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# TRANSIENT RESPONSE OF MULTICONDUCTOR TRANSMISSION LINE EXCITED BY A NONUNIFORM ELECTROMAGNETIC FIELD

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## ABSTRACT

The time-domain transmission-line equations for uniform multiconductor transmission lines in a conductive, homogeneous medium excited by a transient, nonuniform electromagnetic field, are derived from Maxwell's equations. Depending on how the line voltage is defined, two formulations are possible. One of these formulations is considerably more convenient to apply than the other. The assumptions made in the derivation of the transmission-line equations and the boundary conditions at the terminations are discussed. For numerical calculations, the transmission-line equations are represented by finite-difference techniques, and numerical examples are included.

## ACKNOWLEDGMENT

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## SECTION I

## INTRODUCTION

Electronic subsystems on aircrafts, missiles, and ground electronic systems are commonly connected by closely-coupled multiconductor cables. It is possible that these conductors can be illuminated by extraneous electromagnetic fields; i.e., high-power radars, nuclear electromagnetic pulse (EMP), etc. (ref. 1). The excitation can often be nonuniform along the cable. Furthermore, if the cable is close enough to a nuclear detonation, the time-varying conductivity of the medium surrounding the conductors can significantly effect their behavior.

The response of multiconductor transmission lines illuminated by an electromagnetic field has been reported by several investigators (refs. 2 through 9). The frequency-domain solution for the special case of a two conductor line illuminated by a nonuniform electromagnetic field was obtained by Taylor, et al, (ref. 2) and later in a more convenient form by Smith (ref. 3). The case of a uniform plane wave incident on a three-conductor line in the transverse direction (perpendicular to the system's longitudinal axis), with the electric field intensity vector polarized parallel to the line axis, was considered in reference 4. Procedures for extending this result to multiconductor lines were suggested. Transmission line mode response of multiconductor lines in a transient electromagnetic field was obtained in reference 5. Characteristic impedances of each set of isolated conductor pairs were employed in reference 5, and propagation modes between each pair of conductors were defined. Clearly this is an approximation since scalar characteristic impedances do not exist for multiconductor lines; and there are only n fundamental propagation modes in an n+1 conductor line.

The frequency response of multiconductor lines illuminated by a nonuniform electromagnetic field was obtained in reference 6, by extending the results of reference 2 to multiconductor lines. The formulation in reference 6 neglects the conductivity of the medium surrounding the conductors.

The solution of general multiconductor transmission line networks has been considered in references 8 and 9.

In this paper, we derive the transmission line equations for multiconductor lines in a homogeneous medium excited by a transient, nonuniform electromagnetic field. The time-domain formulation is appropriate because of the time-varying conductivity. The time-varying air conductivity precludes the use of Fourier transforms. In the derivation of the first transmission-line equation, we parallel references 2 and 6 to some extent; wherein the equations are derived in the frequency domain. The second transmission line equation in reference 6, however, is not applicable for a medium of finite conductivity. The assumptions made in the derivation of the transmission line equations are discussed in section III.

The transmission-line equations in references 2 and 6 contain both electric and magnetic induced sources; specifically, the time-derivative of the magnetic field appears. In this paper, it is pointed out that two formulations are possible, depending on how the line voltages are defined. One of these formulations requires considerably less computation in the time domain than the other.

## SECTION II

## DERIVATION OF THE TRANSMISSION LINE EQUATIONS FROM MAXWELL'S EQUATIONS

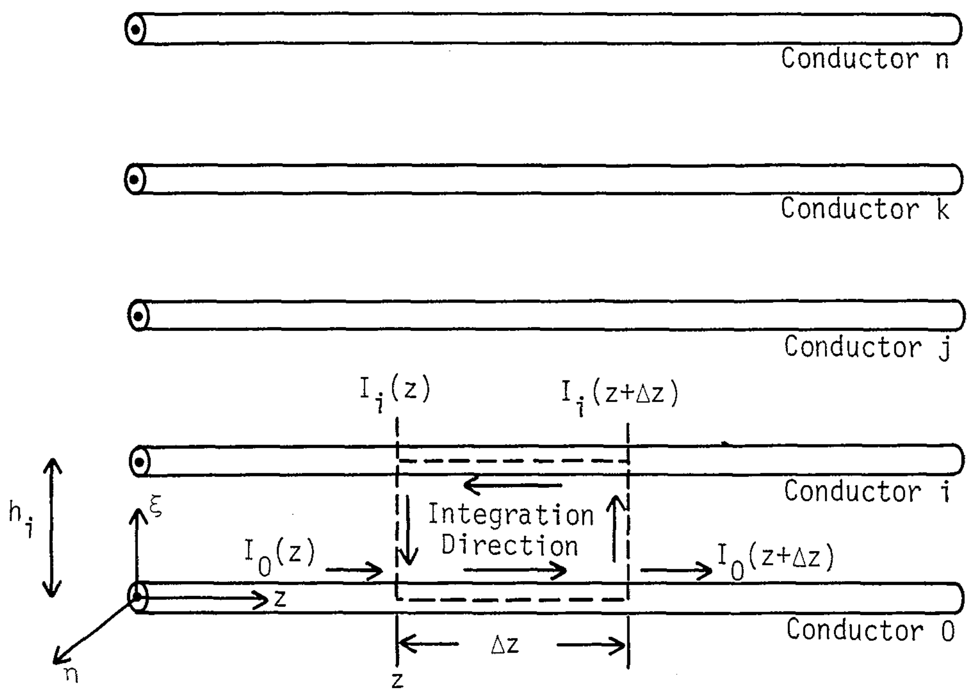
The arrangement considered is indicated in figure 1, which shows a transmission line consisting of n+1 conductors. The conductor labelled zero is the "reference" conductor (usually the ground or shield), so named because it is the reference for all voltages. The line is assumed to be uniform along its length (z direction), but with arbitrary cross section. The $j^{th}$ conductor has a radius $a_j$, and is located a distance $h_j$ from the reference conductor. The medium surrounding the conductors has conductivity, permittivity, and permeability of $\sigma$, $\varepsilon$, and $\mu$, respectively.

No assumptions are made about the exciting field, i.e., the excitation can be any nonuniform field which satisfies Maxwell's equations. The problem of interest is the calculation of the voltages and currents on the line and in the terminations. Each end of the line can be terminated with arbitrary impedances between any two conductors, and between any conductor and the reference. In the derivation of the first transmission line equation, we parallel references 2 and 6 to some extent; however, in these references the equations are derived in the frequency domain.

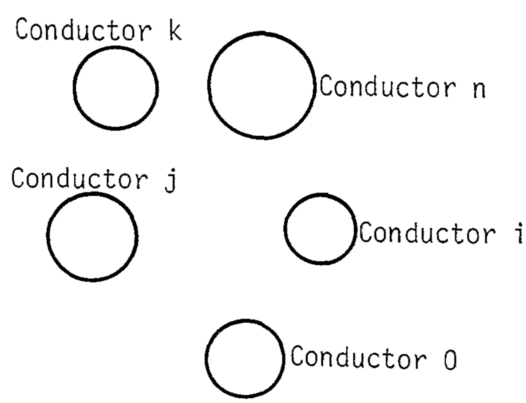
Refer to figure 1 and consider the area enclosed by the dotted lines between conductor i and conductor 0 (reference) and between z and z + $\Delta z$ in the $\xi$-z plane. The starting point in the derivation is the Maxwell equation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} . \qquad (1)$$

Integrate equation 1 over the area enclosed by the dotted lines as shown in figure 1a to obtain




a. Side View




b. Cross-sectional View

Figure 1. n+1 Conductor Transmission Line

$$\int_{S_i} \nabla \times \vec{E} \cdot d\vec{A} = \int_{C_i} \vec{E} \cdot \vec{d\ell} = -\frac{\partial}{\partial t} \int_{S_i} \vec{B} \cdot \hat{n} dA_i . \quad (2)$$

Here $S_i$ is a rectangular surface in the $\xi$-z plane and enclosed by the dotted lines as shown in figure 1a. The unit normal $\hat{n}$ is $\hat{n}=\hat{\eta}$, where $\hat{\eta}$ is the unit vector in the $\eta$-direction (perpendicular to the plane formed by the z-axis and the line joining centers of the $i^{th}$ and the reference conductors), $dA_i = d\xi dz$ and $C_i$ is a contour encircling $S_i$ in the counter clockwise direction. In equation 2, electric and magnetic fields are functions of $\xi$, $\eta$, z, and t, and represent total fields; that is, they consist of the incident fields plus the scattered fields. Here, the scattered fields are the fields due to the induced currents in the conductors. Performing the integration in equation 2 over the area as shown in figure 1, we obtain

$$\int_0^{h_i} E_{\xi_i}(\xi_i, z+\Delta z) d_{\xi_i} - E_{z_i}(h_i)\Delta z - \int_0^{h_i} E_{\xi_i}(\xi_i, z) d_{\xi_i} + E_{z_0}(0)\Delta z$$

$$= -\frac{\partial}{\partial t} \int_z^{z+\Delta z} \int_0^{h_i} B_{\eta_i}(\xi_i, z) d\xi_i dz \quad . \quad (3)$$

In this equation $E_{\xi_i}$ is the $\xi$-component of the total electric field in the direction of the straight line joining the $i^{th}$ and the reference conductors, $E_{z_i}$ is the z-component of the total electric field (line axis) on the $i^{th}$ conductor, and $B_{\eta_i}$ is the $\eta$-component of the total magnetic field density perpendicular to the plane formed by the z-axis and the line joining the centers of the $i^{th}$ and the reference conductors. $h_i$ is the distance between the centers of the $i^{th}$ and the reference conductors. We have suppressed the time dependence of the fields for convenience.

Now divide equation 3 by $\Delta z$ and take the limit as $\Delta z \to 0$; the result is

$$\frac{\partial}{\partial z}\int_0^{h_i} E_{\xi_i}(\xi_i,z)d\xi_i - \left\{E_{z_i}(h_i) - E_{z_0}(0)\right\} = -\frac{\partial}{\partial t}\int_0^{h_i} B_{\eta_i}(\xi_i,z)d\xi_i \quad (4)$$

It is convenient to separate these fields; i.e.,

$$\begin{aligned} E_{\xi_i} &= E^i_{\xi_i} + E^s_{\xi_i} \\ E_{z_i} &= E^i_{z_i} + E^s_{z_i} \\ B_{\eta_i} &= B^i_{\eta_i} + B^s_{\eta_i} \end{aligned} \quad (5)$$

The superscripts i and s refer to incident and scattered, respectively.

In terms of the scattered fields, then, equation 4 can be written as

$$\frac{\partial}{\partial z}\int_0^{h_i} E^s_{\xi_i}(\xi_i,z)d\xi_i + \frac{\partial}{\partial t}\int_0^{h_i} B^s_{\eta_i}(\xi_i,z)d\xi_i - \left\{E_{z_i}(h_i) - E_{z_0}(0)\right\}$$

$$= -\frac{\partial}{\partial z}\int_0^{h_i} E^i_{\xi_i}(x,z)d\xi_i - \frac{\partial}{\partial t}\int_0^{h_i} B^i_{\eta_i}(\xi_i,z)d\xi_i \quad (6)$$

Equation 6 is exact, in that there are no approximations made in its derivation. In order to obtain the transmission line equations, the quantities on the left-hand side must be directly related, point-wise, to voltages and currents on the line. In general, the relationship of electric and magnetic fields to charges and currents is an integral relationship, so that the fields at particular points are related to charges and currents along the entire line. That is, in a plane z = constant, the electric and magnetic fields depend on the charges and currents not only at that point, but also at all other points z.

If the scattered currents flow parallel to the line, the scattered fields will be transverse magnetic. In reference 10 it is shown that for any transverse magnetic wave:

a. The line integral of electric field between any two points in a transverse plane is independent of the path. Therefore, the transverse electric field can be expressed as the gradient of a scalar potential.

b. The magnetic flux through any strip of a cylindrical surface passing through two points in any plane z = constant is independent of the contour of the strip.

These properties indicate that the transverse fields have a static character.

Property a. shows that transverse voltage is single valued. Therefore, anticipating transverse magnetic propagation, the scattered line voltage $V_i^s$ on the $i^{th}$ conductor with respect to the reference conductor is defined as

$$V_i^s(z) = -\int_0^{h_i} E_{\xi_i}^s(\xi_i, z)\,d\xi_i \tag{7}$$

Property b indicates that a constant inductance matrix can be defined. The scattered magnetic flux passing between the two conductors per unit length of line can be directly related to the per-unit-length inductance matrix, and the currents in the conductor at that point. In particular,

$$\int_0^{h_i} B_{\eta_i}^s(\xi_i, z)\,d\xi_i = -[L_{i1}, L_{i2}, \ldots, L_{in}] \begin{bmatrix} I_1(z) \\ I_2(z) \\ \cdot \\ \cdot \\ \cdot \\ I_n(z) \end{bmatrix} \tag{8}$$

where $I_1, \ldots I_n$ are the currents at z in the conductors, and $L_{ij}$'s are the elements of per-unit-length inductance matrix of the line. Of course, if there is a significant internal inductance in the conductors, the relationship is not this simple.

There are n equations like equations 6, 7, and 8; substituting equations 7 and 8 into 6 and writing these in a matrix form, we obtain

$$\frac{\partial}{\partial z}\left[V_i^s(z)\right] + \frac{\partial}{\partial t}[L_{ij}][I_i(z)] + \left\{\left[E_{z_i}(z,h_i) - E_{z_0}(z,0)\right]\right\}$$
$$= \left[\frac{\partial}{\partial z}\int_0^{h_i} E^i_{\xi_i}(\xi_i,z)d\xi_i + \frac{\partial}{\partial t}\int_0^{h_i} B^i_{\eta_i}(\xi_i,z)d\xi_i\right]. \quad (9)$$

It should be pointed out, that the voltage, $V_i^s$, in equation 9 is the scattered (induced) voltage on the $i^{th}$ conductor and not the total voltage. To obtain the total voltage it is necessary to add to $V_i^s$ the voltage of the incident field between the reference and the $i^{th}$ conductor. The right-hand side of equation 9, which is the "source" term, can be expressed in terms of the z components of the electric field along the conductors and the reference conductor; that is, for any incident field

$$\frac{\partial}{\partial z}\int_0^{h_i} E^i_{\xi_i}(\xi_i,z)dx + \frac{\partial}{\partial t}\int_0^{h_i} B^i_{\eta_i}(\xi_i,z)d\xi_i = E^i_{z_i}(z,h_i) - E^i_{z_0}(z,0) \quad (10)$$

Equation 10 is obtained by integrating equation 1 over the area enclosed by the dotted lines as shown in figure 1 for the incident field. If there are no components of incident field parallel to the conductors, or if the z components of the incident fields at the conductors and at the reference conductor (ground) are equal, the source term disappears.[2]

[2] We point this out to show that if there is no component of the incident field parallel to the transmission line, the line is not excited.

The total fields

$$E_{z_i}(z,h_i) - E_{z_0}(z,0)$$

must be related to currents along the line. This is most easily accomplished in the frequency domain, where it is possible to define the conductor and the reference conductor impedances.

$$\tilde{E}_{z_i}(z,h_i) - \tilde{E}_{z_0}(z,0) = Z_i\tilde{I}_i(z,h_i) + Z_g\tilde{I}_1(z,h_1) + Z_g\tilde{I}_2(z,h_2)$$
$$+ Z_g\tilde{I}_3(z,h_3) + \cdots Z_g\tilde{I}_i(z,h_i) + \cdots + Z_g\tilde{I}_n(z,0) \quad (11)$$

Here $\tilde{E}_{z_i}$ and $\tilde{I}_i$ denote Fourier transforms, $Z_i$ is the internal impedance per unit length of the $i^{th}$ conductor, and $Z_g$ is the internal impedance per unit length of the reference conductor, respectively. In equation 11, it is assumed that the sum of the current in all conductors (including the reference conductor) for any position z is zero; that is,

$$\tilde{I}_0(z) = -\sum_{i=1}^{n} \tilde{I}_i(z) \ . \quad (12)$$

There are n equations like equation 11 (i = 1, 2, ... n); writing equation 11 in matrix form, we obtain,

$$\left[\tilde{E}_{z_i}(z,h_i) - \tilde{E}_{z_0}(z,0)\right] = [Z_{ij}]\,[\tilde{I}_i(z)] \quad (13)$$

where the matrices [Z] and the [I] are defined as

$$[Z_{ij}] = \begin{bmatrix} Z_1 + Z_g & Z_g & \cdots & Z_g \\ Z_g & Z_2 + Z_g & \cdots & Z_g \\ \vdots & & & \vdots \\ Z_g & Z_g & \cdots & Z_n + Z_g \end{bmatrix}$$

and

$$[\tilde{I}_i] = \begin{bmatrix} \tilde{I}_1(z) \\ \tilde{I}_2(z) \\ \cdot \\ \cdot \\ \cdot \\ \tilde{I}_n(z) \end{bmatrix} \tag{14}$$

respectively.

There is no difficulty in Fourier transforming equation 6 so that equation 13 makes sense. As will be seen, in the derivation of the second transmission-line equation, the appearance of a product term involving two time functions negates the use of the Fourier transform. Thus, it is necessary to convert equation 13 to the time domain, and techniques to do this will be discussed. With this in mind, equation 9 is one of the transmission-line equations. Substituting, equation 10 in equation 9, we obtain,

$$\frac{\partial}{\partial z}\left[V_i^s(z)\right] + \frac{\partial}{\partial t}[L_{ij}]\,[I_i(z)] + \left\{\left[E_{z_i}(z,h_i)\right] - \left[E_{z_0}(z,0)\right]\right\}$$
$$= \left[E^i_{z_i}(z,h_i) - E^i_{z_0}(z,0)\right] \quad . \tag{15}$$

Note that there are two forms of the source term in the first transmission-line equation, as seen from equation 9 and equation 15, respectively. If it is assumed that the internal impedances of all conductors (including the reference conductor) are constant, so that

$$[Z_{ij}] = [R_{ij}] = \begin{bmatrix} R_1 + R_g & R_g & \cdots & R_g \\ R_g & R_2 + R_g & \cdots & R_g \\ \vdots & & & \vdots \\ R_g & R_g & \cdots & R_n + R_g \end{bmatrix} \tag{16}$$

where $R_1$, $R_2$ ... $R_n$ and $R_g$ are the per-unit-length internal resistances of the conductors and the reference conductor, respectively.

Then, equation 15 becomes

$$\frac{\partial}{\partial z}\left[V_i^s(z)\right] + [L_{ij}]\frac{\partial}{\partial t}[I_i(z)] + [R_{ij}][I_i(z)] = \left[E_{z_i}^i(z,h_i) - E_{z_0}^i(z,0)\right] \tag{17}$$

Since the inductance matrix $[L_{ij}]$ is constant, it has been taken out of the differentation.

Equation 17 is one of the transmission-line equations. This form of the equation, as compared to equation 9, is more convenient to use for numerical computation, which will be discussed later.

The second transmission-line equation is derived from the Maxwell equation

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad . \tag{18}$$

Consider a closed cylindrical surface just outside the $i^{th}$ conductor and of length $\Delta z$ between z and $z + \Delta z$. Then integrating equation 18 over this surface, we obtain

$$\oint \nabla \times \vec{H} \cdot d\vec{A}_i = \oint \vec{J} \cdot d\vec{A}_i + \frac{\partial}{\partial t}\oint \vec{D} \cdot d\vec{A}_i = 0 \tag{19}$$

From the ends of the $i^{th}$ wire at z and $z + \Delta z$

$$\int_{\text{ends}} \vec{J} \cdot d\vec{A}_i = I_i(z + \Delta z) - I_i(z) \tag{20}$$

From the cylindrical portion of the surface,

$$\int_{cyl} \vec{J} \cdot d\vec{A} = \Delta z \int_0^{2\pi} \sigma(a_i,\theta)\, E^s_{r_i}(a_i,\theta)\, a_i d\theta \tag{21}$$

where $\sigma(a_i,\theta)$ is the conductivity of the medium just external to the $i^{th}$ conductor, $E^s_{r_i}(a_i,\theta)$ is the radial component of the scattered field just external to the $i^{th}$ conductor, and $a_i$ is the radius of the $i^{th}$ conductor. Substituting equations 20 and 21 into equation 19 and dividing by $\Delta z$ we obtain,

$$\frac{\partial I_i(z)}{\partial z} + \int_0^{2\pi} \sigma(a_i,\theta)\, E^s_{r_i}(a_i,\theta)\, a_i d\theta + \frac{\partial Q_i}{\partial t} = 0 \tag{22}$$

where $Q_i$ is the total net charge per unit length on the $i^{th}$ conductor. Generally speaking, the air conductivity is a function of time and position, and is cross-sectionally inhomogeneous. Therefore, the product of $\sigma$ and $E^s_{r_i}$ in this equation undermines the use of Fourier transforms. In addition, air conductivity is a strong function of the total electric field, so that equation 22 is nonlinear.

The charge and integral terms in equation 22 must be related to the voltages on the line. To do this, consider the divergence of the scattered electric field vector:

$$\nabla \cdot \vec{E}^s = \nabla_t \cdot \vec{E}^s_t + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon} \tag{23}$$

Equation 23 indicates that only if

$$\frac{\partial E^s_z}{\partial z} \equiv 0 \Rightarrow E^s_z \equiv E^s_z(x,y), \tag{24}$$

will the transverse electric field in any plane z = constant be directly related to the charge at that point. If equation 24 is true, again assuming transverse magnetic propagation, the line voltage and charge are related by the capacitances per unit length; that is,

$$Q_i = C_{i_1} V_1^s + C_{i_2} V_2^s + \cdots + C_{i_n} V_n^s \quad . \tag{25}$$

In matrix form

$$[Q_i] = [C_{ij}]\,[V_i^s] \tag{26}$$

where $[C_{ij}]$ is the capacitance per unit length matrix of the multiconductor line (ref. 11).

Finally, to complete equation 22, it is desirable to relate the integral term to the scattered voltage. If the conductivity is uniform around the conductor, that is, if it does not vary with $\theta$, then

$$\int_0^{2\pi} \sigma(a_i,\theta)\, E_r^s(a_i,\theta)\, a_i d\theta = \sigma \int_0^{2\pi} E_r^s(a_i,\theta)\, a_i d\theta$$

$$= \frac{\sigma}{\varepsilon} \int_0^{2\pi} D_i^s(a_i,\theta) a_i d\theta \tag{27}$$

Thus, if $a_i$ is small or equation 24 is valid, then

$$\int_0^{2\pi} \sigma(a_i,\theta)\, E_r^s(a_i,\theta)\, a_i d\theta = \frac{\sigma}{\varepsilon} Q_i$$

$$= \frac{\sigma}{\varepsilon}\left[C_{i_1} V_1^s + C_{i_2} V_2^s + \cdots + C_{i_n} V_n^s\right] \tag{28}$$

Equation 22 then becomes

$$\frac{\partial I_i(z)}{\partial z} + \frac{\sigma}{\varepsilon}\left[C_{i_1} V_1^s + C_{i_2} V_2^s + \cdots + C_{i_n} V_n^s\right]$$

$$+ \frac{\partial}{\partial t}\left[C_{i_1} V_1^s + C_{i_2} V_2^s + \cdots + C_{i_n} V_n^s\right] = 0 \tag{29}$$

There are n equations like equation 29 for i = 1, 2, ... n. Combining these equations in matrix form we obtain

$$\frac{\partial}{\partial z}[I_i(z)] + \frac{\sigma}{\varepsilon}[C_{ij}]\left[V_i^s(z)\right] + \frac{\partial}{\partial t}[C_{ij}]\left[V_i^s(z)\right] = 0 \tag{30}$$

Here, $\sigma$ is the conductivity of the medium surrounding the conductors, which is assumed to be homogeneous around any cross section.

Define a conductance per unit length matrix as

$$[G_{ij}] = \frac{\sigma}{\varepsilon}[C_{ij}] \quad . \tag{31}$$

The per-unit-length capacitance matrix $[C_{ij}]$ is constant (independent of time) in equation 30. Substituting equation 31 in equation 30 we obtain,

$$\frac{\partial}{\partial z}[I_i(z)] + [G_{ij}]\left[V_i^s(z)\right] + [C_{ij}]\frac{\partial}{\partial t}\left[V_i^s(z)\right] = 0 \tag{32}$$

Equation 32 is the second transmission-line equation. Recall that the first transmission line equation is equation 17:

$$\frac{\partial}{\partial z}\left[V_i^s(z)\right] + [R_{ij}][I_i(z)] + [L_{ij}]\frac{\partial}{\partial t}[I_i(z)] = \left[E_{z_i}^i(z,h_i) - E_{z_0}^i(z,0)\right] \tag{33}$$

In equations 32 and 33, the voltage on the line is the scattered voltage. The total voltage $V_i^T$ on the $i^{th}$ conductor is given by

$$V_i^T(z) = V_i^s(z) + V_i^i(z) = V_i^s(z) - \int_0^{h_i} E_{\xi_i}^i(\xi_i,z)\,d\xi_i \tag{34}$$

where $E_{\xi_i}^i(\xi_i,z)$ is the transverse ($\xi_i$-direction) component of the incident electric field at $(\xi_i,z)$.

Figure 2 shows the equivalent circuit of a small section of length of $\Delta z$ of the multiconductor transmission line. The sources appear only as voltage sources in series with the conductors.

Note that the source term appears only in one transmission-line equation. The transmission-line equations in reference 2 and 6 contain sources in both equations. The solution of equations 32 and 33 gives the scattered voltages and the induced currents on the line. The total voltages on the line are obtained from equation 34. Indeed, the transmission-line equations 32 and 33 are equivalent to those obtained in reference 6, where they are derived in the frequency domain. This can be shown easily by writing equations 32 and 33 in terms of the total voltages. Substituting equation 34 into equations 32 and 33, and equation 10 in equation 33 we obtain,

$$\frac{\partial}{\partial z}[I_i(z)] + [G_{ij}]\left[V_i^T(z)\right] + [C_{ij}]\frac{\partial}{\partial t}\left[V_i^T(z)\right] = -[G_{ij}]\left[\int_0^{h_i} E^i_{\xi_i}(\xi_i,z)d\xi_i\right]$$

$$- [C_{ij}]\frac{\partial}{\partial t}\left[\int_0^{h_i} E^i_{\xi_i}(\xi_i,z)d\xi_i\right] \qquad (35)$$

$$\frac{\partial}{\partial z}\left[V_i^T(z)\right] + [R_{ij}][I_i(z)] + [L_{ij}]\frac{\partial}{\partial t}[I_i(z)]$$

$$= \frac{\partial}{\partial t}\left[\int_0^{h_i} B^s_{\eta_i}(\xi_i,z)d\xi_i\right] \qquad (36)$$

Equations 35 and 36 are the transmission-line equations in terms of total line voltage, and are equivalent to the equations obtained in reference 6. In reference 6, the equations were derived in the frequency domain and the conductivity of the medium was neglected.

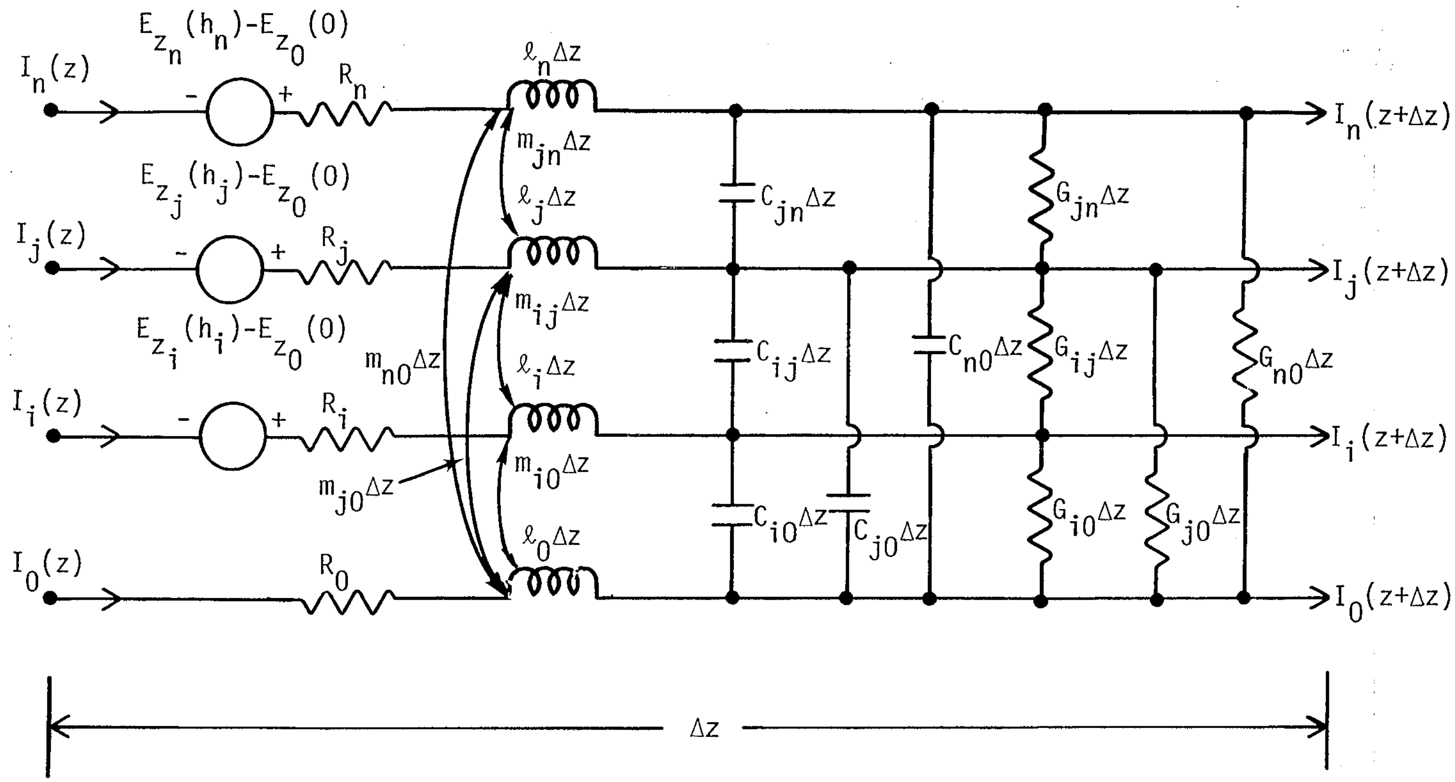



Figure 2. Equivalent Circuit of a Section of n+1 Conductor Line

In equations 32 and 33 (or equivalently equations 35 and 36), the time dependence of voltages, currents, conductivity, and the incident fields has been suppressed for convenience. Since these equations are formulated in the time domain, the time-varying conductivity of the medium poses no problem in the solution. Furthermore, this time-domain formulation is more convenient, if at some future time nonlinear effects are included.

The numerical solution of equations 32 and 33 is numerically more efficient than the solution of equations 35 and 36, since the source term appears in only one equation as the difference of incident tangential electric fields. In equations 35 and 36, the incident fields appear differentiated with respect to time, and in the case of fast-rising waveforms, the differentiated terms will have a faster risetime than the original waveform. Therefore, in the time-domain solution, a finer resolution of the differentiated terms is required, and hence more computation. Thus, equations 32 and 33 are more convenient to apply than equations 35 and 36.

In the following sections, we will consider the validity of approximations used in the derivation of the transmission-line equations, boundary conditions for the two formulations, and the numerical solution of the equations.

## SECTION III

## VALIDITY OF APPROXIMATIONS USED IN TRANSMISSION LINE ANALYSIS

From section II, the major assumptions made in the time-domain transmission line model are,

a. The transmission line response is transverse magnetic.

b. $\partial E_z^S/\partial z$ is small compared to $\nabla_t \cdot \vec{E}_t$.

c. The sum of the induced currents in the conductors and the corresponding z component of current in the reference conductor are equal in magnitude and opposite in sign.

d. The conductivity of the medium surrounding any conductor is uniform.

The validity of these assumptions is considered in this section.

a. Transverse Magnetic Response

If the current in any system of conductors is confined to flow in a single direction, then there can be no component of magnetic field in this direction. This is easily seen from the relation

$$\vec{H} = \nabla \times \vec{A}$$

where $\vec{A}$ is the vector magnetic potential. The direction of $\vec{A}$ coincides with the direction of $\vec{J}$ and thus $\vec{I}$. If all conductors, including reference, are small, then this is the case.

In the case of a transmission-line system (with all conductors) over a perfectly-conducting ground plane, the current in the conductors is also confined to flow in the direction of the conductors. Therefore, the component of vector potential due to this current will always generate a transverse magnetic wave.

In between these extremes; that is, between zero and infinite conductivity of the ground plane, the vector potential must have a vertical component (ref. 13). It follows that finite conductivity of the ground plane prohibits complete transverse magnetic propagation.

It is difficult to determine, without much further analysis, the quantitative effects of the impure transverse magnetic propagation, and the resultant description of wires above a ground plane by transmission-line-type equations. Vance (ref. 14) attempted to handle this question by comparing a transmission-line analysis with the exact solution for a particular case. Unfortunately, he considered a perfectly conducting ground, and as was previously stated, this assumption forces transverse magnetic behavior.

Olsen and Chang (ref. 15), Wait (ref. 16), and Schlessinger (ref. 17) derive analytic expressions for the current induced in an infinite wire above a finitely conducting ground; however, these authors do not consider any comparison to determine relative validity of the transverse magnetic assumption. Indeed, all of these derivations must, of necessity, assume impure transverse magnetic behavior.

One could assume that any transverse electric waves will not propagate well; i.e., that their attenuation will be comparatively high. This indeed may be the case; however, the transverse electric fields will extract energy (particularly over long distances and at the higher frequencies) which could significantly alter the frequency content of the induced current.

It should be pointed out that seemingly minor perturbations in the model can affect significant variations in the frequency domain. For example, Schlessinger (ref. 17) points out that neglect of the multiple reflections, between the wire and ground, of the incident signal can alter the high frequency content of the induced current by an order of magnitude.

b. Neglect of $\partial E_z^s/\partial z$

$\partial E_z^s/\partial z$ is very strongly dependent on the incident field. In fact, if the conductors are perfect, $E_z^s$ is equal in magnitude to $E_z^i$ along the conductors. Thus, it is seen that even for an "ideal" transmission line, the normal transmission line model may not be adequate for a distributed source; that is, it may not be adequate, if $\partial E_z^i/\partial z$ is high compared to $\nabla_t \cdot \vec{E}_t^s$.

As pointed out in the previous section, Vance (ref. 14) attempted to establish the validity of the transmission-line equations by comparing analytic solutions with transmission-line solutions for a particular case.

In his example, not only were perfect conductors assumed, but also the impinging electromagnetic field was vertically incident, with the electric field parallel to the wire. Therefore, along the conductors

$$\frac{\partial E_z^i}{\partial z} = \frac{\partial E_z^s}{\partial z} = 0$$

It is not surprising, then, that a good comparison was obtained.

In reference 18, the analytic solutions for isolated, long wires with transmission-line-like solutions have been compared. The incident electromagnetic wave was homogeneous, planar with the electric field parallel to the wire, and the angle of incidence was varied. Only for grazing angles of incidence, where $E_z^i$ is small, were the solutions comparable.

Like the transverse magnetic approximation, this is a complex problem and a numerical comparison between analytic solutions and transmission line solutions is warranted.

c. Currents in Conductors

If the appropriate impedances of the conductors and the reference can be defined, the tangential electric fields on the conductors can be related to the line currents. The impedances must be converted to the time domain. That is, we must obtain a relationship between currents and tangential electric fields which can be used in the time-domain solution of the transmission line equations. A procedure to do this is outlined here.

Consider any impedance. In general, the series impedance of transmission lines will be irrational; however, this does not alter the procedure. Since interest centers on the time domain, it is desirable to approximate the impulse response of this impedance in the time domain rather than in the frequency domain. The impulse response will be approximated by a finite sum of exponentials, which in turn will allow a lumped, passive network approximation.

The impulse response can be obtained numerically. If more convenient,* the impulse response of the corresponding admittance can be obtained, since the final results will be equivalent. This impulse response will be approximated by the sum

$$h(t) \approx \sum_{0}^{N} A_i \varepsilon^{s_i t} \tag{37}$$

where $h(t)$ is the impulse response,

$s_i$ are complex exponential coefficients.

Prony's method (ref. 19) is particularly convenient for evaluating the coefficients $A_i$ and exponential factors $s_i$ in equation 37. A small extension of Prony's method permits the approimxation of equation 37 to be done in a minimum-mean-square sense. The $A_i$ and $s_i$ allow the lumped-parameter network to be immediately synthesized, so that a set of differential equations relating the electric field and current can be obtained. A running solution of these equations must be accounted at each longitudinal increment along the transmission line, so that the relationship in differential form is most convenient (ref. 20).

d. The Conductivity of the Medium

It was assumed that the conductivity of the medium surrounding any conductor was uniform; that is, it does not vary with $\theta$. The source of conductivity is often nuclear radiation from a weapon. Over distances comparable to a wire spacing, these emissions will essentially be uniform.

The resulting conductivity of the medium is, however, quite nonlinear, due to the strong dependence of electron mobility on the total electric field. Thus the spatial variation of conductivity is related to the spatial variation of total electric field.

*The impedance, for example, may increase indefinitely with frequency.

If the diamter of the conductors is assumed to be small compared to the transverse interspatial wire distances, or if the scattered fields have negligible effect on conductivity, then the medium conductivity will not vary around the wire. Therefore, the assumption of uniform air conductivity made in equation 27 is valid.

If these assumptions are not appropriate, then account should be taken of the conductivity variation with electric field strength. The nonlinear behavior is reasonably easy to incorporate in the numerical solution of the time-domain transmission-line equations. It will be necessary, however, to estimate the scattered electric field in the vicinity of the conductors, so that the net electric field (scattered plus incident) can be used to alter the air conductivity created by the nuclear radiation in the absence of the wire.

## SECTION IV

## BOUNDARY CONDITIONS AND NUMERICAL SOLUTIONS

The coupled set of transmission line equations for multiconductor lines (equations 32 and 33), as derived in section II, are:

$$\frac{\partial}{\partial z}\left[V_i^S(z)\right] + [R_{ij}]\,[I_i(z)] + [L_{ij}]\,\frac{\partial}{\partial t}\,[I_i(z)] = \left[V_{s_i}(z)\right] \qquad (38)$$

$$\frac{\partial}{\partial z}\,[I_i(z)] + [G_{ij}]\left[V_i^S(z)\right] + [C_{ij}]\,\frac{\partial}{\partial t}\left[V_i^S(z)\right] = 0 \qquad (39)$$

In equation 38, $V_{s_i}(z)$ , is the source voltage (per-unit-length) vector defined as

$$\left[V_{s_i}(z)\right] = \left[E_{z_i}(h,z) - E_{z_0}(0,z)\right] .$$

In equation 38 and 39, the time dependence of the scattered voltage, current, source voltage, and conductance have been suppressed. The total voltage at any point on the line is the sum of the scattered voltage and the incident voltage as defined in equation 34.

Equations 38 and 39 are solved using finite-difference techniques. The finite-difference scheme of figure 3 is used in the solution, and follows the development of reference 21. Each conductor is broken up into alternating voltage and current nodes, with the ends of each conductor defined as voltage nodes.

The point-centered finite-difference representations of equations 38 and 39 are:

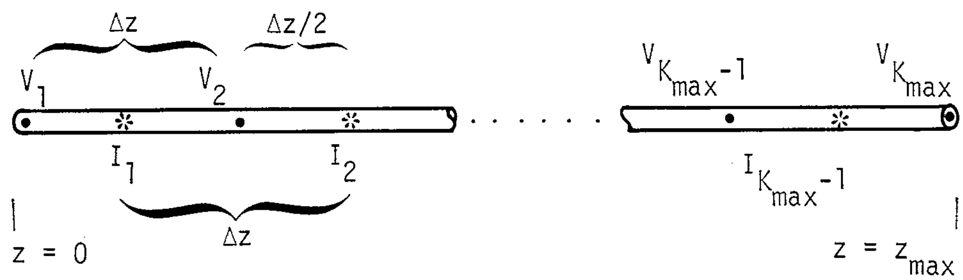



Figure 3. The Finite Difference Gridding Imposed on Each Wire With $z_{max} = (K_{max}-1)\Delta z$

$$\left[\frac{V_{i,k+1}^{n+1} - V_{i,k}^{n+1}}{\Delta z}\right] + [R_{ij}]\left[\frac{I_{i,k}^{n+1} + I_{i,k}^{n}}{2}\right] + [L_{ij}]\left[\frac{I_{i,k}^{n+1} - I_{i,k}^{n}}{\Delta t}\right] = \left[\frac{V_{s_i,k}^{n+1} + V_{s_i,k}^{n}}{2}\right] \tag{40}$$

$$\left[\frac{I_{i,k+1}^{n} - I_{i,k}^{n}}{\Delta z}\right] + \left[G_{ij}^{n}\right]\left[\frac{V_{i,k+1}^{n+1} + V_{i,k+1}^{n}}{2}\right] + [C_{ij}]\left[\frac{V_{i,k+1}^{n+1} - V_{i,k+1}^{n}}{\Delta t}\right] = 0 \tag{41}$$

where subscript k denotes the incremental position and superscript n denotes the incremental time with the following definitions:

$$z_V^k = (k-1)\Delta z; \; z_I^k = (k-1/2)\Delta z$$

$$t_v^n = n\Delta t; \; t_I^n = (n+1/2)\Delta t$$

$$I_{i,k}^n = I_i(z_I^k, t_I^n); \; V_{i,k}^n = V_i(z_V^k, t_V^n)$$

$$V_{s_i,k}^n = V_{s_i}(z_V^k, t_V^n)$$

In equations 40 and 41, the superscript s on the scattered voltage has been suppressed for convenience.

Solution of equations 40 and 41 yields:

$$\left[I^{n+1}_{i,k}\right] = \left[\frac{L_{ij}}{\Delta t} + \frac{R_{ij}}{2}\right]^{-1} \left\{\left[\frac{V^{n+1}_{s_i,k} + V^{n}_{s_i,k}}{2}\right] - \left[\frac{V^{n+1}_{i,k+1} - V^{n+1}_{i,k}}{\Delta z}\right] + \left[\frac{L_{ij}}{\Delta t} - \frac{R_{ij}}{2}\right]\left[I^{n}_{i,k}\right]\right\} \quad (42)$$

For $k = 1, 2, \ldots, k_{max}$

and
$$\left[V^{n+1}_{i,k}\right] = \left[\frac{G^{n}_{ij}}{2} + \frac{C_{ij}}{\Delta t}\right]^{-1} \left\{\frac{I^{n}_{i,k} - I^{n}_{i,k+1}}{\Delta z} + \left[\frac{C_{ij}}{\Delta t} - \frac{G^{n}_{ij}}{2}\right]\left[V^{n}_{i,k}\right]\right\} \quad (43)$$

For $k = 1, 2, \ldots, k_{max}-1$ and $n = 0, 1, \ldots, N_{max}-1$

Note that the scattered voltages at times $(n+1)\Delta t$ can be evaluated from their own value at the earlier time $n\Delta t$ and from the currents at the earlier time $n\Delta t$. Similarly, the currents at times $(n+1)\Delta t$ are evaluated from the scattered voltages at times $(n+1)\Delta t$ and their own values at earlier times $n\Delta t$.

The difference equations are applied at successive discrete time steps to all the spatial points. That is, the equations are assumed to hold for all the points of the one-dimensional space. The size of the spatial increment $\Delta z$ determines the time step $\Delta t$. For stability, $\Delta t$ must satisfy the Courant stability condition, which may be expressed as

$$\Delta t < \frac{\Delta z}{v_p}$$

where $v_p$ is the velocity of propagation in the medium

The boundary conditions must be applied at the terminal ends of the transmission line. This is most easily done in the frequency domain, where it is easier to define a relation between voltages and currents. At the input terminal of the transmission line ($z = 0$), the boundary condition is

$$\left[\tilde{V}^{s}_{i}(z=0) + \tilde{V}^{i}_{i}(z=0)\right] = -\left[Z^{1}_{ij}\right]\left[\tilde{I}_{i}(z=0)\right] \quad (44)$$

and at the load of the transmission line the boundary condition is

$$\left[\tilde{V}_i^s(z=\ell) + \tilde{V}_i^i(z=\ell)\right] = \left[Z_{ij}^2\right]\left[\tilde{I}_i(z=\ell)\right] \tag{45}$$

where $\left[Z_{ij}^1\right]$ and $\left[Z_{ij}^2\right]$ are the impedance matrices of the terminating networks at the input and the load ends of the transmission line; $\tilde{V}_i^s$, $\tilde{V}_i^i$, and $\tilde{I}_i$ denote Fourier transforms. Equations 44 and 45 must be transformed to the time domain in order to be applicable to equations 42 and 43. For a lumped-parameter, RLC network, a relation between the voltage and the current can be easily obtained in the time domain.

Note that the currents are calculated $\Delta z/2$ distance away from the terminal ends as shown in figure 3. The currents at the input terminals ends are obtained by extrapolating the currents at $z = \Delta z/2$ and $z = 3\Delta z/2$; and the current at the load terminal is obtained by extrapolating the currents at $z = (k_{max}-1)\Delta z - \Delta z/2$ and $z - (k_{max}-2)\Delta z - \Delta z/2$.

Finally, the calculation of per-unit-length inductance and capacitance matrices remains. These matrices can be calculated by using either the numerical techniques described in references 22 and 23 or can be determined experimentally (refs. 24 and 25).

## SECTION V

## NUMERICAL RESULTS

To illustrate the numerical solution and the transient response of transmission lines, two examples will be considered in this section. These examples present results for a lossless two-conductor line over a perfectly conducting ground. These results illustrate the propagation of transient voltages and currents for comparison with analytic solutions.

In examples 1 and 2, the transmission line consists of two conductors of 0.2 cm diameter each, 10 meters in length, separated by a distance of 0.5 cm. Both conductors are located 10 cm above a perfectly conducting ground. The per-unit-length inductance and capacitance matrices of the line are:

$$[L_{ij}] = \begin{bmatrix} 1.0596 & 0.7378 \\ 0.7378 & 1.0596 \end{bmatrix} \mu H/m; \quad [C_{ij}] = \begin{bmatrix} 20.3548 & -14.1731 \\ -14.1731 & 20.3548 \end{bmatrix} pF/m$$

Each conductor is terminated to ground with 100 Ω resistors at both ends.

In example 1, conductor one is excited at one end with a voltage source (as shown in figure 4) in series with the 100 Ω termination. The voltage source is a half-sine wave of width 20 ns and peak amplitude of 1 volt. The voltage and current response of the line at both ends is shown in figures 5 and 6, which show both incident and reflected waves at both ends of the line.

In example 2, the transmission line is excited by a parallel, spatially short, pulsed field located at the line center, i.e., z = 5 meters from each end. The time variation of electric field is the same as that shown in figure 4, and has an amplitude of 1 volt/m. The voltage and current response of the line at both ends is shown in figures 7 and 8.

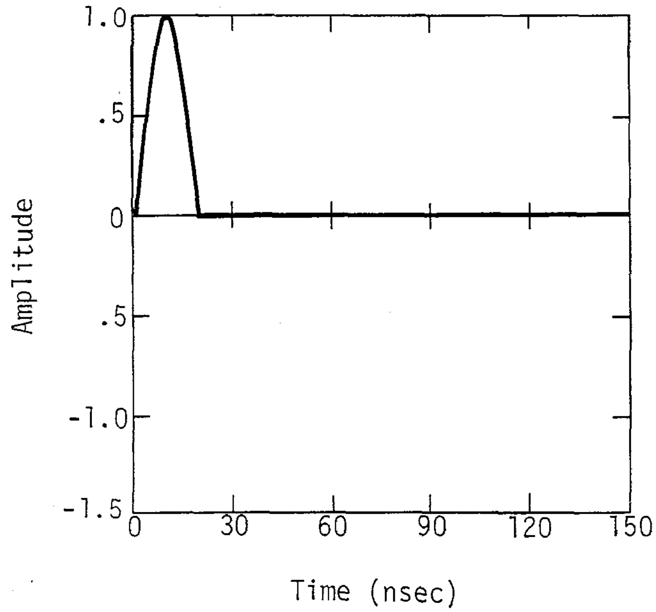
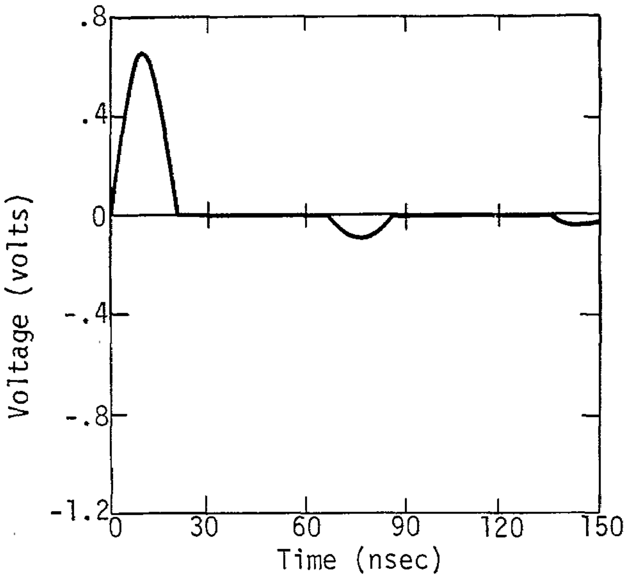
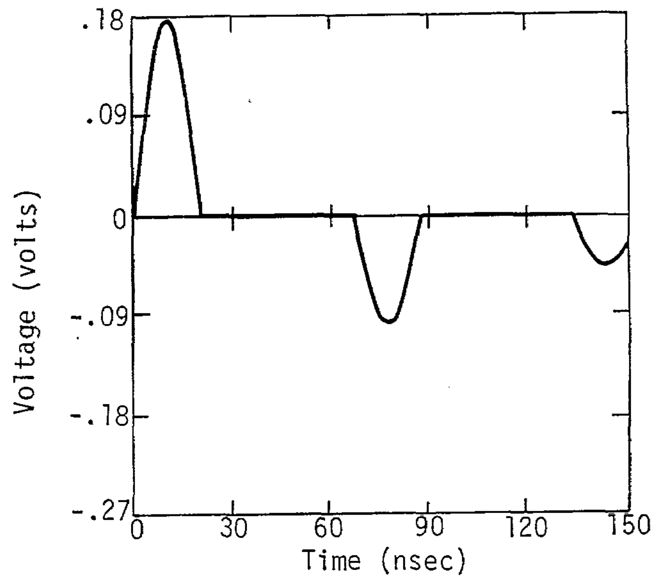



Figure 4. The Driving Pulse Used in Examples 1 and 2.

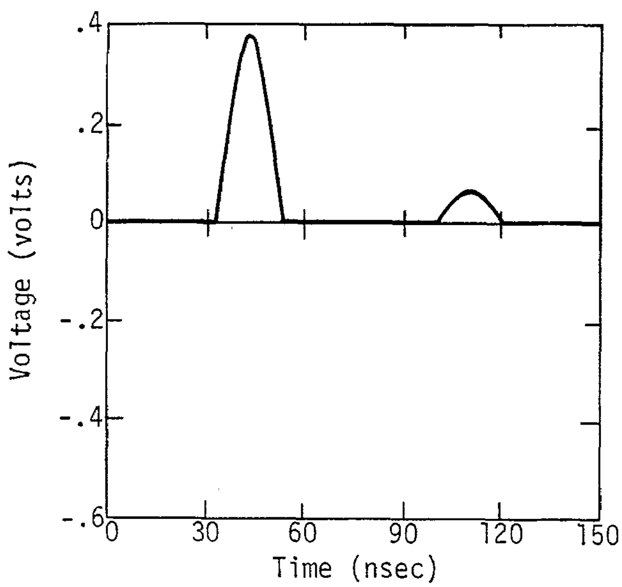
The results of examples 1 and 2 agree with the analytical solutions of the transmission line equations.



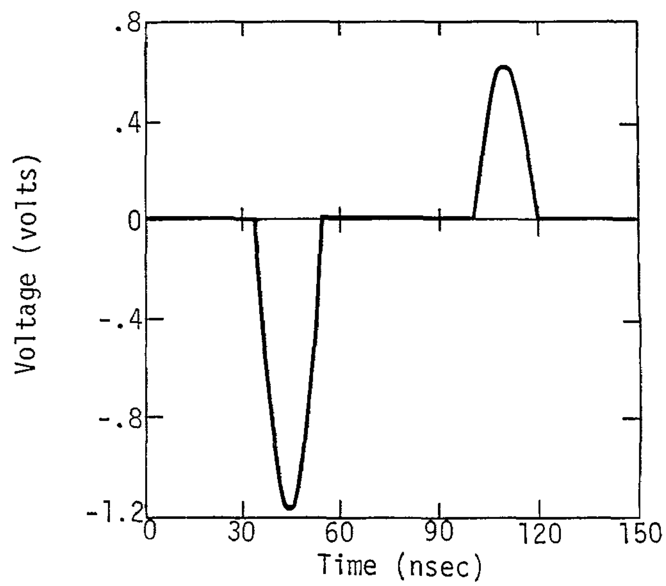

a. Total Voltage on Conductor One at the Input End (z=0)




b. Total Voltage on Conductor Two at the Input End (z=0)

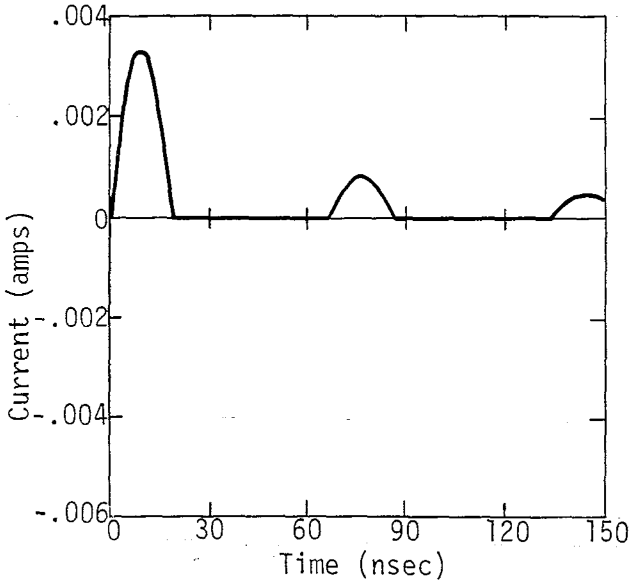



c. Total Voltage on Conductor One at the Load End (z=10 m)

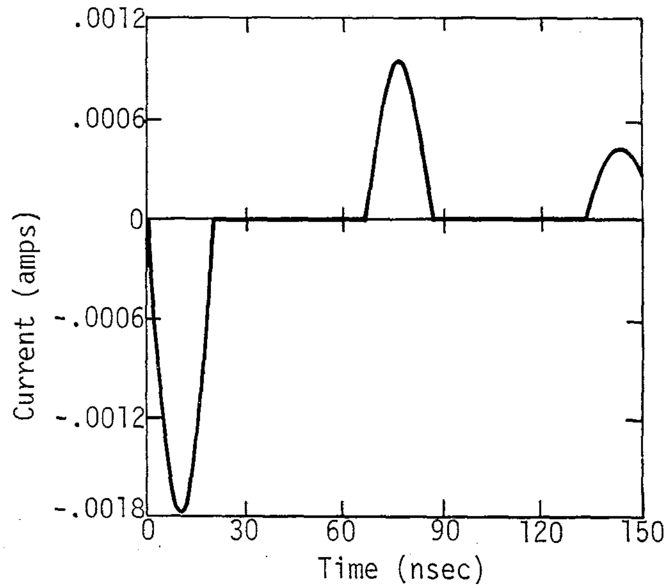



d. Total Voltage on Conductor Two at the Load End (z=10 m)

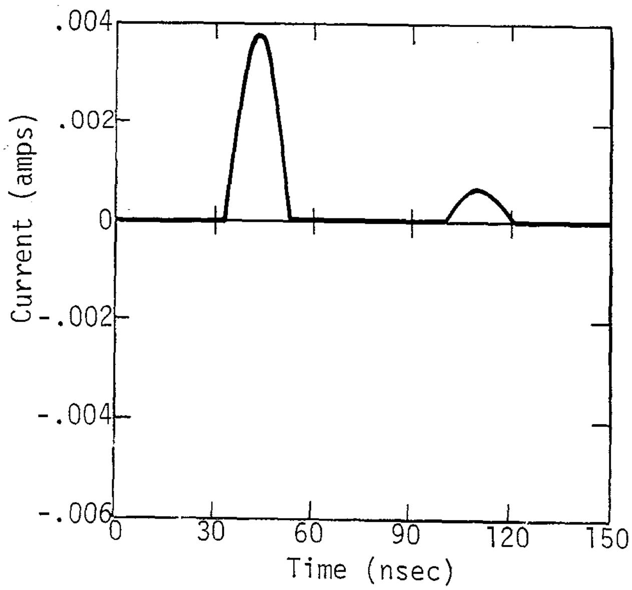
Figure 5. Transient Voltage Response of the Two Conductor Line (Over a Ground Plane) Excited at One End.



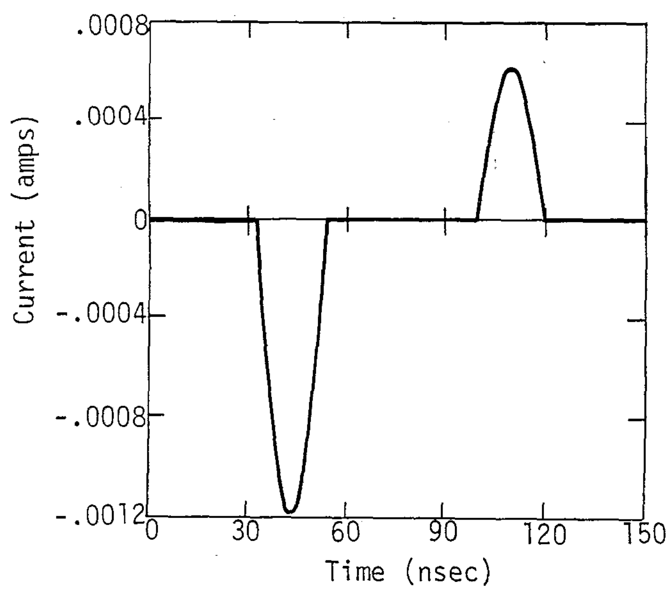

a. Current on Conductor One at the Input End (z=0)




b. Current on Conductor Two at the Input End (z=0)

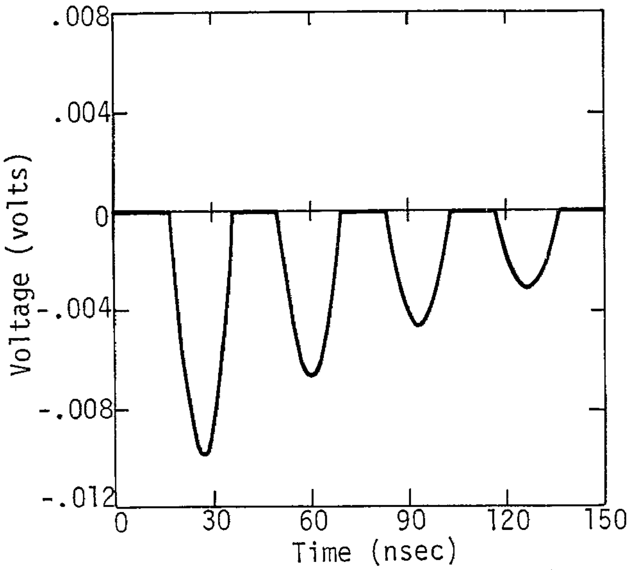



c. Current on Conductor One at the Load End (z=10 m)

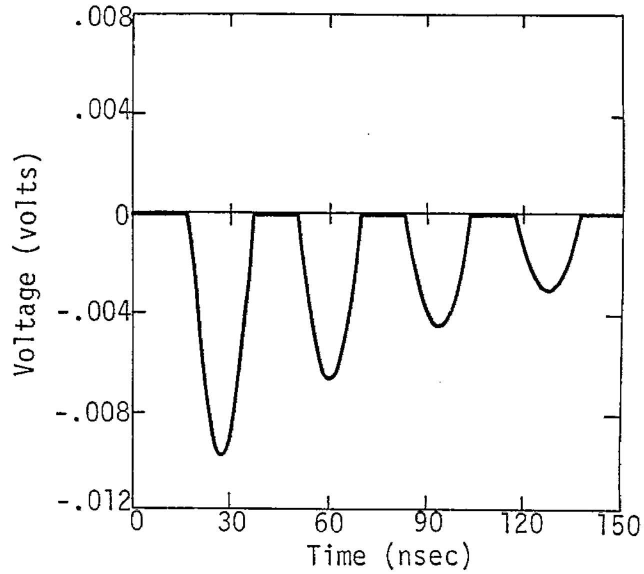



d. Current on Conductor Two at the Load End (z=10 m)

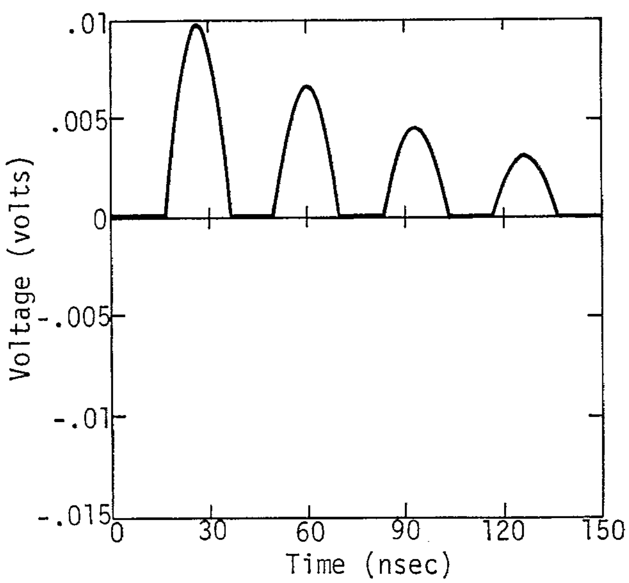
Figure 6. Transient Current Response of the Two Conductor Line (Over a Ground Plane) Excited at One End.



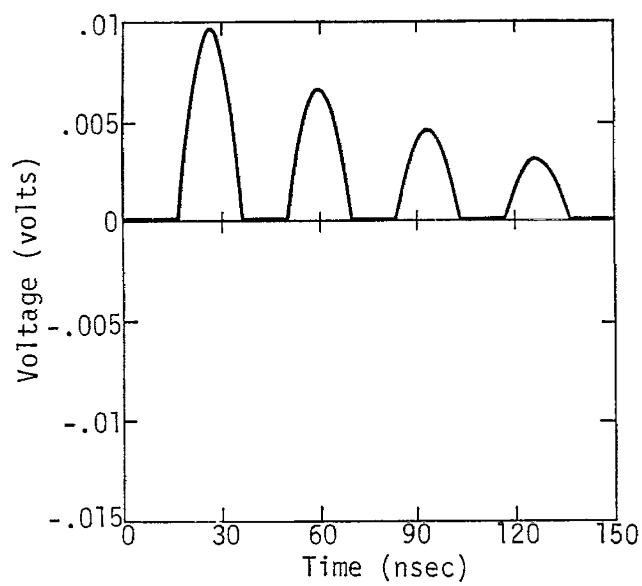

a. Total Voltage on Conductor One at the Input End (z=0)




b. Total Voltage on Conductor Two at the Input End (z=0)

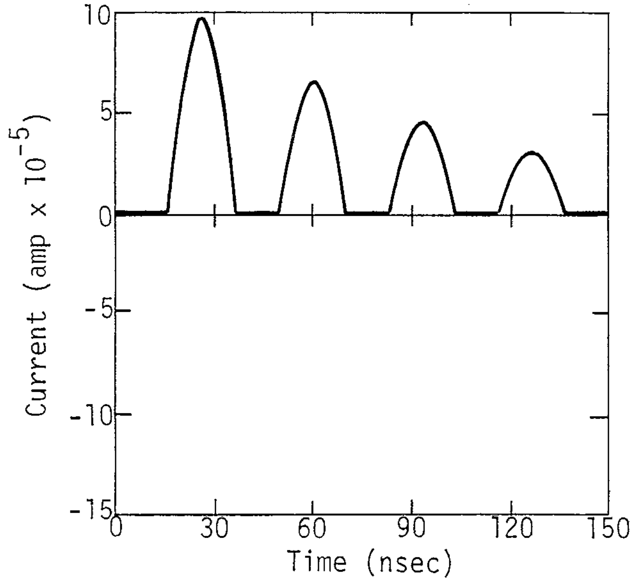



c. Total Voltage on Conductor One at the Load End (z=10 m)

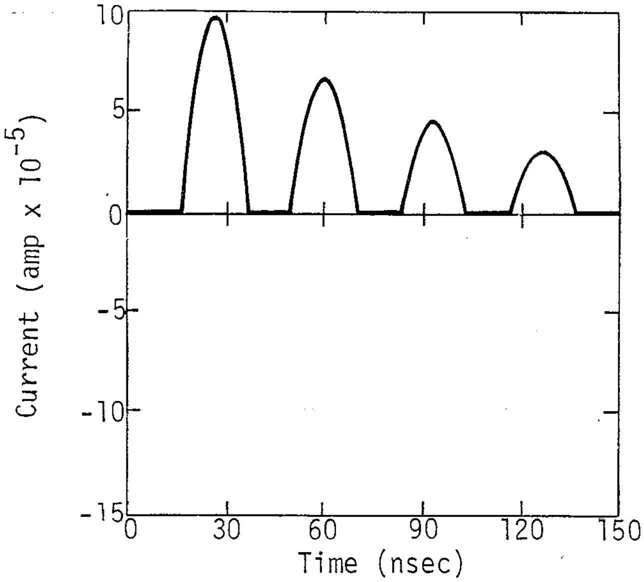



d. Total Voltage on Conductor Two at the Load End (z=10 m)

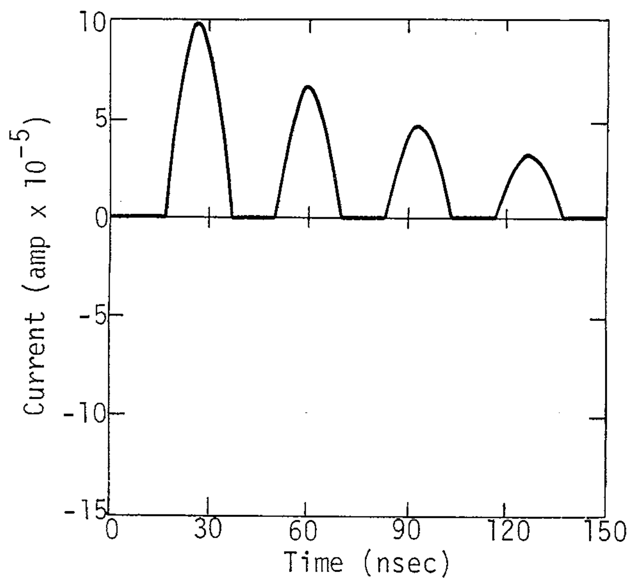
Figure 7. Transient Voltage Response of the Two Conductor Line (Over a Ground Plane) Excited at the Line Center.



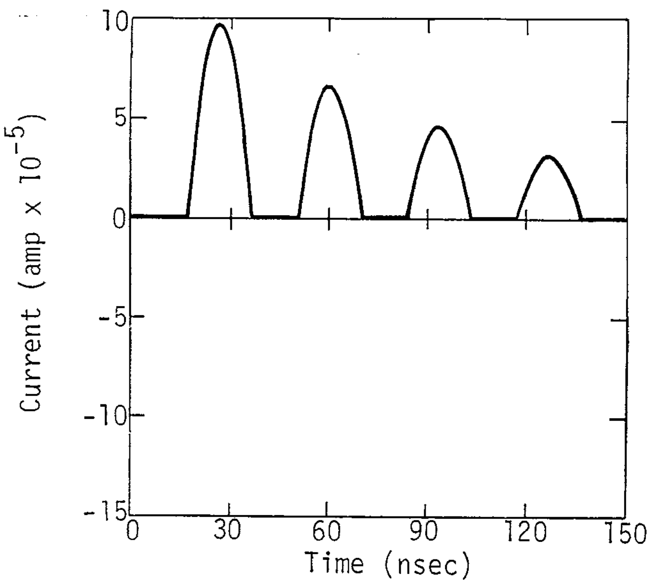

a. Current on Conductor One at the Input End (z=0)




b. Current on Conductor Two at the Input End (z=0)




c. Current on Conductor One at the Load End (z=10 m)




d. Current on Conductor Two at the Load End (z=10 m)

Figure 8. Transient Current Response of the Two Conductor Line (Over a Ground Plane) Excited at the Line Center.

## SECTION VI

## CONCLUSIONS

The equations for determining the time-domain response of multiconductor transmission lines excited by a transient, nonuniform electromagnetic field have been presented. Some of the assumptions made in the derivation have been discussed. The validity of the transmission-line equations rests on the assumptions of both transverse magnetic propagation and a small longitudinal derivative of the longitudinal electric field. Two formulations are presented in this paper: scattered voltage formulation and total voltage formulation. The scattered voltage formulation contains sources only in one equation. Since these sources turn out to be the net incident electric field parallel to the conductors, the numerical solution of the transmission-line equations is more efficient. The driving sources for the total voltage formulation contain time derivatives and spatial integrals of the incident fields. When using the scattered voltage formulation, it must be remembered that the vertical component of the incident electric field appears as a voltage source in the line terminations.

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