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Fundamentals of Steady-State and Transient Electromagnetic Fields in Shielding Enclosures

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Abstract

A simplified theory of shielding of steady-state and transient electromagnetic fields by enclosures of arbitrary shape with conductive and/or magnetic walls is developed. The integral equation for the unknown current in the thin shield is discussed, and an equivalent circuit representation is given. Comparisons with exact solutions are performed.

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Summary

A simplified theory of electromagnetic shielding by enclosures with conductive and/or magnetic walls is presented. The theory is based on a simplified type of boundary condition at the enclosure walls. Theoretical results are compared with exact solutions for steady-state and transient excitation and the approximations turns out to be extremely good in all cases of practical interest. For enclosures of arbitrary shapes, the problem is formulated as a single integral equation in the unknown current in the shield. Numerical solution of the equation is discussed, as well as a possible equivalent circuit representation.

1. The Problem

The problem of computation of shielding efficiency of conductive or magnetic shields on incoming electromagnetic radiation is certainly an old one. The first paper on the argument seems to be that of Larmor [1], published in 1884, and essentially based on a low frequency analysis. If we scan the technical literature on the argument during the last 10 years, it appears that papers can be broadly grouped into three categories: study of shielding cavities with no coupling holes, so that the field which leaks inside is essentially due to the imperfect conductivity of the walls [2-18]; study of shielding cavities with coupling holes [19-28]; transmission through grids [29-33]. Then, there are papers of more general interest, which cover more than one argument [34-37], and papers which are related to the case of shielding using magnetic materials, wherein the non-linearity of the shield is taken into account [38-42]. A number of papers is mainly experimental [43-48]. The above referenced papers are not exhaustive of the bibliography on the argument, but only representative of general research trends in the area. It is also noted that the only available standards for measuring the shielding effectiveness for quasi-static, sinusoidal and transient excitation are still those of the MIL Report of 1956 [49].

In all cases, the theoretical approach of study is based on the solution of Maxwell's equations with the appropriate boundary conditions on the shield surfaces. The excitation is generally assumed to be sinusoidal, with some noticeable exceptions in more recent papers which consider pulsed fields; in some cases quasi-static incident fields are considered, especially in the case of sources in presence of plane screens [10,13-15], which is important for the MIL standard specifications (which are, however, objectionable [50]). A circuit type of approach is also used in some cases [51-54].

Examination of the body of theoretical papers shows that, with the eventual exception of the circuital approach, the mathematical machinery is so heavy that the physical insight is often missed. Most of the papers, if not all of them, require extensive numerical computations, even if the approach is not purely numerical from the onset. In the case of papers which make use of a circuit type analogy, the connection with a rigorous approach is not completely clear either.

From the above considerations, it seems worthy to develop a simplified theory of electromagnetic shielding, in order to obtain simple expressions for the electromagnetic field inside the shielding enclosure. The basic idea which underlies the theory is briefly sketched in this section and is rigorously developed in the following ones. An important feature is that the approximation theory is compared with the exact one in some canonical problems, so that the approximation involved is clearly understood. The result is that the new theory is always an excellent approximation, in all cases of practical interest, for both steady-state and transient excitation. The theory is developed for the case of a shield with no apertures, although this limitation can be relaxed, sometimes very easily, as discussed in the conclusions of the paper. Interesting results are very simple expressions for the transient field inside the shielding enclosure which beautifully check with existing numerical data; the rigorous justification of the equivalent circuits, wherein the heuristic approach similar to that already available [53] is referred to for completeness under Sect. 6; the development of a simple integral equation, valid in any frequency range, whose numerical solution would allow the study of shielding properties of enclosures of arbitrary shape under steady-state or transient excitation. The basic idea on which the theory is based is the following.

When the electromagnetic field inside a shielding enclosure is being computed, a *three-region problem* is studied; the outside region, where the incident field is applied; the inside region, where the shielded field is present; and the region occupied by the shielding material. This last region is generally thin with respect to the incident wavelength or, if a transient field is applied, its thickness is small compared to the pulse spatial width or, eventually, pulse rise time multiplied by the velocity of light. The shielding material is usually highly conductive and may exhibit magnetic properties; it may be inhomogeneous, either due to variable thickness or to the presence of holes, gaskets, etc. These are essentially the reasons why the problem is rather complicated to analyze: a multi-region problem, with at least one region highly lossy and possibly inhomogeneous, wherein different field expansions should be used. And it is easily predictable that results of a theoretical analysis turn out to be so complicated that physical implications are difficult to assess.

For all of the above reasons, it seems worthwhile to try to simplify the mathematical model of the shielding problem. And the idea, that the geometry of the problem naturally suggests, is to substitute to the region occupied by the shielding material something which is equivalent, or nearly equivalent, but mathematically simpler to analyze. The incident field will induce in the shielding material a density current $\underline{J}$ and a density charge ρ, related by the continuity equation. These density current and charge will flow or be present inside the shielding material and will be responsible for the rapid decay of the field in the shield. Letting δ represent the shield thickness, the current and charge per unity length will be $\underline{J}_S = \underline{J}\delta$ and $\rho_S = \rho\delta$ respectively. The basic, intuitive idea is now *to substitute to the real shield*, of thickness δ, *a sort of infinitesimally thin shield on which*, however, *surface density current* $\underline{J}_S = \underline{J}\delta$ *and charge* $\rho_S = \rho\delta$ *are present.* The original three region problem - outside, inside, shield - with everywhere continuous fields (save eventually on edges or tips) has now been replaced by a two-region problem - outside, inside - with partly discontinuous fields on the boundary surface between the two regions. From another viewpoint, *the rapid variation of some field components inside the shield has been approximately accounted for by a discontinuous behavior.*

It should be noted that a new unknown has been introduced in our problem, namely the surface density current $\underline{J}_S$. However, it is related to the field vectors through the properties of the shield. For instance, in the case of a homogeneous conducting shield of conductivity σ, $\underline{J}_S$ is simply equal to $\sigma\delta$ times the tangential component of electric field component, continuous across the shield.

In the presentation of the basic ideas of this theory, reference has been made to an electric type of shield. We will show that similar reasoning applies to the case of a magnetic type of shield, wherein the surface electric density currents and charges are substituted by surface magnetic density currents and charges respectively. The procedure is formalized, for both cases, under Sect. 3.

A key point is obviously the assessment of the limit of validity of this approximate theory. This is made under Sect. 4 for steady-state and Sect. 5 for transient fields, comparing approximate and exact results. Then, the theory is fully developed in the case of an enclosure of arbitrary shape, with the only limitation that no coupling holes are present. The possibility of encompassing these more complicated situations is outlined in the conclusion of the paper.

2. Relevant Formulas and Pertinent Expansions

For the reader's convenience, relevant formulas and pertinent expansions used throughout the paper are hereafter collected.

Bessel and Hankel functions

$$J_n'(x) = \tfrac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x)\right] \qquad (2.1)$$

$$J_n''(x) = \tfrac{1}{4}\left[J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)\right] \qquad (2.2)$$

$$J_n'''(x) = \frac{1}{8}\left[J_{n-3}(x) - 3J_{n-1}(x) + 3J_{n+1}(x) - J_{n+3}(x)\right] \qquad (2.3)$$

and similarly for the Hankel functions. A prime means derivative operation.

$$J_n(x) = \frac{(x/2)^n}{0!n!} - \frac{(x/2)^{n+2}}{1!(n+1)!} + \frac{(x/2)^{n+4}}{2!(n+2)!} - \dots \qquad (2.4)$$

For $x \gg 1$, $x \gg n$:

$$J_n(x) \sim \sqrt{\frac{2}{\pi x}}\cos\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right). \qquad (2.5)$$

Wronskian relation:

$$H_{n-1}^{(2)}(x)J_n(x) - H_n^{(2)}(x)J_{n-1}(x) = H_n^{(2)\prime}(x)J_n(x) - H_n^{(2)}(x)J_n'(x)$$

$$= \frac{2}{\pi i x} . \qquad (2.6)$$

$$J_{-n}(x) = (-)^n J_n(x). \qquad (2.7)$$

For $x \to 0$, $n \geq 1$: $H_n^{(2)}(x) \simeq i\left(\frac{2}{x}\right)^n \frac{(n-1)!}{\pi}$. (2.8)

Spherical Bessel Functions

$$j_n(x) = \sqrt{\frac{\pi}{2x}}\; J_{n+\frac{1}{2}}(x) \qquad (2.9)$$

and similarly for the spherical Hankel function.

$$j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x} = \frac{x}{3} - \frac{x^3}{30} + \frac{x^5}{840} - \cdots \qquad (2.10)$$

$$h_1^{(2)}(x) = -\frac{1}{x}\left(1 - \frac{i}{x}\right) \exp(-ix) = \frac{i}{x^2}\left(1 + \frac{x^2}{2} - \frac{ix^3}{3} - \frac{x^4}{8} + \ldots\right) \qquad (2.11)$$

$$\left[xj_1(x)\right]' = \left(1 - \frac{1}{x^2}\right)\sin x + \frac{1}{x}\cos x = \frac{2}{3}x - \frac{2}{15}x^3 + \frac{1}{168}x^5 - \ldots \qquad (2.12)$$

$$\left[xh_1^{(2)}(x)\right]' = \frac{1}{x}\left(1 - \frac{i}{x}\right) \exp(-ix) = -\frac{i}{x^2}\left(1 - \frac{x^2}{2} + \frac{2ix^3}{3} + \frac{3x^4}{8} - \ldots\right) \qquad (2.13)$$

Fourier Transforms

$$-\int_{-\infty}^{+\infty} \frac{\exp(-i\frac{\omega}{c}x)}{4\pi x} \exp(i\omega t)\, d\omega = \frac{-1}{4\pi x}\,\delta\left(t - \frac{x}{c}\right) \qquad (2.14)$$

$$-\int_{-\infty}^{+\infty} \nabla \frac{\exp(-i\frac{\omega}{c}x)}{4\pi x} \exp(i\omega t)\, d\omega = \nabla\left(\frac{-1}{4\pi x}\right)\left[\delta\left(t - \frac{x}{c}\right) + \frac{x}{c}\,\delta'\left(t - \frac{x}{c}\right)\right] \qquad (2.15)$$

where a prime means derivative operation with respect to t.

3. The Shielding Boundary Conditions

Let us consider an enclosure of finite volume and of large dimensions compared to the wall thickness δ (see fig. 1), with no coupling apertures to the outside. The bounding surfaces, S_e and S_i, are sufficiently *smooth*

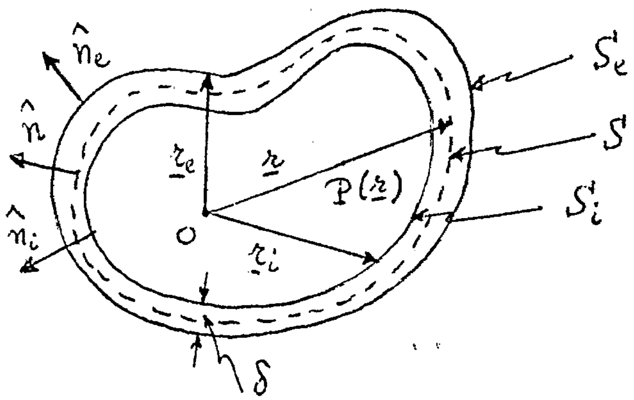

Fig. 1. Geometry of the enclosure.

so that the unit normal vectors $\hat{n}_e$, $\hat{n}_i$ can be unambiguously defined everywhere. Let also S be an *average* surface between S_e and S_i , with unit normal $\hat{n}$. The medium outside and inside the enclosure is assumed to be the same (although this limitation can easily be relaxed) and of electromagnetic parameters ε_o, μ_o. The material of the shield is characterized by permeability μ_o, permittivity $\varepsilon_o\varepsilon$, conductivity σ. Let us further indicate with $(\underline{E}^o, \underline{H}^o)$ the known incident electromagnetic field, $(\underline{E}^e, \underline{H}^e)$ the field outside the enclosure, sum of the incident and scattered field, $(\underline{E}^i, \underline{H}^i)$ the field inside the enclosure and $(\underline{E}, \underline{H})$ the field in the region occupied by the shield material. In transient analysis, fields will be functions of position $\underline{r}$ and time t; in steady-state analysis of position $\underline{r}$ and angular frequency ω, wherein a time dependence $\exp(i\omega t)$ is assumed. The same symbols for fields in time and frequency domain are used except where confusion could occur.

We assume the tangential components of electric field and normal components of magnetic field to be continuous across S; and tangential components of magnetic field and normal components of electric filed to be discontinuous due to equivalent surface current density $\underline{J}_s$ and surface charge density ρ_s respectively, both unknown for the moment, and located on S. Hence:

$$\underline{E}^e - \underline{E}^i = \frac{\rho_S}{\varepsilon_o} \hat{n} \tag{3.1}$$

$$\underline{H}^e - \underline{H}^i = \underline{J} \times \hat{n} \tag{3.2}$$

On the other hand, $\underline{J}_s$ and ρ_s are related by the continuity equation:

$$\nabla \cdot \underline{J}_s(\underline{r}, t) + \frac{\partial \rho_s(\underline{r}, t)}{\partial t} = 0 \tag{3.3}$$

$$\nabla \cdot \underline{J}_s(\underline{r}, \omega) + i\omega\rho_S(\underline{r}, \omega) = 0 \tag{3.4}$$

in time and frequency domain respectively. Equations (3.1-2) will be referred to as *electric shield boundary conditions.*

For the case of a shield with no apertures (which, however, may exhibit a variable conductivity σ or thickness δ) the introduced new vector field $\underline{J}_s$ can be easily related to the electric field. If $\underline{J}$ is the density current induced by the electric field in the shield, we have

$$\underline{J}_s = \underline{J}\delta = (\sigma + i\omega\varepsilon_o\varepsilon)\ \delta\ \hat{n} \times \underline{E} \times \hat{n} \simeq \sigma\delta\hat{n} \times \underline{E}^i \times \hat{n}, \text{ on S} \tag{3.5}$$

since the tangential component of the electric field is assumed to be continuous across S and $\sigma >> i\omega\varepsilon_o\varepsilon$ as usually in good conductors. Accordingly, the electric shield enclosure problem is specified in terms of the following boundary conditions, which stem out from (3.5 and 1.2):

$$\begin{cases} \hat{n} \times \underline{E}^e - \hat{n} \times \underline{E}^i = 0 & (3.6) \\ \hat{n} \times \underline{H}^e - \hat{n} \times \underline{H}^i = \sigma\delta\ \hat{n} \times \underline{E}^i \times \hat{n} & (3.7) \end{cases}$$

Consider now the same geometry of fig. 1, when the shield material is characterized by permittivity ε_o and permeability $\mu_o\mu$. We will assume the tangential components of magnetic field and normal components of electric field to be continuous across s; and tangential components of electric field and normal components of magnetic field to be discontinuous due to equivalent magnetic surface current density $\underline{J}_{ms}$ and surface charge density ρ_{ms} respectively, both unknown for the moment and located on s . Hence

$$\underline{H}^e - \underline{H}^i = \frac{\rho_{ms}}{\mu_o}\hat{n} \tag{3.8}$$

$$\underline{E}^e - \underline{E}^i = -J_{ms}\times\hat{n} \tag{3.9}$$

On the other hand, $\underline{J}_{ms}$ and ρ_{ms} are related by the continuity equation

$$\nabla \cdot \underline{J}_{ms}(\underline{r}, t) + \frac{\partial\rho_{ms}(\underline{r}, t)}{\partial t} = 0 \tag{3.10}$$

$$\nabla \cdot \underline{J}_{ms}(\underline{r}, \omega) + i\omega\rho_{ms}(r, \omega) = 0 \tag{3.11}$$

in time and frequency domain respectively. Equations (3.8-9) will be referred to as *magnetic shield boundary conditions.*

For the same case of shield with no apertures the introduced new vector field $\underline{J}_{ms}$ can be easily related to the magnetic field. As a matter of fact, the magnetic field in the shield, $\underline{H}$, is associated with a magnetic induction, $\underline{B} = \mu_o\mu\ \underline{H}$. Then, use of integral form of Maxwell equations shows that

$$\hat{n}_e \times \underline{E}^e - \hat{n}_i \times \underline{E}^i \approx -i\omega\mu_o\mu\delta\ \hat{n} \times \underline{H} \times \hat{n} \tag{3.12}$$

Comparison of (3.8 and 12) leads to the identification

$$\underline{J}_{ms} = i\omega\mu_o\mu\delta\ \hat{n} \times \underline{H} \times \hat{n} = i\omega\mu_o\mu\delta\ \hat{n} \times \underline{H}^i \times \hat{n} \tag{3.13}$$

The last equality results from the continuity of the tangential component of magnetic field across . Accordingly, the magnetic shield is specified in terms of the following boundary conditions, which stem out from (2.13 and 8-9):

$$\begin{cases} \hat{n} \times \underline{H}^e - \hat{n} \times \underline{H}^i = 0 & (3.14) \\ \hat{n} \times \underline{E}^e - \hat{n} \times \underline{E}^i = -i\omega\mu_0\mu\delta\, \hat{n} \times \underline{H}^i \times \hat{n} & (3.15) \end{cases}$$

It is explicitly noted that in the static case, wherein pure electric or magnetic fields exist, boundary conditions (3.6-7 and 14-15) are not appropriate, and use should be made of (3.1) and (3.8) respectively. Broad-band problems which include static or quasi-static fields require, as a consequence, some attention. A suitable formulation of this problem is given under Sect. 7, wherein a proper normalization for the surface current is introduced.

It is interesting to examine what happens when electric and magnetic shields are used together. When two shields are used, there are two possibilities (see fig. 2): either the magnetic shield can be located outside (case a) or the electric shield can face the incident field (case b). For the former case appropriate boundary conditions are the following:

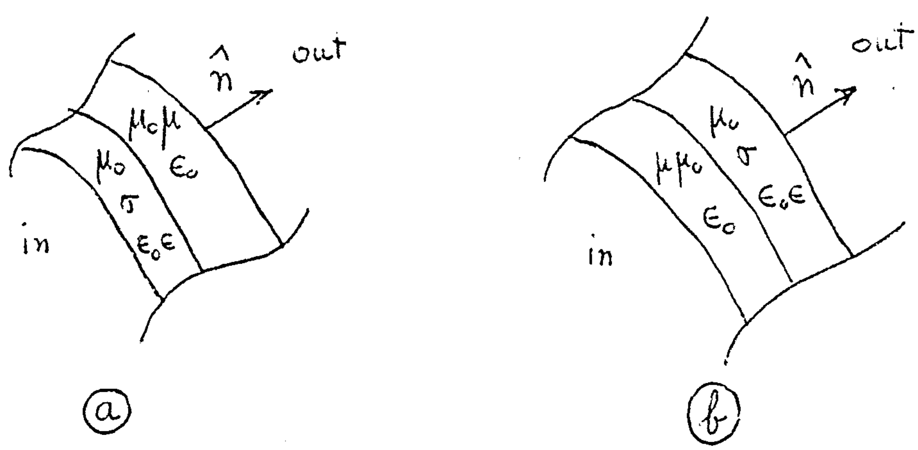

Fig. 2. Possible geometries of two-shield problem.

$$\begin{cases} \hat{n} \times \underline{H}^e - \hat{n} \times \underline{H}^i = \sigma\delta\, \hat{n} \times \underline{E}^i \times \hat{n} & (3.16) \\ \hat{n} \times \underline{E}^e - \hat{n} \times \underline{E}^i = -i\omega\mu_o\mu\delta\, \hat{n} \times \underline{H}^e \times \hat{n} & (3.17) \end{cases}$$

while, for the latter, we have

$$\begin{cases} \hat{n} \times \underline{H}^e - \hat{n} \times \underline{H}^i = \sigma\delta\, \hat{n} \times \underline{E}^e \times \hat{n} & (3.18) \\ \hat{n} \times \underline{E}^e - \hat{n} \times \underline{E}^i = -i\omega\mu_o\mu\delta\, \hat{n} \times \underline{H}^i \times \hat{n} & (3.19) \end{cases}$$

Accordingly, different positioning of the shields is not irrelevant. This problem will be dealt with under Sect. 5.

4. Approximation Study. Steady-State Analysis.

In order to check the validity of approximate boundary conditions on which previous analysis is based, a specific example is worked out. To this end, let us consider a plane wave normally incident on an infinitely long metal cylindrical enclosure (see fig. 3). Let superscripts "i" and "e" be used for fields inside and outside the cylinder, respectively, while no superscript is used for the field in the shieilding material. Let $k_s \simeq \sqrt{-i\omega\mu_o\sigma}$ be the propagation constant in the shield material and $\zeta_s \simeq \sqrt{i\omega\mu_o/\sigma}$ be its intrinsic impedance. Furthermore, k and ζ are propagation constant and intrinsic impedance in free-space inside and outside the enclosure.

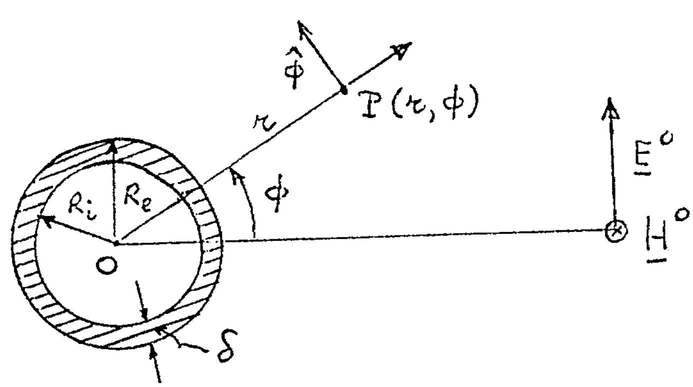

Fig. 3. Plane wave incident on a cylindrical enclosure. H-polarization.

Appropriate expansion of the electromagnetic field in the three regions of fig. 3 are the following:

$$\begin{cases} H^e = H^o \sum_{-\infty}^{\infty}{}_n\, i^n \left[J_n(kr) + a_n H_n^{(2)}(kr)\right] \exp(in\phi) & (4.1) \\ E_\phi^e = i\zeta H^o \sum_{-\infty}^{\infty}{}_n\, i^n \left[J_n'(kr) + a_n H_n^{(2)'}(kr)\right] \exp(in\phi) & (4.2) \end{cases}$$

$$\begin{cases} H = H^o \sum_{-\infty}^{\infty}{}_n\, i^n \left[c_n J_n(k_s r) + d_n Y_n(k_s r)\right] \exp(in\phi) & (4.3) \\ E_\phi = i\zeta_s H^o \sum_{-\infty}^{\infty}{}_n\, i^n \left[c_n J_n'(k_s r) + d_n Y_n'(k_s r)\right] \exp(in\phi) & (4.4) \end{cases}$$

$$\begin{cases} H^i = H^o \sum_{-\infty}^{\infty}{}_n\, i^n\, b_n J_n(kr) \exp(in\phi) & (4.5) \\ E_\phi^i = i\zeta H^o \sum_{-\infty}^{\infty}{}_n\, i^n b_n J_n'(kr) \exp(in\phi) & , (4.6) \end{cases}$$

where a prime means the derivative operation with respect to the argument.

Exact boundary conditions lead to the following 4x4 linear system for the expansion coefficients

$$J_n(kR_e) + a_n H_n^{(2)}(kR_e) = c_n J_n(k_s R_e) + d_n Y_n(k_s R_e) \qquad (4.7)$$

$$b_n J_n(kR_i) = c_n J_n(k_s R_i) + d_n Y_n(k_s R_i) \qquad (4.8)$$

$$J_n'(kR_e) + a_n H_n^{(2)'}(kR_e) = \chi \left[c_n J_n'(k_s R_e) + d_n Y_n'(k_s R_e)\right] \qquad (4.9)$$

$$b_n J_n'(kR_i) = \chi \left[c_n J_n'(k_s R_i) + d_n Y_n'(k_s R_i)\right] \qquad (4.10)$$

when $\chi = \sqrt{i\omega\varepsilon_o/\sigma}$.

On the contrary, the approximate *shield boundary condition* (3.6-7) leads to the 2x2 linear system

$$\begin{cases} -\left[J_n(ka) + \bar{a}_n H_n^{(2)}(ka)\right] + \bar{b}_n J_n(ka) = i\sigma\delta\zeta\bar{b}_n J_n'(ka) & (4.11) \\ J_n'(ka) + \bar{a}_n H_n^{(2)}(ka) = \bar{b}_n J_n'(ka) & (4.12) \end{cases}$$

where $\bar{a}_n$, $\bar{b}_n$ are the new expansion coefficients outside and inside the enclousure, respectively, and $a = (R_e + R_i)/2$. Note, for future reference, that $i\sigma\delta\zeta = -\, k_s\delta/\chi$.

A comparison between the systems (4.7-10) and (4.11-12) is simplified when (4.7) is subtracted from (4.8) and (4.10) from (4.9), thus obtaining

$$\begin{cases} -\left[J_n(kR_e) + a_n H_n^{(2)}(kR_e)\right] + b_n J_n(kR_i) = -c_n\left[J_n(k_sR_e) - J_n(k_sR_i)\right] - \\ \qquad - d_n\left[Y_n(k_sR_e) - Y_n(k_sR_i)\right] & (4.13) \\ J_n'(kR_e) + a_n H_n^{(2)\prime}(kR_e) = b_n J_n'(kR_i) + \chi c_n\left[J_n'(k_sR_e) - J_n'(k_sR_i)\right] + \\ \qquad + \chi d_n\left[Y_n'(k_sR_e) - Y_n(k_sR_i)\right] & (4.14) \end{cases}$$

Equations (4.13-14) will now be compared to (4.11-12), after proper manipulation. To this end, note that

$$J_n(k_sR_e) - J_n(k_sR_i) = J_n'(x)k_s\delta + \frac{J'''(x)}{3!}(k_s\delta)^3 + \dots \qquad (4.15)$$

$$Y_n(k_sR_e) - Y_n(k_sR_i) = Y_n'(x)\, k_s\delta + \frac{Y_n'''(x)}{3!}(k_s\delta)^3 + \dots \qquad (4.16)$$

$$J_n'(k_sR_e) - J_n'(k_sR_i) = J_n''(x)\, k_s\delta + \frac{J_n^{iv}(x)}{3!}(k_s\delta)^3 + \dots \qquad (4.17)$$

$$Y_n'(k_sR_e) - Y_n'(k_sR_i) = Y_n''(x)\, k_s\delta + \frac{Y_n^{iv}(x)}{3!}(k_s\delta)^3 + \dots \qquad (4.18)$$

when $x = k_s a$. Then, using (4.15-18), we have

$$c_n\left[J_n(k_sR_e) - J_n(k_sR_i)\right] + d_n\left[Y_n(k_sR_e) - Y_n(k_sR_i)\right] =$$

$$= \left[c_n J_n'(x) + d_n Y_n'(x)\right] k_s\delta + \left[c_n J_n'''(x) + d_n Y_n''(x)\right] \frac{(k_s\delta)^3}{3!} + \dots \qquad (4.19)$$

$$c_n \left[J_n'(k_s R_e) - J_n'(k_s R_i)\right] + d_n\left[Y_n'(k_s R_e) - Y_n'(k_s R_i)\right] =$$

$$= \left[c_n J_n''(x) + d_n Y_n''(x)\right] k_s\delta + \left[c_n J_n^{iv}(x) + d_n Y_n^{iv}(x)\right] \frac{(k_s\delta)^3}{3!} + \dots \qquad (4.20)$$

We will now explicitly assume that the shield thickness δ is very small compared to the wavelength of the *incident* field, namely

$$k\delta << 1. \qquad (4.21)$$

Accordingly, $kR_e \simeq kR_i \simeq ka$ with an error of order $k\delta << k_s\delta$. An estimate of the quantity $\left[c_n J_n'(x) + d_n Y_n'(x)\right]$ and subsequent derivatives can then be gained from (4.10). Hence,

$$\chi\left[c_n J_n'(x) + d_n Y_n'(x)\right] \simeq b_n J_n'(ka), \qquad (4.22)$$

$$\chi\left[c_n J_n''(x) + d_n Y_n''(x)\right] \simeq \frac{b_n}{k_s} J_n''(ka) \qquad (4.23)$$

and similarly for higher order derivatives. Then, using (4.22) and subsequent derivatives for simplifying (4.19-20) and then substituting in (4.13-14), we get the final result

$$- \left[J_n(ka) + a_n H_n^{(2)}(ka)\right] + b_n J_n(ka) = - \frac{k_s\delta}{\chi} b_n \left[J_n'(ka) + J_n'''(ka) \frac{(k\delta)^2}{3!} + \dots\right] \qquad (4.24)$$

$$J_n'(kR) + a_n H_n^{(2)}(kR) = b_n\left[J_n'(ka) + J_n''(ka)k\delta + \dots\right] \qquad (4.25)$$

The original system (4.7-10) has now been cast in the form (4.24-25) which is similar to (4.11-12) and a comparison is meaningful. The two systems differ in quantities involving $k\delta$, so that it can be anticipated that the approximation in using the shield boundary conditions will not depend on shield material parameters, but only on its thickness compared to the *incident* wavelength.

Since the field inside the enclosure is of interest, the two coefficients, b_n, b_n, should be compared, hence

$$\bar{b}_n = \frac{H_n^{(2)} J_n' - J_n H_n^{(2)'}}{H_n^{(2)} J_n' - J_n H_n^{(2)'} + i\sigma\delta\zeta H_n^{(2)} J_n'} \qquad (4.26)$$

$$b_n = \frac{H_n^{(2)} J_n' - J_n H_n^{(2)'}}{H_n^{(2)} J_n' - J_n H_n^{(2)'} + i\sigma\delta\zeta H_n^{(2)'} J_n' + H_n^{(2)} J_n'' k\delta + i\sigma\delta\zeta H_n^{(2)'} J_n''' \frac{(k\ )^2}{3!} + \dots} \qquad (4.27)$$

where all functions are computed for the argument equal to ka. From (4.25-6) it follows that

$$\frac{\bar{b}_n - b_n}{b_n} \simeq \frac{H_n^{(2)} J_n'' + i\sigma\delta\zeta H_n^{(2)'} J_n''' k\delta/3!}{H_n^{(2)} J_n' - H_n^{(2)'} J_n + i\sigma\delta\zeta H_n^{(2)'} J_n'} k\delta \qquad (4.28)$$

In all practical situations $\sigma\delta\zeta$ is a very large quantity. Accordingly:

$$\frac{\bar{b}_n - b_n}{b_n} \simeq \frac{J_n'''(ka)}{J_n'(ka)} \frac{(k\delta)^2}{3!} \qquad (4.29)$$

which gives the relative error in using the approximate expansion coefficients $\bar{b}_n$ instead of the exact ones b_n.

For ka < 1, it is useful to transform (4.29) using (2.3), thus getting

$$\frac{\bar{b}_n - b_n}{b_n} = \frac{1}{8} (k\delta)^2 + \frac{1}{24} \frac{J_{n-3}(ka) - J_{n+3}(ka)}{J_{n-1}(ka) - J_{n+1}(ka)} (k\delta)^2 \qquad (4.30)$$

Use of (2.4 and 7) shows that the second term at the right hand side of (4.30) is of order $(ka)^2$ for n = 0, 1, and of unity for n = 3. Accordingly, the relave error is of order $(k\delta)^2$. For n > 3, the second term is of order $(n-1)(n-2)(\delta/a)^2$, so that the approximation may become worse for exceedingly large values of n.

For ka > n and large, use of (2.5) shows that

$$\frac{\bar{b}_n - b_n}{b_n} \simeq \frac{(k\delta)^2}{6} \tag{4.31}$$

The conclusion is that *the shield boundary conditions are a very good approximation to the exact ones provided that δ/λ is small, λ being the incident wavelength.* In passing, we note that the field inside the cylindrical enclosure takes the following remarkabl simple expression upon solution of (4.11-12) and use of (2.6):

$$\underline{H}^i = H^o \hat{z} \sum_{-\infty}^{\infty}{}_n \; i^n \frac{J_n(kr)\ \exp(in\phi)}{1 + \frac{\pi}{2}\ \delta\mu_o a\sigma\delta H_n^{(2)'}(ka) J_n'(ka)} \tag{4.32}$$

$$\underline{E}^i = \zeta H^o \sum_{\infty}^{\infty}{}_n \frac{\exp(in\phi)}{1 + \frac{\pi}{2}\omega\mu_o a\sigma\delta H_n^{(2)'}(ka) J_n'(ka)} \left[\frac{n}{kr} J_n(kr)\hat{r} + i\ J_n'(kr)\hat{\phi}\right] \tag{4.33}$$

In particular, at r = 0, only one term of the series is different from zero, and we get

$$H^i(r = 0) = \frac{\underline{H}^o}{1 + \frac{\pi}{2}\omega\mu_o a\sigma\delta H_1^{(2)}(ka) J_1(ka)} \simeq \frac{1}{1 + i\omega \frac{\mu_o a\sigma\delta}{2}} \underline{H}^o \tag{4.34}$$

$$\underline{E}^i(r = 0) = \frac{\underline{E}^o}{1 + \frac{\pi}{2}\omega\mu_o a\sigma\delta H_1^{(2)'}(ka) J'(ka)} \simeq \frac{i\omega \frac{2\varepsilon_o a}{\sigma\delta}}{1 + i\omega \frac{2\varepsilon_o a}{\sigma\delta}} \underline{E}^o \tag{4.35}$$

when the last equalities are valide under the assumption ka < 1 and use has been made of (2.2,4 and 8). Results (4.34-35) will be used in Sect. 6 for a circuital description of the shielding phenomenon.

5. Approximation Study. Transient Analysis.

The introduced shield boundary conditions can be successfully used for studying the performance of a shielding enclosure under transient excitation.

The example of a spherical shielding cavity will be considered (see fig. 4), so that a comparison with the numerical data of ref [18] is possible.

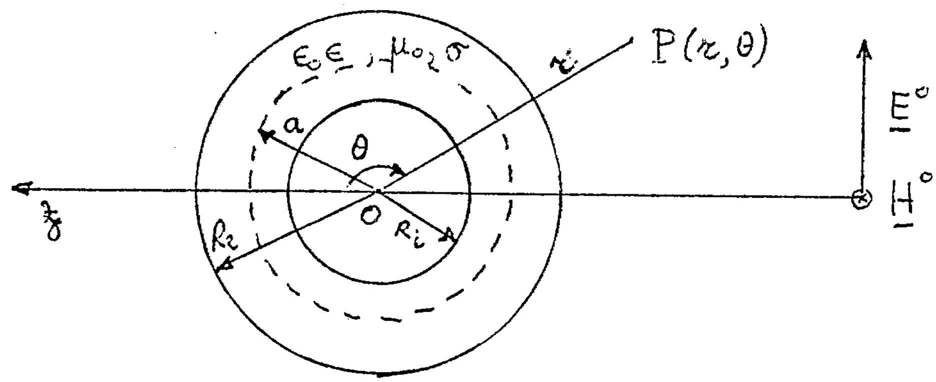

Fig. 4. Transient field incident on a spherical enclosure.

Let us first consider a steady-state incident plane wave, ($\underline{E}^o$, $\underline{H}^o = \hat{z} \times \underline{E}^o/\zeta$). Computation of the field inside the cavity ($\underline{E}^i$, $\underline{H}^i$) is straightforward and parallel to that for the cylindrical case. Only the final result will be hereafter referred to. The reader interested in the *exact* analysis can consult [18].

The field in the *center of the cavity*, i.e., r = 0, is given by

$$\underline{E}^i = \frac{\underline{E}^o}{1 + \sigma\delta\zeta\left[kah_1^{(2)}(ka)\right]'\left[kaj_1(ka)\right]'} \tag{5.1}$$

$$\underline{H}^i = \frac{\underline{H}^o}{1 + \sigma\delta\zeta(ka)^2 h_1^{(2)}(ka)\ j_1(ka)} \tag{5.2}$$

where $h_1^{(2)}(x)$ and $j_1(x)$ are spherical Hankel and Bessel functions respectively, as defined in (2.9). Accordingly, the field in the center of the enclosure has the same polarization of the incident field.

Consider now an incident field which is bandlimited to an upper frequency such that $ka < 1$. Then, a series expansion of the denomination of (5.1-2) is appropriate. Let $\tau = \sqrt{\varepsilon_0 \mu_0}\, a$ and is the transit time along the radius of the cavity and $g = \zeta_0 \sigma \delta$ and is the (normalized) surface conductance of the shield. For ka small we get

$$\underline{E}^i(\omega) \quad \frac{i\omega\frac{3\tau}{2g}}{1 + i\omega\frac{3\tau}{2g}} \underline{E}^o(\omega) \tag{5.3}$$

$$\underline{H}^i(\omega) \quad \frac{1}{1 + i\omega\,\frac{g\tau}{3}} \underline{H}^o(\omega) \tag{5.4}$$

where use has been made of (2.10-13). Equations (5.3-4) provide the transfer function for computing the Fourier transform of the electromagnetic fields in the center of the spherical enclosure excited by a bandlimited incident plane wave.

The time dependent electric field in the center of the cavity, $\underline{E}^i(t)$, is expressed as the convolution product of the inverse Fourier transforms of the two factors appearing in (5.3). Letting U(t) be the unit step function,

$$\underline{E}^i(t) = \underline{E}^o(t) * \left[\delta(t) - \frac{\exp(-2gt/3\tau)}{3\,\tau/2g} U(t)\right] = \frac{d\underline{E}^o(t)}{dt} * \left[\exp(-2gt/3\tau)\, U(t)\right] =$$

$$= \frac{3\tau}{2g}\left\{\frac{d\underline{E}^o(t)}{dt} - \frac{d^2\underline{E}^o(t)}{dt^2} * \left[\exp(-2gt/3\tau)\, U(t)\right]\right\} \tag{5.5}$$

where the asterisk means the convolution operator and successive integration by parts have been performed. Accordingly, the electric field is highly shielding provided that its rate of change with time is small compared to $3\tau/2g$. This is always the case for all practical situations. For instance, an enclosure of radius $a = 1$ m, thickness $\delta = 1$ mm and with copper walls ($\sigma = 5.8 \times 10^7$ siemens/m) has $3\tau/2g \simeq 2 \cdot 10^{-16}$ sec.

Equation (5.5) shows that the *dominant part of the electric field in the center of the enclosure is proportional to the time derivative of the incident field.* This behavior is clearly displayed under figs. 3 through 7 of ref. [29], where a plane wave with a gaussian pulse variation in time has been assumed. Results of ref. [29] have been obtained by numerical Fourier inversion of exact steady state solution.

The time depedent magnetic field in the cavity, $\underline{H}^i(t)$, is similarly expressed as

$$\underline{H}^i(t) = \underline{H}^o(t) * \left[\frac{\exp(-3t/g\tau)}{g\tau/3} U(t)\right] \tag{5.6}$$

For the same enclosure with copper walls and a = 1 m, δ = 1 mm, we have $g\tau/3 \simeq 25$ msec.

An incident field pulsed in time, with a measure of pulse width T, produces a completely different magnetic field inside the enclosure according to the values of T compared to $g\tau/3$. As a matter of fact, integration by parts of (5.6) gives

$$\underline{H}^i(t) = \underline{H}^o(t) - \frac{g\tau}{3}\left\{\frac{d\underline{H}^o}{dt} + \frac{d^2\underline{H}^o}{dt^2} * \left[\exp(-3t/g\tau)U(t)\right]\right\} \tag{5.7}$$

Accordingly, for $T \gg g\tau/3$

$$\underline{H}^i(t) \simeq \underline{H}^o(t) \tag{5.8}$$

and the *magnetic field is not appreciably shielded.* On the contrary, for $T \ll g\tau/3$, we have directly from (5.6)

$$\underline{H}^i(t) = \frac{3}{g\tau}\exp(-3t/g\tau)\int_{-\infty}^{t}\underline{H}^o(u)\exp(3u/g\tau)du \simeq \frac{3}{g\tau}\exp(-3t/g\tau)\int_{-\infty}^{t}\underline{H}^o(u)du \tag{5.9}$$

It follows that the *magnetic field* in the center of the enclosure *is proportional to the time integral of the incident field* and is extinguished with a time constant $g\tau/3$. This behavior is clearly displayed under figs. 11 through 13 of ref. [29].

The case of a magnetic type spherical enclosure can be treated similarly. Relations dual to (5.1-2) are the following

$$\underline{H}^i = \frac{\underline{H}^o}{1 + ik\delta\mu \left[kah_1^{(2)}(ka)\right]' \left[kaj_1(ka)\right]'} \tag{5.10}$$

$$\underline{E}^i = \frac{\underline{E}^o}{1 - ik\,\delta\mu\,(ka)^2 h_1^{(2)}(ka) j_1(ka)} \tag{5.11}$$

And the equivalents of (5.3-4), valid for $ka < 1$, are

$$\underline{H}^i(\omega) \quad \frac{\underline{H}^o(\omega)}{1 + \frac{2\mu\delta}{3a}} \tag{5.12}$$

$$\underline{E}^i(\omega) \quad \frac{\underline{E}^o(\omega)}{1 - \omega^2\,\tau^2\,\frac{\mu\delta}{3a}} \tag{5.13}$$

Accordingly, no significant shielding is provided for the electric field in the low frequency range, while the magnetic field may be appreciably shielded if $2\mu\delta \gg 3a$.

It is interesting to consider the case of two shielding sheets, one electric and one magnetic and one over the other. If the *magnetic sheet* is located *outside* and the *electric one inside*, the expressions equivalent to (5.3-4) are the following

$$\underline{E}^i(\omega) = \frac{i\omega\,\frac{3\tau}{2g}}{1 + i\omega\,\frac{3\tau}{2g} - \omega^2\,\tau^2\,\frac{\mu\delta}{a}}\,\underline{E}^o(\omega) \tag{5.14}$$

$$\underline{H}^i(\omega) = \frac{\underline{H}^o(\omega)}{1 + \frac{2\mu\delta}{3a}} \frac{1}{1 + i\omega \frac{g\tau}{3} \frac{3a + \mu\delta}{3a + 2\mu\delta}} \qquad (5.15)$$

It is seen that the *best* of pure electric and magnetic shield is taken, also with some minor changes in time constants.

Slightly different results are obtained when the *electric shield* is located *outside* and the *magnetic one inside*. Hence:

$$\underline{E}^i(\omega) = \frac{i\omega\frac{3\tau}{2g}}{1 + i\omega \frac{3\tau}{2g} - \omega^2\tau^2 \frac{\mu\delta}{2a}} \underline{E}^o(\omega) \qquad (5.16)$$

$$\underline{H}^i(\omega) = \frac{\underline{H}^o(\omega)}{1 + \frac{2\mu\delta}{3a}} \frac{1}{1 + i\omega\frac{g\tau}{3} \frac{3a + 6\mu\delta}{3a + 2\mu\delta}} \qquad (5.17)$$

It is recognized that the latter configuration produces a better shielding at higher frequencies. The physical reason is that the electric shield has, in this case, a sort of magnetic core so that its efficiency increases. On the contrary, in the other configuration, the magnetic shield has a sort of electric core, whose induced currents tend to counterbalance the external magnetic field, thus decreasing the efficiency of the magnetic shield.

The transient analysis for these other cases parallels that for the electric shield. No significant new results are obtained.

6. The Circuital Viewpoint.

Equation (5.3), as well as (4.35), suggests the possible use of the equivalent circuit of fig. 5a for an estimate of the field inside the shielding enclosure in the case of a low-frequency bandlimited incident fied. For the spherical enclosure,

$$RC = \frac{3\tau}{2g} = \frac{3\varepsilon_o a}{2\sigma\delta} \qquad (6.1)$$

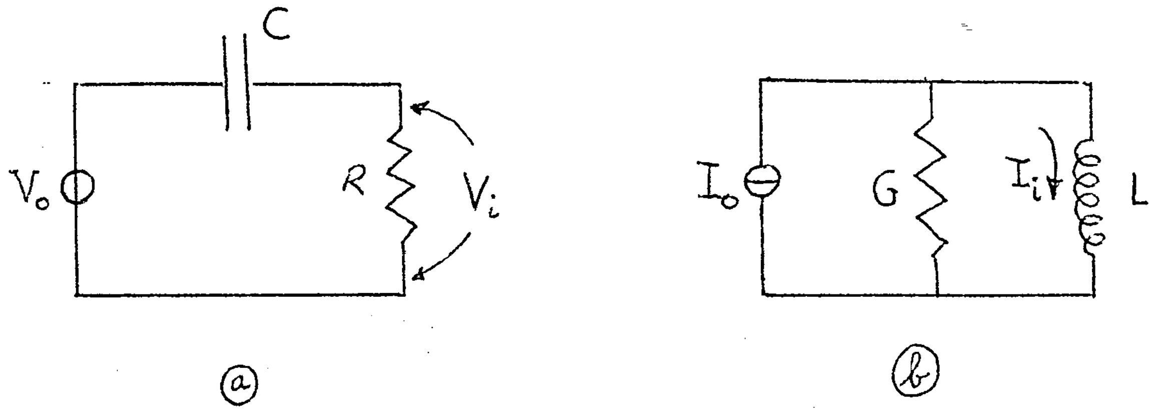

Fig. 5. Equivalent circuit of a shielding enclosure.

and V_o,V_i are yet unspecified input and output voltages respectively.

Similarly, Eq. (5.4), as well as (4.34), suggests the circuit of fig. 5b, wherein I_o,I_i are yet unspecified input and output currents and, for the spherical enclosure,

$$GL = \frac{g\tau}{3} = \frac{\sigma\delta\mu_o a}{3} \qquad (6.2)$$

An heuristic justification of above circuits may be considered to be a narcisistic academic exercise. However, it is not so, since it provides a way to estimate shielding efficiencies for enclosures of either complicated shape or uneasy surface boundary conditions (enclosures with holes, gaskets, etc.). We will justify hereafter the equivalent circuit of fig. 5a. Consider the solution of a metal sphere in a uniform static field $\underline{E}_o$, thus described by an incident potential $\phi^o = -E_o r \cos\theta$ [55]. We will identify the voltage V_o of fig. 5a with the applied potential difference across the sphere, hence

$$V_o = 2E_o a \qquad (6.3)$$

On the other hand, the applied potential ϕ_o will interact with the surface charge distribution $\rho_s = 3\varepsilon_o E_o \cos\theta$ with a resulting work W

$$-W = \frac{1}{2} \int_0^{\pi} a d\theta \int_0^{2\pi} a \sin\theta d\phi \; 3\varepsilon_o E_o^2 \, a \cos^2\theta = 2\pi\varepsilon_o E_o^2 a^3 \tag{6.4}$$

We will assume the negative of this work to be stored in an equivalent capacitance with an applied voltage V_o, i.e.,

$$-W = \frac{1}{2} \frac{4E_o^2 a^2}{C} \tag{6.5}$$

Then, solving for C, we have

$$C = \pi\varepsilon_o a \tag{6.6}$$

The finite (high) conductivity of the spherical shell and the (slow) dynamical behavior of the field will result in a slight perturbation. A density current $\underline{J}$ will flow in the shield, given, via (3.4), by

$$\underline{J} = \frac{-i\omega a 3\varepsilon_o E_o \sin\theta}{\delta} \tag{6.7}$$

Accordingly, we will have a total ohmic power dissipation

$$P = \frac{1}{2} \int_0^{\pi} a d\theta \int_0^{2\pi} a \sin\theta d\phi \; \frac{9\omega^2 a^2 \varepsilon_o^2 E_o^2 \sin^2\theta}{\sigma\delta} = 12\pi \frac{\omega^2 a^4 \varepsilon_o^2 E_o^2}{\sigma\delta} \tag{6.8}$$

The total potential difference due to this current is given by

$$V_R = \int_{\pi}^{0} a d\theta \; \frac{i\omega a 3\varepsilon_o E_o \sin\theta}{\sigma\delta} = \frac{6i\omega a^2 \varepsilon_o E_o}{\sigma\delta} \tag{6.9}$$

We will assume the power P to be dissipated on a resistor R with an applied voltage V_R, i.e.,

$$P = \frac{1}{2} \frac{|V_R|^2}{R} \tag{6.10}$$

Then, solving for R, we have

$$R = \frac{3}{2\pi\sigma\delta} \tag{6.11}$$

Now, it can be immediately checked that (6.6) times (6.11) gives the exact value (6.1) for the time constant of the circuit of fig. 1a. Then, $V_i/2a$ gives an estimate of the field inside the cavity, which is described by a potential equal to V_i times a coordinate function. For instance, for this case of spherical geometry, the potential inside the enclosure would be equal to $- V_i r\cos\theta/2a$. If we assume the above procedure to be valid for an enclosure of arbitrary shape, we conclude that *the solution for the static surface charge on the enclosure may result in valuable information about the intensity of the shielded electric field.* An approach similar to the previous one has been exploited elsewhere [53].

7. Integral Equation Formulation for the Field in the Shielding Enclosure. Steady-State Case.

Let us now consider the general case of an enclosure of arbitrary shape, as depicted in fig. 1. The two surfaces S_e, S_i are sufficiently smooth so that unit normals $\hat{n}_e, \hat{n}_i$ can be unambiguously defined everywhere, as well as the unit normal $\hat{n}$.

Let us first consider the case of an electric shield. The integral equation for the field on the two surfaces S_e, S_i are well-known and are just quoted hereafter with reference to a Green's function:

$$\psi = (\underline{r},\underline{r}') = - \frac{\exp(-ik|\underline{r}-\underline{r}'|)}{4\pi|\underline{r}-\underline{r}'|}, \quad k = \omega\sqrt{\varepsilon_o\mu_o} = \frac{2\pi}{\lambda} \qquad (7.1)$$

They are, for the electric field

$$\frac{1}{2}\underline{E}^e = \underline{E}^o - \iint_{S_e}\left[(\hat{n}_e' \times \underline{E}^e) \times \nabla'\psi + (\hat{n}_e' \cdot \underline{E}^e)\nabla'\psi - i\omega\mu_o(\hat{n}_e' \times \underline{H}^e)\psi\right]dS' \qquad (7.2)$$

$$\frac{1}{2}\underline{E}^i = \iint_{S_i}\left[(\hat{n}_i' \times \underline{E}^i) \times \nabla'\psi + (\hat{n}_i' \cdot \underline{E}^i)\nabla'\psi - i\omega\mu_o(\hat{n}_i' \times \underline{H}^i)\psi\right]dS' \qquad (7.3)$$

In (7.2-3) the prime in the operator, ∇', means that the gradient operation is performed with respect to the primed $\underline{r}'$ coordinate; the integrals are intended as principal values in the Cauchy sense; and $\psi = \psi(\underline{r}_e, \underline{r}_e')$ in (7.2) while $\psi = \psi(\underline{r}_i, \underline{r}_i')$ in (7.3). The similar equations for the magnetic field are easily obtained from (7.2-3) by changing $\underline{E}$ with $\underline{H}$ and vice-versa, and μ_o with $-\varepsilon_o$. Hence:

$$\frac{1}{2}\underline{H}^e = \underline{H}^o - \iint_{S_e}\left[(\hat{n}_e' \times \underline{H}^e) \times \nabla'\psi + (\hat{n}_e' \cdot \underline{H}^e)\nabla'\psi + i\omega\varepsilon_o(\hat{n}_e' \times \underline{E}^e)\psi\right]dS' \tag{7.4}$$

$$\frac{1}{2}\underline{H}^i = \iint_{S_i}\left[(\hat{n}_i' \times \underline{H}^i) \times \nabla'\psi + (\hat{n}_i' \cdot \underline{H}^i)\nabla'\psi + i\omega\varepsilon_o(\hat{n}_i' \times \underline{E}^i)\psi\right]dS' \tag{7.5}$$

From (7.1-2), (3.1,2,4) and (7.4-5), (3.1-2), it follows that

$$\frac{1}{2}(\underline{E}^e + \underline{E}^i) \simeq \underline{E}^o + \iint_S\left[\frac{1}{i\omega\varepsilon_o}\nabla' \cdot \underline{J}_s\nabla'\psi + i\omega\mu_o\underline{J}_s\psi\right]dS', \quad \underline{r} \text{ on } S \tag{7.6}$$

$$\frac{1}{2}(\underline{H}^e + \underline{H}^i) \simeq \underline{H}^o - \iint_S \underline{J}_s \times \nabla'\psi\, dS', \quad \underline{r} \text{ on } S \tag{7.7}$$

Then, from (7.6) and (3.1,4),

$$\underline{E}^i = \underline{E}^o + \frac{1}{2}\frac{\nabla \cdot \underline{J}_s}{i\omega\varepsilon_o}\hat{n} + \iint_S\left[\frac{1}{i\omega\varepsilon_o}\nabla' \cdot \underline{J}_s\nabla'\psi + i\omega\mu_o\underline{J}_s\psi\right]dS', \quad \underline{r} \text{ on } S \tag{7.8}$$

and, from (7.7) and (3.2),

$$\underline{H}^i = \underline{H}^o - \frac{1}{2}\underline{J}_s \times \hat{n} - \iint_S \underline{J}_s \times \nabla'\psi dS', \quad \underline{r} \text{ on } S \tag{7.9}$$

Equations (7.8-9) express the electromagnetic field $(\underline{E}^i, \underline{H}^i)$ in any point $\underline{r}$ of the surface $S \simeq S_i$ and, therefore, completely determine the field inside the enclosure when the surface current $\underline{J}_s$ is known.

The integral equation for the unknown vector field $\underline{J}_s$ can be obtained when $\underline{J}_s$ is related to the fields on S. For an enclosure with no apertures the appropriate relation is (3.5). Then, the projection of (7.8) on the surface S will provide the integral equation for $\underline{J}_s$, hence

$$\frac{1}{\sigma\delta}\underline{J}_s = \hat{n} \times \underline{E}^o \times \hat{n} + \hat{n} \times \iint_S \left[\frac{\nabla' \cdot \underline{J}_s}{i\omega\varepsilon_o} \nabla'\psi + i\omega\mu_o \underline{J}_s \psi\right] dS' \times \hat{n} \qquad (7.10)$$

or, in a convenient normalized form,

$$\eta\underline{F} = \underline{E} + \hat{n} \times \iint_S \left[\nabla' \cdot \underline{F}\nabla'\psi - \beta^2 \underline{F}\psi\right] dS' \times \hat{n} \qquad (7.11)$$

where all lengths are normalized to a typical dimension "a" of the enclosure, $\beta = 2\pi/\lambda$ (remember, all lengths are normalized), $\zeta = \sqrt{\mu_o/\varepsilon_o}$,

$$\underline{E} = a\hat{n} \times \underline{E}^o \times \hat{n} \qquad (7.12)$$

$$\underline{F} = \frac{\zeta\, a\underline{J}_s}{i\beta} = \frac{a\hat{n} \times \underline{E}^i \times \hat{n}}{\eta} \qquad (7.13)$$

$$\eta = \frac{i\omega\varepsilon_o a}{\sigma\delta} \qquad (7.14)$$

The particular normalization for $\underline{F}$ *is suggested by the opportunity to have a vector field which does not vanish as the frequency is reduced to zero.* And this is certainly true for $\underline{F}$, whose divergence is proportional to the induced density charge on the shielding material. Note further that $\underline{E}$ and $\underline{F}$ have the same dimension, i.e., volt.

The case of a magnetic shield can be treated similarly. From (7.4-5), (3.8-9,11) and (7.1-2), (3.8-9), it follows that

$$\frac{1}{2}(\underline{H}^e + \underline{H}^i) = \underline{H}^o + \iint_S \left[\frac{1}{i\omega\mu_o} \nabla' \cdot \underline{J}_{ms}\nabla'\psi + i\omega\varepsilon_o \underline{J}_{ms}\psi\right] dS', \ \underline{r} \text{ on } S \qquad (7.15)$$

$$\frac{1}{2}\left(\underline{E}^e + \underline{E}^i\right) = \underline{E}^o + \iint_S \underline{J}_{ms} \times \nabla'\psi \, dS' , \quad \underline{r} \text{ on } S \tag{7.16}$$

Then, from (7.15) and (3.8,11)

$$\underline{H}^i = \underline{H}^o + \frac{1}{2}\frac{\nabla \cdot \underline{J}_{ms}}{i\omega\mu_o}\hat{n} + \iint_S \left[\frac{\nabla' \cdot \underline{J}_{ms}}{i\omega\mu_o}\nabla'\psi + i\omega\varepsilon_o \underline{J}_{ms}\psi\right] dS' , \quad \underline{r} \text{ on } S \tag{7.17}$$

and from (7.16) and (3.9)

$$\underline{E}^i = \underline{E}^o + \frac{1}{2}\underline{J}_{ms} \times \hat{n} + \iint_S \underline{J}_{ms} \times \nabla'\psi dS' , \quad \underline{r} \text{ on } S \tag{7.18}$$

which are the equations dual to (7.8-9).

Also, in this case, the integral equation for the unknown vector field $\underline{J}_{ms}$ can be obtained when $\underline{J}_{ms}$ is related to the fields on S. For an enclosure with no apertures, the appropriate relation is (3.13). Then, the projection of (7.17) on the surface S will provide the integral equation for $\underline{J}_{ms}$, hence

$$\frac{\underline{J}_{ms}}{i\omega\mu_o\mu\delta} = \hat{n} \times \underline{H}^o \times \hat{n} + \hat{n} \times \iint_S \left[\frac{\nabla' \cdot \underline{J}_{ms}}{i\omega\mu_o}\nabla'\psi + i\omega\varepsilon_o \underline{J}_{ms}\psi\right] dS' \times \hat{n} \tag{7.19}$$

or, in a convenient normalized form,

$$\xi\underline{G} = \underline{\mathcal{H}} + \hat{n} \times \iint_S \left[\nabla' \cdot \underline{G}\nabla'\psi - \beta^2\underline{G}\psi\right] dS' \times \hat{n} \tag{7.20}$$

where all lengths are normalized to a typical dimension "a" of the enclosure,

$$\underline{\mathcal{H}} = a\hat{n} \times \underline{H}^o \times \hat{n} \tag{7.21}$$

$$\underline{G} = \frac{a\underline{J}_{ms}}{i\beta\zeta} = a\,\frac{\hat{n} \times \underline{H} \times \hat{n}}{\xi} \tag{7.22}$$

$$\xi = \frac{a}{\mu\sigma} \tag{7.23}$$

Note that the vector field $\underline{G}$ does not vanish as the frequency approaches zero, since its divergence is proportional to the induced magnetic density charge on the screen; and that $\underline{G}$ and $\underline{H}$ have the same dimensions, i.e., ampere. Equation (7.20) is the dual of (7.11).

8. Integral Equation Formulation for the Field in the Shielding Enclosure. Transient Case.

Equations (7.11) and (7.20) allow, in principle, the computation of steady-state surface electric or magnetic current on the electric or magnetic shield respectively. Then, use of inverse Fourier operator will allow the computation of the transient surface currents. Alternatively, an integral equation formulation for time-dependent surface currents is possible. These time-dependent integral equations can be obtained starting from the time-dependent integral equation corresponding to (7.2-5); and then following the same procedure of Sect. 9. Or, more simply, eqs. (7.11 and 20) can be directly transformed in time-domain. Note that the ω-integration can be inverted with the spatial integration, since it does not change the singular behavior of the integrand at $\underline{r} = \underline{r}'$.

In order to get the final results in a convenient form, the time is normalized to $\tau = \sqrt{\varepsilon_o \mu_o}\, a = a/c$, while all lengths are still normalized to "a". The retarded time t^* is defined according to

$$t^* = t - |\underline{r} - \underline{r}'| \tag{8.1}$$

(remember that all quantitites are normalized). Then, a time-independent Green's function

$$\overline{\psi}(\underline{r}, \underline{r}') = -\frac{1}{4\pi|\underline{r} - \underline{r}'|} \tag{8.2}$$

is introduced, and results (2.14-15) used. We get

$$\frac{1}{\sigma\delta\zeta}\frac{\partial \underline{F}(\underline{r},t)}{\partial t} = \underline{E}(\underline{r},t) + \hat{n} \times \iint_S \left\{ \nabla'\bar{\psi}\nabla' \cdot \left[\underline{F}(\underline{r}',t^*) - |\underline{r} - \underline{r}'| \frac{\partial \underline{F}(\underline{r}',t^*)}{\partial t^*} \right] + \bar{\psi}\frac{\partial^2 \underline{F}(\underline{r}', t^*)}{\partial t^{*2}} \right\} dS' \times \hat{n} \qquad (8.3)$$

which is analogous to (7.11); and, for the equation corresponding to (7.20),

$$\frac{a}{\mu\delta}\underline{G}(\underline{r},t) = \underline{H}(r,t) + \hat{n} \times \iint_S \left\{ \nabla'\bar{\psi}\nabla' \cdot \left[\underline{G}(\underline{r}',t^*) - |\underline{r} - \underline{r}'| \frac{\partial \underline{G}(\underline{r}', t^*)}{\partial t^*} \right] + \bar{\psi}\frac{\partial^2 \underline{G}(\underline{r}', t^*)}{\partial t^{*2}} \right\} dS' \times \hat{n} \qquad (8.4)$$

9. Considerations About the Solution of the Integral Equations.

Equations (7.11 and 20), or their time-dependent counterparts, should be solved numerically in all cases of practical interest. Formally, the equations are of the second kind due to the presence of terms $\eta\underline{F}$ and ξG and, as a consequence, their numerical solution is stable. However, when the order of magnitude of the two parameters η and ξ is observed in practical situations, it follows that the above statement is really true only for eq. (7.20). Unfortunately, η is so small that the term $\eta\underline{F}$ cannot play any significant role in the numerical solution of (7.11). Accordingly, some effort seems necessary in order to overcome this difficulty.

The form of eq. (7.11) suggests the expansion of the unknown field vector $\underline{F}$ as follows:

$$\underline{F} = \underline{F}_o + \eta\underline{F}_1 + \eta^2\underline{F}_2 + \ldots \qquad (9.1)$$

Then, substituting in eq. (7.11) and equating terms of equal power in η leads to the following integral equations in the vector coefficients of the expansion (9.1):

$$\underline{E} + L_e(\underline{F}_o) = 0 \tag{9.2'}$$

$$-F_o + L_e(\underline{F}_1) = 0 \tag{9.2''}$$

......

where L_e represents symbolically the operator

$$\hat{n} \times L_e \to \hat{n} \times \iint_S dS' \left[\nabla' \psi \nabla' \cdot - \beta^2 \psi \right] \tag{9.3}$$

Eq. (9.2) *is the usual* EFIE (electric field integral equations) *to which* (7.11) *reduces when* $\sigma\delta \to \infty$. It is important to enquire if also (7.9), projected on S, reduces to the usual MFIE (magnetic field integral eqation) in the same $\sigma\delta \to \infty$ limit. We have

$$\hat{n} \times \underline{H}^i = \hat{n} \times \underline{H}^o - \frac{i\omega\varepsilon_o}{2} \underline{F} + i\omega\varepsilon_o L_m(\underline{F}) \tag{9.4}$$

$$L_m \to \hat{n} \times \iint_S dS[\nabla' \psi \times] \tag{9.5}$$

When $\omega = 0$, $\hat{n} \times \underline{H}^i = \hat{n} \times \underline{H}^o$, *which states the obvious result that the static magnetic field is not shielded by a metal enclosure. Eq.* (9.4) *reduces to the* MFIE *only if the additional condition* $\hat{n} \times \underline{H}^i = 0$ *is imposed.* On the other hand, solution (5.4) for the spherical case shows that

$$\underline{H}^i(r = 0) \simeq \frac{\underline{H}^o}{i\omega\mu_o a\sigma\delta/3} \simeq 0 \tag{9.6}$$

only if $\omega\mu_o a\sigma\delta/3 >> 1$. For instance, for a = 1 m, δ = 1 mm, and a shield with copper walls, we get the following lower bound for the incident frequency: f >> 6 Hz. Then, even at very small frequencies we can assume the dominant term for $\underline{H}^i$ in (9.4) to be of order η. Substituting (9.1) in (9.4) and equating terms of equal powers in η, we get

$$\hat{n} \times \underline{H}^o = \frac{i\omega\varepsilon_o}{2} \underline{F}_o - i\omega\varepsilon_o L_m(\underline{F}_o) \tag{9.7}$$

$$\hat{n} \times \underline{H}^i = \frac{i\omega\varepsilon_o}{2} (\eta \underline{F}_1 + \dots) - i\omega\varepsilon_o L_m(\eta \underline{F}_1 + \dots) \tag{9.8}$$

Eq. (9.7) is the usual form of the MFIE for a body of infinite conductivity and can be conveniently used for computing the term $\underline{F}_o$ of the expansion (9.1), wherein the tangential component of the electric field on $S_i \simeq S$ is given, via (7.13), by

$$\hat{n} \times \underline{E}^i = \frac{\eta}{a} \hat{n} \times (\underline{F}_o + \eta \underline{F}_1 + \dots) \simeq \frac{\eta}{a} \hat{n} \times \underline{F}_o \tag{9.9}$$

The computation of the dominant term for the tangential component of the magnetic field on $S_i \simeq S$, i.e.,

$$\hat{n} \times \underline{H}^i \simeq i\omega\varepsilon_o \eta \left[\frac{1}{2} \underline{F}_1 - L_m(\underline{F}_1)\right] \tag{9.10}$$

requires, unfortunately, the solution of eq. (9.2"), which is of first kind and, consequently, possibly unstable from a numerical viewpoint. For the stabilization of the solution, several techniques can be used [56-57].

10. Conclusions and Recommendations.

A simplified theory of electromagnetic shielding by metal and/or magnetic type enclosures has been presented. The study of the approximation involved has proved that the *theory is absolutely reliable up to incident wavelengths, or spatial pulse widths, larger than the shield thickness.*

Most of the theory has been presented with reference to continuous shields, i.e., shields with no apertures. However, the theory is capable of handling the most general case of *discontinuous shields* (shields with apertures, gaskets, etc.) provided that *the proper relationship between tangential fields and currents on the shield is given.* This relationship is already known for a number of practical situations, e.g., for the case of a shield of coplanar conductors or of a wire grid [29-33]; and can be obtained with reference to a number of canonical problems.

In all these cases of complicated configuration and, consequently, difficult analytical solution, a first estimate of the field inside the enclosure can be obtained by using the equivalent circuit point of view exploited under Sect. 6.

References

[1] J. Larmor, Phyl. Mag., p. 4, Jan. 1884; reprinted in 1924, Mathematical and Physical Papers, Cambridge Univ. Press, 1924.

[2] F.A.M. Rizt, "Low-frequency shielding effectiveness of a double cylinder enclosure," IEEE Trans. on EMC, EMC-19, 14-21, 1977.

[3] T.K. Wu, L.L. Tsai, "Time domain shielding properties of conducting cylindrical shells," IEEE Trans. on EMC, EMC-17, 191-193, 1975.

[4] T.K. Wu, L.L. Tsai, "Shielding properties of thick conducting cylindrical shells with an obliquely incident plane wave," IEEE Trans. on EMC, EMC-17, 189-191, 1975.

[5] T.K. Wu, L.L. Tsai, "Shielding properties of thick conducting cylindrical shells," IEEE Trans. on EMC, EMC-16, 201-204, 1974.

[6] V.I. Zaniyatin, "The defraction of electromagnetic waves by radio transparent layers of finite thickness," Radio Eng. Eelctronic Phys., 19, 5-12, Nov. 1974.

[7] D. Shieber, "Shielding performance of metallic cylinders," IEEE Trans. on EMC, EMC-15, 12-16, 1973. Correction same year, 88.

[8] L.F. Babcock, "Direct graphical determination of optimum shielding and suppression," IEEE Trans. on EMC, EMC-12, 9-14, 1969.

[9] I.V. Lyndell, "Minimum attenuation of spherical shields," IEEE Trans. Antennas Propagat., AP-16, 369-371, 1968.

[10] P.R. Bannister, "New theoretical expressions for predicting shielding effectiveness for the plane shield case," IEEE Trans. on EMC, EMC-10, 2-7, 1968.

[11] W.W. Cooley, "Low frequency shielding effectiveness of nonuniform enclosures," IEEE Trans. on EMC, EMC-10, 34-43, 1968.

[12] S. Shenfeld, "Shielding of cylindrical tubes," IEEE Trans. on EMC, EMC-10, 29-34, 1968.

[13] A.P. Ferro, R.C. Rustay, "Transient plane wave magnetic field attenuation through an infinite plate using a simplified Laplace transfer function," IEEE Trans. Antennas Propagat., AP-15, 448-452, 1967.

[14] C.M. Ryan, "Computer expression for predicting shielding effectiveness for the low-frequency plane shield case," IEEE Trans. on EMC, EMC-9, 83-94, 1967.

[15] J.R. Moser, "Low-frequency shielding of a circular loop electromagnetic source," IEEE Trans. on EMC, EMC-9, 6-18, 1967.

[16] D.A. Miller, J.E. Bridges, "Geometrical effects on shielding effectiveness at low frequencies," IEEE Trans. on EMC, EMC-8, 174-186, 1966. Correction on EMC-9, 22.

[17] C.W. Harrison, Jr., M.L. Houston, R.W.P. King, T.T. Wu, "The propagation of transient electromagnetic fields into a cavity formed by two imperfectly conducting sheets," IEEE Trans. Antennas Propagat., AP-13, 149-158, 1965, and also Interaction Note 31, April 1964.

[18] C.W. Harrison, Jr., C.H. Papas, "Attenuation of transient fields by spherical shell," IEEE Trans. on Antennas Propagat., AP-13, 960-966, 1965, and also Interaction Note 34, July 1964.

[19] Y. Rahmat-Samii, R. Mittra, "Electromagnetic coupling through small apertures in a conducting screen," IEEE Trans. Antennas Propagat., AP-25, 180-187, 1977.

[20] T.Y. Chou, A. Taylor Adams, "The coupling of electromagnetic waves through long slots," IEEE Trans. on EMC, EMC-19, 65-73, 1977.

[21] S. Safavi-Naiui, S.W. Lee, R. Mittra, "Transmission of an EM wave through the aperture of a cylindrical cavity," IEEE Trans. on EMC, EMC-19, 74-81, 1977.

[22] P.L.E. Uslenghi, "A simple model for electromagnetic field penetration through gaskets and seams," Alta Frequenza, 45, 616-620, 1976.

[23] T.B.A. Senior, "Electromagnetic field penetration into a cylindrical cavity," IEEE Trans. on EMC, EMC-18, 71-73, 1976, and also Interaction Note 221, January 1975.

[24] B.D. Graves, T.T. Crow, C.D. Taylor, "On the electromagnetic field penetration through apertures," IEEE Trans. on EMC, EMC-18, 154-162, 1976, and also Interaction Note 199, August 1974.

[25] T.B.A. Senior, G.A. Desjardins, "Electromagnetic field penetration into a spherical cavity," IEEE Trans. on EMC, EMC-16, 205-210, 1974, and also Interaction Note 142, August 1973.

[26] D.R. Wilton, C.M. Butler, K.R. Umashankar, "Penetration of EM fields through small apertures in planar screens: Selected data," Interaction Note 213, Air Force Weapons Lab, KAFB, 1974.

[27] L.O. Wilson, "The shielding of a plane wave by a cylindrical array of infinitely long thin waves," IEEE Trans. Antennas Propagat., AP-22, 689-696, 1974.

[28] C.D. Taylor, "EM pulse penetration through small apertures," IEEE Trans. on EMC, EMC-15, 17-26, 1973, and also Interaction Note 74, March 1971.

[29] M.I. Kontorovich, V.P. Akimov, "Shielding characteristics of a spherical grid," Radio Eng. Electronic Phys., 19, 11-16, 1974.

[30] M.I. Kontorovich, A.D. Zuchov, "Average boundary conditions for a two-dimensional orthogonal grid of thin conductors," Radio Eng. Electronic Phys., 18, 1793-1799, Dec. 1973.

[31] M.I. Kontorovich, "Averaged boundary conditions for a grid consisting of nonparallel and nonrectilinear conductors placed on a nonplane surface," Radio Eng. Electronic Phys., 17, 902-910, June 1972.

[32] M.I. Kontorovich, V. Yu. Petrunkiu, N.A. Yesepkina, M.I. Astrakhau, "The coefficient of reflection of a plane electromagnetic wave from a plane wire mesh," Radio Eng. Electronic Phys., 7, 222-231, 1962.

[33] M.I. Kontorovich, "Averaged boundary conditions at the surface of a grating with square mesh," Radio Eng. Electronic Phys., 8, 1446-1454, 1963.

[34] R.B. Schultz, "Shielding in practical design for electromagnetic compatibility," R.F. Ficchi, Ed., 69-92, New York, Hayden, 1971.

[35] R.B. Schultz, G.C. Huang, W.L. Williams, "RF shielding design," IEEE Trans. EMC, EMC-10, 168-175, 1968.

[36] R.B. Schultz, Ed., Special Issue on Shielding, IEEE Trans. on EMC, EMC-10, 1-175, 1968.

[37] R.B. Schultz, V.C. Plautz, D.R. Brush, "Shielding theory and practice," Proc. 9th Tri-Serivce Conf. on Electromagnetic Compatibility, Chicago, 1963.

[38] D.A. Still, "Interaction of magnetic fields and ferromagnetic shields," IEEE Trans. on EMC, No. 2, EMC-13, 45-50, 1971.

[39] D.E. Merewether, "Design of shielded cables using saturable ferromagnetic materials," IEEE Trans. on EMC, EMC-12, 138-141, 1970, and also Interaction Note 68, August 1970.

[40] D.E. Merewether, "Analysis of shielding characteristics of saturable ferromagnetic cable shields," IEEE Trans. on EMC, EMC-12, 134-137, 1970, and also Interaction Note 67, August 1970.

[41] D.E. Merewether, "EM pulse transmission through a thin sheet of saturable ferromagnetic material of infinite surface area," IEEE Trans. on EMC, EMC-11, 139-143, 1969, Interaction Note 38, November 1969.

[42] R.B. Schultz, "ELF and VLF shielding effectiveness of high-permeability materials," IEEE Trans. on EMC, EMC-10, 95-100, 1968.

[43] S.Y. Liao, "RF shielding effectiveness and light transmission of copper or silver film coating on plastic substrate," IEEE Trans. EMC, EMC-18, 148-153, 1976.

[44] E.D. Knowels, "Cable shielding effectiveness testing," IEEE Trans. on EMC, EMC-16, 16-23, 1974.

[45] R.M. Whitner, "Cable shielding performance and CW response," IEEE Trans. on EMC, EMC-15, 180-187, 1973.

[46] W. Jarva, "Shielding tests for cables and small enclosures in the 1-10 GHz band," IEEE Trans. on EMC, EMC-12, 12-24, 1970.

[47] H.A. Mendez, "Meaningful EMC measurements in shielding enclosures," IEEE International Symposium on EMC, Asbury Park, New Jersey, USA, 1969.

[48] V.C. Plautz, R.B. Schultz, R. Goldman, "Shielding density: An electromagnetic shielding design concept for weight sensitive applications," IEEE Trans. on EMC, EMC-10, 120-129, 1968.

[49] "Attenuation measurements for enclosures, electromagnetic shielding, for electronic test purposes, Methods of," MIL-STD-285, June 25, 1956.

[50] G. Franceschetti, "Are the shielding standard tests meaningful?," to be published.

[51] C.P. Bates, G.T. Hawley, "A model for currents and voltages induced within long transmission cables by an EM wave," IEEE Trans. on EMC, EMC-13, No. 4, 18-31, 1971.

[52] W.H. Haynes, C.L. Wilkerson, "Investigation of a dipole model for transient analysis of electromagnetic field coupling into long cables," IEEE Trans. on EMC, EMC-12, 112-117, 1970.

[53] D.A. Miller, J.E. Bridges, "Review to circuit approach to calculate shielding effectiveness," IEEE Trans. on EMC, EMC-10, 52-62, 1968.

[54] L.F. Babcock, "Shielding circuits from EM pulses," IEEE Trans. on EMC, EMC-9, 45-48, 1967.

[55] J.A. Stratton, Electromagnetic Theory, McGraw-Hill Book Co., New York, Ch. 3, 21, 1941.

[56] L.P. Grabar, "Stabilization and numerical solution of unstable problems encountered in the theory of antenna synthesis," Radio Eng. Electronic. Phys., 14, 1352-1359, 1969.

[57] A. Ya. Fel'd, I.V. Sukharevskiy, "Reduction of diffraction problems on non-closed surfaces to integral equations of the second kind," Radio Eng. Electronic Phys., 11, 1017-1024, 1966.