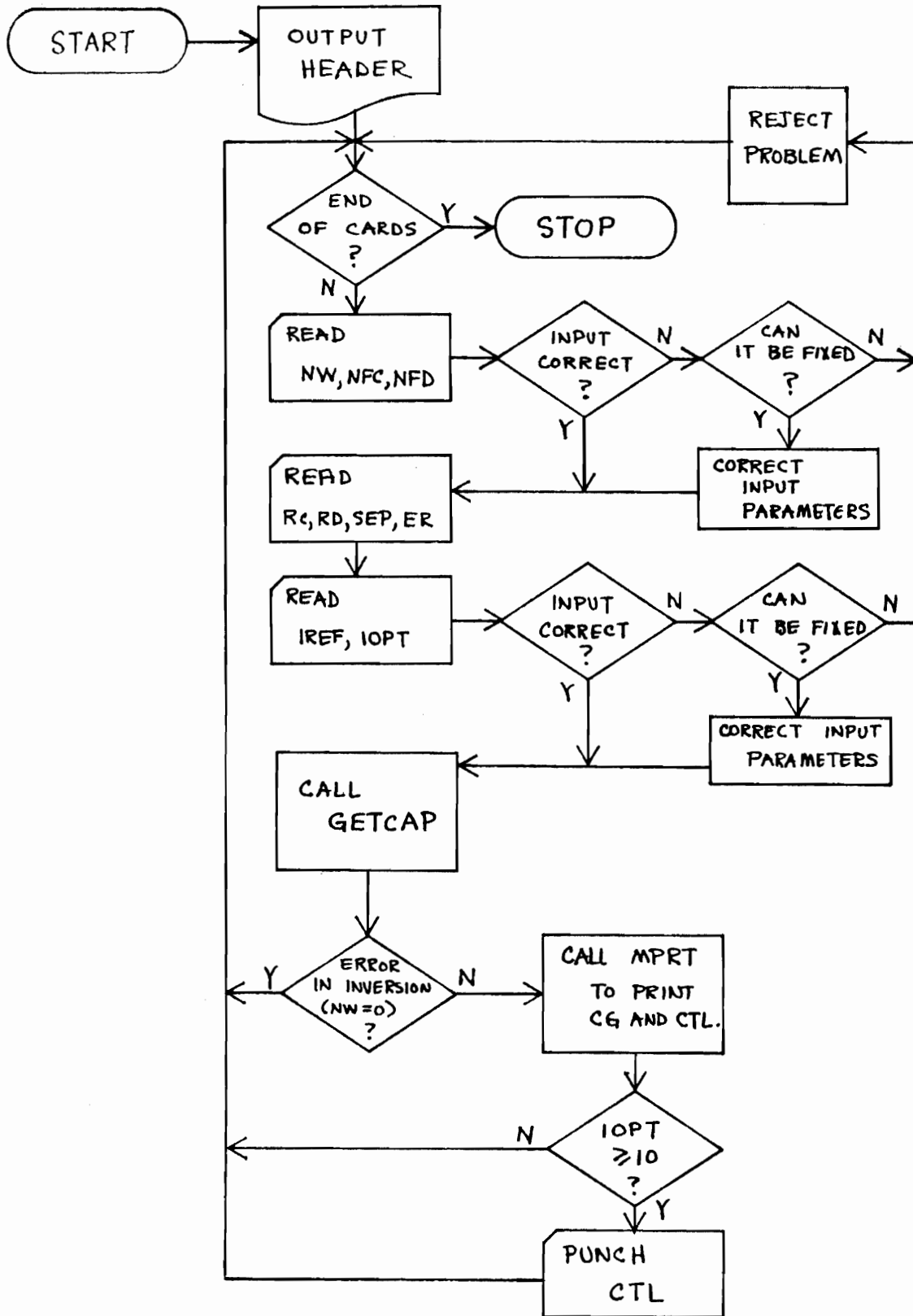
Figure 6.1.   Flowchart of MAIN program.

$$C1((NW-1)^2)$$

$$D(NF^2)$$

$$D1((NW\ NF)^2)$$

$$LT(2\ NF\ NW)$$

$$SCR((NF\ NW\ +\ 1)/2)\quad.$$

Note that these items are stored in vector format. This dimensioning will allow all problems up to the maximum specified to be considered. When the GETCAP subroutine returns to the main program, a printed output of the generalized and transmission line capacitance matrices is generated by calling subroutine MPRT.

Subroutine GETCAP is the computational part of the GETCAP program. All input data, output data, and working vectors are passed through the argument list as shown below.

SUBROUTINE GETCAP (NW,NFC,NFD,NF,RC,RD,CCSEP,ER,IREF,
                   IOPT,CG,CTL,D,D1,SCR,LT)

The first ten arguments are the input variables described earlier. CG and CTL are the resulting per-unit-length generalized and transmission-line capacitance matrices, respectively. The last four arguments are working vectors used by GETCAP. D1 is matrix $\underset{\sim}{D}$ in Eq. (2.28) stored in vector format.

Subroutine GETCAP operates as shown in the flowchart of Fig. 6.2. The first step is a scaling of the input wire dimensions to prevent numerical underflow and overflow by the computer in generating the inverse of the D1 matrix. Equations (2.17) and (2.18) are then used to compute the

81
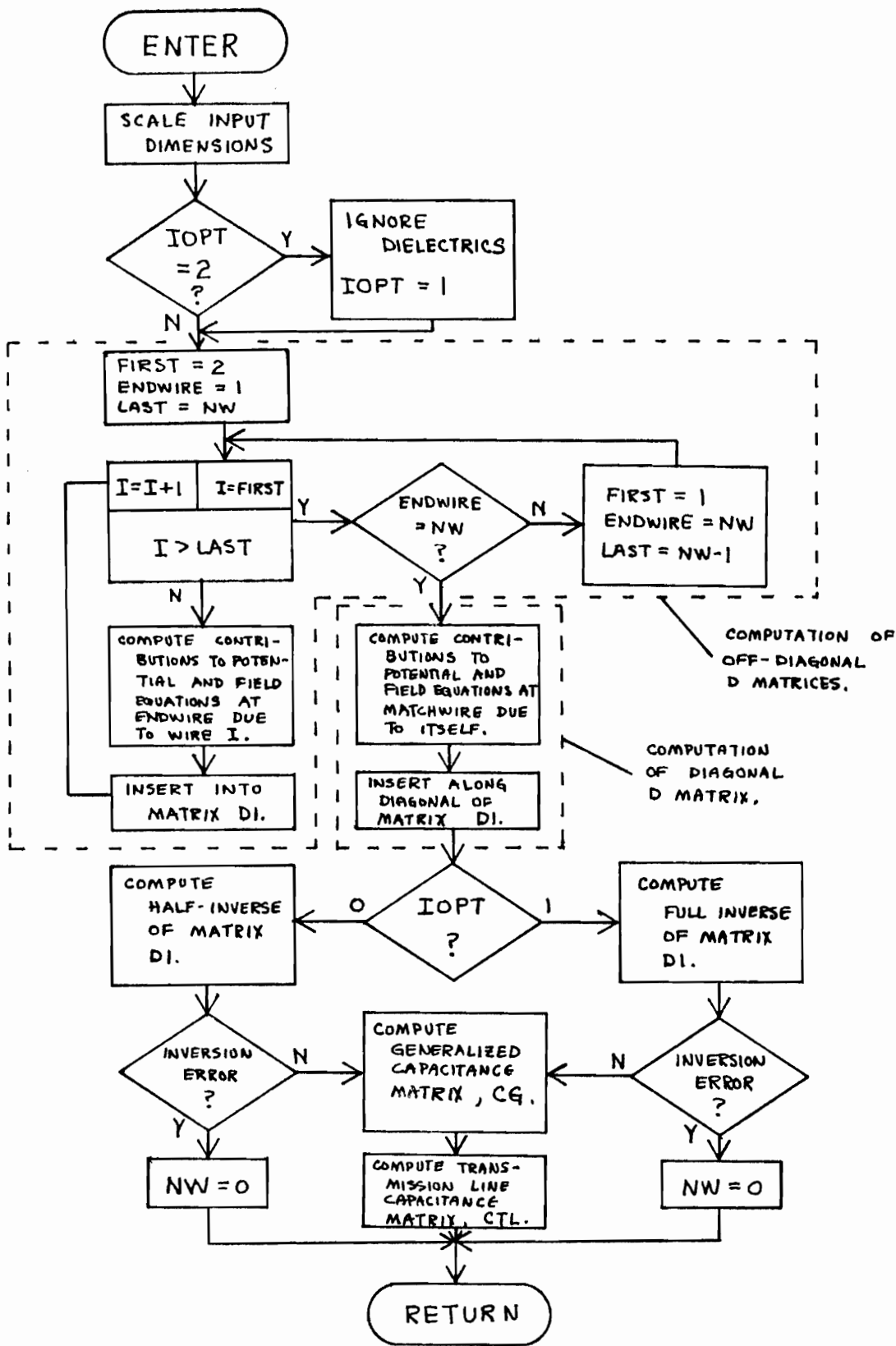
Figure 6.2. Basic Flowchart of Subroutine GETCAP.

82

off-diagonal submatrices $D_{\sim n}$, for n = ±1,..., ±(n-1), shown in Eq. (2.28). These submatrices are then inserted into the large matrix D1 according to the relationship of Eq. (2.28).

Equations (2.17), (2.18), (2.19), and (2.20) are then used to generate the diagonal block submatrix $D_{\sim 0}$ of Eq. (2.28). $D_{\sim 0}$ is then inserted in the diagonal blocks of the D1 matrix. Computer matrix D is used to hold each of the block submatrices $D_{\sim n}$, n=0,±1,...,±(n-1) for insertion into D1.

It should be noted that when IOPT = 0 (or 10), the $D_{\sim}$ matrix in Eq. (6.1) is stored in the computer in the vector D1, such that the first $(NFxNW/2)^2$ locations correspond to matrix $P_{\sim}$, the next $(NFxNW/2)^2$ locations correspond to matrix $Q_{\sim}$, followed by $(NFxNW/2)^2$ locations for $R_{\sim}$, and then $(NFxNW/2)^2$ locations for $S_{\sim}$. In this case, the submatrices in $D_{\sim}$ are inserted into the D1 matrix to take into account this unusual storage order.

When IOPT=2 (or 12), the dielectrics are ignored. Thus, equations (2.18) and (2.20) for the electric field vector at a point are not used in generating the $D_{\sim}$ submatrices. Also, since the dielectric surfaces are ignored, the contributions to the conductor potentials due to the charges on these surfaces are ignored.

Following the generation of matrix D1, it is inverted according to the method designated by IOPT. If IOPT=1 (or 11), subroutine MINV inverts the entire D1 matrix. If IOPT=0 (or 10), the method described by Eq. (6.3) is used to

83

compute half of the inverse of D1.

The matrix $\underset{\sim}{D}$ represented by Eq. (6.2) is stored in D1
and inverted according to the following procedure:

1)   Invert matrix $\underset{\sim}{S}$ using MINV. $\underset{\sim}{S}$ is replaced
           by its inverse.

2)   Multiply $\underset{\sim}{S}^{-1}$ $\underset{\sim}{R}$, and insert the result in $\underset{\sim}{S}$
           using MPC.

3)   Multiply $\underset{\sim}{Q}$ by $\underset{\sim}{S}^{-1}$ using MPC and insert the
           result ($\underset{\sim}{Q}\ \underset{\sim}{S}^{-1}\ \underset{\sim}{R}$) in $\underset{\sim}{Q}$.

4)   Subtract $\underset{\sim}{Q}\underset{\sim}{S}^{-1}\underset{\sim}{R}$ from $\underset{\sim}{P}$ and insert the result
           ($\underset{\sim}{P}-\underset{\sim}{Q}\underset{\sim}{S}^{-1}\underset{\sim}{R}$) into $\underset{\sim}{P}$.

5)   Invert ($\underset{\sim}{P}-\underset{\sim}{Q}\underset{\sim}{S}^{-1}\underset{\sim}{R}$) using MINV. Store the result
           in $\underset{\sim}{P}$.

6)   Multiply $\underset{\sim}{S}^{-1}\underset{\sim}{R}$ by $(\underset{\sim}{P}-\underset{\sim}{Q}\underset{\sim}{S}^{-1}\underset{\sim}{R})^{-1}$ using MPC, then
           multiply the result by -1 and store in $\underset{\sim}{R}$.

Half of the inverse of D1 then lies in partitions $\underset{\sim}{P}$ and $\underset{\sim}{R}$.
This method takes slightly over half the time required to
invert the entire D1 matrix.

Once the D1 matrix is inverted, the elements of the
generalized capacitance matrix, $\underset{\sim}{C}$, are computed from the
elements of the inverse of D1 by Eq. (2.43) and stored in
the computer as matrix CG. When IOPT=1 (or 11), all the
elements of $\underset{\sim}{C}$ are computed. For IOPT=0 (or 10), the sym-
metry conditions as described in Section III.3 are utilized
and only a portion of the elements of $\underset{\sim}{C}$ are computed, as
indicated by Eq. (3.8). The rest of the elements are
assumed to be duplicates of those computed.

The transmission line capacitance matrix, $\underset{\sim}{C}$, is then
computed from the generalized capacitance matrix by fol-
lowing the algorithm in Section V.2, and the elements of $\underset{\sim}{C}$

are stored in the computer as matrix CTL. Subroutine GETCAP then returns to the main program with both the per-unit-length generalized capacitance matrix, CG, and the per-unit-length transmission line capacitance matrix, CTL.

Subroutine MINV is a standard matrix inversion routine included in the IBM Scientific Subroutine Package [16]. Its argument list is shown below:

SUBROUTINE MINV(A, N, D, L, M)

The arguments have the following meaning:

    A - An input matrix which is destroyed in computation
        and replaced by the inverse of A.

    N - The order of matrix A.

    D - The resultant determinant of A.

    L - A working vector of length N.

    M - A working vector of length N.

Subroutine MPC is used to find the product of two general matrices and to multiply the resulting matrix product by a constant. The argument list for MPC is shown below.

SUBROUTINE MPC (A, B, R, S, L, M, N, C)

These arguments have the following significance:

    A - The first matrix, dimensioned L by M.

    B - The second matrix, dimensioned M by N.

    R - The resulting matrix, dimensioned L by N.

    S - A scratch vector, length L.

    L - The number of rows in A and R.

    M - The number of columns in A (number of rows in
        B).

85

N - The number of columns in B and R.

C - The constant by which R is multiplied.

The multiplication takes the form:

$$R = CAB \quad .$$

The subroutine was written so that neither A nor B are destroyed by the computation. In addition, if the dimensions of matrix R are large enough (N greater than or equal to M), the use of scratch vector S provides that A and R can share the same storage locations. This is useful when two large matrices are to be multipled, and the first one is no longer needed after the multiplication, thus saving the amount of storage needed by the matrices. In the GETCAP program, the large size of the D1 matrix makes this savings considerable. The flowchart of Fig. 6.3 illustrates operation of the MPC subroutine.

Subroutine MPRT is used for printing a matrix in matrix format. Elements are printed in scientific notation rounded to three digits. The rows and columns are numbered on the printout to provide easy identification of any element.

The argument list of MPRT is shown below:

SUBROUTINE MPRT (A, M, N, B, J)

where the arguments have the following meaning:

A - The matrix to be printed.

M - The number of rows in A.

N - The number of columns in A.

B - The title of the matrix in Hollerith format.

86
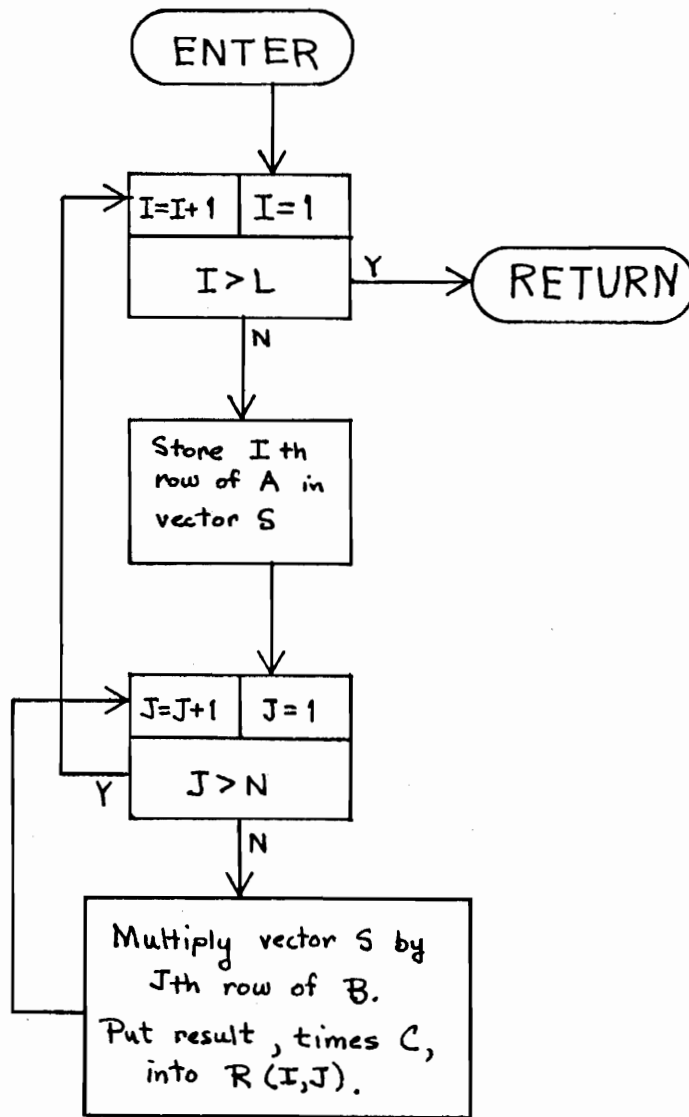
Figure 6.3.   Flowchart of Subroutine MPC.

J - The number of characters in the title.

The title, given in B, is printed on the header of the first page of the printout. The first twelve columns are then printed on page two, and so forth, until the entire matrix has been printed. The printout can then be separated, and page two connected to the right of page one, page three to the right of page two, etc., so that the printed matrix can be viewed in its normal matrix form. An example of a general matrix printed with MPRT is given in Fig. 6.4. The calling statement was

CALL MPRT(A,6,7,'TYPICAL OUTPUT OF A',19)

with the contents of A stored as shown in the printout.

The listing of the program found in Appendix B includes comment statements to provide an interested user the ability to check or find any particular portion of the program. The program was designed to operate in as little time as possible, using as little duplicated storage as possible. The object code consumes approximately 167K bytes for the capability to work with up to twenty wires and a total of ten Fourier series terms per wire. Run times can be significantly reduced by running the program on the FORTRAN G or H compilers with an object deck. Compilation is no longer necessary, and execution time is much faster than on diagnostic compilers such as WATFIV available at the University of Kentucky. Typical run times on the IBM System 370/165 at the University of Kentucky using the FORTRAN G compiler with no printout of the source listing are shown below.

88

TYPICAL OUTPUT OF A MATRIX - - PAGE 1

COLUMN	1	2	3	4	5	6	7
ROW	******	******	******	******	******	******	******
1	1.00D 00	2.00D 00	3.00D 00	4.00D 00	5.00D 00	6.00D 00	7.00D 00
2	8.00D 00	9.00D 00	1.00D 01	1.10D 01	1.20D 01	1.30D 01	1.40D 01
3	1.50D 01	1.60D 01	1.70D 01	1.80D 01	1.90D 01	2.00D 01	2.10D 01
4	2.20D 01	2.30D 01	2.40D 01	2.50D 01	2.60D 01	2.70D 01	2.80D 01
5	2.90D 01	3.00D 01	3.10D 01	3.20D 01	3.30D 01	3.40D 01	3.50D 01
6	3.60D 01	3.70D 01	3.80D 01	3.90D 01	4.00D 01	4.10D 01	4.20D 01

Figure 6.4   Sample Output Generated by Subroutine MPRT.

VI.4   Verification of Program Operation

Unfortunately, there is no way of comparing these computed results to known data on the capacitance matrices of ribbon cables since none apparently exists. Comparisons can be made, however, to known exact and approximate formulas

for the capacitance where the dielectric effect is ignored; i.e., bare wires.

The data in Table 8 show how the capacitance for the two bare-wire case converges rapidly to the exact value found by the equation [5]

$$C_0 = \frac{\pi \varepsilon_v}{\cosh^{-1}\left(\frac{d}{2r_c}\right)} \quad , \quad\quad\quad (6.7)$$

where d is the center-to-center separation of the wires, and r_c is the radius of the conductor. The per-unit-length inductance L is related to C_0 by Eq. (5.16). Ribbon cables typically have a conductor separation greater than four times the conductor radius. In this range, results for the per-unit-length inductance for a two-wire cable are extremely accurate with even small numbers of Fourier series terms.

Results from GETCAP for a multiwire bare cable assembly can be compared to the approximate transmission line capacitance matrix derived in Chapter V. These results are shown

---

*20 wires, 10 coefficients per wire, IOPT=0.

Separation:	2.1	2.5	3.0
Exact:	88.3266336390219	40.1303567097695 3	28.9022860200871 8
Program:			
NF = 1	32.9527718698 7344	28.0830115932 7916	24.1608822126 7191
2	53.7641176185 8517	36.9182514747 1941	28.2445064456 6931
3	67.8341137979 5934	39.3296935189 0125	28.8126010349 6420
4	76.6677236582 4638	39.9339781312 6190	28.8898761431 3759
5	81.8678589192 7163	40.0822203891 5016	28.9005458289 6143
6	84.8046455177 6120	40.1185286423 2260	28.9020397179 3245
7	86.4232783056 1355	40.1274430236 0560	28.9022509317 6778
8	87.3032068599 8552	40.1296374818 3029	28.9022809978 9533
9	87.7779071148 7611	40.1301788800 27915	28.9022852987 5102
10	88.0329061802 1939	40.1303126830 5180	28.9022859162 0657

Separation:	4.0	10.0
Exact:	21.1215891359 8862	12.1339466635 2968
Program:		
NF = 1	19.6358270488 8583	12.0543953323 5852
2	21.0318046826 3799	12.1333346906 16347
3	21.1158228210 8362	12.1339412967 1470
4	21.1212030393 1120	12.1339466125 6600
5	21.1215626795 7364	12.1339466630 3271
6	21.1215872983 4855	12.1339466635 2477
7	21.1215890072 5528	12.1339466635 2964
8	21.1215891269 1928	12.1339466635 2969
9	21.1215891353 4718	12.1339466635 2969
10	21.1215891359 4313	12.1339466635 2969

Conductor radius,
$r_c = 1.0$

All elements $*10^{-12}$, pF/M

Table 8   Comparison of exact and results from GETCAP for the transmission-line capacitance of a two-wire cable with no dielectric.

91

reference conductor

$r_c = 1.0$

sep=10.0
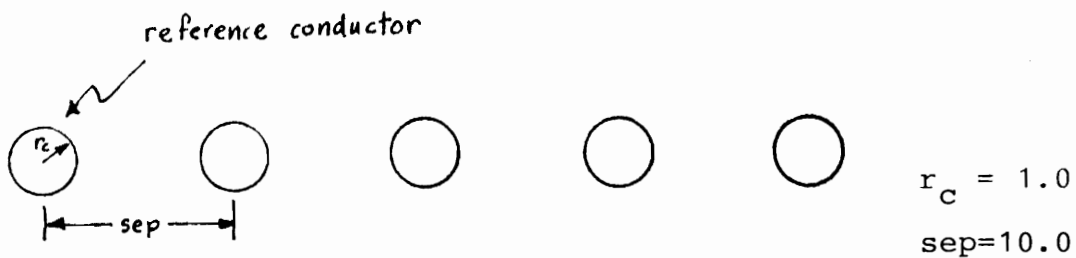
Figure 6.5 Five wire bare ribbon cable.

in Table 9 for the cable shown in Fig. 6.5. The approximate results agree with the GETCAP results to at least two digits, and the largest error is approximately 2% between the two sets of results.

Another check was made of the results from GETCAP by checking the rate of convergence of the elements in the first row of the generalized capacitance matrix for a five wire ribbon cable (with the dielectric included) as shown in

reference conductor

$r_c = 1.0$

$r_d = 2.0$

sep= 4.0

$\varepsilon_r = 4.0$
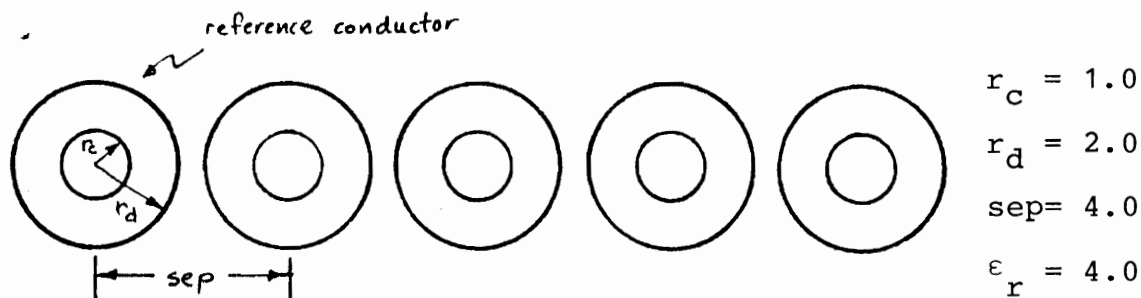
Figure 6.6 Five wire flatpack cable.

Fig. 6.6. The results in Fig. 6.7 indicate smooth convergence to the final values for increasing values of NF. (The magnitudes of the elements are plotted since the off-diagonal terms of $\underset{\sim}{C}$ are negative.) These results are for the worst case problem (dielectric boundaries touching, high relative dielectric constant).

92

Elements of the Transmission-Line Capacitance
Matrix for a Five-wire Bare Cable

(Elements $*10^{-12}$, pF/M)

Element	GETCAP Results	Approximate
C(1,1)	18.87646053717670	18.78148468923442
C(1,2)	-6.851495768047740	-6.854631737467468
C(1,3)	-2.129410716129314	-2.066002017837609
C(1,4)	-1.843114316184374	-1.826053886227327
C(2,2)	19.14682911214455	19.03688696351830
C(2,3)	-6.851495768052782	-6.854631737467466
C(2,4)	-2.721918788019494	-2.663811744291685
C(3,3)	18.87646053717669	18.78148468923442
C(3,4)	-8.052439736815269	-8.034797047702019
C(4,4)	14.81610887311056	14.72743429621767

Physical conditions: $r_c = 1.0$

sep $= 10.0$

End wire chosen as reference.

Table 9   Comparison of GETCAP results to the approximate formula derived in Chapter V for the capacitance matrix of multi-conductor bare wire.
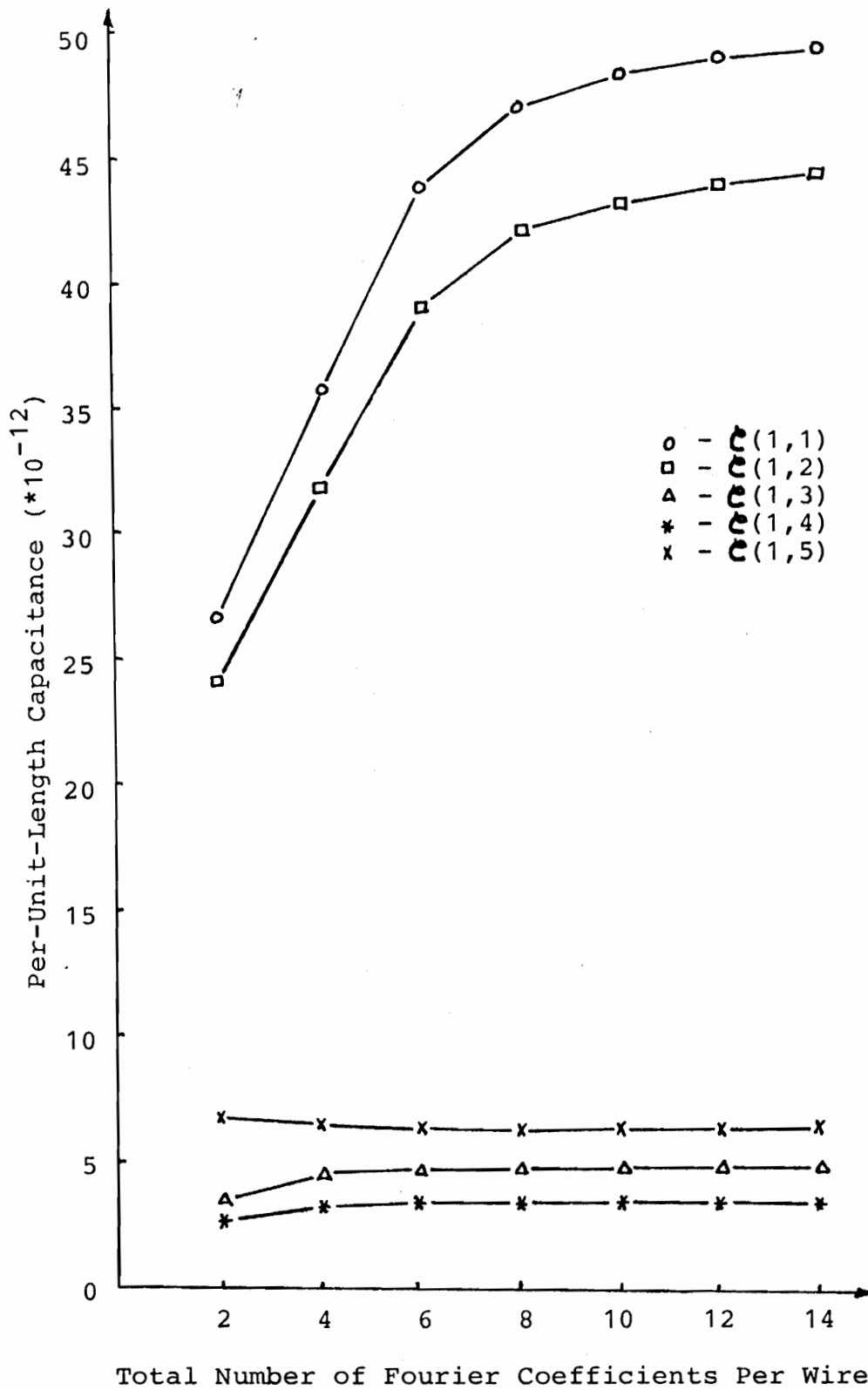
Figure 6.7.   Convergence of the elements of the
              generalized capacitance matrix.

CHAPTER VII

USING GETCAP

VII.1 Introduction

This chapter will describe the use of the GETCAP
program for two different purposes. Section VII.2 will
consider use of the GETCAP subroutine for cases where it
will be required to compute and deliver the generalized and
transmission line capacitance matrices to other program
units written by the user. More commonly, though, the
transmission line capacitance matrix of the ribbon cable is
used as input data for another program, such as in a cross-
talk analysis. Section VII.3 will describe the entire
GETCAP program with its facilities for outputting the
elements of the transmission line capacitance matrix in
punched card format.

VII.2 Using Subroutine GETCAP

Subroutine GETCAP is the workhorse of the GETCAP
program. It is used by calling the subprogram from another
FORTRAN program unit, using the following statement and
argument list:

        CALL GETCAP (NW, NFC, NFD, NF, RC, RD, CCSEP, ER,
                     IREF, IOPT, CG, CTL, D, D1, SCR, LT)

As described in Chapter VI, the arguments have the following

95

meaning:

NW   - Number of wires in the cable.

NFC   - Number of Fourier series terms to be used
to represent the charge distributions
around the dielectric surfaces. Note:
the charge distribution is assumed to be
of the form

$$\sigma_i(\theta) = \sum_{m=0}^{(NFC-1)} \sigma_{im}\cos m\theta \quad \text{for } i=1,\ldots,NW.$$

NFD   - Number of Fourier series terms to be used
to represent the charge distributions around
the dielectric surfaces. Note: the charge
distribution is assumed to be of the form

$$\sigma'_i(\theta) = \sum_{m=0}^{(NFD-1)} \sigma'_{im}\cos m\theta \quad \text{for } i=1,\ldots,NW.$$

(See Section III.3 for a discussion of how to
choose these.)

NF   - Total number of Fourier series terms per
wire. (NF = NFC + NFD).

RC   - Radii of the conductors.

RD   - Radii of the outer dielectric insulation
surfaces.

CCSEP - Center-to-center separation of any two
adjacent conductors.

(These cable dimensions must appear in the same
units: meters, mils, inches, etc.)

ER   - Relative dielectric constant of the insulation
material (relative to free space).

IREF   - The reference conductor for the transmission
line voltages. ($1 \leq IREF \leq NW$)

IOPT   - Option selector.
      0 - fast solution
      1 - long matrix inversion
      2 - dielectric insulations removed
          (bare wire case)

CG   - Holds the computed per-unit-length generalized
capacitance matrix in vector format. (dimension
NW^2)

96

CTL — Holds the computed per-unit-length transmission line capacitance matrix in vector format. (dimension $(NW-1)^2$)

D — A working matrix of dimension NF x NF.

D1 — A working vector of length $(NF \cdot NW)^2$.

SCR — A scratch vector of length $\dfrac{NF(NW+1)}{2}$.

LT — A scratch vector of length $2 \cdot NF \cdot NW$.

All that is required in using subroutine GETCAP is to provide for and properly dimension these required matrices and vectors in the calling program. One must be especially careful about the size of vector D1. A typical case of twenty wires and ten Fourier coefficients per wire requires that D1 be dimensioned to provide $(20 \times 10)^2 = 40,000$ storage locations. The user should note that D1 is dimensioned in the main program as a matrix D1(NF·NW,NF·NW). In subroutine GETCAP, however, D1 is stored in vector format.

GETCAP requires two other subroutines. They are:

MINV — A standard IBM SSP matrix inversion program [16]

and MPC — The product of two matrices with a scalar constant multiplying the result.

Both of these routines are included in the program listing found in Appendix B.

VII.3 Using the GETCAP Program

The full GETCAP program is a user-oriented method to find the capacitance matrices of ribbon cables. The input to

97

the program is through three data cards, (1) a problem
description card, (2) a physical characteristics card, and
(3) an option card.   Their format is shown below.

(1)   Problem description card:

    Cols. 1-3,   NW
    Cols. 4-6,   NFC
    Cols. 7-9,   NFD

Example:
  Assume a fifteen wire flat pack cable,
  four expansion terms for the conductor
  charge distribution and eight expansion
  terms for the charge distribution
  around the dielectric surface.

The problem descriptor card would look like:

    <u>Col. 1</u>   <u>Col. 4</u>   <u>Col. 7</u>

      ø15øø4øø8
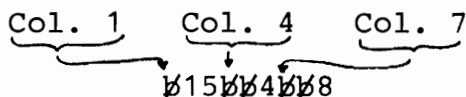
where ø denotes a blank item.

(2)   Physical characteristics card:

    Cols. 1-10,   RC
    Cols. 11-20, RD
    Cols. 21-30, CCSEP
    Cols. 31-40, ER

The dimensions of RC, RD, and CCSEP can be
in any units convenient to the user.   Each
of these dimensions, however, must be in the
same units.

Example:
  Assume an orientation as shown in Fig 7.1.
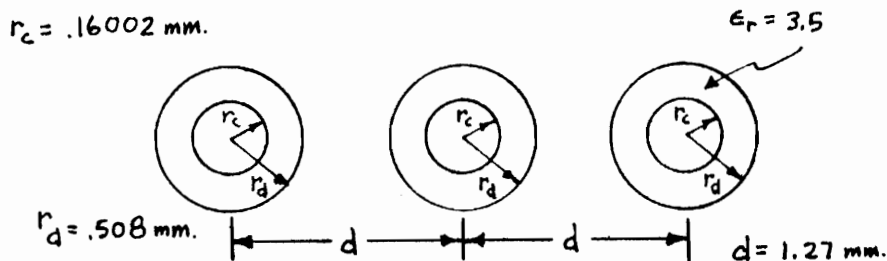
Fig. 7.1   Typical physical characteristics
98

The physical characteristics card would be:

Col. 1   Col. 11   Col. 21   Col. 31

.16002øøøø.508øøøøøø1.27øøøøøø3.5øøøøøøø

(3) Option card:

Cols. 1-3, IREF -- the reference conductor.

Note:
Conductor numbering is sequential from
the end of the cable, as illustrated
below:
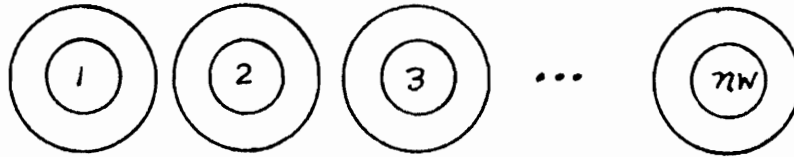
Therefore, IREF can range from 1 to NW.

Cols. 4-6, IOPT -- option.

IOPT = 0   fast solution
IOPT = 1   long inversion
IOPT = 2   dielectric insulation removed
IOPT = 10⎱ ⎰same as 0, 1, and 2 above except
IOPT = 11⎰ ⎱that the upper triangle of the
IOPT = 12⎰   transmission-line capacitance
             matrix is punched on cards.

Example:
Assume conductor 1 chosen as reference and
execute in fastest time, with a punched
copy of the transmission line capacitance
matrix generated.

Then IOPT = 10 and the option card is:

Col. 1   Col. 4

øø1ø10

Typical output from the GETCAP program is shown in Fig.
7.2. Fig. 7.2a shows the header page which is printed at
the beginning of a batch of problems to the program. Fig.
7.2b is the first page of output associated with a problem.

```
************************************************************************

        GGG    EEEEE  TTTTT   CCC    AAA    PPPP
       G   G   E        T    C   C  A   A   P   P
       G       E        T    C      A   A   P   P
       G  GG   EEEE     T    C      AAAAA   PPPP
       G   G   E        T    C      A   A   P
       G   G   E        T    C   C  A   A   P
        GGGG   EEEEE    T     CCC   A   A   P


************************************************************************

        GGG    EEEEE  TTTTT   CCC    AAA    PPPP
       G   G   E        T    C   C  A   A   P   P
       G       E        T    C      A   A   P   P
       G  GG   EEEE     T    C      AAAAA   PPPP
       G   G   E        T    C      A   A   P
       G   G   E        T    C   C  A   A   P
        GGGG   EEEEE    T     CCC   A   A   P
```

GENERALIZED AND TRANSMISSION LINE CAPACITANCE MATRICES

OF RIBBON CABLES

```
************************************************************************
```

B A T C H   R U N

Fig. 7.2a Typical Output from the GETCAP program.

100

```
*****************************************************************

GENERALIZED AND TRANSMISSION LINE CAPACITANCE MATRICES OF "FLAT-PACK" CABLE

CONDITIONS:
    5 WIRES,    3 FOURIER COEFFICIENTS AROUND THE CONDUCTOR, AND    7 AROUND THE DIELECTRIC

PHYSICAL CHARACTERISTICS:
    CONDUCTOR RADIUS= 1.6002E-04 (METERS)
    DIELECTRIC RADIUS= 5.0800E-04 (METERS)
    CENTER-TO-CENTER SEPARATION= 1.2700E-03 (METERS)
    RELATIVE DIELECTRIC CONSTANT=   3.500E 00

REFERENCE CONDUCTOR IS NO.    1
OPTION NUMBER IS    0

*****************************************************************
```

Fig. 7.2b Typical output from the GETCAP program.

101

GENERALIZED CAPACITANCE MATRIX -- PAGE 1

```
COLUMN      1            2            3            4            5

ROW   ********************************************************************
      *
  1 * 1.85E-11    -2.35E-11    -7.31E-12    -6.43E-12    -1.20E-11
      *
  2 * -2.35E-11    3.88E-11    -1.90E-11    -4.54E-12    -6.43E-12
      *
  3 * -7.31E-12   -1.90E-11    3.95E-11    -1.90E-11    -7.31E-12
      *
  4 * -6.43E-12   -4.45E-12   -1.90E-11    3.88E-11    -2.35E-11
      *
  5 * -1.20E-11   -6.43E-12   -7.31E-12   -2.35E-11    1.85E-11
```

Fig. 7.2c Typical output from the GETCAP program.

102

TRANSMISSION LINE CAPACITANCE MATRIX - - PAGE 1

COLUMN   1   2   3   4
     ****
     *
1 * 4.08E-11 -1.71E-11 -2.46E-12 -2.07E-12
     *
2 * -1.71E-11   4.11E-11 -1.71E-11 -3.41E-I2
     *
3 * -2.46E-12 -1.71E-11   4.08E-11 -1.92E-11
     *
4 * -2.07E-12 -3.41E-12 -1.92E-11   2.76E-11

ROW

Fig. 7.2d Typical output from the GETCAP program.

All of the input data are printed as they are read in, and any errors in the data are flagged on this page. The item, (METERS), on this printout is used to show that all wire dimensions must be in the same units (meters, inches, mils, etc.). Fig. 7.2c is the output of the generalized capacitance matrix associated with the ribbon cable described by the data in Fig. 7.2b. The elements have units of Farads per meter. Fig. 7.2d, the transmission line capacitance matrix with a conductor chosen as reference for the line voltages, is the final page of output for each problem.

VII.4 Format of the Output of the Transmission Line Capacitance Matrix

An algorithm was developed in Chapter V which is used in the computer program to determine the transmission line capacitance matrix from the generalized capacitance matrix; any of the conductors in the ribbon cable can be chosen as the reference conductor in this algorithm.

In the printout of the transmission line capacitance matrix, the row and column indices are numbered from 1 to (NW-1). Thus, if the last wire, NW, is chosen as the reference, the indices will correspond to the elements correctly. However, if another conductor is chosen, the user will have the responsibility for correctly interpreting the results. For example, the case of a 10 wire cable can be considered. If conductor 5 is chosen as reference, $CTL_{5,1}$ is actually the element of $\underset{\sim}{C}$ describing the capacitance between wires 6 and 1. Similarly, $CTL_{5,5}$ describes

104

the self capacitance of wire 6.

The punched card output is of a similar format. One upper triangular element is punched per card, with the row and column indices of the element punched as well. Suppose the capacitance matrices were determined for a twenty wire ribbon cable with the 20-th wire chosen as reference. A sample punched card would look as follows:

C   10   3   -4.384692E-10

This element would be $C_{10,3}$ with value $-4.384692 \times 10^{-10}$. The user has the responsibility for correctly interpreting the results if any wire other than the last wire is chosen as the reference conductor.

CHAPTER VIII

SUMMARY

A method for computing the capacitance matrix for dielectric-coated conductors was applied to the case of ribbon cables. Simplifications in the method were made possible by the symmetry of the cable dimensions; the radii of the conductors are all identical, the radii of the dielectric insulations are all identical, and the center-to-center spacing of adjacent wires is identical. In addition, the wires are oriented in a horizontal plane which is maintained throughout the length of the cable. An attempt was made to optimize the selection of matchpoints to ensure valid results and reduce computation time.

Using the generalized capacitance matrix, a technique for obtaining the transmission line capacitance and inductance matrices was developed; these matrices are used in frequency response and crosstalk analyses of cable systems. An approximate method for determining the transmission line inductance matrix was also developed.

Computer program GETCAP was written to utilize the results of this report. Given the wire dimensions and the number of wires in the cable, GETCAP will compute the per-unit-length generalized capacitance matrix, and from that

106

determine the per-unit-length transmission line capacitance matrix for a ribbon cable. Results indicate rapid convergence of the matrices to accurate values. The GETCAP program is written to be useful even to the person who is not a computer specialist, with simple input formats and error correcting facilities.

Ribbon cables are now widely used in the interconnection of electronic systems. The ability to compute the transmission line capacitance matrix for such cables enables the multiconductor transmission line equations to be solved. This in turn will enable a more precise analysis of ribbon cable systems through a detailed analysis of crosstalk.

APPENDIX A

The purpose of this appendix is to provide justification for omitting the reference potential terms in the potential expressions in Chapter II. Consider Fig. A.1(a) in which infinitesimal line charges lie on a cylindrical surface of radius r'. The potential $\emptyset(r,\theta)$ with respect to the potential reference point due to one of the line charges is (reference [7], pp. 91-92)

$$\emptyset(r,\theta) = - \frac{\lambda}{2\pi\varepsilon_v} \ln\left(\frac{d}{d_0}\right) \tag{A.1}$$

where the distances from the line charge to the potential and reference points are given by

$$d^2 = r^2 + r'^2 - 2rr'\cos(\theta'-\theta) \tag{A.2a}$$

and

$$d_0^2 = r_0^2 + r'^2 - 2r_0r'\cos(\theta'-\theta_0) \ . \tag{A.2b}$$

If the cylindrical surface supports a per-unit-length charge distribution of the form

$$\sigma(\theta') = \sigma_0 + \sum_{m=1}^{k} (\sigma_m\cos m\theta' + \sigma_m'\sin m\theta') \tag{A.3}$$

then the potential $\emptyset(r,\theta)$ can be obtained as the limiting case of an infinite number of infinitesimal line charges with appropriate weighting given in Eq. (A.3) as [4]

108

(a)

(b)

Fig. A.1.

109
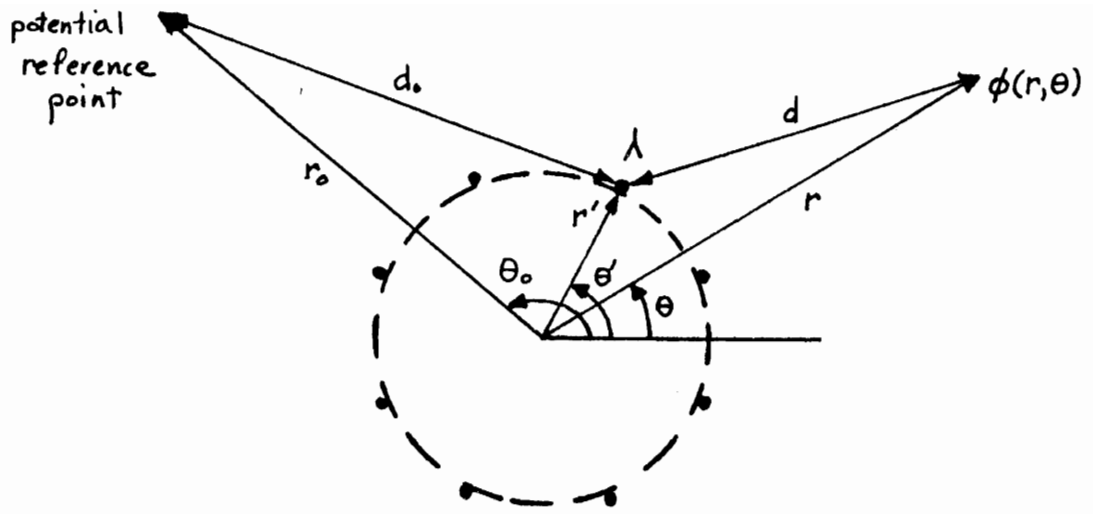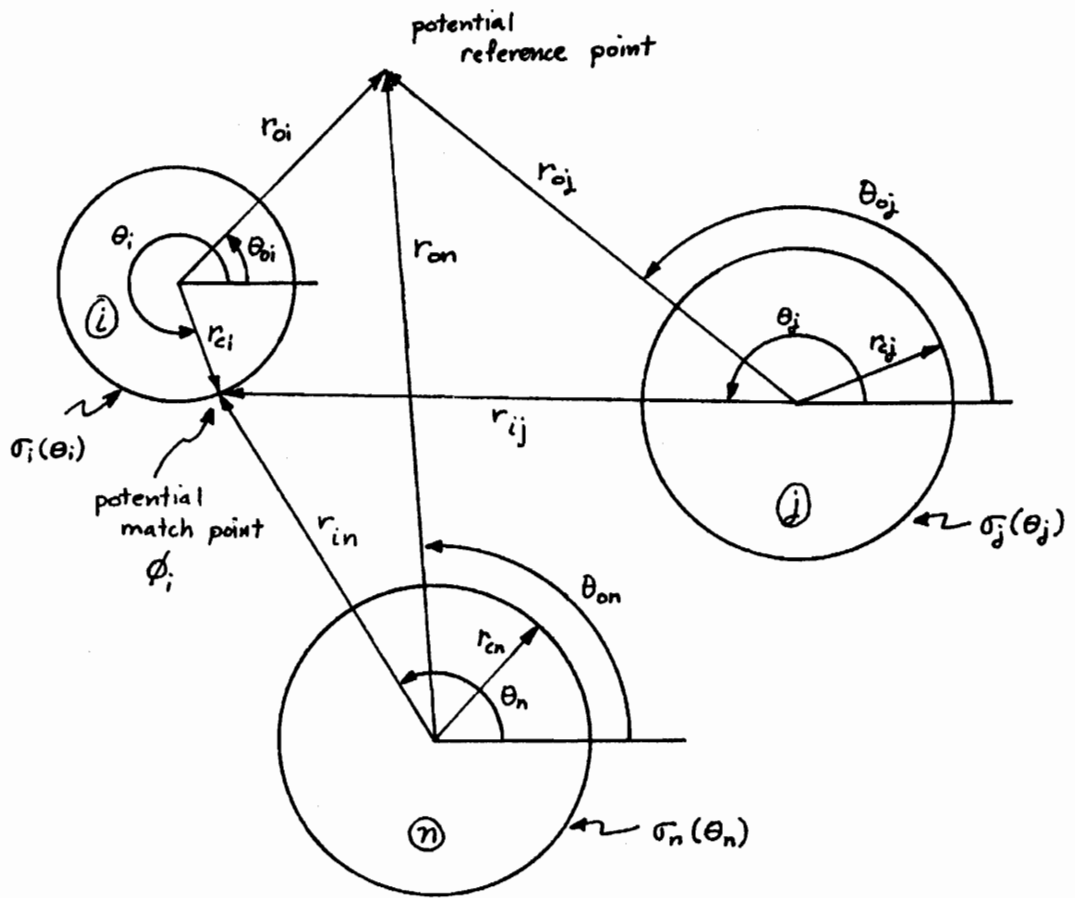
$$\emptyset(r,\theta) = -\frac{\sigma_0}{2\pi\epsilon_v} \int_0^{2\pi} \ln\left(\frac{d}{d_0}\right) r' \, d\theta'$$

$$- \frac{1}{2\pi\epsilon_v} \sum_{m=1}^{k} \left\{ \int_0^{2\pi} \sigma_m \cos m\theta' \, \ln\left(\frac{d}{d_0}\right) r' \, d\theta' \right.$$

$$\left. + \int_0^{2\pi} \sigma_m' \sin m\theta' \, \ln\left(\frac{d}{d_0}\right) r' \, d\theta' \right\} . \qquad (A.4)$$

Substituting the expressions for d and d_0 given in Eq. (A.2) into Eq. (A.4) yield integrals which can be evaluated in closed form. The result is [4]

$$\emptyset(r,\theta) = \sigma_0 D_0(r,\theta) - \sigma_0 D_0(r_0,\theta_0) \qquad (A.5)$$

$$+ \sum_{m=1}^{k} \left\{ \sigma_m D_m(r,\theta) - \sigma_m D_m(r_0,\theta_0) \right.$$

$$\left. + \sigma_m' D_m'(r,\theta) - \sigma_m' D_m'(r_0,\theta_0) \right\}$$

where

$$D_0(r,\theta) = \begin{cases} -\dfrac{r' \ln r}{\epsilon_v} & r \geq r' \\[4mm] -\dfrac{r' \ln r'}{\epsilon_v} & r \leq r' \end{cases} \qquad (A.6a)$$

$$D_m(r,\theta) = \begin{cases} \dfrac{(r')^{m+1}}{2\,\epsilon_v m r^m} \cos m\theta & r \geq r' \\[4mm] \dfrac{(r)^m}{2\epsilon_v m (r')^{m-1}} \cos m\theta & r \leq r' \end{cases} \qquad (A.6b)$$

110

$$D'_m(r,\theta) = \begin{cases} \dfrac{(r')^{m+1}}{2\varepsilon_v m r^m} \quad \text{sinm}\theta & r \geqslant r' \\[4mm] \dfrac{(r)^m}{2\varepsilon_v m (r')^{m-1}} \text{sinm}\theta & r \leq r' \end{cases} \tag{A.6c}$$

The terms $D_0(r_0,\theta_0)$, $D_m(r_0,\theta_0)$ and $D'_m(r_0,\theta_0)$ are the reference potential terms which were omitted in the potential expressions in Chapter II.

Consider a typical system of n circular conductors shown in Fig. A.1(b) bearing per-unit-length charge distributions of the form

$$\sigma_i(\theta_i) = \sigma_{i0} + \sum_{m=1}^{k_i} (\sigma_{im}\text{cosm}\theta_i + \sigma'_{im}\text{sinm}\theta_i) \tag{A.7}$$

A typical expression for the potential at a matchpoint on the i-th conductor in Fig. A.1(b) is

$$\emptyset_i = \cdots + \sigma_{i0}D_{i0}(r_{ci},\theta_i) - \sigma_{i0}D_{i0}(r_{0i},\theta_{0i})$$

$$+ \sum_{m=1}^{k_i}\left\{\sigma_{im}D_{im}(r_{ci},\theta_i) - \sigma_{im}D_{im}(r_{0i},\theta_{0i})\right.$$

$$\left. + \sigma'_{im}D'_{im}(r_{ci},\theta_i) - \sigma'_{im}D'_{im}(r_{0i},\theta_{0i})\right\}$$

$$\cdots + \sigma_{j0}D_{j0}(r_{ij},\theta_j) - \sigma_{j0}D_{j0}(r_{0j},\theta_{0j})$$

$$+ \sum_{m=1}^{k_j}\left\{\sigma_{jm}D_{jm}(r_{ij},\theta_j) - \sigma_{jm}D_{jm}(r_{0j},\theta_{0j})\right.$$

$$\left. + \sigma'_{jm}D'_{jm}(r_{ij},\theta_j) - \sigma'_{jm}D'_{jm}(r_{0j},\theta_{0j})\right\}$$

$$\cdots + \sigma_{n0}D_{n0}(r_{in},\theta_n) - \sigma_{n0}D_{n0}(r_{0n},\theta_{0n})$$

$$+ \sum_{m=1}^{k_n}\left\{\sigma_{nm}D_{nm}(r_{in},\theta_n) - \sigma_{nm}D_{nm}(r_{0n},\theta_{0n})\right.$$

$$\left. + \sigma'_{nm}D'_{nm}(r_{in},\theta_n) - \sigma'_{nm}D'_{nm}(r_{0n},\theta_{0n})\right\}$$

$$(A.8)$$

If we allow the reference point for the potentials to move to infinity, the reference potential terms for the $\sigma_{im}\cos m\theta_i$ and $\sigma'_{im}\sin m\theta_i$ expansion terms go to zero as is clear from Eq. (A.6b) and (A.6c). If the total per-unit-length charge on the system of conductors is zero, then the reference potential terms due to the constant expansion terms may also be removed. This can be shown in the following manner. The total per-unit-length charge on the i-th

112

conductor is

$$q_i = \int_0^{2\pi} \sigma_i(\theta_i) r_{ci} d\theta_i$$

(A.9)

$$= 2\pi r_{ci} \sigma_{i0}$$

and the portion of the potential expression in Eq. (A.8) consisting of the reference potential terms due to the constant charge expansion terms is

$$\cdots -\sigma_{i0} D_{i0}(r_{0i}, \theta_{0i}) \quad \cdots \quad -\sigma_{j0} D_{j0}(r_{0j}, \theta_{0j})$$

$$\cdots -\sigma_{n0} D_{n0}(r_{0n}, \theta_{0n})$$

(A.10)

$$= -\sum_{m=1}^{n} \sigma_{m0} D_{m0}(r_{0m}, \theta_{0m})$$

Utilizing the expression for $D_{m0}(r_{0m}, \theta_{0m})$ of the form given in Eq. (A.6a), Eq. (A.10) can be written as

$$-\sum_{m=1}^{n} \sigma_{m0} \left(-\frac{r_{cm} \ln(r_{0m})}{\epsilon_v} \right).$$

(A.11)

With the expression for the total per-unit-length charge on the i-th conductor given in Eq. (A.9), Eq. (A.11) can be written as

$$\sum_{m=1}^{n} q_m \frac{\ln(r_{0m})}{2\pi\epsilon_v} .$$

(A.12)

Requiring the system to be electrically neutral, i.e.,

113

$$\sum_{m=1}^{n} q_m = 0 \quad , \tag{A.13}$$

Eq. (A.12) can be written as

$$-\left(\sum_{m=2}^{n} q_m\right) \frac{\ln(r_{01})}{2\pi\varepsilon_v} + \sum_{m=2}^{n} q_m \frac{\ln(r_{0m})}{2\pi\varepsilon_v} \quad . \tag{A.14}$$

By combining associated terms, Eq. (A.14) can be written as

$$\sum_{m=2}^{n} \frac{q_m}{2\pi\varepsilon_v} \ln\left(\frac{r_{0m}}{r_{01}}\right) \quad . \tag{A.15}$$

As the reference potential point moves to infinity, the distance from the centers of the conductors to the reference point become the same, i.e., $r_{01} = r_{02} = \cdots = r_{0n}$, and Eq. (A.15) approaches zero. Therefore, the reference potential terms in the potential expressions may be omitted. Implicit in this is the fact that the potentials, $\emptyset_i$, are with respect to infinity which is permissible only if the net per-unit-length charge on the system is zero, i.e.,

$$\sum_{m=1}^{n} q_m = 0 \quad . \tag{A.16}$$

APPENDIX B

Listing of Computer Program GETCAP.

The listing will be divided into its individual program parts:

MAIN   -   main program controlling input of data and output of results

GETCAP -   subroutine which computes the per-unit-length generalized and transmission line capacitance matrices for ribbon cables of the specified dimensions

MINV   -   subroutine which computes the inverse of a matrix (from the IBM Scientific Subroutine Package [16])

MPC   -   subroutine which multiplies two matrices and then multiplies the result by a constant

MPRT   -   subroutine which prints a matrix in standard matrix format

```
C*************************************************************      MAIN  10
C*************************************************************MAIN  20
C                                                                   MAIN  30
C                                                                   MAIN  40
C                                                                   MAIN  50
C         GGG   EEEEE  TTTTT  CCC   AAA   PPPP                       MAIN  60
C        G   G  E        T   C     A   A  P   P                      MAIN  70
C        G      EEEE     T   C     A   A  P   P                      MAIN  80
C        G  GG  EEEE     T   C     AAAAA  PPPP                       MAIN  90
C        G   G  E        T   C     A   A  P                          MAIN 100
C        G   G  E        T   C   C A   A  P                          MAIN 110
C         GGGG  EEEEE    T    CCC  A   A  P                          MAIN 120
C                                                                   MAIN 130
C           V   V   1            4                                   MAIN 140
C           V   V  11           44                                   MAIN 150
C           V   V   1          4 4                                   MAIN 160
C            V V    1    XXX   44444                                 MAIN 170
C            V V    1            4                                    MAIN 180
C             V     1            4                                    MAIN 190
C             V   11111          4                                    MAIN 200
C                                                                   MAIN 210
C                                                                   MAIN 220
C  PROGRAM TO COMPUTE THE GENERALIZED AND TRANSMISSION              MAIN 230
C  LINE CAPACITANCE MATRICES FOR RIBBON OR                          MAIN 240
C  "FLAT-PACK" CABLES                                               MAIN 250
C                                                                   MAIN 260
C                                                                   MAIN 270
C  WRITTEN BY ARTHUR E. FEATHER                                     MAIN 280
C             ELECTRICAL ENGINEERING DEPARTMENT                     MAIN 290
C             UNIVERSITY OF KENTUCKY                                MAIN 300
C             LEXINGTON, KENTUCKY  40506                            MAIN 310
C                                                                   MAIN 320
C*************************************************************MAIN 330
C                                                                   MAIN 340
C                                                                   MAIN 350
C                                                                   MAIN 360
```

116

```
C.........................................................MAIN 370
C                                                         MAIN 380
C     DIMENSIONING INFORMATION:                           MAIN 390
C         C(NW**2)                                        MAIN 400
C         C1((NW-1)**2)                                   MAIN 410
C         D(NF**2)                                        MAIN 420
C         D1(((NW*NF)**2)                                 MAIN 430
C         LT(2*NF*NW)                                     MAIN 440
C         SCR((NF*NW+1)/2)                                MAIN 450
C              WHERE:   NW IS THE NUMBER OF WIRES         MAIN 460
C                       NF IS THE TOTAL NUMBER OF FOURIER COEFF.  MAIN 470
C                          I.E., NF=NFC+NFD               MAIN 480
C                                                         MAIN 490
C     THIS DIMENSION PROVIDES FOR UP TO 20 WIRES AND UP TO    MAIN 500
C     10 FOURIER COEFFICIENTS PER WIRE                    MAIN 510
C     DIMENSION C(400),C1(361),D(10,10),D1(200,200),LT(400),SCR(100)  MAIN 520
C                                                         MAIN 530
C                                                         MAIN 540
      DIMENSION C(25),C1(16),D(20,20),D1(100,100),LT(200),SCR(50)  MAIN 550
C                                                         MAIN 560
C                                                         MAIN 570
C.........................................................MAIN 580
C                                                         MAIN 590
C     SCR AND LT ARE SCRATCH VECTORS OF DIFFERENT TYPE BUT CAN   MAIN 600
C     SHARE STORAGE LOCATIONS                            MAIN 610
      EQUIVALENCE (SCR(1),LT(1))                          MAIN 620
      WRITE(6,46)                                         MAIN 630
      WRITE(6,47)                                         MAIN 640
    1 READ(5,30,END=11,ERR=9)NW,NFC,NFD                   MAIN 650
      WRITE(6,31)NW,NFC,NFD                               MAIN 660
C                                                         MAIN 670
C     CALL FOR ERROR IF NW IS LESS THAN 2                 MAIN 680
C                                                         MAIN 690
      IF(NW-1)2,2,3                                        MAIN 700
    2 WRITE(6,32)                                          MAIN 710
      GO TO 10                                             MAIN 720
```

```
C                                                                      MAIN 730
C           CALL FOR ERROR IF NFC IS GREATER THAN NFD                  MAIN 740
C                                                                      MAIN 750
3         IF(NFD-NFC)4,5,5                                             MAIN 760
4         WRITE(6,33)NFC                                              MAIN 770
          NFD=NFC                                                     MAIN 780
5         READ(5,34,END=8,ERR=7)RC,RD,SEP,ER                          MAIN 790
          WRITE(6,35)RC,RD,SEP,ER                                     MAIN 800
          READ(5,30,END=6,ERR=6)IREF,IOPT                             MAIN 810
          WRITE(6,36)IREF,IOPT                                        MAIN 820
C                                                                      MAIN 830
C           MAKE SURE IREF IS WITHIN RANGE                            MAIN 840
C                                                                      MAIN 850
6         IF(IREF-1)12,16,13                                          MAIN 860
          WRITE(6,39)                                                 MAIN 870
          IREF=1                                                      MAIN 880
          IOPT=0                                                      MAIN 890
          GO TO 21                                                    MAIN 900
7         READ(6,41,END=8)                                            MAIN 910
8         WRITE(6,40)                                                 MAIN 920
          GO TO 1                                                     MAIN 930
9         WRITE(6,43)                                                 MAIN 940
10        READ(5,41,END=11)                                           MAIN 950
          READ(5,41,END=11)                                           MAIN 960
          GO TO 1                                                     MAIN 970
11        WRITE(6,42)                                                 MAIN 980
          STOP                                                        MAIN 990
12        IREF=1                                                      MAIN1000
          GO TO 15                                                    MAIN1010
13        IF(NW-IREF)14,16,16                                         MAIN1020
14        IREF=NW                                                     MAIN1030
15        WRITE(6,37)IREF                                             MAIN1040
C                                                                      MAIN1050
C           CHECK TO SEE IF IOPT IS WITHIN RANGE                      MAIN1060
C                                                                      MAIN1070
16        IPCH=0                                                      MAIN1080
```

118

```fortran
      IOPT10=IOPT-10                                                    MAIN1090
      IF(IOPT10)18,17,17                                                MAIN1100
   17 IPCH=1                                                            MAIN1110
      IOPT=IOPT10                                                       MAIN1120
   18 IF(IOPT-1)19,24,21                                                MAIN1130
   19 IF(IOPT)20,24,24                                                  MAIN1140
   20 IOPT=0                                                            MAIN1150
   21 IF(IOPT-2)23,24,22                                                MAIN1160
   22 IOPT=2                                                            MAIN1170
   23 WRITE(6,38)IOPT                                                   MAIN1180
C                                                                       MAIN1190
C     GENERATE CONSTANTS NEEDED TO PASS TO SUBROUTINE                   MAIN1200
C                                                                       MAIN1210
   24 NF=NFC                                                            MAIN1220
      IF(IOPT-2)25,26,25                                                MAIN1230
   25 NF=NF+NFD                                                         MAIN1240
   26 CALL GETCAP(NW,NFC,NFD,NF,RC,RD,SEP,ER,IREF,IOPT,C,C1,D,D1,SCR,LT)MAIN1250
C                                                                       MAIN1260
C     CHECK FOR ERROR RETURN                                           MAIN1270
C                                                                       MAIN1280
      IF(NW)27,1,27                                                     MAIN1290
C                                                                       MAIN1300
C     RESULTS HAVE BEEN CALCULATED- -MATRICES ARE PRINTED OUT           MAIN1310
C                                                                       MAIN1320
   27 CALL MPRT(C,NW,NW,'GENERALIZED CAPACITANCE',23)                   MAIN1330
      NW1=NW-1                                                          MAIN1340
      CALL MPRT(C1,NW1,NW1,'TRANSMISSION LINE CAPACITANCE',29)          MAIN1350
      IF(IPCH)1,1,28                                                    MAIN1360
   28 DO 29 I=1,NW1                                                     MAIN1370
      II=(I-1)*NW1                                                      MAIN1380
      DO 29 J=1,I                                                       MAIN1390
   29 WRITE(7,44)J,I,C1(II+J)                                           MAIN1400
      WRITE(6,45)                                                       MAIN1410
      GO TO 1                                                           MAIN1420
C                                                                       MAIN1430
C     ERROR MESSAGES AND OUTPUT INFORMATION                             MAIN1440
```

119

```
C                                                                     MAIN1450
30    FORMAT(3I3)                                                     MAIN1460
31    FORMAT('1',100('*'),//,6X,'GENERALIZED AND TRANSMISSION LINE CAPACMAIN1470
     1ITANCE MATRICES OF "FLAT-PACK" CABLE',//,6X,'CONDITIONS:',/,11X,I3MAIN1480
     1,' WIRES, ',I3,' FOURIER COEFFICIENTS AROUND THE CONDUCTOR, AND ',MAIN1490
     1I3,' AROUND THE DIELECTRIC',/)                                    MAIN1500
32    FORMAT(/,' ***ERROR*** NUMBER OF WIRES LESS THAN TWO.  WILL SKIP  MAIN1510
     1THIS PROBLEM.',/)                                                 MAIN1520
33    FORMAT(/,' ***ERROR*** NUMBER OF COEFFICIENTS AROUND CONDUCTOR GIMAIN1530
     1VEN GREATER THAN NUMBER AROUND DIELECTRIC.',//,14X,'BOTH SET EQUAL MAIN1540
     1TO ',I3,'.',/)                                                    MAIN1550
34    FORMAT(4E10.0)                                                   MAIN1560
35    FORMAT(/,6X,'PHYSICAL CHARACTERISTICS:',/,10X,'CONDUCTOR RADIUS= 'MAIN1570
     1,1PE11.4,' (METERS)',/,10X,'DIELECTRIC RADIUS= ',E11.4,' (METERS)'MAIN1580
     1,/,10X,'CENTER-TO-CENTER SEPARATION= ',E11.4,' (METERS)',/,10X,'REMAIN1590
     1LATIVE DIELECTRIC CONSTANT= ',E11.4,/)                           MAIN1600
36    FORMAT(/,6X,'REFERENCE CONDUCTOR IS NO. ',I3,/,6X,'OPTION NUMBER IMAIN1610
     1S ',I3,//,' ',100('*'))                                          MAIN1620
37    FORMAT(/,' ***ERROR*** REFERENCE CONDUCTOR GIVEN OUT OF RANGE.  SMAIN1630
     1ET EQUAL TO ',I3,'.',/)                                          MAIN1640
38    FORMAT(/,' ***ERROR*** OPTION NUMBER OUT OF RANGE.  SET EQUAL TO MAIN1650
     1',I3,/)                                                          MAIN1660
39    FORMAT(/,' ***ERROR*** CARD WITH REFERENCE AND OPTION IN ERROR ORMAIN1670
     1 MISSING.  DEFAULT IREF=1, IOPT=0.',/)                           MAIN1680
40    FORMAT(/,' ***ERROR*** PHYSICAL CHARACTERISTICS CARD MISSING OR IMAIN1690
     1N ERROR.  WILL SKIP THIS PROBLEM.',/)                            MAIN1700
41    FORMAT(' ')                                                     MAIN1710
42    FORMAT(/,' ***ERROR*** CARDS MISSING.  END OF RUN.',/)          MAIN1720
43    FORMAT(/,' ***ERROR*** FIRST CARD IN ERROR.  WILL SKIP THIS PROBLMAIN1730
     1EM.',/)                                                          MAIN1740
44    FORMAT('C',2(2X,I3),1PE15.6)                                    MAIN1750
45    FORMAT(///,' UPPER TRIANGLE OF TRANSMISSION LINE CAPACITANCE MATRIMAIN1760
     1X WAS PUNCHED',///)                                             MAIN1770
46    FORMAT('1',///,2(10X,70('*'),///,22X,'GGG    EEEEE TTTTT  CCC    MAIN1780
     1AAA    PPPP',/,21X,'G   G E',8X,'T   C   A   A P   P',/,21X,'G   MAIN1790
     1G   EEEE  T   C   A   A P   P',//,21X,'G  GG EEEE  T         MAIN1800
```

120

```
1 C         AAAAA  PPPP',/,21X,'G   G    G   E',8X,'T    C       A   A   P','/MAIN1810
1,21X,'G   G   E',8X,'T   C   C   A   A   P','/,22X,'GGGG  EEEEE    T MAIN1820
1    CCC    A   A   P',///))                                          MAIN1830
47   FORMAT(18X,' GENERALIZED AND TRANSMISSION LINE CAPACITANCE MATRICEMAIN1840
1S',/,'+',18X,'    ',14X,'   ',17X,'   ',//,35X,'OF RIBBON CABLES',///MAIN1850
1//,11X,70('*'),7//////,30X,'B A T C H   R U N')                     MAIN1860
     END                                                             MAIN1870
```

121

```
C************************************************************************   GETC   10
C************************************************************************   GETC   20
C                                                                          GETC   30
C                                                                          GETC   40
C        SUBROUTINE GETCAP                                                  GETC   50
C                                                                          GETC   60
C     PURPOSE:                                                             GETC   70
C        TO COMPUTE THE GENERALIZED CAPACITANCE MATRIX, AND FROM           GETC   80
C        THAT THE TRANSMISSION LINE CAPACITANCE MATRIX FOR A               GETC   90
C        "FLAT-PACK" OR RIBBON CABLE WITH N IDENTICALLY ORIENTED           GETC  100
C        DIELECTRIC COVERED WIRES IN A HORIZONTAL PLANE.                   GETC  110
C                                                                          GETC  120
C     USAGE:                                                              GETC  130
C        CALL GETCAP(NW,NFC,NFD,NF,RC,RD,CCSEP,ER,IREF,IOPT,              GETC  140
C                    CG,CTL,D,D1,SCR,LT)                                   GETC  150
C                                                                          GETC  160
C     DESCRIPTION OF PARAMETERS:                                          GETC  170
C        NW  -  NUMBER OF WIRES IN THE PLANE                               GETC  180
C        NFC -  NUMBER OF FOURIER TERMS DESIRED AROUND THE CONDUCTOR       GETC  190
C        NFD -  NUMBER OF FOURIER TERMS AROUND THE DIELECTRIC (MUST        GETC  200
C               BE GREATER THAN OR EQUAL TO NFC)                           GETC  210
C        NF  -  NFC+NFD   (FOR IOPT=2, CAN BE EQUAL TO NFC)                GETC  220
C        RC  -  RADIUS OF THE CONDUCTOR                                    GETC  230
C        RD  -  RADIUS OF THE DIELECTRIC FROM THE CENTER OF THE WIRE       GETC  240
C        CCSEP- CENTER-TO-CENTER SPACING BETWEEN THE CONDUCTORS            GETC  250
C               NOTE:  ALL DIMENSIONS MUST BE IN THE SAME UNITS.           GETC  260
C        ER  -  RELATIVE DIELECTRIC CONSTANT OF THE INSULATION             GETC  270
C        IREF-  NUMBER OF THE REFERENCE CONDUCTOR ("1" INDICATES THE       GETC  280
C               FIRST WIRE.)                                               GETC  290
C        IOPT-  OPTION SELECTOR--                                          GETC  300
C               IOPT=0    ONLY HALF OF D1 INVERTED - FAST METHOD           GETC  310
C               IOPT=1    FULL INVERSION OF D1 - LONG METHOD               GETC  320
C               IOPT=2    DIELECTRIC INSULATION IGNORED -                  GETC  330
C                         BARE WIRE PROBLEM                                GETC  340
C        CG  -  HOLDS GENERALIZED CAPACITANCE MATRIX ON RETURN             GETC  350
C               DIMENSION IS NW BY NW.                                     GETC  360
```

122

```
C      CTL  - HOLDS TRANSMISSION LINE CAPACITANCE MATRIX ON RETURN   GETC 370
C              DIMENSION IS (NW-1) BY (NW-1).                         GETC 380
C      D    - WORKING SQUARE MATRIX OF ORDER (NFC+NFD)                GETC 390
C      D1   - WORKING VECTOR OF DIMENSION  ((NFC+NFD)*NW)**2          GETC 400
C      SCR  - SCRATCH VECTOR OF DIMENSION (NF*NW+1)/2                 GETC 410
C      LT   - SCRATCH VECTOR OF DIMENSION 2*NF*NW                     GETC 420
C                                                                     GETC 430
C      SUBROUTINES REQUIRED:                                         GETC 440
C      MINV - MATRIX INVERSION ROUTINE (FROM IBM SSP)                GETC 450
C      MPC  - MATRIX MULTIPLICATION WITH A CONSTANT MULTIPLIED       GETC 460
C              BY THE RESULT                                          GETC 470
C                                                                     GETC 480
C*********************************************************************GETC 490
C                                                                     GETC 500
       SUBROUTINE GETCAP(NW,NFC,NFD,NF,RC,RD,CCSEP,ER,IREF,IOPT,     GETC 510
      1                  CG,CTL,D,D1,SCR,LT)                          GETC 520
       COMPLEX   MP, CMPLX                                            GETC 530
       DIMENSION CG(NW,NW),CTL(1),D(NF,NF),SCR(1),LT(1),D1(1)         GETC 540
C                                                                     GETC 550
C      CONSTANTS AND COMMON COMPUTATIONS                             GETC 560
C                                                                     GETC 570
       PI=3.141593E0                                                  GETC 580
       EPS=8.854185E-12                                               GETC 590
       NW1=NW-1                                                       GETC 600
       NW12=NW1*NW1                                                   GETC 610
       NFC1=NFC+1                                                     GETC 620
       NFD1=NFD+1                                                     GETC 630
C      NUMBER OF COSINE TERMS NEEDED                                 GETC 640
       NCC=NFC-1                                                      GETC 650
       NCD=NFD-1                                                      GETC 660
       ER1=ER-1.0E0                                                   GETC 670
       ER2=ER+1.0E0                                                   GETC 680
C      ANGLE BETWEEN MATCH POINTS                                    GETC 690
       AC=2.0E0*PI/NFC                                                GETC 700
       AD=2.0E0*PI/NFD                                                GETC 710
C      ANGULAR ROTATION OF MATCHPOINTS FROM 0 DEGREES               GETC 720
```

123

```
      DELTC=PI/(2.0E0*NFC)                                          GETC 730
      DELTD=PI/(2.0E0*NFD)                                          GETC 740
C     CHECK FOR NO DIELECTRIC CASE                                  GETC 750
      IOPT2=IOPT-2                                                  GETC 760
      IF(IOPT2)2,1,2                                                GETC 770
1     NF=NFC                                                        GETC 780
      IOPT=1                                                        GETC 790
C     ORDER OF "D1" MATRIX                                          GETC 800
2     LD=NF*NW                                                      GETC 810
C     ORDER OF THE PARTITIONS OF THE "D1" MATRIX                    GETC 820
      LP1=LD/2                                                      GETC 830
      LP2=LD-LP1                                                    GETC 840
      LS1=LP1*LP1                                                   GETC 850
      LX=LP1*LP2                                                    GETC 860
C                                                                   GETC 870
C******  SCALE INPUT DATA WITH RC BETWEEN 0.1 AND 1.0 TO PREVENT ******GETC 880
C              COMPUTER NUMERICAL INCONSISTENCIES                   GETC 890
C                                                                   GETC 900
      J=-ALOG10(RC)                                                 GETC 910
      A1=10**J                                                      GETC 920
      RC=RC*A1                                                      GETC 930
      RD=RD*A1                                                      GETC 940
      CCSEP=CCSEP*A1                                                GETC 950
C                                                                   GETC 960
C******  COMPUTATION OF THE OFF-DIAGONAL "D" SUBMATRICES  ******    GETC 970
C                                                                   GETC 980
      IW=1                                                          GETC 990
      MW=1                                                          GETC1000
      LU=NW                                                         GETC1010
      GO TO 4                                                       GETC1020
C                                                                   GETC1030
C     MW IS THE PRESENT MATCH WIRE                                  GETC1040
C     IW INDICATES FROM WHICH END OF THE CABLE THE PRESENT POSITION GETC1050
C        IS COMPUTED.                                               GETC1060
C     MPP IS THE PRESENT MATCH POINT ON THE BOUNDARY                GETC1070
C     XSEP IS THE SEPARATION DISTANCE FROM MW TO IW                 GETC1080
```

124

```
C                                                                      GETC1090
3     IW=NW                                                            GETC1100
      MW=0                                                             GETC1110
      LU=NW-1                                                          GETC1120
4     MW=MW+1                                                          GETC1130
      XSEP=(IW-MW)*CCSEP                                               GETC1140
C                                                                      GETC1150
C          THIS SECTION COMPUTES THE POTENTIALS ON THE MATCH WIRE DUE TO    GETC1160
C          THE WIRE AND DIELECTRIC OF THE END WIRE                     GETC1170
C                                                                      GETC1180
C                                                                      GETC1190
      DO  13 MPP=1,NFC                                                 GETC1200
      ANG=(MPP-1)*AC+DELTC                                             GETC1210
      CANG= COS(ANG)                                                   GETC1220
      SANG= SIN(ANG)                                                   GETC1230
      MP= CMPLX(RC*CANG+XSEP,RC*SANG)                                  GETC1240
      RO= CABS(MP)                                                     GETC1250
      AO= ATAN2(AIMAG(MP), REAL(MP))                                   GETC1260
      A1=ALOG(RO)                                                      GETC1270
      D(MPP,1)=-RC*A1                                                  GETC1280
5     IF(IOPT2)5,6,5                                                   GETC1290
6     D(MPP,NFC1)=-RD*A1                                               GETC1300
      A2=RC                                                            GETC1310
      A3=RD                                                            GETC1320
      A11=RO*2.0E0                                                     GETC1330
      IF(NCC)10,10,7                                                   GETC1340
7     DO  9  J=1,NCC                                                   GETC1350
      J1=J+1                                                           GETC1360
      J2=NFC+J1                                                        GETC1370
      A1= COS(J*AO)/(J*A11)                                            GETC1380
      A2=A2*RC                                                         GETC1390
      A3=A3*RD                                                         GETC1400
      A11=A11*RO                                                       GETC1410
      IF(IOPT2)8,9,8                                                   GETC1420
8     D(MPP,J2)=A1*A3                                                  GETC1430
9     D(MPP,J1)=A1*A2                                                  GETC1440
      IF(IOPT2)10,13,10
```

125

```
10      IF(NFC-NCD)11,11,13                                          GETC1450
11      DO 12 J=NFC,NCD                                              GETC1460
        J2=J+NFC1                                                    GETC1470
        A1= COS(J*AO)/(J*A11)                                        GETC1480
        A3=A3*RD                                                     GETC1490
        A11=A11*RO                                                   GETC1500
12      D(MPP,J2)=A1*A3                                              GETC1510
13      CONTINUE                                                     GETC1520
        IF(IOPT2)14,22,14                                            GETC1530
C                                                                    GETC1540
C          THIS SECTION COMPUTES THE ELECTRIC FIELD COMPONENTS ON THE   GETC1550
C          MATCH WIRE DUE TO BOTH THE CONDUCTOR AND DIELECTRIC OF THE   GETC1560
C          END WIRE                                                  GETC1570
C                                                                    GETC1580
14      DO 21 MPP=1,NFD                                              GETC1590
        ANG=(MPP-1)*AD+DELTD                                         GETC1600
        CANG= COS(ANG)                                               GETC1610
        SANG= SIN(ANG)                                               GETC1620
        MP= CMPLX(RD*CANG+XSEP,RD*SANG)                              GETC1630
        RO= CABS(MP)                                                 GETC1640
        AO= ATAN2(AIMAG(MP), REAL(MP))                               GETC1650
        A1=(CANG* COS(AO)+SANG* SIN(AO))*ER1/RO                      GETC1660
        JJ=NFC+MPP                                                   GETC1670
        D(JJ,1)=RC*A1                                                GETC1680
15      D(JJ,NFC1)=RD*A1                                             GETC1690
        A2=RC                                                        GETC1700
        A3=RD                                                        GETC1710
        A11=RO*2.0E0                                                 GETC1720
        IF(NCC)18,18,16                                              GETC1730
16      DO 17 J=1,NCC                                                GETC1740
        J1=J+1                                                       GETC1750
        J2=NFC+J1                                                    GETC1760
        A2=A2*RC                                                     GETC1770
        A3=A3*RD                                                     GETC1780
        A11=A11*RO                                                   GETC1790
        A1=(CANG* COS(J1*AO)+SANG* SIN(J1*AO))*ER1/A11               GETC1800
```

```
      D(JJ,J1)=A1*A2                                           GETC1810
      D(JJ,J2)=A1*A3                                           GETC1820
   17 IF(NFC-NCD)19,19,21                                      GETC1830
   18 DO 20 J=NFC,NCD                                          GETC1840
   19 J1=J+1                                                   GETC1850
      J2=NFC1+J                                                GETC1860
      A3=A3*RD                                                 GETC1870
      A11=A11*RO                                               GETC1880
      A1=(CANG* COS(J1*AO)+SANG* SIN(J1*AO))*ER1/A11           GETC1890
   20 D(JJ,J2)=A1*A3                                           GETC1900
   21 CONTINUE                                                 GETC1910
C                                                              GETC1920
C      THIS SECTION TELLS WHERE TO PUT THE SUBMATRIX "D" IN THE LARGE  GETC1930
C      MATRIX "D1"                                             GETC1940
C                                                              GETC1950
   22 MM=IW                                                    GETC1960
      NN=MW                                                    GETC1970
      GO TO 89                                                 GETC1980
   23 IF(IW-1)24,24,26                                         GETC1990
   24 IF(NW-NN)28,28,25                                        GETC2000
   25 MM=MM+1                                                  GETC2010
      NN=NN+1                                                  GETC2020
      GO TO 89                                                 GETC2030
   26 IF(NN-1)28,28,27                                         GETC2040
   27 MM=MM-1                                                  GETC2050
      NN=NN-1                                                  GETC2060
      GO TO 89                                                 GETC2070
   28 IF(MW-LU)4,29,29                                         GETC2080
   29 IF(IW-1)3,3,30                                           GETC2090
C                                                              GETC2100
C******  COMPUTATION OF DIAGONAL "D" MATRIX    ******         GETC2110
C                                                              GETC2120
C                                                              GETC2130
C      THIS SECTION COMPUTES THE POTENTIALS ON THE CONDUCTORS DUE TO   GETC2140
C      THEMSELVES AND ALSO THE DIELECTRIC SURROUNDING THEM     GETC2150
C                                                              GETC2160
```

127

```
30    A1=-RC*ALOG(RC)                                      GETC2170
      DO 31 J=1,NFC                                        GETC2180
31    D(J,1)=A1                                            GETC2190
      IF(IOPT2)32,35,32                                    GETC2200
32    A2=-RD*ALOG(RD)                                      GETC2210
      DO 33 J=1,NFC                                        GETC2220
33    D(J,NFC1)=A2                                         GETC2230
      A1=ER1*RC/RD                                         GETC2240
      DO 34 J=NFC1,NF                                      GETC2250
      D(J,1)=A1                                            GETC2260
34    D(J,NFC1)=-1.0E0                                     GETC2270
35    A1=RC/RD                                             GETC2280
      DO 43 MPP=1,NFC                                      GETC2290
      ANG=(MPP-1)*AC+DELTC                                 GETC2300
      A3=1.0E0                                             GETC2310
      IF(NCC)39,39,36                                      GETC2320
36    DO 38 J=1,NCC                                        GETC2330
      J1=J+1                                               GETC2340
      D(MPP,J1)=RC* COS(J*ANG)/2.0E0/J                     GETC2350
      IF(IOPT2)37,38,37                                    GETC2360
37    D(MPP,J1+NFC)=D(MPP,J1)*A3                           GETC2370
38    A3=A3*A1                                             GETC2380
39    IF(IOPT2)40,43,40                                    GETC2390
40    IF(NFC-NCD)41,41,43                                  GETC2400
41    DO 42 J=NFC,NCD                                      GETC2410
      D(MPP,J+1+NFC)=RC*A3* COS(J*ANG)/2.0E0/J             GETC2420
42    A3=A3*A1                                             GETC2430
43    CONTINUE                                             GETC2440
      IF(IOPT2)44,51,44                                    GETC2450
C                                                          GETC2460
C         THIS SECTION COMPUTES THE FIELD ON THE DIELECTRIC DUE TO THE  GETC2470
C         DIELECTRIC ITSELF AND ALSO THE CONDUTORS INSIDE THEM GETC2480
C                                                          GETC2490
44    A12=A1*A1                                            GETC2500
      DO 50 MPP=NFC1,NF                                    GETC2510
      ANG=(MPP-NFC1)*AD+DELTD                              GETC2520
```

128

```
      A3=1.0E0                                              GETC2530
      IF(NCC)47,47,45                                       GETC2540
45    DO 46 J=1,NCC                                         GETC2550
      J1=J+1                                                GETC2560
      A2= COS(J*ANG)/2.0E0                                  GETC2570
      D(MPP,J1)=ER1*A3*A12*A2                               GETC2580
      D(MPP,J1+NFC)=-ER2*A2                                 GETC2590
46    A3=A3*A1                                              GETC2600
47    IF(NFC-NCD)48,48,50                                   GETC2610
48    DO 49 J=NFC,NCD                                       GETC2620
49    D(MPP,J1+NFC)=-ER2* COS(J*ANG)/2.0E0                  GETC2630
50    CONTINUE                                              GETC2640
C                                                           GETC2650
C         THIS SECTION INSERTS THE SUBMATRIX ON THE DIAGONAL OF THE    GETC2660
C         LARGE "D1" MATRIX NW TIMES                        GETC2670
C                                                           GETC2680
51    MM=1                                                  GETC2690
      NN=1                                                  GETC2700
      GO TO 89                                              GETC2710
52    MM=MM+1                                               GETC2720
      NN=NN+1                                               GETC2730
      IF(NW-MM)53,89,89                                     GETC2740
C                                                           GETC2750
C******* INVERSION OF "D1" AND COMPUTATION OF THE GENERALIZED *******  GETC2760
C                  CAPACITANCE MATRIX CG                    GETC2770
C                                                           GETC2780
C         IF IOPT=1, A FULL MATRIX INVERSION OF THE "D1" MATRIX IS DONE GETC2790
C         THEN THE TERMS OF THE GENERALIZED CAPACITANCE MATRIX ARE     GETC2800
C         COMPUTED                                          GETC2810
C                                                           GETC2820
53    A3=2.0E0*PI*EPS                                       GETC2830
      IF(IOPT-1)61,54,61                                    GETC2840
54    CALL MINV(D1,LD,DET,LT(1),LT(LD+1))                   GETC2850
      ISTP=1                                                GETC2860
      IF(DET)55,102,55                                      GETC2870
55    DO 60 I=1,NW                                          GETC2880
```

129

```
      IROW=(I-1)*NF+1                                                GETC2890
      DO 60 J=1,NW                                                   GETC2900
      IL=(J-1)*LD*NF+IROW                                            GETC2910
      A1=0.0E0                                                       GETC2920
      A2=0.0E0                                                       GETC2930
      DO 57 K=1,NFC                                                  GETC2940
C                                                                    GETC2950
C     TERMS OF THE CAPACITANCE MATRIX ARE DERIVED FROM THE SUM OF    GETC2960
C     CERTAIN ROWS OF THE MATRIX                                     GETC2970
C                                                                    GETC2980
      A1=A1+D1(IL)                                                   GETC2990
      IF(IOPT2)56,57,56                                              GETC3000
   56 A2=A2+D1(IL+NFC)                                               GETC3010
   57 IL=IL+LD                                                       GETC3020
      IF(IOPT2)59,58,59                                              GETC3030
   58 CG(I,J)=A1*RC*A3                                               GETC3040
      GO TO 60                                                       GETC3050
   59 CG(I,J)=(A1*RC+A2*RD)*A3                                       GETC3060
   60 CONTINUE                                                       GETC3070
      GO TO 79                                                       GETC3080
C                                                                    GETC3090
C     WHEN IOPT DOES NOT EQUAL 1, THE PROGRAM PARTITONS THE "D1"     GETC3100
C     MATRIX AND COMPUTES ONLY HALF OF THE INVERSE OF "D1".   SINCE  GETC3110
C     THE GENERALIZED CAPACITANCE MATRIX IS IDEALLY "SUPER-SYMMETRIC"GETC3120
C     ONLY A FEW OF THESE TERMS ARE ACTUALLY COMPUTED.   THE REST AREGETC3130
C     COPIED FROM THOSE WHICH ARE DONE                               GETC3140
C                                                                    GETC3150
   61 NP=1                                                           GETC3160
      NQ=NP+LS1                                                      GETC3170
      NR=NQ+LX                                                       GETC3180
      NS=NR+LX                                                       GETC3190
      CALL  MINV(D1(NS),LP2,DET,LT(1),LT(LP2+1))                     GETC3200
      ISTP=2                                                         GETC3210
      IF(DET)62,102,62                                               GETC3220
   62 CALL  MPC(D1(NS),D1(NR),D1(NS),SCR,LP2,LP2,LP1,1.0E0)          GETC3230
      CALL  MPC(D1(NQ),D1(NS),SCR,LP1,LP2,LP1,1.0E0)                 GETC3240
```

130

```
            II=0
      DO 64  J=1,LP1
      DO 63  I=1,LP1
      I2=II+I
  63  D1(I2)=D1(I2)-D1(I2+LS1)
  64  II=II+LP1
      CALL  MINV(D1(NP),LP1,DET,LT(1),LT(LP1+1))
      ISTP=3
      IF(DET)65,102,65
  65  CALL  MPC(D1(NS),D1(NP),D1(NR),SCR,LP2,LP1,LP1,-1.0E0)
      IA=(LD-1)*LP1
      IL=1
      IH=NW
  66  ICOL=(IL-1)*NF
      DO 73  I=IL,IH
      IROW=(I-1)*NF+1
      IROW1=IROW+NFC
      IF(LP1-IROW)69,67,67
  67  INC=LP1
      IS=ICOL*LP1+IROW
      IF(LP1-IROW1)70,68,68
  68  INC1=LP1
      IS1=IS+NFC
      GO TO 71
  69  INC=LP2
      IS=IA+ICOL*LP2+IROW
  70  INC1=LP2
      IS1=IA+ICOL*LP2+IROW1
      IS=IS+INC
  71  A1=0.0E0
      A2=0.0E0
      DO 72  J=1,NFC
      A1=A1+D1(IS)
      A2=A2+D1(IS1)
      IS=IS+INC
  72  IS1=IS1+INC1
  73  CG(I,IL)=(A1*RC+A2*RD)*A3
```

GETC3250
GETC3260
GETC3270
GETC3280
GETC3290
GETC3300
GETC3310
GETC3320
GETC3330
GETC3340
GETC3350
GETC3360
GETC3370
GETC3380
GETC3390
GETC3400
GETC3410
GETC3420
GETC3430
GETC3440
GETC3450
GETC3460
GETC3470
GETC3480
GETC3490
GETC3500
GETC3510
GETC3520
GETC3530
GETC3540
GETC3550
GETC3560
GETC3570
GETC3580
GETC3590
GETC3600

131

```
      IL=IL+1                                            GETC3610
      IH=IH-1                                            GETC3620
      IF(IH-IL)74,66,66                                  GETC3630
C                                                        GETC3640
C     THIS SECTION FILLS OUT THE REST OF THE "C" MATRIX  GETC3650
C                                                        GETC3660
74    NW2=NW/2                                           GETC3670
      NP=NW+1                                            GETC3680
      NS=2                                               GETC3690
      NQ=NW                                              GETC3700
      DO 76 I=1,NW2                                       GETC3710
      DO 75 J=NS,NQ                                       GETC3720
75    CG(NQ,J)=CG(NP-J,I)                                GETC3730
      NS=NS+1                                            GETC3740
76    NQ=NQ-1                                            GETC3750
      NS=2                                               GETC3760
      DO 78 I=1,NW1                                       GETC3770
      DO 77 J=NS,NW                                        GETC3780
77    CG(I,J)=CG(J,I)                                    GETC3790
78    NS=NS+1                                            GETC3800
C                                                        GETC3810
C****** THIS SECTION COMPUTES THE TRANSMISSION LINE CAPACITANCE ******GETC3820
C       MATRIX FROM THE GENERALIZED CAPACITANCE MATRIX   GETC3830
C                                                        GETC3840
C       A1 IS THE MATRIX SUM                             GETC3850
C       A2 IS THE ROWSUM                                 GETC3860
C       A3 IS THE COLUMN SUM- - STORED IN D1(1-NW1)      GETC3870
C                                                        GETC3880
79    A1=0.0E0                                           GETC3890
      DO 80 I=1,NW                                        GETC3900
80    A1=A1+CG(IREF,I)                                   GETC3910
      II=1                                               GETC3920
      DO 84 I=1,NW                                        GETC3930
      IF(I-IREF)81,84,81                                 GETC3940
81    A2=0.0E0                                           GETC3950
      A3=0.0E0                                           GETC3960
```

132

```
         DO 82 J=1,NW                                        GETC3970
         A2=A2+CG(I,J)                                       GETC3980
         A3=A3+CG(J,I)                                       GETC3990
         A1=A1+A2                                            GETC4000
         D1(II)=A3                                           GETC4010
   82    DO 83 J=II,NW12,NW1                                 GETC4020
         CTL(J)=A2                                           GETC4030
         II=II+1                                             GETC4040
   83    CONTINUE                                            GETC4050
   84    II=1                                                GETC4060
         JJ=1                                                GETC4070
         DO 88 J=1,NW                                        GETC4080
         IF(J-IREF)85,88,85                                  GETC4090
   85    DO 87 I=1,NW                                        GETC4100
         IF(I-IREF)86,87,86                                  GETC4110
   86    CTL(II)=CG(I,J)-CTL(II)*D1(JJ)/A1                   GETC4120
         II=II+1                                             GETC4130
   87    CONTINUE                                            GETC4140
         JJ=JJ+1                                             GETC4150
   88    CONTINUE                                            GETC4160
C                                                            GETC4170
C        AFTER THE PER-UNIT-LENGTH GENERALIZED AND TRANSMISSION LINE   GETC4180
C        CAPACITANCE MATRICES HAVE BEEN CALCULATED, CONTROL RETURNS    GETC4190
C        TO THE CALLING PROGRAM                             GETC4200
C                                                            GETC4210
         RETURN                                              GETC4220
C                                                            GETC4230
C******  THIS SECTION INSERTS THE SMALL "D" MATRICES INTO THE   ****** GETC4240
C                   LARGE "D1" MATRIX                         GETC4250
C                                                            GETC4260
C        WHEN IOPT IS 1, THE MATRICES ARE INSERTED DIRECTLY.  GETC4270
C        WHEN IOPT IS 0, "D1" IS PARTITIONED AND "D" IS INSERTED  GETC4280
C              ACCORDINGLY.                                   GETC4290
C                                                            GETC4300
   89    IF(IOPT-1)90,99,90                                  GETC4310
   90    ICB=(NN-1)*NF                                       GETC4320
```

133

```
        IRB=(MM-1)*NF                                              GETC4330
        IRX=IRB-LP1                                                GETC4340
        IF(IRX)91,94,94                                            GETC4350
91      NP=ICB*LP1+IRB                                            GETC4360
        NQ=MIN0(NF,-IRX)                                           GETC4370
        DO 93 J=1,NF                                               GETC4380
        DO 92 I=1,NQ                                               GETC4390
92      D1(NP+I)=D(I,J)                                            GETC4400
93      NP=NP+LP1                                                  GETC4410
        NP=-NQ                                                     GETC4420
        IF(NQ-NF)95,98,98                                          GETC4430
94      NQ=0                                                       GETC4440
        NP=IRX                                                     GETC4450
95      NQ=NQ+1                                                    GETC4460
        NP=NP+LD*LP1+ICB*LP2                                       GETC4470
        DO 97 J=1,NF                                               GETC4480
        DO 96 I=NQ,NF                                              GETC4490
96      D1(NP+I)=D(I,J)                                            GETC4500
97      NP=NP+LP2                                                  GETC4510
98      IF(MM-NN)23,52,23                                          GETC4520
99      NP=NF*(MM-1+(NN-1)*LD)                                     GETC4530
        DO 101 I=1,NF                                              GETC4540
        DO 100 J=1,NF                                              GETC4550
100     D1(NP+J)=D(J,I)                                            GETC4560
101     NP=NP+LD                                                   GETC4570
        GO TO 98                                                   GETC4580
C                                                                  GETC4590
C****** ERROR RETURN *******                                       GETC4600
C                                                                  GETC4610
102     WRITE(6,103)ISTP                                           GETC4620
        NW=0                                                       GETC4630
        RETURN                                                     GETC4640
103     FORMAT(' ** SINGULAR MATRIX AT STEP ',I1)                  GETC4650
        END                                                        GETC4660
```

```
C                                                                 MINV   10
C     .............................................................MINV   20
C                                                                 MINV   30
C        SUBROUTINE   MINV                                        MINV   40
C                                                                 MINV   50
C        PURPOSE                                                  MINV   60
C           INVERT A MATRIX                                       MINV   70
C                                                                 MINV   80
C        USAGE                                                    MINV   90
C           CALL  MINV(A,N,D,L,M)                                 MINV  100
C                                                                 MINV  110
C        DESCRIPTION OF PARAMETERS                                MINV  120
C           A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY MINV 130
C               RESULTANT INVERSE.                                MINV  140
C           N - ORDER OF MATRIX A                                 MINV  150
C           D - RESULTANT DETERMINANT                             MINV  160
C           L - WORK VECTOR OF LENGTH N                           MINV  170
C           M - WORK VECTOR OF LENGTH N                           MINV  180
C                                                                 MINV  190
C        REMARKS                                                  MINV  200
C           MATRIX A MUST BE A GENERAL MATRIX                     MINV  210
C                                                                 MINV  220
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED            MINV  230
C           NONE                                                  MINV  240
C                                                                 MINV  250
C        METHOD                                                   MINV  260
C           THE STANDARD GAUSS-JORDAN METHOD IS USED.  THE DETERMINANT MINV 270
C           IS ALSO CALCULATED.  A DETERMINANT OF ZERO INDICATES THAT MINV 280
C           THE MATRIX IS SINGULAR.                              MINV  290
C                                                                 MINV  300
C     .............................................................MINV  310
C                                                                 MINV  320
      SUBROUTINE  MINV (A,N,D,L,M)                                MINV  330
      DIMENSION A(1),L(1),M(1)                                    MINV  340
C                                                                 MINV  350
C        SEARCH FOR LARGEST ELEMENT                               MINV  360
```

135

```
C                                                         MINV 370
      D=1.0                                               MINV 380
      NK=-N                                               MINV 390
      DO 19 K=1,N                                         MINV 400
      NK=NK+N                                             MINV 410
      L(K)=K                                              MINV 420
      M(K)=K                                              MINV 430
      KK=NK+K                                             MINV 440
      BIGA=A(KK)                                          MINV 450
      DO 3 J=K,N                                          MINV 460
      IZ=N*(J-1)                                          MINV 470
      DO 3 I=K,N                                          MINV 480
      IJ=IZ+I                                             MINV 490
1     IF( ABS(BIGA) - ABS(A(IJ))) 2,3,3                   MINV 500
2     BIGA=A(IJ)                                          MINV 510
      L(K)=I                                              MINV 520
      M(K)=J                                              MINV 530
3     CONTINUE                                            MINV 540
C                                                         MINV 550
C     INTERCHANGE ROWS                                    MINV 560
C                                                         MINV 570
      J=L(K)                                              MINV 580
      IF(J-K) 6,6,4                                       MINV 590
4     KI=K-N                                              MINV 600
      DO 5 I=1,N                                          MINV 610
      KI=KI+N                                             MINV 620
      HOLD=-A(KI)                                         MINV 630
      JI=KI-K+J                                           MINV 640
      A(KI)=A(JI)                                         MINV 650
5     A(JI) =HOLD                                         MINV 660
C                                                         MINV 670
C     INTERCHANGE COLUMNS                                 MINV 680
C                                                         MINV 690
6     I=M(K)                                              MINV 700
      IF(I-K) 9,9,7                                       MINV 710
7     JP=N*(I-1)                                          MINV 720
```

```
      DO 8 J=1,N                                              MINV 730
      JK=NK+J                                                 MINV 740
      JI=JP+J                                                 MINV 750
      HOLD=-A(JK)                                             MINV 760
      A(JK)=A(JI)                                             MINV 770
    8 A(JI) =HOLD                                             MINV 780
C                                                             MINV 790
C     DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS MINV 800
C     CONTAINED IN BIGA)                                      MINV 810
C                                                             MINV 820
    9 IF(BIGA) 11,10,11                                       MINV 830
   10 D=0.0                                                   MINV 840
      RETURN                                                  MINV 850
   11 DO 13 I=1,N                                             MINV 860
      IF(I-K) 12,13,12                                        MINV 870
   12 IK=NK+I                                                 MINV 880
      A(IK)=A(IK)/(-BIGA)                                     MINV 890
   13 CONTINUE                                                MINV 900
C                                                             MINV 910
C     REDUCE MATRIX                                           MINV 920
C                                                             MINV 930
      DO 16 I=1,N                                             MINV 940
      IK=NK+I                                                 MINV 950
      HOLD=A(IK)                                              MINV 960
      IJ=I-N                                                  MINV 970
      DO 16 J=1,N                                             MINV 980
      IJ=IJ+N                                                 MINV 990
      IF(I-K) 14,16,14                                        MINV1000
   14 IF(J-K) 15,16,15                                        MINV1010
   15 KJ=IJ-I+K                                               MINV1020
      A(IJ)=HOLD*A(KJ)+A(IJ)                                  MINV1030
   16 CONTINUE                                                MINV1040
C                                                             MINV1050
C     DIVIDE ROW BY PIVOT                                     MINV1060
C                                                             MINV1070
      KJ=K-N                                                  MINV1080
```

```
        DO 18 J=1,N                            MINV1090
        KJ=KJ+N                                MINV1100
17      IF(J-K) 17,18,17                       MINV1110
18      A(KJ)=A(KJ)/BIGA                       MINV1120
        CONTINUE                               MINV1130
C                                              MINV1140
C       PRODUCT OF PIVOTS                      MINV1150
C                                              MINV1160
        D=D*BIGA                               MINV1170
C                                              MINV1180
C       REPLACE PIVOT BY RECIPROCAL            MINV1190
C                                              MINV1200
        A(KK)=1.0/BIGA                         MINV1210
19      CONTINUE                               MINV1220
C                                              MINV1230
C       FINAL ROW AND COLUMN INTERCHANGE       MINV1240
C                                              MINV1250
        K=N                                    MINV1260
20      K=(K-1)                                MINV1270
        IF(K) 27,27,21                         MINV1280
21      I=L(K)                                 MINV1290
        IF(I-K) 24,24,22                       MINV1300
22      JQ=N*(K-1)                             MINV1310
        JR=N*(I-1)                             MINV1320
        DO 23 J=1,N                            MINV1330
        JK=JQ+J                                MINV1340
        HOLD=A(JK)                             MINV1350
        JI=JR+J                                MINV1360
        A(JK)=-A(JI)                           MINV1370
23      A(JI) =HOLD                            MINV1380
24      J=M(K)                                 MINV1390
        IF(J-K) 20,20,25                       MINV1400
25      KI=K-N                                 MINV1410
        DO 26 I=1,N                            MINV1420
        KI=KI+N                                MINV1430
        HOLD=A(KI)                             MINV1440
```

138

```
        JI=KI-K+J          MINV1450
26      A(KI)=-A(JI)       MINV1460
        A(JI) =HOLD        MINV1470
        GO TO 20           MINV1480
27      RETURN             MINV1490
        END                MINV1500
```

```
C************************************************************  MPC   10
C                                                             MPC   20
C     SUBROUTINE MPC                                          MPC   30
C                                                             MPC   40
C     PURPOSE                                                 MPC   50
C        TO MULTIPLY TWO GENERAL MATRICES, AND THEN MULTIPLY THAT  MPC   60
C        RESULT BY A CONSTANT                                 MPC   70
C                                                             MPC   80
C                                                             MPC   90
C     USAGE                                                   MPC  100
C        CALL MPC(A,B,R,S,L,M,N,C)                            MPC  110
C                                                             MPC  120
C     DESCRIPTION OF PARAMETERS                               MPC  130
C        A - FIRST MATRIX, L BY M                             MPC  140
C        B - SECOND MATRIX, M BY N                            MPC  150
C        R - RESULTANT MATRIX (CAN BE SAME AS A), L BY N      MPC  160
C        S - WORKING VECTOR, DIMENSION M                      MPC  170
C        L - NUMBER OF ROWS IN A AND R                        MPC  180
C        M - NUMBER OF COLUMNS IN A AND ROWS IN B             MPC  190
C        N - NUMBER OF COLUMNS IN B AND R                     MPC  200
C        C - CONSTANT BY WHICH R IS MULTIPLIED                MPC  210
C                                                             MPC  220
C     REMARKS                                                 MPC  230
C        NOTE THAT A AND R CAN BE THE SAME MATRIX IF DIMENSIONED  MPC  240
C        CORRECTLY                                            MPC  250
C                                                             MPC  260
C************************************************************  MPC  270
C                                                             MPC  280
      SUBROUTINE MPC(A,B,R,S,L,M,N,C)                         MPC  290
      DIMENSION A(L,M),B(M,N),R(L,N),S(M)                     MPC  300
      DO 3 I=1,L                                              MPC  310
      DO 1 J=1,M                                              MPC  320
    1 S(J)=A(I,J)                                             MPC  330
      DO 3 J=1,N                                              MPC  340
      SUM=0.0E0                                               MPC  350
      DO 2 K=1,M                                              MPC  360
```

140

```
      SUM=SUM+S(K)*B(K,J)                         MPC   370
2     R(I,J)=SUM*C                                MPC   380
3     RETURN                                      MPC   390
      END                                         MPC   400
```

```
C***********************************************************************MPRT  10
C***********************************************************************MPRT  20
C                                                                      MPRT  30
C        SUBROUTINE MPRT                                               MPRT  40
C                                                                      MPRT  50
C        PURPOSE:                                                      MPRT  60
C           TO PRINT OUT A MATRIX IN MATRIX FORM WITH LABELING         MPRT  70
C                                                                      MPRT  80
C        USAGE:                                                        MPRT  90
C           CALL MPRT(A,M,N,B,J)                                       MPRT 100
C                                                                      MPRT 110
C        DESCRIPTION OF PARAMETERS:                                    MPRT 120
C           A - INPUT MATRIX                                           MPRT 130
C           M - NUMBER OF ROWS IN A                                    MPRT 140
C           N - NUMBER OF COLUMNS IN A                                 MPRT 150
C           B - LITERAL INPUT OF TITLE.  HOLLERITH FORM.               MPRT 160
C           J - NUMBER OF CHARACTERS, INCLUDING SPACES, ETC. IN B      MPRT 170
C                                                                      MPRT 180
C***********************************************************************MPRT 190
C                                                                      MPRT 200
      SUBROUTINE MPRT(A,M,N,B,J)                                       MPRT 210
      DIMENSION A(M,N),B(J),C(18)                                      MPRT 220
      LOGICAL*1 B,C/' ',' ','M','A','T','R','I','X',' ',' ','-',' ','P',MPRT 230
     1'A','G','E',' ','1'/                                             MPRT 240
      WRITE(6,3)B,C                                                    MPRT 250
      I=2                                                              MPRT 260
      LL=1                                                             MPRT 270
      LU=MIN0(LL+10,N)                                                 MPRT 280
    1 WRITE(6,6)(L,L=LL,LU)                                            MPRT 290
      WRITE(6,7)                                                       MPRT 300
      DO 2 K=1,M                                                       MPRT 310
    2 WRITE(6,4)K,(A(K,L),L=LL,LU)                                     MPRT 320
      IF(LU.EQ.N)RETURN                                                MPRT 330
      WRITE(6,5)I                                                      MPRT 340
      LL=LU+1                                                          MPRT 350
      I=I+1                                                            MPRT 360
```

142

```
      GO TO 1                                              MPRT 370
3     FORMAT('1',132A1)                                    MPRT 380
4     FORMAT(9X,'*',/,4X,I3,2X,'*',1P11E11.2)              MPRT 390
5     FORMAT('1PAGE ',I2,' RIGHT - - - -.')                MPRT 400
6     FORMAT(/,5X,'COLUMN',11(4X,I3,4X))                   MPRT 410
7     FORMAT(/,3X,'ROW',3X,123('*'))                       MPRT 420
      END                                                  MPRT 430
```

BIBLIOGRAPHY

[1] Digital Equipment Corporation, PDP8/E, PDP8/M, PDP8/F
    Small Computer Handbook. Maynard, Massachusetts, 1973.

[2] C. R. Paul, "Efficient Numerical Computation of the
    Frequency Response of Cables Illuminated by an Elec-
    tromagnetic Field", IEEE Transactions on Microwave
    Theory and Techniques, Vol. MTT-22, pp. 454-457, April
    1974.

[3] W. B. Boast, Vector Fields. New York: Harper and Row,
    1964.

[4] J. C. Clements, Computation of the Capacitance Matrix
    for Dielectric-Coated Wires. MSEE Thesis, University
    of Kentucky, Lexington, Kentucky, December 1973.

[5] W. H. Hayt, Engineering Electromagnetics. New York:
    McGraw-Hill, 1967.

[6] E. C. Jordan and K. G. Balmain, Electromagnetic Waves
    and Radiating Systems. Englewood Cliffs, N.J.:
    Prentice-Hall, 1967.

[7] A. T. Adams, Electromagnetics for Engineers. New York:
    Ronald Press, 1971.

[8] A. T. Adams and J. R. Mautz, "Computer Solutions of
    Electrostatic Problems by Matrix Inversion", Proceed-
    ings 1969 National Electronics Conference, pp. 198-201.

[9] J. C. Clements, C. R. Paul and A. T. Adams, "Compu-
tation of the Capacitance Matrix for Systems of Di-
electric-Coated Cylindrical Conductors", IEEE Trans-
actions on Electromagnetic Compatibility. Vol. EMC-17.
pp. 238-248, November 1975.

[10] R. F. Harrington, Field Computation by Moment Methods.
New York: Macmillan, 1968.

[11] R. M. Fano, L. J. Chu and R. B. Adler, Electromagnetic
Fields, Energy and Forces. New York: Wiley, 1960.

[12] B. Noble, Applied Linear Algebra. New Jersey:
Prentice-Hall, 1969.

[13] C. R. Paul, "Useful Matrix Chain Parameter Identities
for the Analysis of Multiconductor Transmission
Lines", IEEE Transactions on Microwave Theory and
Techniques, Vol. MTT-23, No. 9, pp. 756-760, September
1975.

[14] W. E. Cramer, Inversion of Block Toeplitz Matrices for
Electrostatic Problems. MSEE Thesis, Syracuse
University, May 1973.

[15] W. T. Weeks, "Multiconductor Transmission Line Theory
in the TEM Approximation", IBM Journal of Research
and Development, pp. 604-611, November 1972.

[16] International Business Machines, System/360 Scientific
Subroutine Package, Version III, Programmer's Manual.
White Plains, N.Y., 1968.

[17] D. H. Sinnott, "Matrix Analysis of Linear Antenna
Arrays of Equally Spaced Elements", IEEE Transactions

on Antennas and Propagation, Vol. AP-21, No. 3, pp.
385-386, May 1973.

[18] S. Ligh, "Centro-Symmetric Matrices", Undergraduate
Mathematics Journal, Vol. 3, No. 4, pp. 33-37, Fall
1973.

[19] C. R. Paul, Applications of Multiconductor Transmission
Line Theory to the Prediction of Cable Coupling. Vol. I,
Multiconductor Transmission Line Theory, Technical Report,
Rome Air Development Center, Griffis AFB, N.Y., to appear.