INTERACTION NOTES

Note 332

February 1978

EXPERIMENTAL CHARACTERIZATION OF MULTICONDUCTOR TRANSMISSION LINES IN INHOMOGENEOUS MEDIA USING TIME DOMAIN TECHNIQUES

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ABSTRACT

An effective method for the time domain characterization of lossless multiconductor transmission lines with cross-sectionally inhomogeneous dielectrics is presented. Lines of this type are characterized by multiple propagation modes having different velocities. Time domain reflectometry is used to obtain the characteristic impedance and the modal velocities of the line. A pulse or step function response of the line is used to obtain the modal amplitudes which in turn determine the velocity matrix. The appropriate multiconductor transmission line equations are solved to obtain the per-unit-length inductance and capacitance matrices in terms of the measured characteristic impedance and velocity matrices. The method is concise and complete and identifies the propagation modes in a way that permits direct physical interpretation of the results. The time domain experimental results for a four-conductor transmission line are presented and are found to be in good agreement with independent frequency domain measurements.

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I. INTRODUCTION

The problem of multiconductor transmission lines characterization has been a topic of interest for many years. Previous work (Refs. 1 and 2) provides methods of time domain characterization for multiconductor transmission lines in homogeneous media where all the propagation modes have the same velocity. In general, for a multiconductor line (N conductors plus a ground reference) in cross-sectionally inhomogeneous media, there will be N propagation modes each having a different velocity. A knowledge of the modal velocities and the modal amplitudes is needed in order to obtain the per-unit-length inductance and capacitance matrices from the characteristic impedance matrix (Ref. 3). The propagation modes for multiconductor transmission lines with inhomogeneous dielectrics are discussed in reference 4.

In a recent paper (Ref. 5), a method for the characterization of multiconductor transmission lines in inhomogeneous media in the frequency domain was given. The present paper describes a method of characterizing lossless parallel multiconductor transmission lines in cross-sectionally inhomogeneous media using only time domain techniques. Time domain reflectrometry (TDR) (Ref. 2) is used to determine the characteristic impedance matrix and modal velocities of the multiconductor line.

For a multiconductor transmission line in a homogeneous media, the inductance and the capacitance parameters can be obtained from the knowledge of the characteristic impedance and the velocity of propagation, which is identical for all modes, using the relations (Ref. 2)

$$[Z_{c_{nm}}] = v[L'_{nm}]$$

$$[Z_{c_{nm}}] = \frac{1}{v}[C'_{nm}]^{-1}$$

where $[Z_{c_{nm}}]$ is the characteristic impedance matrix of the transmission line system, $[L'_{nm}]$ and $[C'_{nm}]$ are the per-unit-length inductance and capacitance matrices, and v is the velocity of propagation. For inhomogeneous dielectrics there will, in general, be N distinct velocities of propagation for N modes of propagation and v will be a N×N matrix. The above equations are not valid in that case.

The technique described in this paper prescribes the determination of the velocity matrix for the cross-sectionally inhomogeneous dielectric case. This requires measuring the pulse or step function response of the line. As the wave propagates along the line, the different modes will arrive at different times at the load (Ref. 4). The amplitudes of the different modes arriving at the load are identified as the elements of the eigenvectors of the velocity matrix. The eigenvalues of the velocity matrix (the velocities of propagation of the modes) are obtained from the TDR. Thus, from the eigenvalues and eigenvectors the velocity matrix can be constructed. Finally, the relationships for inductance and capacitance matrices are derived to obtain the per-unit-length parameters.

II. DETERMINATION OF THE VELOCITY MATRIX

Consider the lossless line formed by N conductors, plus a reference conductor (ground). The line is assumed to be uniform along its length (z coordinate), but with arbitrary cross-section. In general, the dielectric surrounding the line is inhomogeneous (e.g., cable made of insulated conductors having different geometries and dielectric materials).

In the presence of materials of different dielectric constants, the propagation can not strictly be TEM. However, the low frequency propagation may be considered "quasi-TEM" (Refs. 6 and 7), and the analysis can proceed from the generalized telegrapher's equations. These equations for the lossless case are (Refs. 6 and 8),

$$\frac{\partial}{\partial z}[V_n(z,t)] = -[L'_{nm}]\frac{\partial}{\partial t}[I_m(z,t)] \tag{1}$$

$$\frac{\partial}{\partial z}[I_n(z,t)] = -[C'_{nm}]\frac{\partial}{\partial t}[V_m(z,t)] \tag{2}$$

with $n = 1,2,\text{---}N$

$m = 1,2,\text{---}N$

Where V_m and I_m represent the voltage with respect to the reference conductor and current on the m th conductor, respectively, as a function of distance z along the line at time, t. $[L'_{nm}]$ and $[C'_{nm}]$ are respectively per-unit-length coefficients of inductance and capacitance matrices of N×N size. The diagonal elements are self and the off-diagonal elements are mutual quantities. Both $[L'_{nm}]$ and $[C'_{nm}]$ are real, symmetric and dominant. The elements of the capacitance matrix $[C'_{nm}]$ and inductance matrix $[L'_{nm}]$ are further characterized by the following properties (Ref. 9):

$$L'_{nm} \geq 0 \text{ for all } n \text{ and } m$$

$$C'_{nn} \geq 0 \text{ for all } n$$

$$C'_{nm} \leq 0 \text{ for all } n \neq m \tag{3}$$

$$\sum_{m=1}^{N} C'_{nm} \geq 0 \text{ for all } n$$

$$\sum_{n=1}^{N} C'_{nm} \geq 0 \text{ for all } m$$

The voltage and current vectors in Eq. (1) and (2) can be written as (Ref. 4)

$$[V_n(z,t)] = [V_n]\ f(z-vt) \tag{4}$$

$$[I_n(z,t)] = [I_n]\ f(z-vt) \tag{5}$$

where $[V_n]$ and $[I_n]$ are the constant vectors. From Eqs. (1), (2), (4) and (5) the eigenvalue equation for $[V_n]$ can be written as

$$[L'_{nm}][C'_{nm}][V_n]_i = 1/v_i^2 \, [V_n]_i \tag{6}$$

where $1/v_i^2$ is an eigenvalue of the matrix $[L'_{nm}][C'_{nm}]$, and $[V_n]_i$ the associated voltage eigenvector. In the case of inhomogeneous dielectrics there will in general be N distinct eigenvalues. Associated with the eigenvalues $1/v_i^2$, i=1,---N, there are also current eigenvectors $[I_n]_i$. The $[I_n]_i$ are the eigenvectors of the adjoint matrix $[C'_{nm}][L'_{nm}]$ and have the same eigenvalues $1/v_i^2$ (Ref. 4). The eigenvalue equation for this case can be written as

$$[C'_{nm}][L'_{nm}][I_n]_i = 1/v_i^2 \, [I_n]_i \tag{7}$$

$1/v_i^2$ are the eigenvalues of the matrix $[L'_{nm}][C'_{nm}]$ or $[C'_{nm}][L'_{nm}]$, where v_i (i=1,---N) are the eigenvalues of the velocity matrix $[v_{nm}]$. If the eigenvalues $1/v_i^2$ and eigenvectors $[V_n]_i$ are known, then using the similarity transformations, the matrix $[L'_{nm}][C'_{nm}]$ can be constructed as

$$[L'_{nm}][C'_{nm}] = [V_{nm}][I/v_i^2][V_{nm}]^{-1} \tag{8}$$

where $[V_{nm}]$ is the matrix formed by the eigenvectors $[V_n]_i$ as its columns and $[1/v_i^2]$ is a diagonal matrix whose elements are $1/v_i^2$. The matrix $[C'_{nm}][L'_{nm}]$ can be constructed in a similar fashion using the current eigenvectors $[I_n]_i$. A velocity matrix can be constructed using the eigenvalues as v_i's and eigenvectors as either $[V_n]_i$ or $[I_n]_i$. In both cases the eigenvalues of the velocity matrix are the same, but the eigenvectors are different.

It can be shown that in order for the modes to represent unattenuated traveling waves, the velocities must be real, i.e., the eigenvalues $1/v_i^2$ must be real and positive (Ref. 4). The v_i's are the eigenvalues of the velocity matrix and represent the velocities of N propagating modes.

a. Determination of the Eigenvalues of the Velocity Matrix

Time domain reflectometry is employed to determine the velocities of the propagation modes. This method is an extension of the conventional method for a two-wire line in determining the velocity of propagation using a time domain reflectometer. In this method, the return travel time for a step function input signal is measured on a line with an open or short circuit load. The ratio of the length of the line to one-half of the total travel time gives the velocity of propagation on the line.

In the case of a multiconductor transmission line of N+1 conductors with inhomogeneous dielectrics, there are N propagating modes traveling at different velocities. These propagating modes are orthogonal to each other (Ref. 4). One conductor of the multiconductor line can be excited with a step function which in turn generates N propagating modes. These modes become separated in time as they travel along the line. If the other end of the line is open circuited or short circuited, then the reflection coefficient for all modes will be +1 or -1 and there will be no mode conversion at the load. These reflected modes can then be recorded at the driven end using the TDR. For this case, the propagation velocities for all modes can be determined from the measured round trip travel time for each mode and the length of the line. These velocities are the eigenvalues of the velocity matrix.

b. Determination of the Eigenvectors of the Velocity Matrix

The eigenvectors of the velocity matrix are determined from the measured propagating modal amplitudes on the conductors using the procedure described in this section. $[V_n]_i$ is the i th eigenvector of the matrix $[L'_{nm}][C'_{nm}]$ corresponding to the eigenvalue $1/v_i^2$. Since the similar matrices have the same eigenvectors, the eigenvectors of the matrix $[L'_{nm}][C'_{mm}]$ and the velocity matrix $[v_{nm}]$ (whose eigenvalues are v_i, i=1,---N) are the same. Thus, the velocity matrix $[v_{nm}]$ can be obtained from the eigenvalues and eigenvectors of the matrix $[L'_{nm}][C'_{nm}]$.

Since the modes of propagation are orthogonal to each other, the eigenvectors form a set of linearly independent vectors, an arbitrary vector can be represented as a sum of voltage eigenvectors in the form (Ref. 4)

$$[E_n] = [V_{nm}][A_m] \tag{9}$$

where $[A_m]$ is a vector.

Let a wave traveling in the forward direction be characterized at some point in space and time by the voltage vector $[V_{f_n}(z,t)]$ which can be expressed in terms of the voltage eigenvectors as

$$[V_{f_n}(z,t)] = [V_{nm}][A_m(t)] \tag{10}$$

where the vector $[A_m(t)]$ is unknown.

Consider a line of length ℓ connected to arbitrary terminal networks at each end (Figure 1), and excited at the end z=0. The different modes propagate at different velocities, so that the knowledge of $[V_{f_n}(z,t)]$ at one time is not sufficient to obtain the $[V_{f_n}(z,t)]$ at other times. Also, knowing $[V_{f_n}(z,t)]$ at one point on the line (e.g., driving end) is not sufficient to obtain the $[V_{f_n}(z,t)]$ at any other point on the line because of the different propagating velocities of the modes. So, $[V_{f_n}(z,t)]$ at z=0 must be decomposed into eigenvectors

$$[V_{f_n}(0,t)] = [V_{nm}][A_m(t)]$$

or

$$V_{f_n}(0,t) = \sum_{m=1}^{N} V_{nm}A_m(t) \tag{11}$$

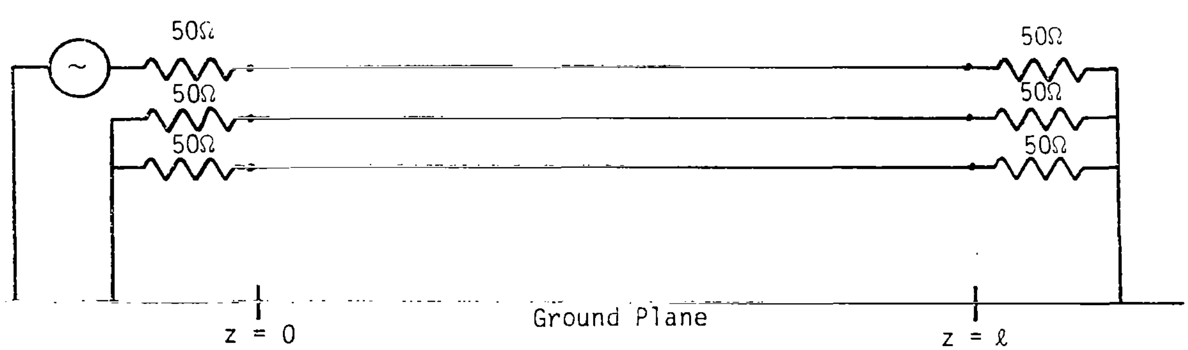


Figure 1. A 3-wire Line Over a Ground Plane

The forward traveling voltage vector at the load $z=\ell$, can be determined as follows.

Define the transit time for each mode as

$$\tau_i = \ell/v_i, \quad i=1,\ 2,---N$$

The desired voltage vector $[V_{f_n}(z,t)]$ at $z=\ell$ is obtained from $[V_{f_n}(o,t)]$ by adding eigenvectors at the appropriate transit time after leaving the point z=0

$$V_{f_n}(\ell,t) = \sum_{m=1}^{N} V_{nm}A_m(t-\tau_m) \tag{12}$$

where $V_{f_n}(\ell,t)$ is the n th component of the vector $[V_{f_n}(\ell,t)]$. The voltage $V_{f_n}(\ell,t)$ will have N components due to N modes. Thus, $[V_{f_n}(\ell,t)]$ can be represented as N×N matrix whose rows are the components of the elements of $[V_{f_n}(\ell,t)]$. This is illustrated by considering an example of four conductors. For this case, Eq. (12) can be written as

$$\begin{bmatrix} V_{f_1}(\ell,t) \\ V_{f_2}(\ell,t) \\ V_{f_3}(\ell,t) \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix} \begin{bmatrix} A_1(t-\tau_1) \\ A_2(t-\tau_2) \\ A_3(t-\tau_3) \end{bmatrix} \tag{13}$$

$$V_{f_1}(\ell,t) = V_{11}\cdot A_1(t-\tau_1) + V_{12}\cdot A_2(t-\tau_2) + V_{13}\cdot A_3(t-\tau_3)$$

Similarly, the other two components can be expressed. If we can rearrange Eq. (13) so that

$$\begin{bmatrix} V_{f_1}(\ell,t) \\ V_{f_2}(\ell,t) \\ V_{f_3}(\ell,t) \end{bmatrix} = \begin{bmatrix} V_{11}\cdot A_1(t-\tau_1) & V_{12}\cdot A_2(t-\tau_2) & V_{13}\cdot A_3(t-\tau_3) \\ V_{21}\cdot A_1(t-\tau_1) & V_{22}\cdot A_2(t-\tau_2) & V_{23}\cdot A_3(t-\tau_3) \\ V_{31}\cdot A_1(t-\tau_1) & V_{32}\cdot A_2(t-\tau_2) & V_{33}\cdot A_3(t-\tau_3) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{14}$$

Then note that the elements of each column are multiplied by the same constant. The elements in the above matrix represent the actual voltages incident at the load at $z=\ell$. Since an eigenvector of a matrix multiplied by a constant is also an eigenvector of the same matrix, the above voltage matrix respresents the voltage eigenvector matrix $[V_{nm}]$ of the multiconductor system.

Thus, the voltage eigenvector matrix can be identified as the incident voltage matrix at the load when the components of the voltages on the conductors are represented as the rows of the eigenvector matrix.

To obtain the incident voltages at the load, the load voltages are measured for all the conductors. Since the different modes arrive at the load at different times, the mode amplitudes on each conductor can be identified by their time of arrival. This will become clear from the experimental results presented in the next section. The load voltages and the incident voltages are related by the following relation

$$[V_{L_n}(\ell,t)]_m = 2[Z_{L_{nm}}][Z_{L_{nm}} + Z_{c_{nm}}]^{-1}[V_{f_n}(\ell,t)]_m \tag{15}$$

where $[V_{L_n}(\ell,t)]_m$ is the load voltage vector for the m th mode, $[Z_{L_{nm}}]$ and $[Z_{c_{nm}}]$ are the load and the characteristic impedance matrices respectively, and $[V_{f_n}(\ell,t)]_m$ is the incident voltage vector at the load for the m th mode. The incident load voltage vector at the load can be obtained from Eq. (15) as

$$[V_{f_n}(\ell,t)]_m = \frac{1}{2}[Z_{L_{nm}} + Z_{c_{nm}}][Z_{L_{nm}}]^{-1} [V_{L_n}(\ell,t)]_m \tag{16}$$

Thus, Eq. (16) gives the eigenvectors of the velocity matrix. The incident voltage vector for a mode represents the eigenvector corresponding to that mode.

We now will proceed in Section 3 to represent the line constants in terms of experimentally measured quantities, e.g., velocity matrix and the characteristic impedance or admittance matrix. The detailed procedure for the measurement of characteristic impedance matrix is described in reference 2 and some experimental results will be presented in Section 5.

III. DERIVATION OF INDUCTANCE AND CAPACITANCE MATRICES

The velocity matrix derived from the voltage eigenvectors is related to the inductance and capacitance matrices by the following relation,

$$[v_{nm}]^{-2} = [L'_{nm}][C'_{nm}] \tag{17}$$

The characteristic admittance matrix of any multiconductor line is related to the inductance and capacitance matrices by the following relation (Ref. 4),

$$[Y_{c_{nm}}][L'_{nm}][Y_{c_{nm}}] = [C'_{nm}] \tag{18}$$

From Eqs. (17) and (18)

$$[Y_{c_{nm}}][L'_{nm}][Y_{c_{nm}}] = [L'_{nm}]^{-1}[v_{nm}]^{-2}$$

or,

$$[L'_{nm}][Y_{c_{nm}}] = [v_{nm}]^{-1}$$

$$[L'_{nm}] = [v_{nm}]^{-1}[Y_{c_{nm}}]^{-1} \tag{19}$$

$$[L'_{nm}] = [v_{nm}]^{-1}[Z_{c_{nm}}]$$

substituting $[L'_{nm}]$ from Eq. (19) into Eq. (18), we obtain

$$[C'_{nm}] = [Y_{c_{nm}}][v_{nm}]^{-1} \tag{20}$$

Thus, Eqs. (18) and (19) relate the per-unit-length inductance and capacitance matrices in terms of the velocity matrix and the characteristic impedance or admittance matrix, which can be obtained from the measurements using the time domain techniques.

It is important to note that the above parameters can also be obtained in terms of the current eigenvectors instead of the voltage eigenvectors. The current eigenvectors are the eigenvectors of the matrix product $[C'_{nm}][L'_{nm}]$ and the velocity matrix can be obtained using the current eigenvectors. Note that the eigenvalues of the matrix $[L'_{nm}][C'_{nm}]$ and $[C'_{nm}][L'_{nm}]$ are the same. The per-unit-length inductance and capacitance matrices are related to the characteristic impedance matrix and the velocity matrix (obtained from the current eigenvectors) by the following relations:

$$[L'_{nm}] = [Z_{c_{nm}}][v'_{nm}]^{-1} \tag{21}$$

$$[C'_{nm}] = [v'_{nm}]^{-1}[Y_{c_{nm}}] \tag{22}$$

where $[v'_{nm}]$ is the velocity matrix obtained from current eigenvectors.

IV. EXPERIMENTAL METHODS

The measurements to be described characterize a multiconductor transmission line in terms of the characteristic admittance, the modal velocities and the eigenvectors of the velocity matrix. From these parameters the per-unit-length inductance and capacitance matrices can be obtained.

The transmission line configuration studied consists of a bundle of insulated wires near a ground reference plane. If there are large differences in the effective dielectric constants of the insulation separating the wires in the bundle, propagation on the cable system is in the form of discrete eigenmodes. An N+1 conductor cable can support N- nondegenerate eigenmodes, each with a discrete modal velocity. In the case of a homogeneous dielectric in the space surrounding the wires, the eigenmodes are degenerate and propagate with the same velocity.

Methods for the characterization of the degenerate case using time domain reflectometry (TDR) have been reported (Ref. 2). The TDR measurements described here are an extension of this methodology to the nondegenerate case. The eigenvectors are determined from measurements of the modal amplitudes present on each wire of the bundle when a single wire is excited with a step or impulse voltage source.

The following sections describe the methods for the measurement of characteristic admittance, modal velocities and the modal amplitudes.

a. Characteristic Admittance

The TDR method used to determine the characteristic admittance matrix of an N+1 conductor cable is given in reference 2. The cable is treated as an N-port network with an input admittance matrix equal to the characteristic admittance matrix of the line, since these parameters are equal for times less than the round trip travel time on the line. Formulas for the diagonal and off-diagonal terms of the characteristic admittance in terms of measured impedance values are given as

$$Y_{ii} = 1/Z_{ii}^{m} \qquad \text{(diagonal)}$$

$$Y_{ij} = Y_{ji} = \tfrac{1}{2}\left(1/Z_{ij}^{m} - 1/Z_{ii}^{m} - 1/Z_{jj}^{m}\right) \qquad \text{(off-diagonal)} \tag{23}$$

where Z_{ii}^{m} is the measured impedance of wire i with all other wires grounded at the input and Z_{ij}^{m} is the measured impedance of wires i and j connected in parallel at the input with all other wires grounded at the input.

The procedures for the measurement of the modal velocities (the eigenvalues of the velocity matrix) and the modal amplitudes (the eigenvectors of the velocity matrix) have been described in Section 2. The experimental results will be presented in the next section.

V. EXPERIMENTAL RESULTS

For the purposes of demonstrating the validity of the methods described, a 3-wire cable (over a ground plane) 20 meters in length was constructed using wires insulated with solid polyethylene, neoprene, and rubber. The wires were wrapped with a dielectric tape to insure a constant cable cross-section over the length of the cable. The cable was supported with stryofoam blocks above an aluminum ground plane in the configuration shown in Figure 2.

TDR recordings obtained by driving each wire in turn with the others grounded at the input end and with the load end open, are presented in Figure 3. Similar data (not shown) was recorded with wires 1 and 2, 2 and 3, and 1 and 3 connected in parallel at the input. The results were used in Eq. (23) to obtain the diagonal and off-diagonal terms of the characteristic admittance matrix. The reflected pulses shown in Figures 3a, 3b and 3c each exhibit three time delayed step functions corresponding to the three discrete propagation modes on the line. The measured round trip travel time of each mode corresponds to propagation velocities of 2.772×10^8, 2.187×10^8 and 2.028×10^8 meters per second.

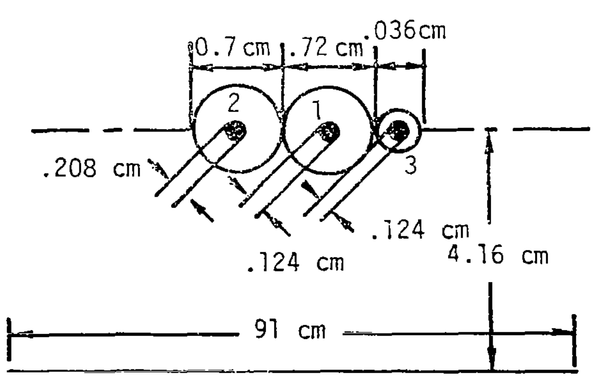
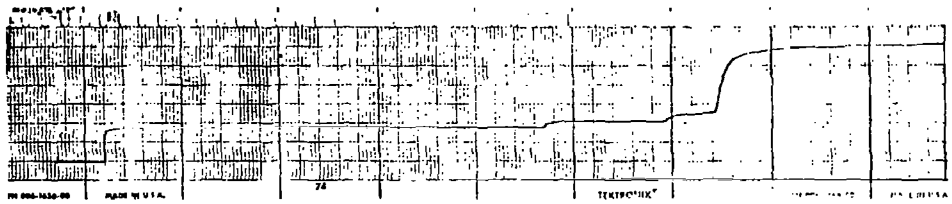
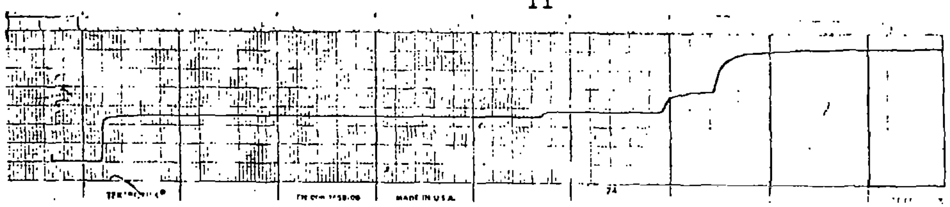


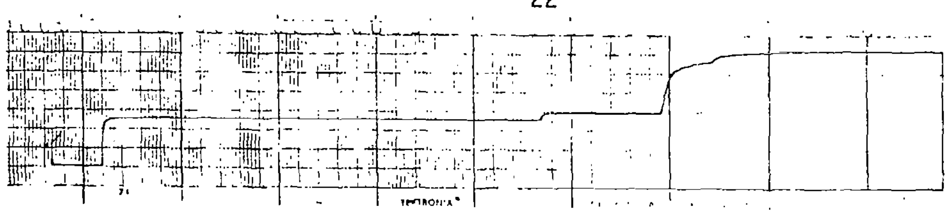
Figure 2. 3-Wire Cable (Over A Ground Plane) Geometry



(a) Z_{11}^{m}



(b) Z_{22}^{m}



(c) Z_{23}^{m}

Figure 3. Waveforms measured with a time domain reflectometer to determine the impedance Z^{m}_{ii} and the modal velocities. The vertical scale is 200 mρ/div; horizontal scale is 6 ns/div.

Measurement of the eigenvectors was accomplished by driving one of the wires with a short duration pulse from a 50 ohm source and terminating the ends of each wire in 50 ohm resistive loads. The output voltage pulses on each wire were recorded using a high impedance voltage probe and a 200 MHz oscilloscope. The modal amplitudes were computed from the measured load voltages using Eq. (16). Only one set of data is required to determine the modal matrix of line; however, three sets were obtained by driving each wire in turn to demonstrate the consistency of the measurements. The recorded pluse data are shown in Figure 4.

The eigenvector matrix, the modal velocities and the characteristic impedance matrix as obtained were used to compute the per-unit-length inductance and capacitance matrices using Eqs. (19) and (20). The inductance, capacitance and eigenvector matrices so obtained are:

(a) Wire #1 Driven

$$[L'_{nm}] = \begin{bmatrix} 0.895 & 0.468 & 0.544 \\ 0.455 & 0.924 & 0.359 \\ 0.537 & 0.359 & 1.01 \end{bmatrix} \mu H/m; [C'_{nm}] = \begin{bmatrix} 44.33 & -18.88 & -20.19 \\ -19.40 & 32.55 & -3.54 \\ -20.21 & -3.77 & 30.47 \end{bmatrix} pF/m$$

$$[V_{nm}] = \begin{bmatrix} 1 & 1 & 1 \\ .918 & 4.311 & -1.14 \\ 1.006 & -5.462 & -0.157 \end{bmatrix}$$

(b) Wire #2 Driven

$$[L'_{nm}] = \begin{bmatrix} 0.893 & 0.467 & 0.539 \\ 0.456 & 0.927 & 0.359 \\ 0.542 & 0.360 & 1.00 \end{bmatrix} \mu H/m; [C'_{nm}] = \begin{bmatrix} 44.21 & -18.89 & -20.28 \\ -19.46 & 32.67 & -3.50 \\ -19.94 & -3.85 & 30.41 \end{bmatrix} pF/m$$

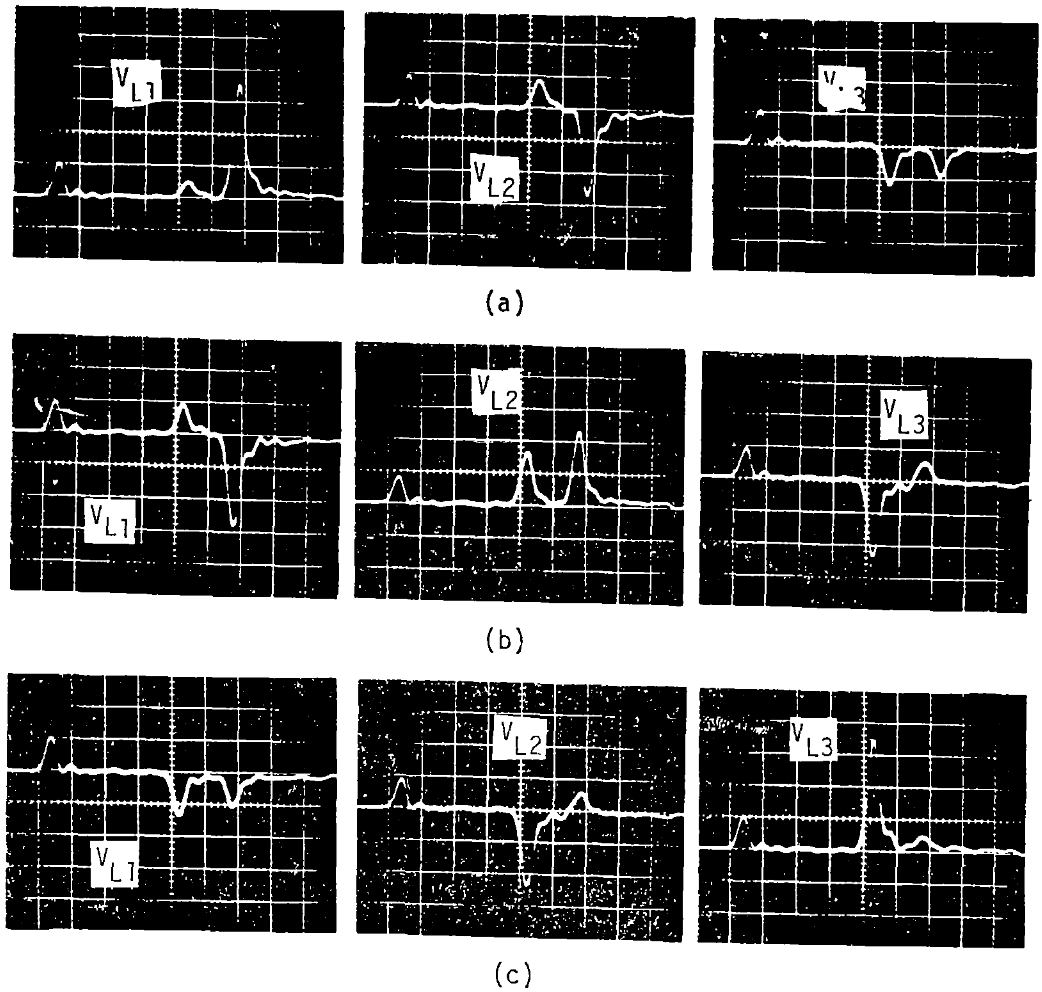


Figure 4. Voltage Waveform At The Load End.(a) Wire 1 Driven, (b) Wire 2 Driven. (c) Wire 3 Driven. Vertical Scale is 0.2 V/div; Horizontal Scale is 5 ns/div.

$$[V_{nm}] = \begin{bmatrix} 1 & 1 & 1 \\ 0.9042 & 3.891 & -1.246 \\ 0.983 & -4.635 & -0.034 \end{bmatrix}$$

(c) Wire #3 Driven

$$[L'_{nm}] = \begin{bmatrix} 0.888 & 0.469 & 0.547 \\ 0.454 & 0.924 & 0.363 \\ 0.524 & 0.366 & 1.01 \end{bmatrix} \mu H/m; \quad [C'_{nm}] = \begin{bmatrix} 44.35 & -18.97 & -20.18 \\ -19.14 & 32.35 & -3.58 \\ -20.99 & -3.27 & 30.8 \end{bmatrix} pF/m$$

$$[V_{nm}] = \begin{bmatrix} 1 & 1 & 1 \\ 0.912 & 5.32 & -1.004 \\ 1.005 & -7.401 & -0.701 \end{bmatrix}$$

Independent measurements of the per-unit-length parameters were also carried out in the frequency domain using the technique described in reference 5. The line parameters determined from the time domain techniques presented here are compared with the frequency domain results in Table 1. The parameters obtained from time domain method are averaged for 3-driving conditions and then the off-diagonal terms are averaged. The non-symmetric off-diagonal terms above result from the measurement and data reduction errors.

VI. CONCLUDING REMARKS

A measurement technique for the characterization of parallel multiconductor transmission line in cross-sectionally inhomogeneous media has been presented. The method uses only time domain techniques and leads to the physical interpretation of the measured quantities, e.g., modal velocities and eigenvectors. A velocity matrix has been introduced

Table 1. Comparison of Per-Unit-Length Parameters From Frequency Domain and Time Domain Measurements

Parameter	Frequency Domain	Time Domain
$[L'_{nm}]$ (μH/m)	$\begin{bmatrix} 0.884 & 0.484 & 0.535 \\ 0.484 & 0.940 & 0.379 \\ 0.535 & 0.379 & 0.992 \end{bmatrix}$	$\begin{bmatrix} 0.892 & 0.461 & 0.538 \\ 0.461 & 0.925 & 0.365 \\ 0.538 & 0.365 & 1.006 \end{bmatrix}$
$[C'_{nm}]$ (pF/m)	$\begin{bmatrix} 46.48 & -20.91 & -20.55 \\ -20.91 & 33.83 & -4.15 \\ -20.55 & -4.15 & 31.10 \end{bmatrix}$	$\begin{bmatrix} 44.30 & -19.12 & -20.30 \\ -19.12 & 32.52 & -3.58 \\ -20.30 & -3.58 & 30.56 \end{bmatrix}$
$[Z_{c_{nm}}]$ (Ω)	$\begin{bmatrix} 232.4 & 148.4 & 159.8 \\ 148.4 & 237.9 & 121.3 \\ 159.8 & 121.3 & 258.3 \end{bmatrix}$	$\begin{bmatrix} 231.4 & 139.2 & 157.2 \\ 139.2 & 230.6 & 112.5 \\ 157.2 & 112.5 & 257.6 \end{bmatrix}$
v_1 m/s	2.829×10^8	2.772×10^8
v_2 m/s	2.178×10^8	2.187×10^8
v_3 m/s	2.009×10^8	2.028×10^8

The above comparison shows a close agreement between the results obtained from two methods.

for this case which can be obtained from either current or voltage eigenvectors where the relation between the per-unit-length parameters and the velocity matrix differ, depending which eigenvector is used to obtain the velocity matrix. Thus, the velocity matrix as used here is a mathematical tool where its physical interpretation is somewhat obscure.

The results obtained from this method are found to be in good agreement with frequency domain results. Also, it is found that the results obtained from this method self consistent with respect to differing conditions.

It should be possible to extend the method given here to the partially degenerate case, where several, but not all, modes have the same velocities. This will be the subject of further investigation.

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