Interaction Notes

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On the Electromagnetic Field Excitation of Unshielded Multiconductor Cables

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Abstract

Unshielded multiconductor cables are considered to be illuminated by electromagnetic plane wave fields. A study of the currents, that are induced in the terminations, is made using quasistatic circuit theory, transmission line theory and wire antenna theory. Questions of accuracy, ranges of validity and general trends are addressed.

INTRODUCTION

In the construction of facilities with many interconnecting electrical systems, unshielded multiconductor cables are used extensively. Generally the routing and branching of the cables are selected for convenience with no definite pattern intended. However under the illumination of intense electromagnetic fields deleterious voltages and/or currents may be induced in the cable terminations.

The response of multiconductor cables to an electromagnetic illumination may be determined by using multiconductor transmission line theory.[1,2] But this technique requires that the relative orientation of the wires within the cable remain fixed over the cable run or at least maintain some identifiable weave pattern. However, the typical cable bundle is not intended to be a multiconductor transmission line, and generally the relative orientation of wires varies considerably and without pattern. Because of this difficulty, the cable bundle is often treated as a single wire with some equivalent radius and then transmission line theory is used to obtain the so-called "core current" or common mode current among the cable wires.[3]

This paper considers a few simple cable configurations and determines the response under different load conditions while examining the common mode currents as related to the differential mode currents. To obtain the results three separate solution techniques are employed. Firstly, standard coupled circuit analysis is used with Faraday's law--the quasistatic formulation. Secondly, transmission line theory is used and thirdly, wire antenna theory is utilized to determine the wire currents. Inasmuch as transmission line theory neglects reradiation from the wire configuration, a comparison of the aforementioned three results gives an indication of the accuracy and range of validity of the transmission line formulation.

ANALYSIS

1. Single Wire Cable Over a Ground Plane

The first configuration to be considered is a finite length wire parallel to a perfectly conducting ground plane. Extending from $z = 0$ to $z = \ell$ the wire is terminated with respect to the ground in the impedances Z_o at $z = 0$ and Z_1 at $z = \ell$ (see figure 1). The wire together with its electromagnetic image forms a two wire transmission line provided that the wire height h satisfies, $h \ll \ell$ and $h \ll \lambda$. From transmission line theory the currents induced in the terminations are[4]*

$$
\begin{aligned}
I_o = {} & \frac{1}{D}\int_0^{\ell} K(z)[Z_c \cosh \gamma(z-\ell) - 2Z_1 \sinh \gamma(z-\ell)]dz \\
& - \frac{2Z_c}{D}\int_0^h [E_x^{inc}(x,\ell) + E_x^{ref}(x,\ell)]dx \\
& + 2\frac{Z_c \cosh \gamma\ell + 2Z_1 \sinh \gamma\ell}{D}\int_0^h [E_x^{inc}(x,0) + E_x^{ref}(x,0)]dx
\end{aligned} \quad \ldots \quad (1)
$$

$$
\begin{aligned}
I_1 = {} & \frac{1}{D}\int_0^{\ell} K(z)[Z_c \cosh \gamma z + 2Z_o \sinh \gamma z]dz \\
& + \frac{2Z_c}{D}\int_0^h [E_x^{inc}(x,0) + E_x^{ref}(x,0)]dx \\
& -2\frac{Z_c \cosh \gamma\ell + 2Z_o \sinh \gamma\ell}{D}\int_0^h [E_x^{inc}(x,\ell) + E_x^{ref}(x,\ell)]dx
\end{aligned} \quad \ldots \quad (2)
$$

where

$$K(z) = [E_z^{inc}(x,z) + E_z^{ref}(x,z)]_{x=-h}^{x=h} \quad (3)$$

$$D = 2Z_c(Z_0 + Z_1)\cosh \gamma\ell + (Z_c^2 + 4Z_0Z_1)\sinh \gamma\ell \quad (4)$$

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*Some modification of Smith's formulas are made to fit the situation being considered here.

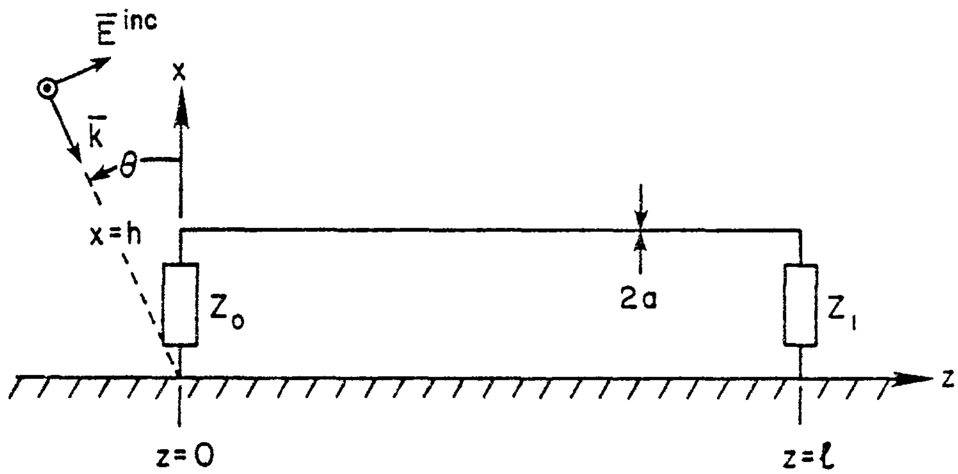

Figure 1: Finite Length Wire Parallel to a Ground Plane and Terminated in Impedances Z_o and Z_1 with Plane Wave Illumination.

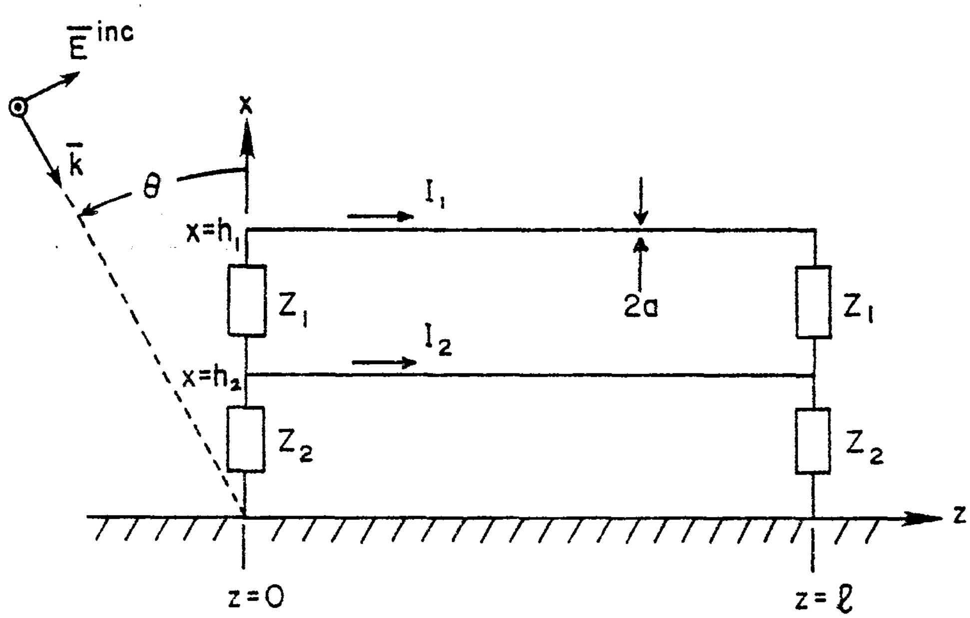

Figure 2: Two Wire Transmission Line Oriented Parallel to a Ground Plane with Plane Wave Illumination.

$$\gamma^2 = jk^2(z^i + j\omega\ell^e)/\omega\ell^e \tag{5}$$

$$Z_c = \sqrt{(z^i + j\omega\ell^e)(\omega\ell^e)/(jk^2)} \tag{6}$$

$$k = \omega\sqrt{\mu_o\varepsilon_o} = 2\pi/\lambda$$

$$\ell^e = (\mu_o/\pi)\ell n(2h/a)$$

$$\mu_o = 4\pi \times 10^{-7} \text{ H/m}$$

z^i is the distributed series resistance of the two wire line (Ω/m)

Here E_x^{inc} and E_z^{inc} are components of the incident field and E_x^{ref} and E_z^{ref} are the components of the ground reflected field or the image source field.

The termination current can also be obtained by circuit theory and Faraday's Induction law (quasistatic approximation). For the foregoing wire configuration, the impedance of the circuit (with image) is

$$Z = 2(Z_o + Z_1) + j\omega L \tag{7}$$

where the inductance of a rectangular loop of wire is[5]

$$L = \frac{\mu_o}{\pi}\left[\ell\,\ell n\,\frac{4h\ell}{a(\ell+d)} + 2h\,\ell n\,\frac{4h\ell}{a(2h+d)} + 2d - \frac{7}{4}(\ell+2h)\right] \tag{8}$$

$$d = \sqrt{(2h)^2 + \ell^2}$$

The induced voltage in the circuit is

$$e_i \approx -j\omega(2B_y^{inc})(2h\ell)$$

Hence the currents in the terminations are

$$I_o = I_1 = \frac{e_i}{Z} = -j\omega \frac{4h\ell\, B_y^{inc}}{2(Z_o + Z_1) + j\omega L} \qquad (9)$$

The foregoing circuit theory is expected to be accurate whenever $h, \ell << \lambda$, and the transmission line theory result is expected to be accurate when $\ell >> h$ and $h << \lambda$. However neither accounts for reradiation from the wire structure. Since reradiation should be most important when the structure resonates and the terminations are shorted, these conditions will be considered explicitly to ascertain the accuracy of the transmission line theory result.

For convenience the illumination on the configuration is considered to be plane wave with the plane of incidence coincident with the plane of the loop and with the incident magnetic field perpendicular to the plane of incidence. When the terminations are also shorted, $Z_o = Z_1 = 0$, then (1) and (2) yield

$$I_o = \frac{4E_o h}{Z_c} \qquad (10)$$

$$I_1 = \frac{4E_o h}{Z_c} e^{-jk\ell \sin\theta} \qquad (11)$$

where θ is the angle of incidence and E_o is the amplitude of the incident plane wave. It is readily noted that (10) and (11) do not exhibit a resonant effect. It may also be noted that if $\ell >> h$ and $h >> a$ then (8) yields

$$L \simeq \ell^e \ell$$

and correspondingly (9) agrees exactly with (10).

2. Two Wire Cable Over a Ground Plane

The second configuration to be considered consists of two parallel wires oriented over a ground plane so that the plane of the wires is normal to the ground. As before the wires are considered to be finite length extending from $z = 0$ to $z = \ell$ with terminations as shown in figure 2.

Together with their electromagnetic images the two wires form a 4 wire configuration, and the current that is induced on the wires by an electromagnetic field may be obtained by using the formulation that was developed by Paul[2] for multiconductor lines. Of course the inherent approximations are those of standard transmission line theory. It is convenient to use the wire currents that are obtained to define differential mode and common mode currents,

$$I_D(z) = I_1(z) - I_2(z) \tag{12}$$

and

$$I_C(z) = I_1(z) + I_2(z) \tag{13}$$

respectively. The common mode current (sometimes referred to as the bulk cable current) can be easily measured without disturbing the cable configuration. Moreover a knowledge of the common mode and differential mode current provides insight into the nature of the cable response. Also the common mode current can be used to obtain an approximation for the individual wire currents.[3]

The wire currents can also be determined by using circuit theory as presented in the previous section. Beginning with the equivalent circuit shown in figure 3, the circuit equations for the wire currents are

$$e_1 = [4(Z_1 + Z_2) + j\omega L_1]I_1 + [4Z_2 + j\omega M]I_2 \tag{14}$$

$$e_2 = [4Z_2 + j\omega M]I_1 + [4Z_2 + j\omega L_2]I_2 \tag{15}$$

where

$$e_1 = -j\omega(2B_y^{inc})(2h_1\ell) \tag{16}$$

$$e_2 = -j\omega(2B_y^{inc})(2h_2\ell) \tag{17}$$

Here L_1 and L_2 can be obtained using (8) by letting $h = h_1$ and $h = h$, respectively. Provided $\ell^2 >> h_1^2 >> a^2$ the expression for the mutual inductance becomes

$$M = \frac{\mu_o}{\pi}\left\{\ell \ \ell n \frac{h_1 + h_2}{h_1 - h_2} + (h_1 + h_2)\left[\ell n \frac{2(h_1 + h_2)}{a} - 2\right] - (h_1 - h_2)\left[\ell n \frac{2(h_1 - h_2)}{a} - 2\right]\right\} \tag{18}$$

Solving (14) and (15) for I_C and I_D yields

$$I_C = \frac{4Z_1 e_2 + j\omega[e_1(L_2 - M) + e_2(L_1 - M)]}{[4(Z_1 + Z_2) + j\omega L_1][4Z_2 + j\omega L_2] - [4Z_2 + j\omega M]^2} \tag{19}$$

$$I_D = \frac{e_1[8Z_2 + j\omega(L_2 + M)] - e_2[8Z_2 + 4Z_1 + j\omega(L_1 + M)]}{[4(Z_1 + Z_2) + j\omega L_1][4Z_2 + j\omega L_2] - [4Z_2 + j\omega M]^2} \tag{20}$$

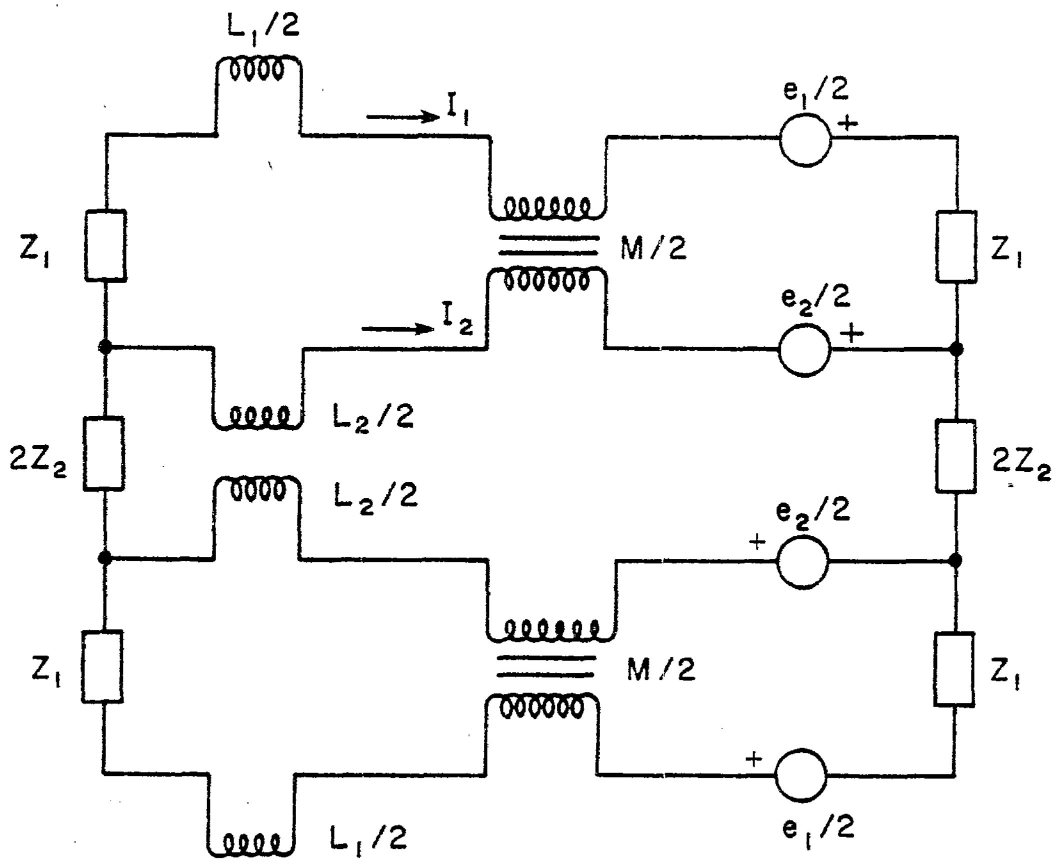

Figure 3: Equivalent Circuit for the Quasistatic Analysis of the Cable Configuration of Figure 2.

Of course the circuit theory result is valid only for low frequency excitation, i.e., when h_1, h_2, $\ell << \lambda$. The transmission line result is expected to be valid for $h_1 << \lambda$ and $\ell >> h_1$. Normally these conditions are satisfied for typical multiconductor cables and frequencies of interest.

Another termination configuration for a two wire cable is shown in figure 4. If $\Delta << \ell$ the mutual and self inductances are virtually the same as the corresponding inductances that are obtained for the cable configuration shown in figure 2. However, the circuit equations are different. They are

$$e_1 = (4Z_1 + j\omega L_1)I_1 + j\omega M I_2$$

$$e_2 = j\omega M I_1 + (4Z_2 + j\omega L_2)I_2$$

Solving these equations yields

$$I_1 = \frac{e_1(4Z_2 + j\omega L_2) - e_2(j\omega M)}{(4Z_1 + j\omega L_1)(4Z_2 + j\omega L_2) + (\omega M)^2} \qquad (21)$$

$$I_2 = \frac{-e_1(j\omega M) + e_2(4Z_1 + j\omega L_1)}{(4Z_1 + j\omega L_1)(4Z_2 + j\omega L_2) + (\omega M)^2} \qquad (22)$$

From the foregoing equations for the wire currents certain general observations are possible. It is noted that if

$$|4Z_1|^2 >> (\omega L_1 - \frac{e_1}{e_2}\omega M)^2$$

and

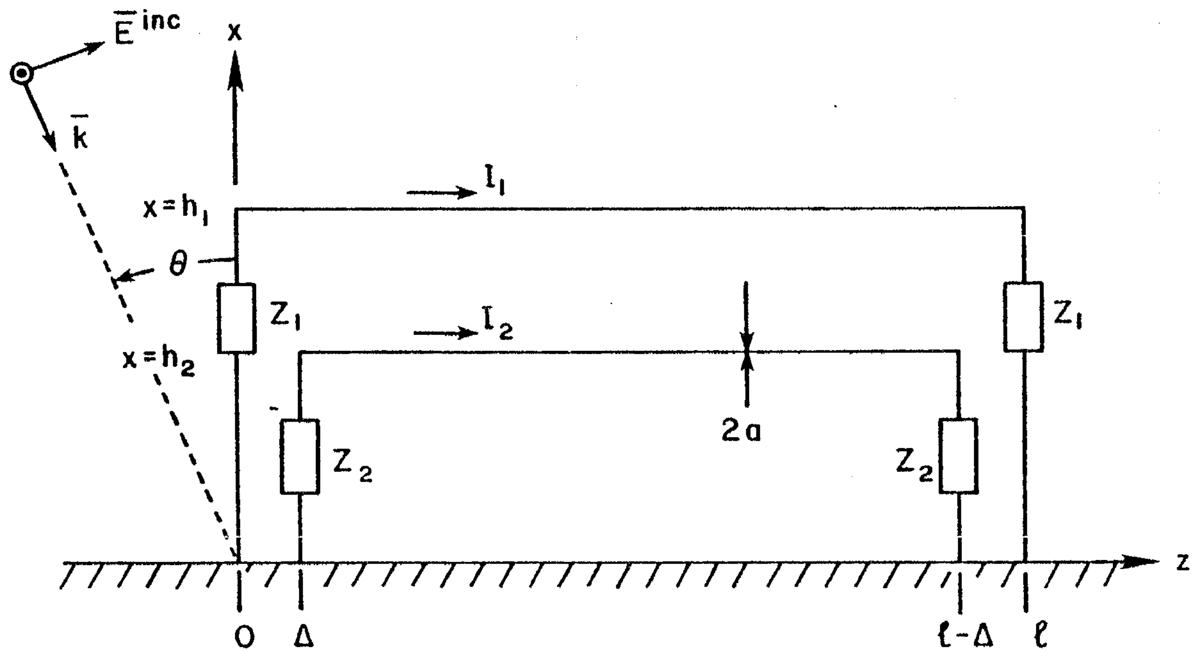

Figure 4: Two Wire Transmission Line Oriented Parallel to a Ground Plane with Plane Wave Illumination.

$$|4Z_2|^2 >> (\omega L_2 - \frac{e_2}{e_1}\omega M)^2$$

then

$$\left|\frac{I_1}{I_2}\right|^2 \approx \frac{h_1}{h_2}\left|\frac{Z_2}{Z_1}\right| \qquad (23)$$

This simple result should be valid over a wide range of parameters and frequencies.

3. Wire Antenna Theory

The Numerical Electromagnetic Code, NEC-(1,2)A[6] was used to obtain the response of the wire problems treated in this paper. The computer code is based on the electric-field-integral equation (EFIE) for thin wire structures. Assuming that: (a) transverse currents are small relative to axial currents, (b) the variation of axial current is uniform around the periphery of the conductors, (c) the total current can be approximated by a current filament on the wire axis and (d) the electric field boundary condition is enforced in the axial direction only, the EFIE for the thin wire becomes

$$-\hat{s}\cdot E^{inc}(\vec{r}) = \frac{-j}{4\pi k}\int_L I(s')\left(k^2\hat{s}\cdot\hat{s}' - \frac{\partial^2}{\partial s\partial s'}\right)g(\vec{r},\vec{r}')ds' \qquad (24)$$

where $E^{inc}(r)$ is the incident electric field,

$I(s)$ is the induced axial current ,

$$g(\vec{r},\vec{r}') = \exp\left(-jk\,|\vec{r}-\vec{r}'|\right)\Big/|\vec{r}-\vec{r}'| \;,$$

$$k = \omega\sqrt{\mu_o\,\varepsilon_o}$$

$$\eta = \sqrt{\mu_o/\varepsilon_o} \quad ,$$

$\hat{s}$ is the unit vector tangent to the wire at $\vec{r}$,

s and s' are the distances along the wire at $\vec{r}$, and $\vec{r}'$, respectively.

Equation (24) is solved by using the method of moments. For the geometry shown in Figures 1, 2, 4, the horizontal wires were divided into 51 segments each and the vertical wires into single segments. In applying the method of moments convenient basis functions and weight functions must be selected. The NEC-(1,2)A code employs basis functions for the axial current of the form

$$I_j(s) = A_j + B_j \sin[k(s - s_j)] + C_j \cos[k(s - s_j)] \qquad (25)$$

where A, B, and C are constants to be determined. The weight functions selected are a set of delta functions:

$$W_i(\vec{r}) = \delta(\vec{r} - \vec{r}_i) \quad . \qquad (26)$$

The sample point, $\vec{r}_i$, is taken at the center of each segment. At a junction of two or more wires of the same radius the conditions of current and charge continuity are imposed. The NEC-(1,2)A code allows the calculation of currents or charge at any segment of a wire. Impedance loading at any segment is also allowed. The perfectly conducting ground plane is included in the model via the method of images.

The results of NEC-(1,2)A are expected to be valid for

$$\Delta/a \geq 4$$

and

$$(ka)^2 << 1 \quad ,$$

where Δ is the length of a segment. The radii for the wires making up the transmission lines were all equal.

NUMERICAL RESULTS

The application of the various formulations that are presented in the foregoing provides information not only for the cable configurations but also provides a means of determining the accuracy of the results obtained from the formulations. For example, the single wire cable configuration of figure 1 provides a test of the predictions that are obtained using transmission line approximations. Considering the cable to be electrically shorted at both ends, data are obtained from transmission line theory and wire antenna theory and presented in figure 5. There is very good agreement over most of the frequency range with the exceptions of the neighborhoods of the loop resonances at f = 7.5 MHz and f = 15 MHz. Note that the results from transmission line theory failed to exhibit the resonances fo the structure. This result is explained in the appendix. Also it should be mentioned that the wire antenna results that are shown in figure 5 do not necessarily include the peak current values that occur.

The aforementioned single wire cable was also considered to be illuminated with $\theta = 0°$. As seen from (10) the magnitudes of the transmission line currents are independent of θ. The wire antenna results differ from the transmission line current only in the neighborhood of the f = 7.5 resonance. Since the excitation of the cable for $\theta = 0°$ is symmetric about $z = \ell/2$ and the resonant current mode at f = 7.5 MHz is antisymmetric about $z = \ell/2$, this resonant current mode is not excited.[7]

For pulse excitation the response can be obtained by using frequency domain results (such as those considered) and Fourier frequency

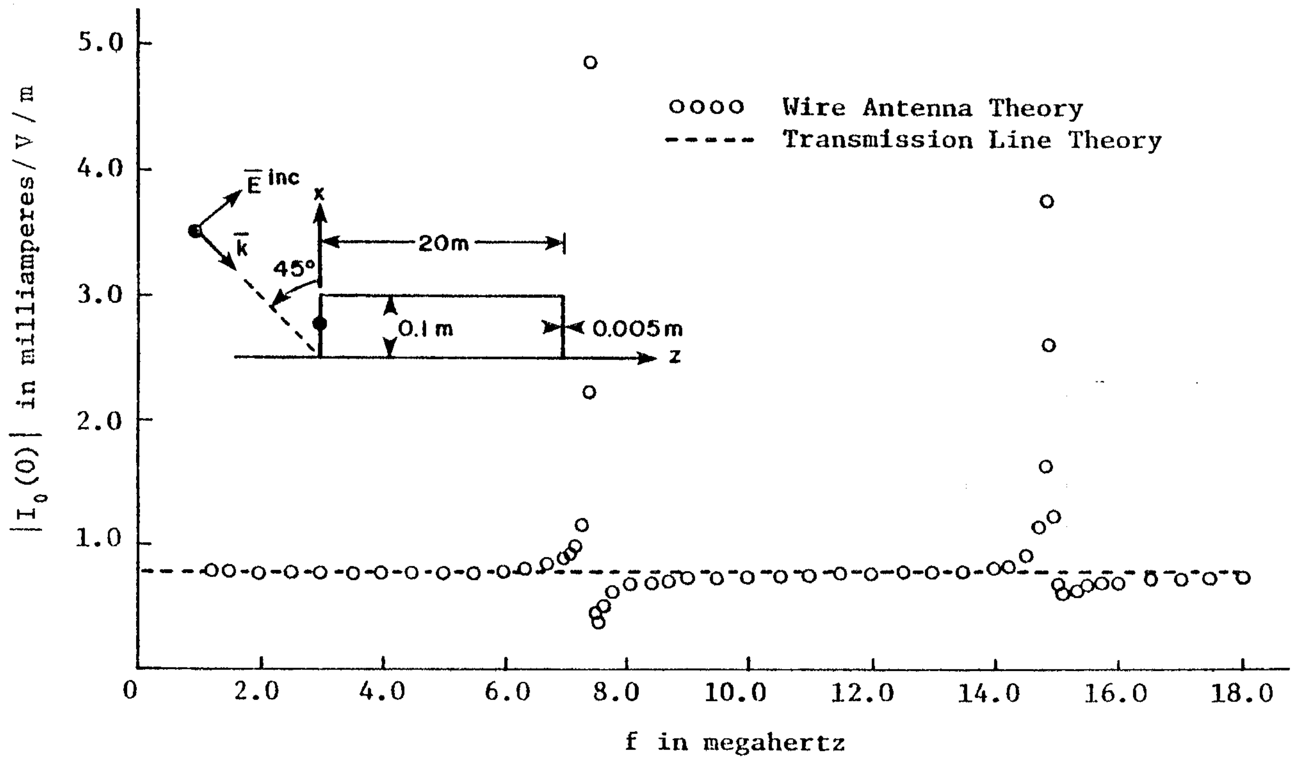

Figure 5: Termination Current for a Rectangular Loop Oriented Normal to a Perfect Ground Plane and Illuminated by a Plane Electromagnetic Wave.

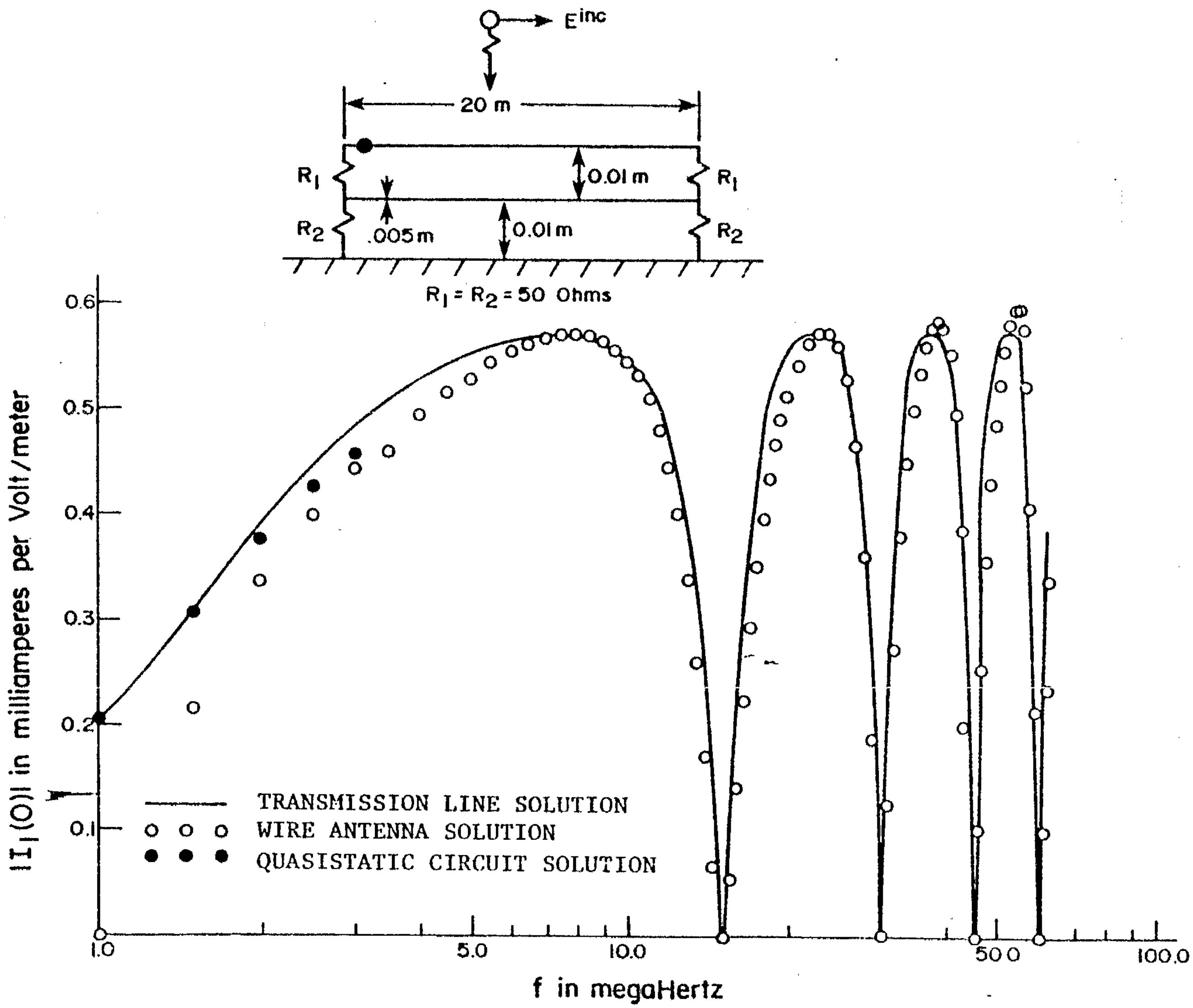

Figure 6: Two-Wire Transmission Line Oriented Parallel to a Perfect Ground Plane and Illuminated by a Plane Electromagnetic Wave.

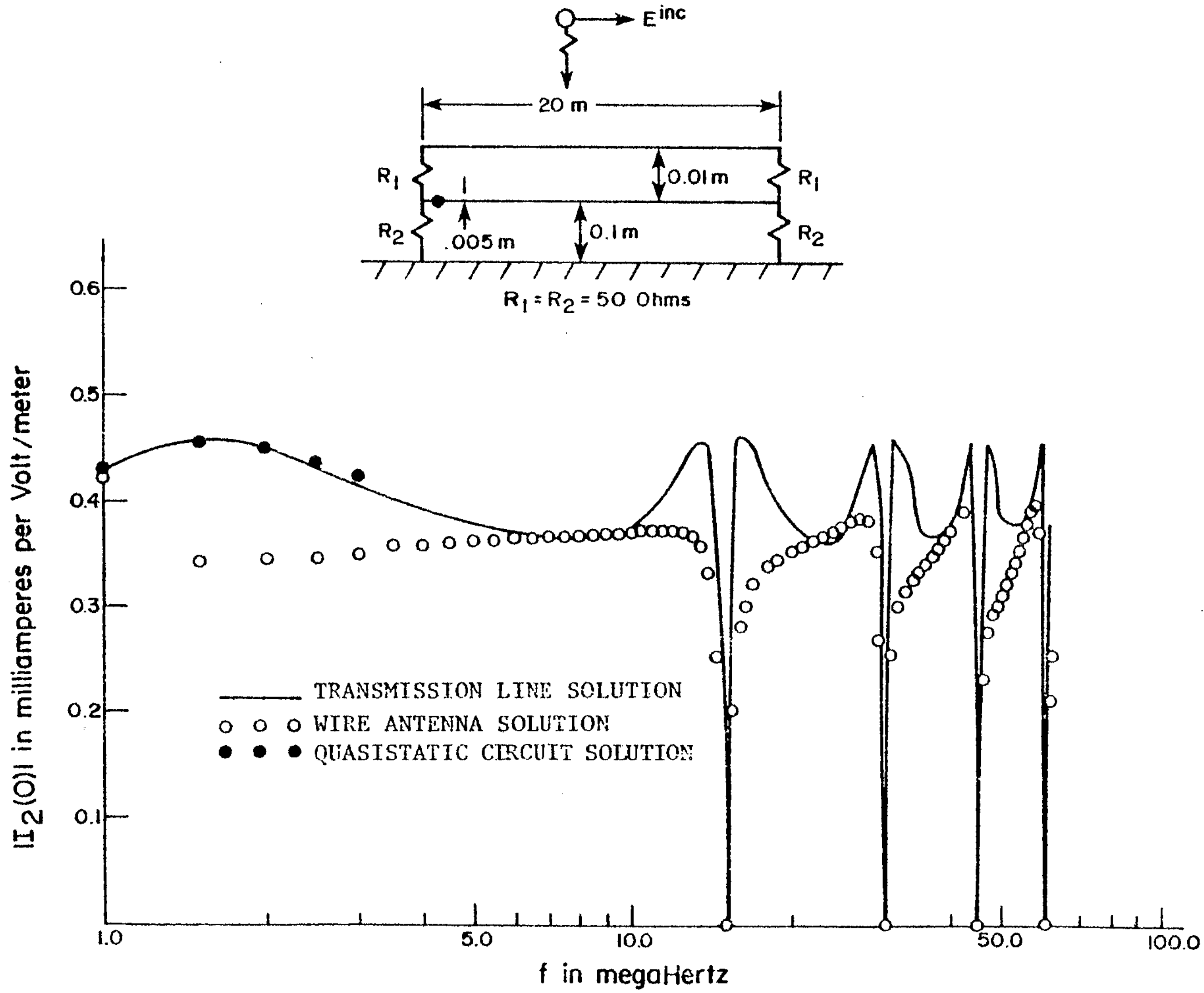

Figure 7: Two-Wire Transmission Line Oriented Parallel to a Perfect Ground Plane and Illuminated by a Plane Electromagnetic Wave.

superposition. Hence the differences between the wire antenna results and the transmission line results for the shorted single wire cable would affect the time domain results. Accordingly the transmission line results obtained for this structure must be interpreted and used very carefully.

It may be noted that there is no wire antenna data shown in figure 5 for frequencies below about 1.5 MHz. Below this frequency the numerical solution technique becomes unstable and erratic results are obtained. This generally occurs when wire antenna currents are obtained by solving the appropriate electric field integral equation using the method of moments.

Results for the two wire cable oriented over a ground plane as shown in figure 2, with $\theta = 0°$, are presented in figures 6 and 7*. Three formulations are used for these data. For the current on the top wire, figure 6, there is reasonable agreement among the results from the three. Up to about 3 MHz results from the circuit solution and the transmission line solution agree extremely well and they represent the most accurate data. In the neighborhood of the resonances the integral equation solution (wire antenna theory) agrees quite well with the transmission line solution. As the frequency is increased the accuracy of the wire antenna theory deteriorates slightly due to an insufficient number of wire segments. Note that the results from transmission line theory does exhibit resonances for this cable configuration.

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*The transmission line theory data were provided by Dr. C. R. Paul from a computer code described in reference 8.

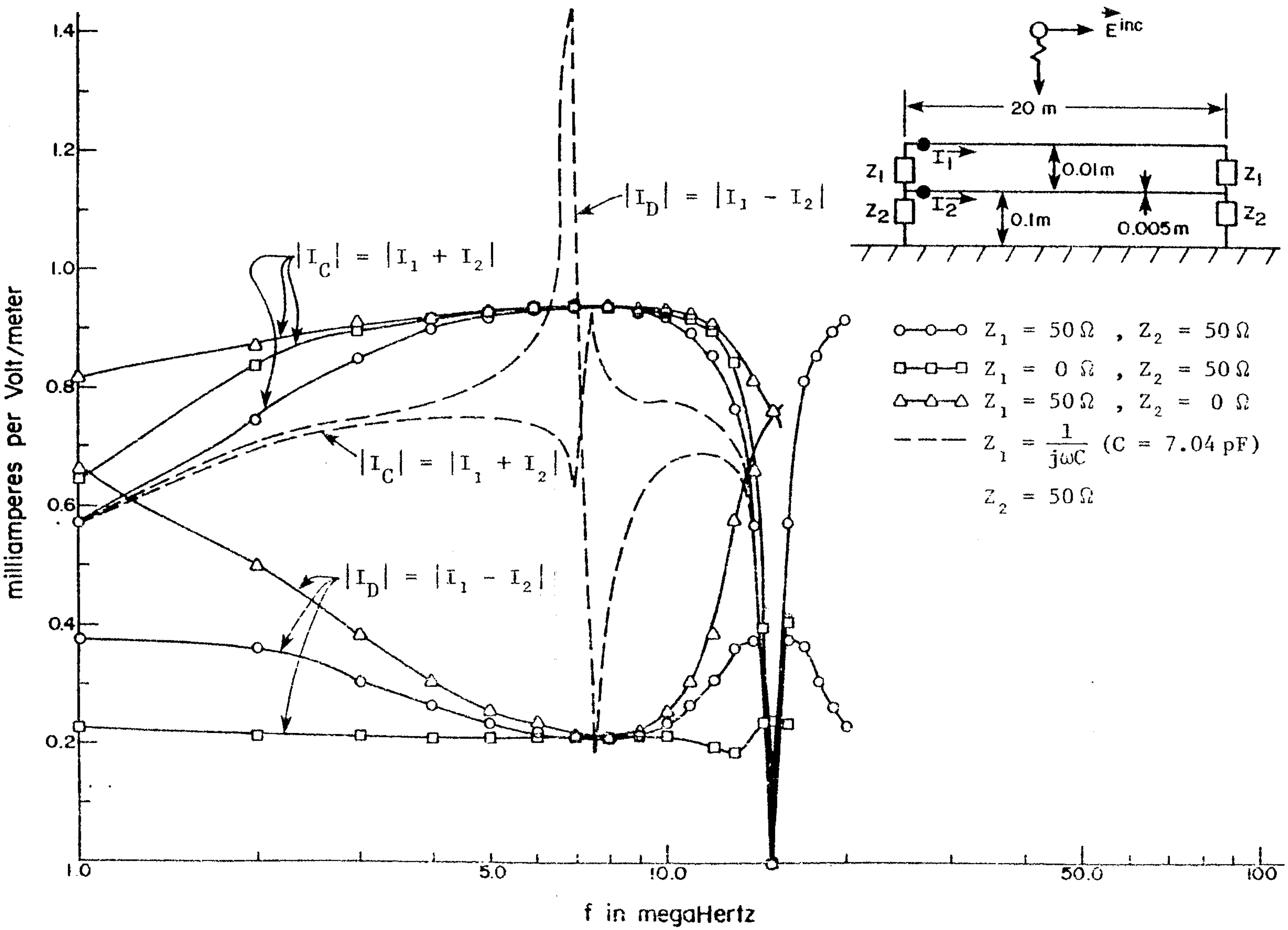

Figure 8: Two-Wire Transmission Line Oriented Parallel to a Perfect Ground Plane and Illuminated by a Plane Electromagnetic Wave.

In figure 7 the current induced in the lower wire of the two wire cable of figure 2 is shown for $\theta = 0°$ plane wave excitation. The agreement among the three formulations used to obtain the induced current is not as good as that exhibited in figure 6 for the top wire. At low frequency the circuit solution agrees quite well with the transmission line solution. Note that the results from wire antenna agrees with the transmission line theory results only in the immediate vicinity of the wire resonances. It is not clear which results are more accurate.

The wire currents of a multiconductor cable are expected to depend on the termination impedances. This is exhibited in figure 8 where the currents I_C and I_D obtained from transmission line theory is shown as a function of frequency for a few termination configurations. From these data it is seen that the relative magnitudes of the differential and common mode currents vary not only with the termination configuration but also may vary with frequency. And clearly the differential mode cannot be neglected, particulary when capacitive loads may occur. The value of capacitance used in figure 8 was chosen to resonate with the loop inductance, as predicted by (8), at about 7 MHz.

CONCLUSIONS

A few simple multiconductor cable configurations are considered to determine the accuracy of various formulations and to examine the relative importance of the common mode and differential mode currents induced by plane wave illumination. Three independent theoretical formulations are used to study the induced cable currents with several general observations resulting.

1. Transmission line theory does not predict resonances in the current induced at the terminations of a transmission line that is shorted at both ends and illuminated by a plane wave electromagnetic field. With the exception of the shorted transmission line, the transmission line theory result agreed reasonably well with the wire antenna result.

2. The application of wire antenna theory to multiwire cables encounters difficulty in the low frequency regime due to numerical instability and in the high frequency regime due to convergence problems.

3. The relative importance of the common mode and the differential mode depends upon the terminations and may be a function of frequency.

4. In general the differential mode currents induced on a multiconductor cable by electromagnetic illumination cannot be neglected in comparison with the common mode currents.

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APPENDIX

Contrary to normal expectations the transmission line theory predictions for the single wire parallel to the ground did not exhibit resonances in the termination currents. Which brings up two important questions, (1) Why are resonances not predicted by the analysis? and (2) Do resonances actually occur? This appendix is written to respond to these questions.

First consider the situation where $Z_1 = 0$ and $Z_o \neq 0$ (see figure 1). Accordingly (1) yields, for a lossless line,

$$I_o = \frac{j\,4\,h\,Z_c\,E_o\,\sin k\ell}{D} \tag{A1}$$

where

$$D = j\,Z_c^2\,[\sin k\ell - j\,2\,\frac{Z_o}{Z_c}\cos k\ell\,] \tag{A2}$$

The zeros of I_o in the complex s-plane are those values of $s = j\omega = j\,k\,c$ for which

$$\sin k\ell = 0 \tag{A3}$$

Hence the zeros are

$$s_z = j\,n\,\pi\,\frac{c}{\ell} \qquad n = 1, 2, 3, \ldots \tag{A4}$$

Note that the zeros are independent of Z_o.

The poles of I_o in the complex s-plane are those values of s for which $D = 0$. These are for $Z_o = \delta Z_c$, where $\delta^2 << 1$,

$$s_p = [\,-2\,\delta + j m\pi]\,\frac{c}{\ell} \qquad m = 1, 2, 3, \ldots \tag{A5}$$

The resonant frequencies are therefore

$$\omega_{res} = |s_p|$$

Hence, if $Z_o \neq 0$, transmission line theory predicts resonances. However if $\delta \to 0$ then the poles coalesce with the zeros and the resonances thereby are canceled. The result is frequency independent as exhibited in figure 5. It should be pointed out that the zeros of the wire currents are position dependent. That is, resonances are predicted by transmission line theory at other points along the wire when $Z_o = Z_1 = 0$.

The first question posed at the outset has been answered. In answer to the second question regarding the existance of resonances on an actual structure, one must conclude that actual resonances do occur just as is predicted by the wire antenna theory analysis, since the poles for the actual configuration can never lie on the $j\omega$ axis of the complex s-plane and thereby be canceled by the zeros.