Interaction Notes

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A Wire Passing by a Circular Aperture in an Infinite Ground Plane

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Abstract

A procedure is given to calculate the elements of an equivalent circuit representation of an aperture in an infinite ground plane for a cable passing by the aperture and parallel to the ground plane. For an electrically small hole whose linear dimensions are much smaller than the shortest distance between the hole's center and the wire, explicit simple working formulas are obtained for all the elements (impedances and generators) in the equivalent circuit. When the restriction on the hole's size relative to the distance from the wire is removed, explicit but somewhat complicated expressions for the equivalent generators are derived. These expressions are plotted against various pertinent length parameters of the geometry of the problem.

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I. INTRODUCTION

One of the most common interaction problems found on an aircraft is a cable passing by an aperture, such as a window, the seams of a passenger door, or an open wheel well. The main concern is that large EMP related currents may be induced on a cable passing near these kinds of apertures and eventually propagate to mission-critical equipment. In order to perform a reliable vulnerability assessment of such equipment the equivalent circuit representation of the aperture must be known from which it is then a straightforward matter to calculate the induced cable currents from familiar transmission-line equations. The present report is devoted to the determination of an equivalent lumped network representation of an aperture lying in an infinite, perfectly conducting plane with an infinitely long wire running parallel to the plane (Fig. 1).

The interaction problem between a cable and an aperture has been treated in the past, almost exclusively for a coaxial cable with apertures in the cable shield [1 - 5]. Reference [6] appears to be the only article dealing with the geometry shown in Fig. 1, but it considers only the two equivalent sources of a small circular hole and leaves out the impedance elements in the lumped network representation.

In Section II, an integral equation for the aperture electric field is first formulated under the thin-wire assumption. The resulting equation is further simplified through a quasi-static approximation and supplemented by another integral equation relating the aperture electric field and the short-circuited electric field on the plane. Section III treats the case of a small hole in which the shortest distance between the hole's center and the wire is much larger than the hole's linear dimension. Explicit simple expressions are obtained for all the lumped elements (impedances and generators) in the network representation. In Section IV, the restriction on the hole's size relative to the distance from the wire is removed. Explicit but somewhat complicated expressions are derived for the equivalent voltage and current sources; data calculated from these expressions are presented graphically for various relevant length parameters involved in Fig. 1.

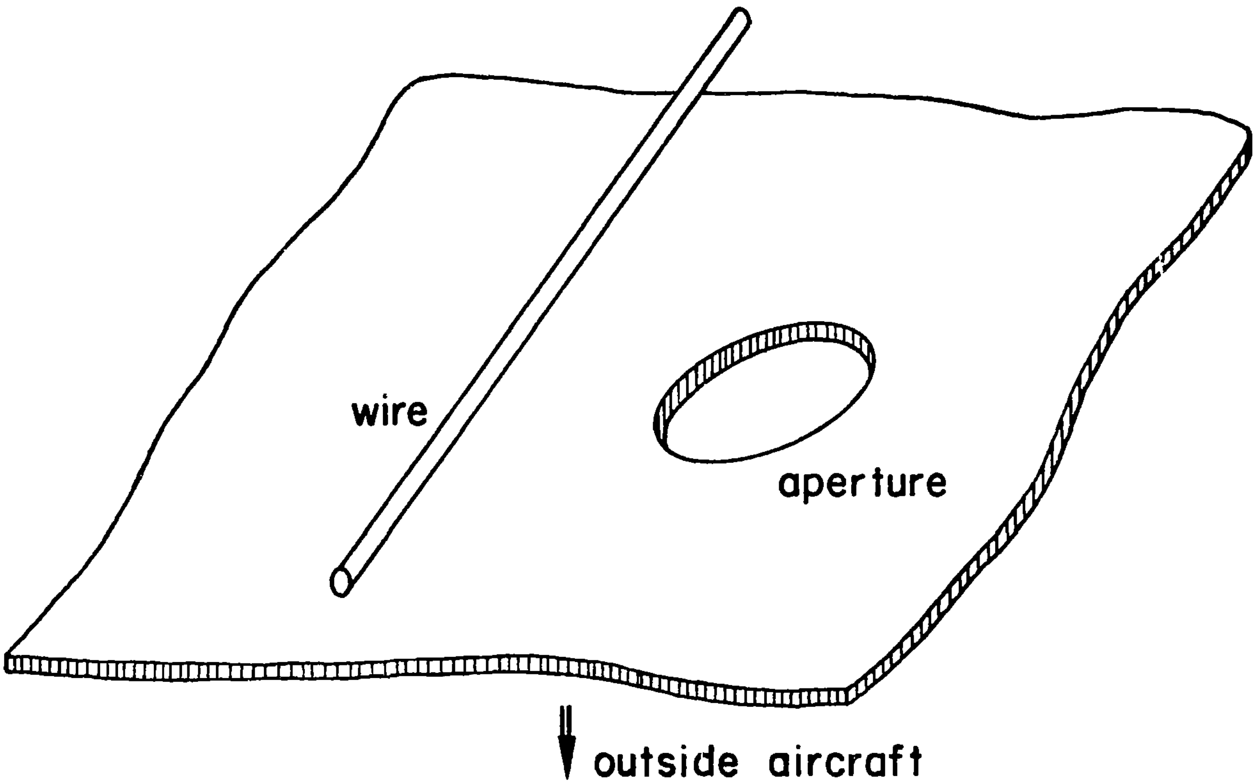


Figure 1. A wire passing by an aperture lying in an infinite ground plane.

II. FORMULATION OF THE PROBLEM

In this section we will first set up the integral equation for the tangential electric field in the aperture (Fig. 2) under the assumption that the wire is very thin compared to the distance between the wire and the conducting plate. The resulting equation will be simplified further by additional physically reasonable assumptions and an approximate analytical solution will be obtained.

Immediately below the plate $(y = 0-)$ the scattered tangential magnetic field is given by, with time convention $e^{-i\omega t}$ suppressed throughout,

$$\hat{y} \times \underline{H}(x,0-,z) = 2i\omega\varepsilon\, \hat{y} \times (\underline{\underline{I}} + \frac{1}{k^2}\nabla\nabla)\cdot \iint_A G(x,0-,z;\, x',0,z')[\hat{y} \times \underline{E}(x',z')]dx'dz'$$

where the surface integral is taken over the aperture A and the free-space Green's function is

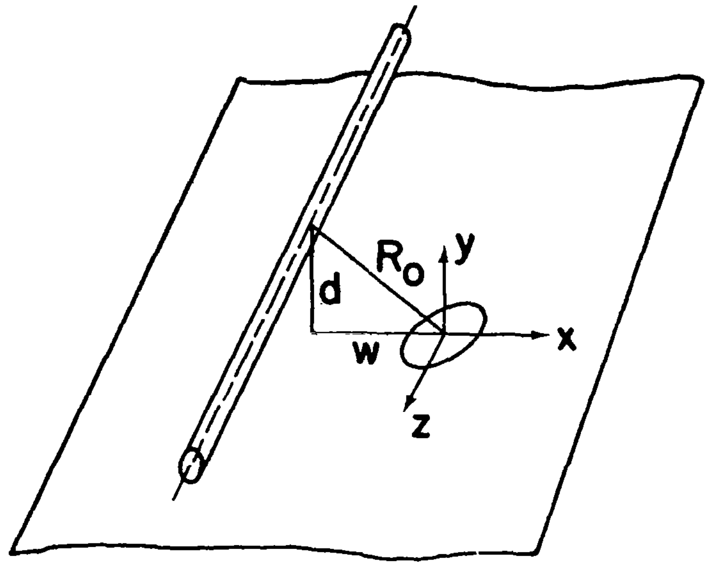
$$G(x,y,z;\, x',y',z') = \frac{1}{4\pi} \frac{e^{ik\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$$

Immediately above the plate $(y = 0+)$ the expression of the scattered field is more complicated and is given by (see Appendix A)

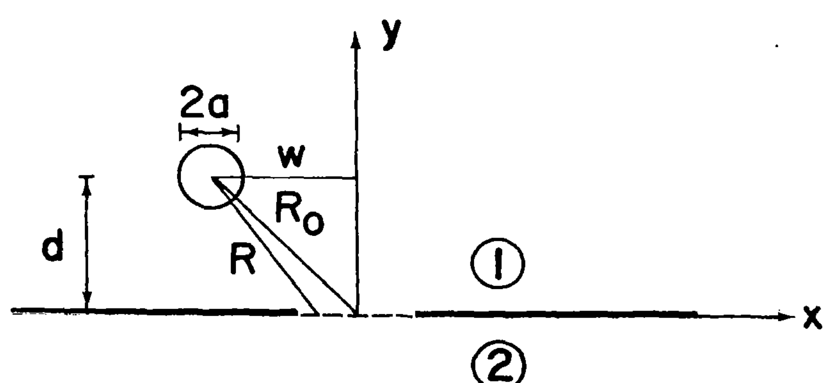
$$\hat{y} \times \underline{H}(x,0+,z) = -\hat{z}H_x^w$$
$$-2i\omega\varepsilon\, \hat{y} \times (\underline{\underline{I}} + \frac{1}{k^2}\nabla\nabla)\cdot \iint_A G(x,0+,z;\, x',0,z')[\hat{y} \times \underline{E}(x',z')]dx'dz'$$

where H_x^w is the magnetic field at $y = 0+$ due to the wire current and is given by

$$H_x^w = -i\omega\varepsilon \iint_A K(R,z;\, R',z')E_z(x',z')dx'dz' \qquad (1)$$



2(a)



2(b)

Figure 2(a). Coordinate system of the geometry of the problem.
(b). Cross-sectional view of Figure 2(a).

with

$$K(R,z;\ R',z') = \frac{i}{2\pi}\frac{d^2}{RR'}\int_{-\infty}^{\infty} \frac{H_o^{(1)'}(\zeta R)H_o^{(1)'}(\zeta R')}{H_o^{(1)}(2\zeta d) - H_o^{(1)}(\zeta a)}\, e^{ih(z-z')}dh$$

$$R = \sqrt{d^2 + (x+w)^2}\ , \quad R' = \sqrt{d^2 + (x'+w)^2}\ , \quad \zeta = \sqrt{k^2 - h^2}$$

Finally, we match the tangential magnetic field across the aperture and the following integro-differential equation for the tangential aperture electric field results:

$$4\,\hat{y} \times (\underline{\underline{I}} + \frac{1}{k^2}\nabla_t\nabla_t)\cdot\iint_A G(x,0,z;\ x',0,z')[\hat{y} \times \underline{E}(x',z')]dx'dz' \pm \frac{1}{i\omega\varepsilon}\,\hat{y} \times \underline{H}_{sc}$$

$$= -\hat{z}\,\frac{1}{i\omega\varepsilon}\,H_x^w \tag{2}$$

where $+\underline{H}_{sc}$ $(-\underline{H}_{sc})$ is the short-circuited magnetic field when the sources are in region 2 (region 1).

In what follows, we will simplify (2) under certain physically reasonable assumptions. First, let us estimate the order of magnitude of the term on the right-hand side of (2). Obviously, the main contribution of $K(R,z;\ R',z')$ will come from the neighborhood where h is close to k, i.e., $|\zeta| << 1$. Thus, one can use the small-argument expansions for the Hankel function and obtains

$$K(R,z;\ R',z') \sim \frac{-1}{\pi^2 \ell n(2d/a)}\,\frac{d^2}{R^2R'^2}\int_{-\infty}^{\infty}\frac{1}{k^2 - h^2}\, e^{ih(z-z')}dh$$

$$= \frac{i}{\pi k\ \ell n(2d/a)}\,\frac{d^2}{R^2R'^2}\, e^{ik|z-z'|}$$

Thus, H_x^w is given by, with $Z_o = \sqrt{\mu/\varepsilon}$,

$$H_x^w \sim \frac{1}{Z_o\pi \;\ell n(2d/a)} \frac{d^2}{R^2} \iint_A \frac{1}{R'^2} E_z(x',z')e^{ik|z-z'|}dx'dz'$$

from which one immediately deduces that

$$H_x^w \propto \frac{1}{\ell n(2d/a)} \frac{d^2 A}{R_o^4} H_{sc}$$

with $R_o^2 = d^2 + w^2$. Thus, if either $A \ll R_o^2$ or $d \ll R_o$ or $\ell n(2d/a) \gg 1$, the right-hand side of (2) can be neglected, implying that the effect of the wire on the aperture field is negligible.

Next, we make the assumption that the wavelength is much larger than all the cross-sectional dimensions of Fig. 2. Equation (2) is finally simplified to the following form:

$$\nabla_t \nabla_t \cdot \iint_A \frac{\hat{y} \times \underline{E}(x',z')}{\sqrt{(x-x')^2 + (z-z')^2}} dx'dz' = \pm\; i\omega\mu\pi \underline{H}_{sc} \tag{3}$$

It is to be noted that equation (3) alone is not enough to determine the total tangential electric field in the aperture. This is not unexpected because under the quasi-static approximation the electric and magnetic problems become uncoupled. To get another equation we turn our attention to the incident electric field. To this end one goes through the same procedure that leads to (2) and obtains the integro-differential equation by matching the normal electric field at the aperture,

$$\begin{aligned} -4\,\hat{y} \cdot \nabla_t \times \iint_A G(x,0,z;\, x',0,z')[\hat{y} \times \underline{E}(x',z')]dx'dz' \pm \hat{y} \cdot \underline{E}_{sc} \\ = E_y^w \end{aligned} \tag{4}$$

where $\underline{E}_{sc}$ is the short-circuited electric field; E_y^w is the electric field at $y = 0$ due to the wire current and is given by

$$E_y^w = \frac{\partial}{\partial z} \iint\limits_A K(R,z;\ R',z')E_z(x',z')dx'dz' \tag{5}$$

By the same argument as above the right-hand side of (4) can be neglected. After making the quasi-static assumption equation (4) is simplified to

$$\hat{y}\cdot\nabla_t\times\iint\limits_A \frac{\hat{y}\times\underline{E}(x',z')}{\sqrt{(x-x')^2+(z-z')^2}}\,dx'dz' = \pm\pi\hat{y}\cdot\underline{E}_{sc} \tag{6}$$

Equations (1), (3), (5) and (6) constitute the formulation of the problem under the quasi-static and thin-wire approximations. It should be pointed out that if one wants to obtain the first two terms of the aperture magnetic current $\hat{y}\times\underline{E}$ in the power series expansion in frequency, then one has also to keep two terms in the power series expansion in frequency of $\underline{E}_{sc}$ in equation (6). In Appendix B, equations (3) and (6) are discussed in more detail.

III. SMALL-HOLE APPROXIMATION

If the characteristic length of the aperture is much smaller than the distance between the wire and the center of the aperture, one can easily obtain from (1), (3), (5) and (6) the network representation of the aperture for the TEM mode of the wire-plate geometry shown in Fig. 2.

With the aperture <u>closed</u> the normalized field distributions of the TEM mode for the geometry of Fig. 2 are given by

$$\underline{e}_o = \frac{1}{\sqrt{N}} \frac{2(x+w)yd\hat{x} - [(x+w)^2 - y^2 + d^2]d\hat{y}}{\pi[(x+w)^2 + (y-d)^2][(x+w)^2 + (y+d)^2]}$$

$$\underline{h}_o = \hat{z} \times \underline{e}_o \tag{7}$$

where $N = \ln(2d/a)/(2\pi)$ is the normalization factor. In conformity with the definitions of Ref. [1], we have

$$\underline{E}_t(x,y,z) = V_o(z)\underline{e}_o(x,y) , \qquad \underline{H}_t(x,y,z) = I_o(z)\underline{h}_o(x,y)$$

$$V_o(z) = V(z)/\sqrt{N} , \qquad I_o(z) = I(z)\sqrt{N} \tag{8}$$

and

$$\frac{dV}{dz} = i\omega LI , \qquad \frac{dI}{dz} = i\omega CV$$

where

$$L = \mu \ln(2d/a)/(2\pi) , \qquad C = 2\pi\varepsilon/\ln(2d/a)$$

and V_o , I_o , V , I are respectively the mode voltage, mode current, line voltage and line current.

With the aperture open, the current on the wire will be perturbed. The perturbation on the TEM mode can be calculated by examining the fields far away from the aperture. Clearly, the fields of interest are the fields which arise from the wire current. From (1) one has

$$H_x^w = -i\omega\varepsilon \iint_A K(R,z;\ R',z')E_z(x',z')dx'dz' \ , \qquad \text{at} \quad y = 0+$$

which gives, for $z \to \infty$,

$$H_x^w \sim \frac{\sqrt{\varepsilon/\mu}}{\pi \ \ln(2d/a)} \frac{d^2}{d^2 + (x+w)^2} \iint_A \frac{1}{d^2 + (x'+w)^2} E_z(x',z')e^{ik(z-z')}dx'dz' \qquad (9)$$

By virtue of (7) and (8) the corresponding line current is obtained from (9) to be

$$I(z) = \frac{1}{2\pi Z_c} \iint_A \frac{d}{d^2 + (x'+w)^2} E_z(x',z')e^{ik(z-z')}dx'dz' \ , \qquad z \to \infty \qquad (10)$$

with $Z_c = \sqrt{L/C} = \sqrt{\mu/\varepsilon}\ \ln(2d/a)/(2\pi)$. Similarly, one has from (5)

$$\begin{aligned} E_y^w &= \frac{\partial}{\partial z} \iint_A K(R,z;\ R',z')E_z(x',z')dx'dz' \\ &\sim \frac{-1}{\pi \ \ln(2d/a)} \frac{d^2}{d^2 + (x+w)^2} \iint_A \frac{1}{d^2 + (x'+w)^2} E_z(x',z')e^{ik(z-z')}dx'dz' \end{aligned} \qquad (11)$$

and

$$V(z) = \frac{1}{2\pi} \iint_A \frac{d}{d^2 + (x'+w)^2} E_z(x',z')e^{ik(z-z')}dx'dz' \ , \qquad z \to \infty \qquad (12)$$

Note that (10) and (12) can also be obtained from the approach suggested in Ref. [1]. When the aperture is electrically small, equation (10) becomes

$$I(z) = \frac{1}{2\pi Z_c} e^{ikz} \iint_A \frac{d}{d^2 + (x'+w)^2} E_z(x',z')(1 - ikz')dx'dz' \tag{13}$$

and (12) becomes

$$V(z) = \frac{1}{2\pi} e^{ikz} \iint_A \frac{d}{d^2 + (x'+w)^2} E_z(x',z')(1 - ikz')dx'dz' \tag{14}$$

So far the hole has been assumed only electrically small. The unknown aperture field E_z in (13) and (14) is in general to be determined by solving (3) and (6). We now assume the aperture to be very small compared to $(d^2 + w^2)^{\frac{1}{2}}$. Then equation (13) can be approximated as

$$\begin{aligned} I(z) &\sim \frac{1}{2\pi Z_c} \frac{d}{d^2 + w^2} e^{ikz} \iint_A E_z(x',z')(1 - ikz')dx'dz' \\ &= \frac{\exp(ikz)}{2} \left[\frac{i\omega\mu}{\pi Z_c} (d/R_o^2)\underline{m}\cdot\hat{x} - \frac{i\omega Z_w}{\pi Z_c} (d/R_o^2)\underline{p}\cdot\hat{y} \right] \end{aligned} \tag{15}$$

where

$$\iint_A \hat{y} \times \underline{E}(\underline{r}'_s)dx'dz' = i\omega\mu\underline{m}$$

$$\frac{\varepsilon}{2} \iint_A \underline{r}'_s \times [\hat{y} \times \underline{E}(\underline{r}'_s)]dx'dz' = \underline{p}$$

$$\underline{r}'_s = \hat{x}\, x' + \hat{z}\, z' \,, \qquad Z_w = \sqrt{\mu/\varepsilon}$$

Similarly, equation (14) gives

$$V(z) \sim \frac{\exp(ikz)}{2}\left[\frac{i\omega\mu}{\pi}\frac{d}{R_o^2}\,\underline{m}\cdot\hat{x} - \frac{i\omega Z_w}{\pi}\frac{d}{R_o^2}\,\underline{p}\cdot\hat{y}\right] \tag{16}$$

Equations (15) and (16) can be easily shown to satisfy the following transmission-line equations:

$$\begin{aligned}\frac{dV}{dz} &= i\omega LI + i\omega\mu\frac{d}{\pi R_o^2}\,\hat{x}\cdot\underline{m}\,\delta(z)\\ \frac{dI}{dz} &= i\omega CV - i\omega\frac{Z_w}{Z_c}\frac{d}{\pi R_o^2}\,\hat{y}\cdot\underline{p}\,\delta(z)\end{aligned} \tag{17}$$

With equation (17) we are in a position to find the lumped network representation of the aperture for the TEM mode of the geometry shown in Fig. 2. To find the equivalent sources we assume there are $\underline{E}_{sc}$ and $\underline{H}_{sc}$ externally driving the aperture from the $y < 0$ region. Then, using the definitions

$$\underline{p} = \epsilon\alpha_e\underline{E}_{sc}\ , \qquad \underline{m} = -\underline{\underline{\alpha}}_m\cdot\underline{H}_{sc}$$

where α_e and $\underline{\underline{\alpha}}_m$ are the electric and the magnetic polarizabilities, we have from (17)

$$\begin{aligned}\frac{dV}{dz} &= i\omega LI + V_{eq}\delta(z)\\ \frac{dI}{dz} &= i\omega CV + I_{eq}\delta(z)\end{aligned} \tag{18}$$

where the equivalent voltage V_{eq} and the equivalent current I_{eq} are given by

$$V_{eq} = -\, i\omega\mu \frac{d}{\pi R_o^2}\, \hat{x} \cdot \underline{\underline{\alpha}}_m \cdot \underline{H}_{sc}$$

$$I_{eq} = -\, i\omega\varepsilon\alpha_e \frac{Z_w}{Z_c} \frac{d}{\pi R_o^2}\, \hat{y} \cdot \underline{E}_{sc} \tag{18.a}$$

which agrees with the results reported in Ref. [6].

To calculate the lumped impedance elements of the aperture, we assume a TEM mode propagating along the wire. Then, we have

$$\underline{p} = -\, \varepsilon\alpha_e \underline{E}_t$$

$$= -\, \varepsilon\alpha_e V \underline{e}_o / \sqrt{N} = \frac{\varepsilon\alpha_e}{N} \frac{d}{\pi R_o^2} V\, \hat{y} \tag{19}$$

$$\underline{m} = \underline{\underline{\alpha}}_m \cdot \underline{H}_t$$

$$= \sqrt{N}\, I\, \underline{\underline{\alpha}}_m \cdot \underline{h}_o = \frac{d}{\pi R_o^2}\, I\, \underline{\underline{\alpha}}_m \cdot \hat{x}$$

Substitution of (19) in (17) gives

$$\frac{dV}{dz} = i\omega[L + L_a\delta(z)]I$$

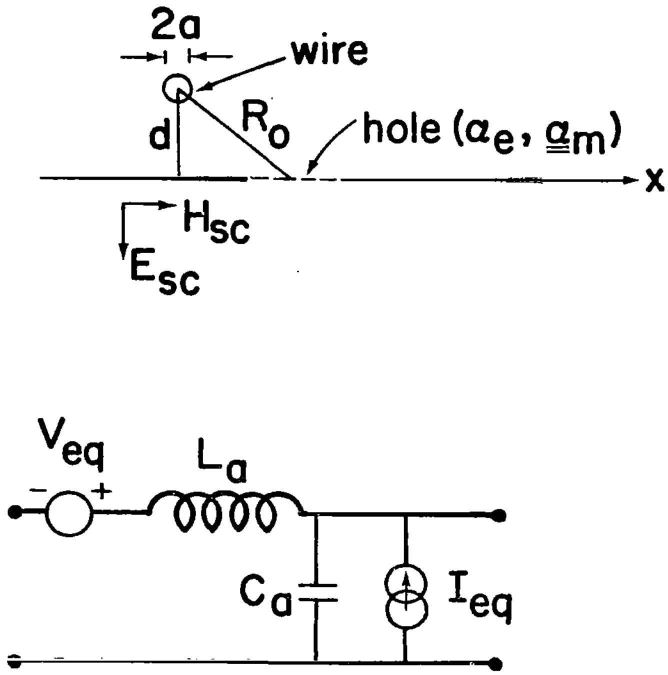
$$\frac{dI}{dz} = i\omega[C + C_a\delta(z)]V \tag{20}$$

where

$$L_a = \mu \left(\frac{d}{\pi R_o^2}\right)^2 \hat{x} \cdot \underline{\underline{\alpha}}_m \cdot \hat{x}$$

$$C_a = -\varepsilon\alpha_e \left(\frac{Z_w}{Z_c} \frac{d}{\pi R_o^2}\right)^2 \tag{20.a}$$

Equations (18) and (20) enable one to draw the network representation of the aperture as shown in Fig. 3. In the next section we will remove the restriction that the hole size is small compared to the distance between the wire and the hole.



$$V_{eq} = j\omega\mu\left(\frac{d}{\pi R_o^2}\right)\hat{x}\cdot\underline{\underline{\alpha}}_m\cdot\underline{H}_{sc}\,, \quad I_{eq} = j\omega\epsilon\left(\frac{d}{\pi R_o^2}\right)\frac{\alpha_e Z_w}{Z_c}\,E_{sc}$$

$$L_a = \mu\,\alpha_{m,xx}\left(\frac{d}{\pi R_o^2}\right)^2 \qquad -C_a = \mu\frac{\alpha_e}{Z_c^2}\left(\frac{d}{\pi R_o^2}\right)^2$$

$$Z_c = \frac{1}{2\pi}Z_w\cosh^{-1}(d/a) \simeq \frac{1}{2\pi}Z_w\ln(2d/a)\,, \qquad Z_w = (\mu/\epsilon)^{1/2}$$

Figure 3. Equivalent circuit of a small hole (R_o >> hole's dimensions). Here, j replaces -i in the text.

IV. MODERATE-SIZED HOLE

In Section III the line current and the line voltage are derived under the long wavelength approximation. Then, the hole is taken to be much smaller than the distance between the wire and the hole and a lumped network representation is obtained for the hole. In some practical cases the wire may be close to the hole and the results obtained in Section III for the small hole will no longer apply. In this section we will derive the equivalent sources for a moderate-sized circular hole and leave out from our consideration the much less important elements of the equivalent network representation, namely, the impedances of the hole. By a moderate-sized circular hole it is meant that the hole's diameter is comparable to the distance between the wire and the hole's center.

The starting points are equations (13) and (14) in which the unknown quantity is the aperture field E_z, which satisfies (3) and (6). Assuming that $\underline{E}_{sc}$ and $\underline{H}_{sc}$, which excite the circular hole from the $y < 0$ region, are uniform, one has [7,8]

$$\begin{aligned} E_z = &- (\rho/\pi)\,(b^2 - \rho^2)^{-\frac{1}{2}} \cos\phi \; \hat{y}\cdot\underline{E}_{sc} \\ &+ \frac{2i\omega\mu}{3\pi}\,(b^2-\rho^2)^{-\frac{1}{2}}\left[(2b^2 - 2\rho^2 + \rho^2\cos^2\phi)\;\hat{x}\cdot\underline{H}_{sc} - \rho^2 \sin\phi\cos\phi\;\hat{z}\cdot\underline{H}_{sc}\right] \end{aligned} \tag{21}$$

where b is the radius of the circular hole and ρ, ϕ are the polar coordinates with respect to the hole with

$$\begin{aligned} z' &= \rho\cos\phi \\ x' &= \rho\sin\phi \end{aligned} \tag{22}$$

One can convince oneself that (21) indeed satisfies (3) and (6).

After substituting (21) and (22) into (13), it is seen that the integrals whose integrands are independent of ω vanish and the remaining terms are given by

$$I(z) = \frac{d}{2\pi Z_c} e^{ikz} \left\{ \frac{2i\omega\mu}{3\pi} \hat{x}\cdot \underline{H}_{sc} \int_0^b \int_0^{2\pi} \frac{2b^2 - 2\rho^2 + \rho^2 \cos^2\phi}{\sqrt{b^2-\rho^2}[(\rho \sin\phi + w)^2 + d^2]} \rho d\phi d\rho \right.$$

$$\left. + \frac{i\omega\sqrt{\mu\varepsilon}}{\pi} \hat{y}\cdot \underline{E}_{sc} \int_0^b \int_0^{2\pi} \frac{\rho^2 \cos^2\phi}{\sqrt{b^2-\rho^2}[(\rho \sin\phi + w)^2 + d^2]} \rho d\phi d\rho \right\}$$

It should be pointed out that this expression can be directly written down from the general theory of [1]. After some lengthy manipulation the integrals are evaluated to give

$$I(z) = \frac{e^{ikz}}{2} \left[\frac{2i\omega\mu b^2}{\pi Z_c} \hat{x}\cdot \underline{H}_{sc} F + \frac{i\omega\varepsilon Z_w b^2}{\pi Z_c} \hat{y}\cdot \underline{E}_{sc} F \right] \tag{23}$$

where

$$F = (1 + A)\sin^{-1}\beta + B \,\ell n(\alpha + \sqrt{\alpha^2 - 1}) - d/b$$

$$A = (d^2 - w^2)/b^2 , \qquad B = 2dw/b^2$$

$$\alpha = \frac{1}{2}\sqrt{(X+1)^2 + Y^2} + \frac{1}{2}\sqrt{(X-1)^2 + Y^2}$$

$$\beta = \frac{1}{2}\sqrt{(X+1)^2 + Y^2} - \frac{1}{2}\sqrt{(X-1)^2 + Y^2}$$

$$X = \frac{1}{\sqrt{2}} \sqrt{\frac{\sqrt{(1+A)^2 + B^2} + (1+A)}{\sqrt{(1+A)^2 + B^2}}}$$

$$Y = \frac{1}{\sqrt{2}} \sqrt{\frac{\sqrt{(1+A)^2 + B^2} - (1+A)}{\sqrt{(1+A)^2 + B^2}}}$$

Similarly, the evaluation of (14), as expected, gives

$$V(z) = Z_c I(z) \tag{24}$$

Equations (23) and (24) will satisfy (18) provided that the equivalent voltage source V_{eq} and the current source I_{eq} are given by

$$\begin{aligned} V_{eq} &= f_s V_{eq,\ \text{small hole}} \\ I_{eq} &= f_s I_{eq,\ \text{small hole}} \end{aligned} \tag{25}$$

where

$$f_s = \frac{3R_o^2}{2bd} F$$

We leave it to the interested reader to show that f_s indeed reduces to unity when $R_o >> 2b$. Curves for the factor f_s as a function of $2b/R_o$ and d/R_o are given in Figs. 4 and 5.

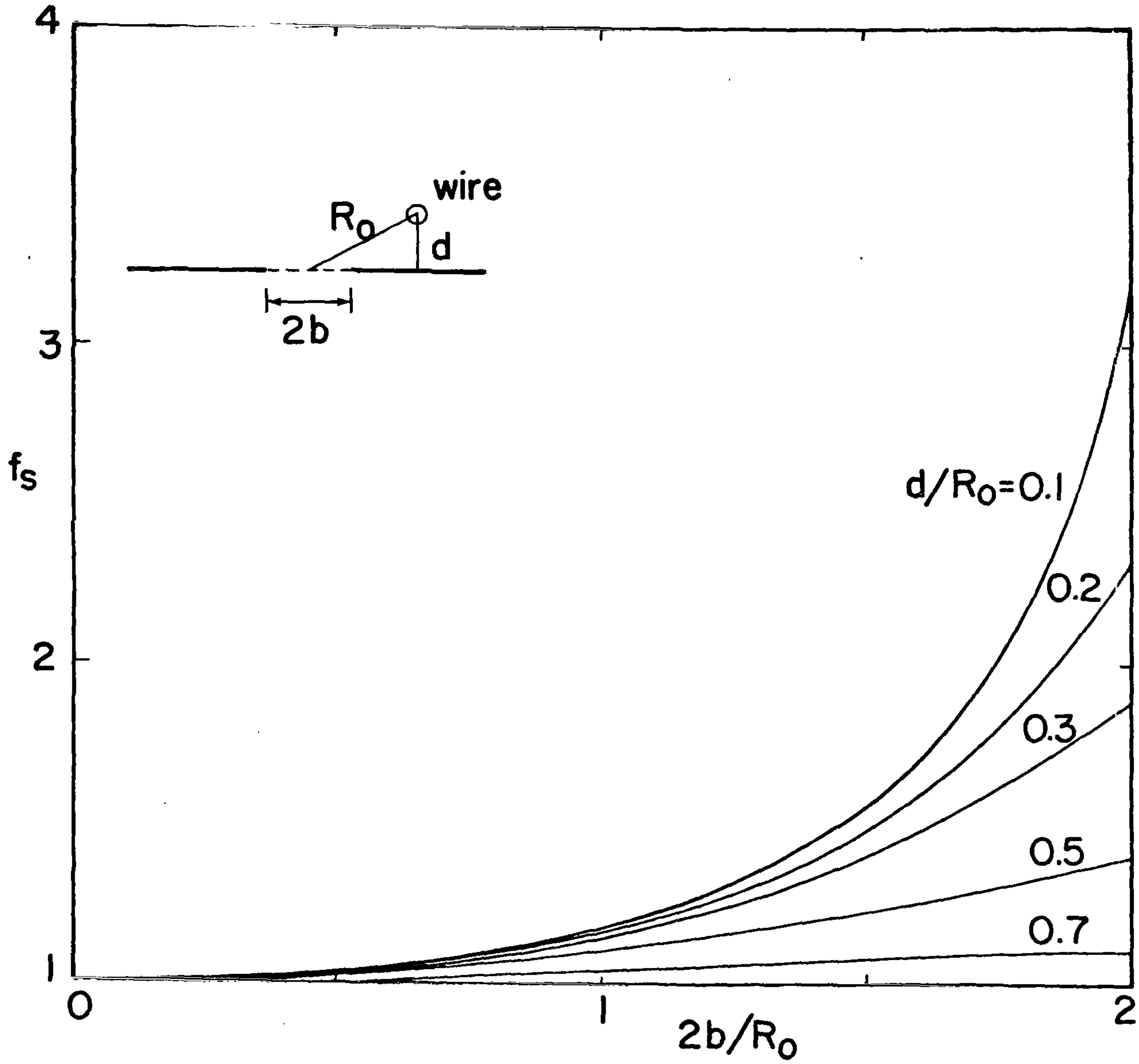


Figure 4. Effect of hole size on the source factor f_s, where f_s is defined as $V_{eq} = f_s V_{eq,\ small\ hole}$, $I_{eq} = f_s I_{eq,\ small\ hole}$.

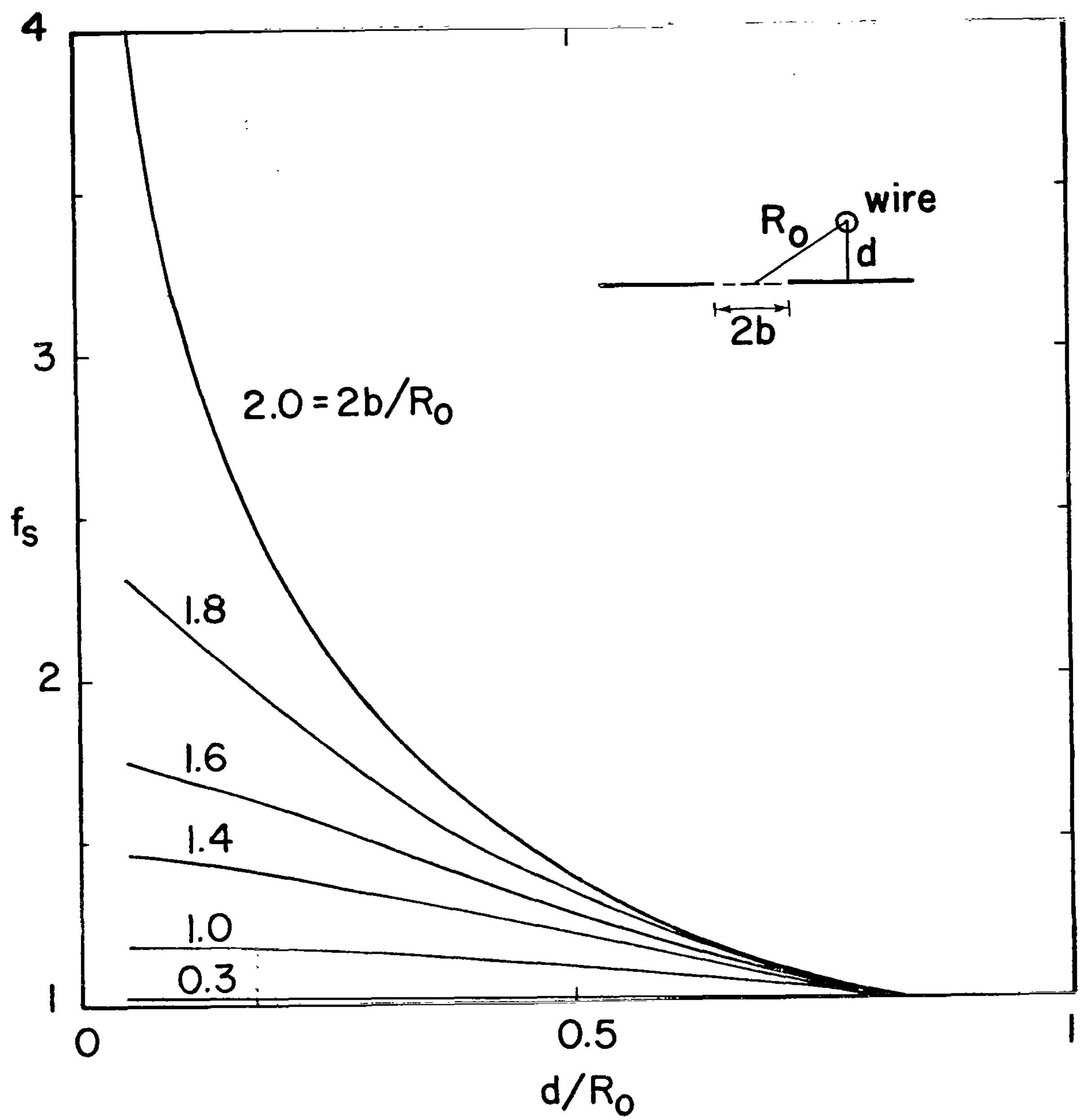


Figure 5. Effect of the distance of the wire from the hole and/or ground plane on the source factor f_s, where f_s is defined in Figure 4.

APPENDIX A

In this appendix expressions for calculating the electric and magnetic fields generated by the current on the wire (Fig. 2) will be derived.

We first assume the aperture tangential $\underline{E}$ field to be a delta function, that is, $\hat{y} \times \underline{E} = \hat{t}\delta(x-x')\delta(z-z')$, $\hat{t}$ being a unit vector. The result will then be used to obtain the solution for a general aperture field distribution. Let the wire current δI induced by this delta-function aperture source be

$$\delta \underline{I} = \hat{z}\,\frac{1}{2\pi}\int_{-\infty}^{\infty} \delta\tilde{I}(h)e^{ihz}dh$$

where $\delta\tilde{I}(h) = \int_{-\infty}^{\infty} \delta I(z)e^{-ihz}dz$ is the Fourier transform of $\delta I(z)$.

At the surface of the wire, the z-component of the electric field δE_z^w produced by the wire current and its image is

$$\delta E_z^w = -\frac{1}{8\pi\omega\varepsilon}\int_{-\infty}^{\infty} \delta\tilde{I}(h)\zeta^2[H_o^{(1)}(2\zeta d) - H_o^{(1)}(\zeta a)]e^{ihz}dh \qquad \text{(A-1)}$$

with $\zeta^2 = k^2 - h^2$.

The electric field $\delta\underline{E}^a$ due to a delta-function aperture field $\hat{y} \times \underline{E} = \hat{t}\delta(x-x')\delta(z-z')$ is simply

$$\delta\underline{E}^a = 2\nabla\times\hat{t}G(x,y,z;\ x',0,z')$$

$$= \frac{i}{4\pi}\,\nabla\times\left\{\hat{t}\int_{-\infty}^{\infty} H_o^{(1)}[\zeta\sqrt{(x-x')^2+y^2}]e^{ih(z-z')}dh\right\}$$

where the unit vector $\hat{t} = \hat{x}\, t_x + \hat{z}\, t_z$. Then the electric field δE_z^a on the surface of the wire is

$$\delta E_z^a = -\frac{i}{4\pi}\frac{dt_x}{R'}\int_{-\infty}^{\infty} \zeta H_o^{(1)'}(\zeta R')e^{ih(z-z')}dh \tag{A-2}$$

with $R'^2 = (w+x')^2 + d^2$. From the requirement that the total tangential electric field vanishes on the surface of the wire, one has from (A-1) and (A-2)

$$\delta\tilde{I}(h) = i\omega\varepsilon \frac{2dt_x}{R'} \frac{H_o^{(1)'}(\zeta R')e^{-ihz'}}{\zeta[H_o^{(1)}(2\zeta d) - H_o^{(1)}(\zeta a)]} \tag{A-3}$$

Since the magnetic field in free space from a wire current $\delta\tilde{I}(h)$ is given by the formula

$$\delta\underline{H}^w = -\frac{i}{8\pi}\hat{\phi}\int_{-\infty}^{\infty} \zeta\delta\tilde{I}(h)H_o^{(1)'}(\zeta\rho)e^{ihz}dh$$

with $\rho^2 = (x+w)^2 + (y-d)^2$, the total tangential magnetic field arising from this current and its image at $y = 0+$ is

$$\delta H_x^w(x,0+,z) = -\frac{i}{4\pi}\frac{d}{R}\int_{-\infty}^{\infty} \delta\tilde{I}(h)\zeta H_o^{(1)'}(\zeta R)e^{ihz}dh$$

$$= \frac{\omega\varepsilon}{2\pi}\frac{d^2}{RR'}t_x\int_{-\infty}^{\infty}\frac{H_o^{(1)'}(\zeta R')H_o^{(1)'}(\zeta R)}{H_o^{(1)}(2\zeta d) - H_o^{(1)}(\zeta a)}e^{ih(z-z')}dh$$

where $R = \sqrt{(x+w)^2 + d^2}$. The total tangential magnetic field at $y = 0+$ due to the wire current from a general distribution of $\hat{y} \times \underline{E}$ in the aperture is found by superposition and the result is

$$H_x^w(x,0+,z) = \frac{\omega\varepsilon}{2\pi}\frac{d}{R}\iint_A \frac{d}{R'}\,E_z(x',z')\int_{-\infty}^{\infty}\frac{H_o^{(1)'}(\zeta R')H_o^{(1)'}(\zeta R)}{H_o^{(1)}(2\zeta d) - H_o^{(1)}(\zeta a)}\,e^{ih(z-z')}dh\,dx'dz' \qquad \text{(A-4)}$$

Similarly, the normal electric field at $y = 0+$ arising from the wire current is given by

$$E_y^w(x,0+,z) = -\frac{1}{2\pi}\frac{d}{R}\iint_A \frac{d}{R'}\,E_z(x',z')\int_{-\infty}^{\infty} h\,\frac{H_o^{(1)'}(\zeta R')H_o^{(1)'}(\zeta R)}{H_o^{(1)}(2\zeta d) - H_o^{(1)}(\zeta a)}\,e^{ih(z-z')}dh\,dx'dz' \qquad \text{(A-5)}$$

APPENDIX B

The exact integral equations for the tangential electric field in an aperture lying in an infinite perfectly conducting plane are given by equations (2) and (4) with the right-hand sides set equal to zero, viz.,

$$(\nabla_t \nabla_t + k^2 \underline{\underline{I}}) \cdot \int_A G(\underline{r}_t - \underline{r}_t') \hat{y} \times \underline{E}(\underline{r}_t') dS' = \pm \frac{i\omega\mu}{4} \underline{H}_{sc} \tag{B-1}$$

$$\hat{y} \cdot \nabla_t \times \int_A G(\underline{r}_t - \underline{r}_t') \hat{y} \times \underline{E}(\underline{r}_t') dS' = \pm \frac{1}{4} \hat{y} \cdot \underline{E}_{sc} \tag{B-2}$$

where $\underline{r}_t$ and $\underline{r}_t'$ are position vectors lying in the plane of the aperture. If terms higher than ω are neglected, equation (B-1) reduces to

$$\nabla_t \int_A \frac{1}{|\underline{r}_t - \underline{r}_t'|} \nabla_t' \cdot (\hat{y} \times \underline{E}) dS' = \pm i\omega\mu\pi \underline{H}_{sc} \tag{B-3}$$

which is equivalent to equation (3) in the text. If one introduces the magnetic scalar potential ψ via

$$\underline{H} = -\nabla\psi \, , \qquad \underline{H}_{sc} = -\nabla\psi_o$$

$$\nabla_t \cdot (\hat{y} \times \underline{E}) = -i\omega\mu H_y = i\omega\mu \frac{\partial\psi}{\partial y}$$

into (B-3) one obtains

$$\int_A \frac{1}{|\underline{r}_t - \underline{r}_t'|} \frac{\partial\psi}{\partial y'} dS' = \pm \pi\psi_o \tag{B-4}$$

which is the familiar integral equation for planar apertures in magnetostatics.

If one takes the static limit of (B-2) one obtains

$$\nabla_t \cdot \int_A \frac{\underline{E}_t(\underline{r}_t')}{|\underline{r}_t - \underline{r}_t'|} dS' = \pm \pi \hat{y} \cdot \underline{E}_{sc} \qquad (\underline{E}_t = \underline{E} - \hat{y} E_y) \tag{B-5}$$

With $\underline{E}_t = -\nabla_t \varphi$, equation (B-5) gives

$$\nabla_t^2 \int_A \frac{\varphi(\underline{r}_t')}{|\underline{r}_t - \underline{r}_t'|} dS' = \mp \pi \hat{y} \cdot \underline{E}_{sc} \tag{B-6}$$

which is the familiar integro-differential equation for planar apertures in electrostatics.

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