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An Electric Model for a Cable Clamp on a Single Wire Transmission Line

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ABSTRACT

This report describes the development of a lumped element model for a grounded cable clamp on a single wire transmission line. With such a model, it will be possible to increase the accuracy of existing single line, internal EMP interaction computer codes by accounting for local perturbations in the transmission line geometry. It is shown that a typical cable clamp can be represented by a Tee network, having either inductances or capacitances as elements. The determination of these circuit elements is shown to be straightforward, as they are simply related to the geometry of the cable clamp.

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SECTION I

INTRODUCTION

The calculation of electromagnetic pulse (EMP) energy propagation on complicated, multiconductor cables within a complex system, such as an aircraft, is often simplified by considering a single conductor transmission line model. Such a model assumes an infinite ground plane as a current return path and typically ignores local perturbations in the transmission line geometry.

This is the first of a series of reports which discuss possible improvements in the single line model. One type of improvement involves representing the effect of local perturbations in the transmission line geometry by lumped impedances along the transmission line (in series and in parallel).

As an illustration of this process, consider a uniform transmission line passing over a ground plane and near a field perturbing obstacle, as shown in Figure 1. In the vicinity of the obstacle, the capacitive and inductive parameters of the line are modified. For an obstacle small compared with a wavelength, the effects of the discontinuity on the propagation of voltage and current waves on the line can be found by inserting a lumped element network into the otherwise unperturbed transmission line, as shown in Figure 2.

The most fundamental forms for the two-port network representing the discontinuity are known to be a Pi or a Tee configured circuit. In the study of various canonical problems pertaining to the single transmission line model, we will deal primarily with the Tee network, as in Figure 3, and will evaluate the parameters Z_1, Z_2 and Z_3 as a function of the transmission line geometry near the obstacle.

The specific problem treated in this report is the calculation of the effects of cable clamps on the single line model. As is evident from a visual inspection of the interior of aircraft, cables are often fastened to the interior walls (the "groundplane") by a clamp consisting of a grounded metallic strap. Figure 4 shows some typical cables and clamps found in the Advanced Airborne Command Post (AABNCP). This report describes the analysis of this type of cable clamp, and presents explicit formulas for the impedance elements of the two port network representing the effects of this clamp on EMP energy propagation.

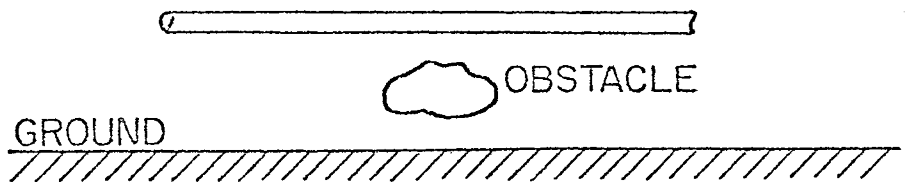

Figure 1. Uniform Transmission Line Passing Near Obstacle

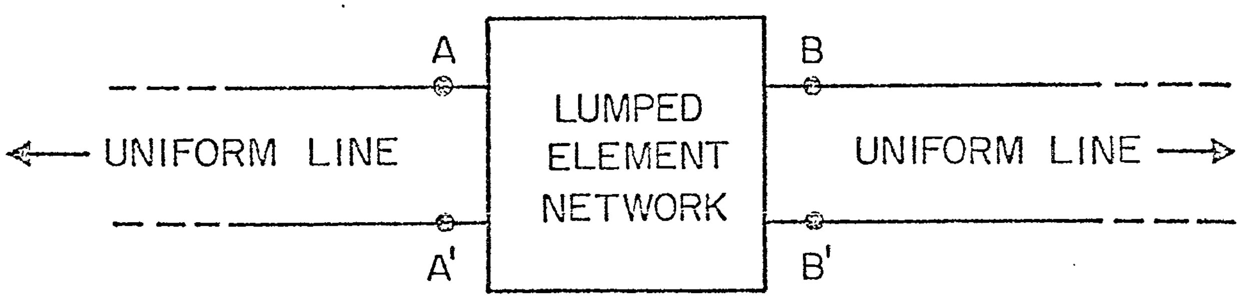

Figure 2. Two Port Network Representing Effect of Obstacle

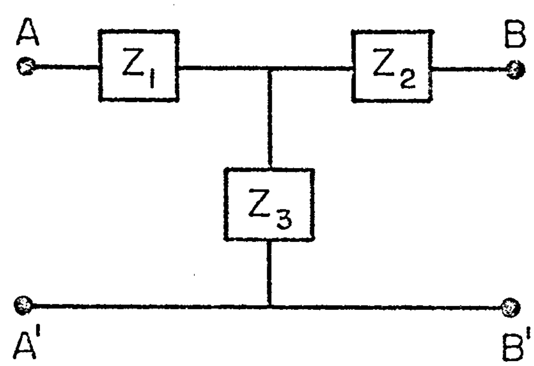

Figure 3. Tee Configuration for the Two Port Network of Figure 2

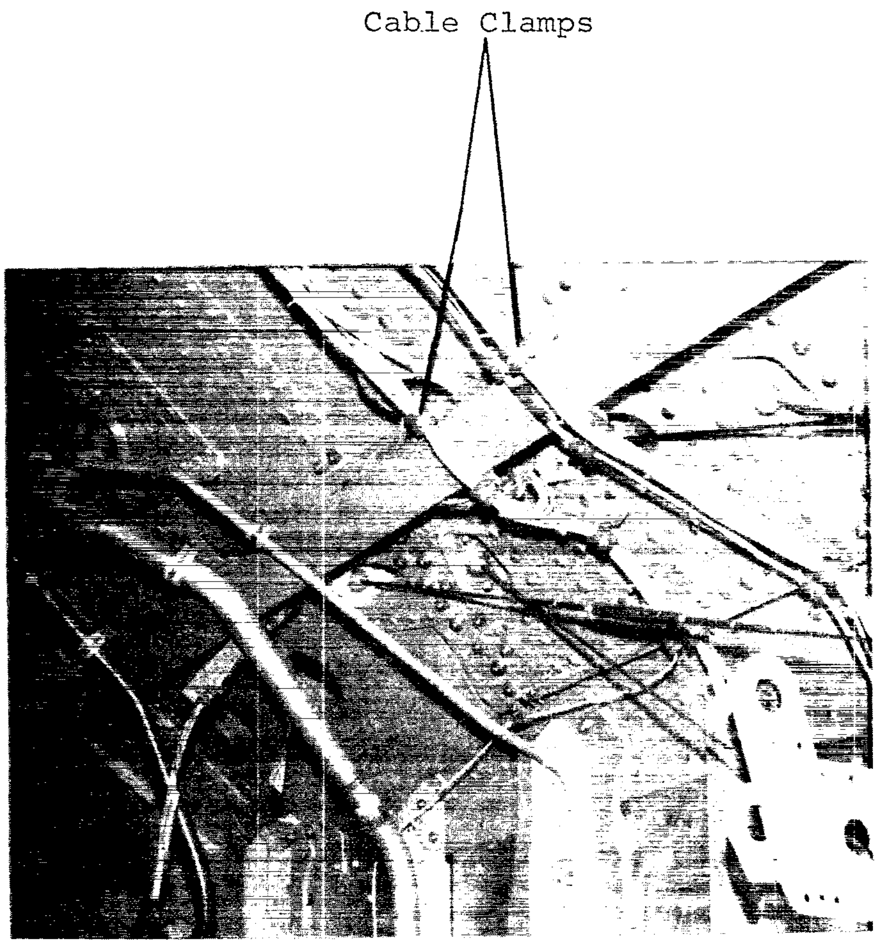

Figure 4. Typical Cable Clamps Found in the AABNCP

SECTION II

DETERMINATION OF EQUIVALENT CIRCUIT FOR CLAMP

The first step in determining the lumped parameters for a cable clamp is to choose a reasonable physical model for the clamp geometry. Such an idealized geometry should be reasonably simple to analyze, yet general enough to represent a wide variety of different clamp styles and dimensions.

One physical model for the clamp which has these features is shown in Figure 5. This model consists of a metallic cylinder of radius b and length ℓ surrounding the transmission line and connected to the ground plane. The radius of the transmission line is a, and its height above the ground plane is h. The material between the wire and outer cylinder representing the clamp is a dielectric and has constitutive parameters μ_o and $\varepsilon = \varepsilon_r \varepsilon_o$, where $\mu_o = 4\pi \times 10^{-7}$ henry/meter, $\varepsilon_o = 8.854 \times 10^{-12}$ farad/meter and ε_r is the relative dielectric constant. The connection between the cylinder and ground plane is represented by a thin, conducting strap of thickness t and length $(h-b)$. This strap, as well as the cylindrical sheath, is assumed to be perfectly conducting. In addition, the thickness of the outer cylinder will be assumed to be negligible.

For this model, the length of the clamp, ℓ , is usually constrined to be much less than the electrical wavelength on the transmission line. This constraint also applies to the other dimensions of the problem. In the analysis which follows, however, no such restrictions will be placed on ℓ. After the impedance elements for an arbitrary length are determined, they will be simplified for small clamp lengths, thereby yielding simple inductances and capacitances for the impedance elements of the equivalent circuit.

To develop an electrical model from the geometrical model of the clamp, it is convenient to decompose the problem into simpler parts. First, consider the cylindrical sleeve around the wire. As shown in Figure 6, this is equivalent to a coaxial transmission line of length ℓ. The terminals A-A′ and B-B′ represent the ends of this transmission line.

The analysis of such a transmission line is given in detail in many texts. King (ref. [1]), for example, shows how the section of uniform transmission line between the terminals A-A' and B-B' can be represented by the equivalent circuit shown in Figure 7, with

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1. King, R.W.P., Transmission Line Theory, Dover 1965.

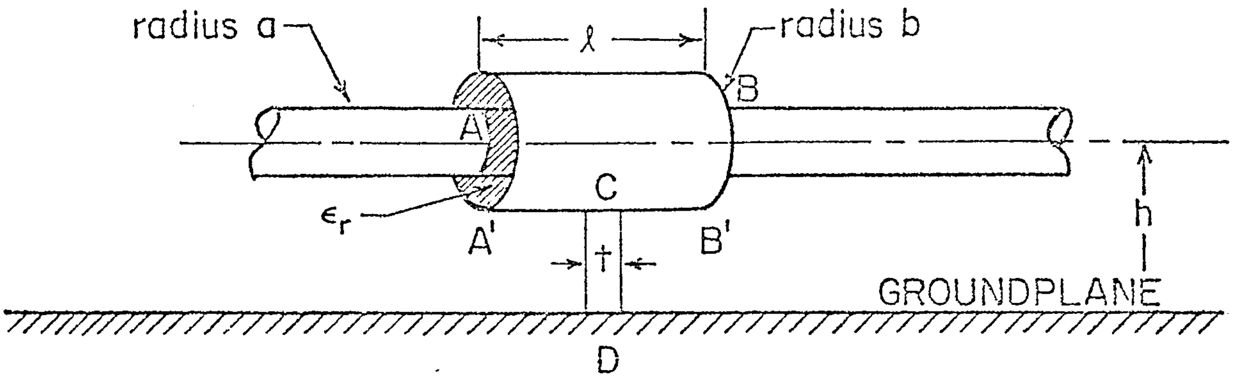

Figure 5. Physical Model for Cable Clamp

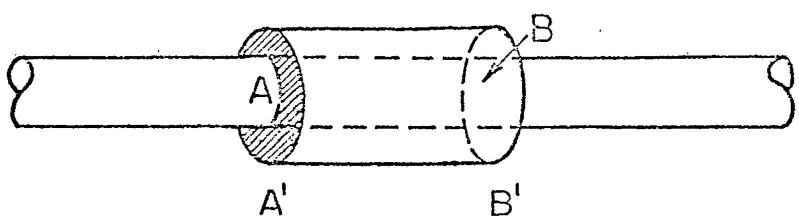

Figure 6. Coaxial Portion of Clamp

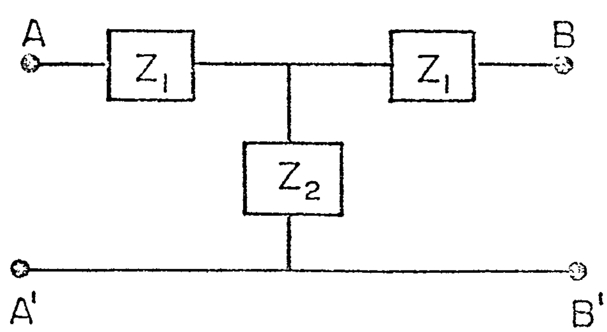

Figure 7. Equivalent Circuit for Structure in Figure 6.

$$Z_1 = Z_c \tanh(\gamma\ell/2) \quad (1)$$

$$Z_2 = Z_c/\sinh(\gamma\ell) \quad (2)$$

and the characteristic impedance, Z_c, being given by

$$Z_c = \frac{Z_o \, \ell n(b/a)}{2\pi\sqrt{\varepsilon_r}} \quad (3)$$

The term Z_o is the characteristic impedance of free space and is expressed as

$$Z_o = \sqrt{\mu_o/\varepsilon_o} \simeq 120\pi \text{ ohms} \quad (4)$$

The term γ is the complex propagation constant in the dielectric medium and, assuming no dielectric conductivity, is given by

$$\gamma = j\,\frac{\omega}{c}\,\sqrt{\varepsilon_r} \quad (5)$$

where ω is the radian frequency and $c = 3\times10^8$ meters/sec is the speed of light in free space.

In a similar manner, it is possible to model the outer surface of the clamp as a cylindrical transmission line over a ground plane as in Figure 8. Since a conducting strap connects points C and D, it is necessary to actually consider two separate transmission lines, one to the left and another to the right of the terminals C-D.

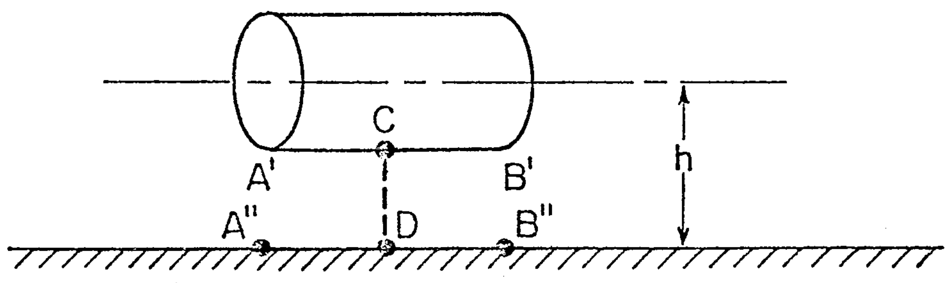

Figure 8. Cylindrical Line over Ground

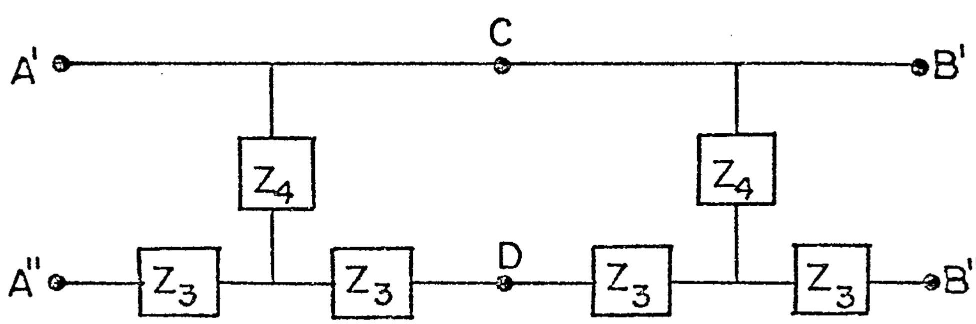

Figure 9. Equivalent Circuit for Cylindrical Line between Terminals A'-A" and B'-B"

Neglecting for the moment the effect of the short between C-D , it is possible to employ two equivalent circuits to model the portion of the problem between A' and B' and the ground plane. The resulting circuit is illustrated in Figure 9. Note that since the terminals A'-B' were used as common terminals for the equivalent circuit of the coaxial portion of the clamp, it is required that these terminals also serve as the common terminals for the cylinder in Figure 8. Thus, the equivalent circuit in Figure 9 is inverted in relation to the usual equivalent circuit for this geometry.

For this circuit, the elements are again given by King (ref. 1) as:

$$Z_3 = Z_c' \tanh(\gamma' \ell/4) \tag{6}$$

and

$$Z_4 = Z_c'/\sinh(\gamma' \ell/2) \tag{7}$$

with the characteristic impedance being that of a cylindrical line of radius b over a ground plane,

$$Z_c' = \frac{Z_o}{2\pi} \cosh^{-1}(h/b) = \frac{Z_o}{2\pi} \ell n \left(\frac{h}{b} + \sqrt{\left(\frac{h}{b}\right)^2 - 1} \right) \tag{8}$$

For this case, the propagation constant γ' is given simply by

$$\gamma' = \frac{j\omega}{c} \tag{9}$$

The effect of a conducting strap from point C to D can be represented primarily in terms of an inductance, since the length of the strap, h-b , is small compared with a wavelength. To find this inductance, the thin strap is first replaced by a wire of diameter d , where d = t/2 (ref. 2). As shown in Figure 10, the current flowing through this wire can be divided into two parts, I_1 flowing through the circuit on the left and I_2 flowing on the right.

Consider first the circuit on the left. The total inductance of this circuit cannot be determined until details are obtained of how the circuit is structured to the left of terminals A-A". Nevertheless, the contributions to the total loop inductance of the segments A"-D, D-C and C-A' can be computed. In fact, the contributions from A"-D and C-A' have been already included in the transmission line model for this circuit, and are given by the impedance elements Z_3 of Figure 9.

The inductance of the wire from C to D can also be computed, not knowing details of the complete circuit. The magnetic flux density B , produced by an infinitesimal current element Idy, is known to be (ref. 3)

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2. Plonsey, R., and R.E. Collin, Principles and Applications of Electromagnetic Fields, McGraw-Hill, 1961.

3. Tai, C.T., "Cylindrical Dipoles," Ch. 3 of Antenna Engineering Handbook, ed. by H. Jasik, McGraw-Hill, 1961.

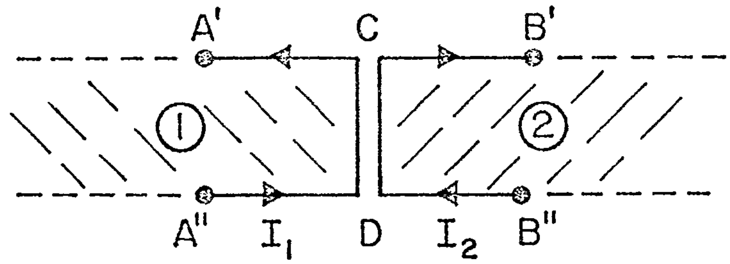

Figure 10. Decomposition of Strap Current

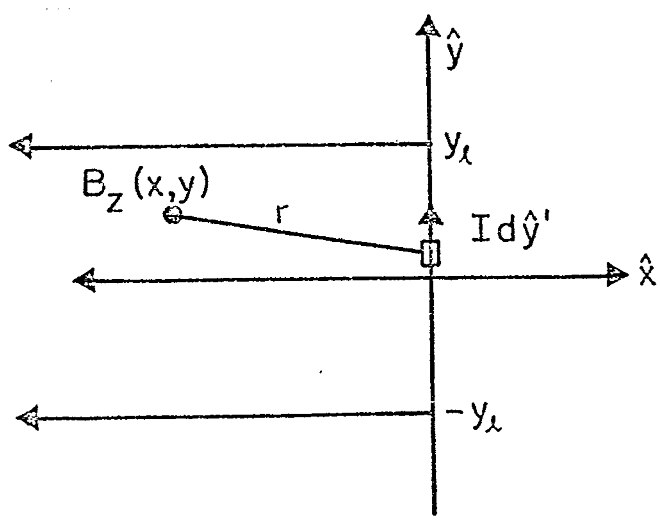

Figure 11. Geometry for Calculation of Strap Inductance

$$d\overline{B} = \frac{\mu_o}{4\pi} \frac{Idy}{r^3} (\hat{y} \times \overline{r}) \qquad (10)$$

where $\overline{r}$ is the vector distance from the current element to the point at which the field is evaluated. As seen from Figure 11, this distance is expressed as

$$r = \sqrt{x^2 + (y-y')^2} \qquad (11)$$

for the source element at y'.

Integrating this last relation between the limits $y_\ell = \pm(h-b)/2$ will give the magnetic field at any point (x,y). This field is normal to the (x,y) plane and is given by

$$B_z(x,y) = \frac{-\mu_o x}{4\pi} \int_{\frac{-(h-b)}{2}}^{\frac{h-b}{2}} \frac{I_1 dy'}{\left(x^2+(y-y')^2\right)^{3/2}} = \frac{\mu_o I}{4\pi x} \left[\frac{y + (h-b)/4}{\left[x^2 + \left(y + \frac{h-b}{4}\right)^2\right]^{1/2}} - \frac{y-(h-b)/4}{\left[x^2 + \left(y - \frac{h-b}{4}\right)^2\right]^{1/2}} \right] \qquad (12)$$

The inductance of this segment of the circuit enclosing area 1 will be related to the total flux passing through the surface. This is calculated as

$$\phi_1 = \int_{-\frac{(h-b)}{2}}^{\frac{h-b}{2}} \int_{-\infty}^{d/2} B_z(x,y)\,dxdy$$

$$= \frac{\mu_o I_1}{4\pi}\left[\frac{d}{2} - \sqrt{\frac{d^2}{4} + (h-b)^2} + (h-b)\ln\left(\frac{h-b + \sqrt{(h-b)^2 + \frac{d^2}{4}}}{d/2}\right)\right] \qquad (13)$$

where $d = t/2$.

The inductance of this part of the strap is then given by

$$L_1 = \frac{\phi_1}{I_1} \qquad (14)$$

A similar set of calculations can be made for the circuit on the right of the strap C-D. This yields an inductance L_2, which is equal to L_1 by symmetry.

The total inductance of the strap is then determined by combining L_1 and L_2. Since these inductances are connected in parallel across C-D, the total strap inductance is then given by:

$$L_s = \frac{L_1}{2} = \frac{\mu_o}{4\pi}\left[\frac{t}{4} - \sqrt{\left(\frac{t}{4}\right)^2 + (h-b)^2} + (h-b)\ln\left(\frac{(h-b) + \sqrt{(h-b)^2 + \left(\frac{t}{4}\right)^2}}{t/4}\right)\right] \qquad (15)$$

With the determination of the strap inductance, the equivalent circuits of Figures 7 and 9 can be combined to form a lumped equivalent network representing the total effect of the clamp. This is illustrated in Figure 12.

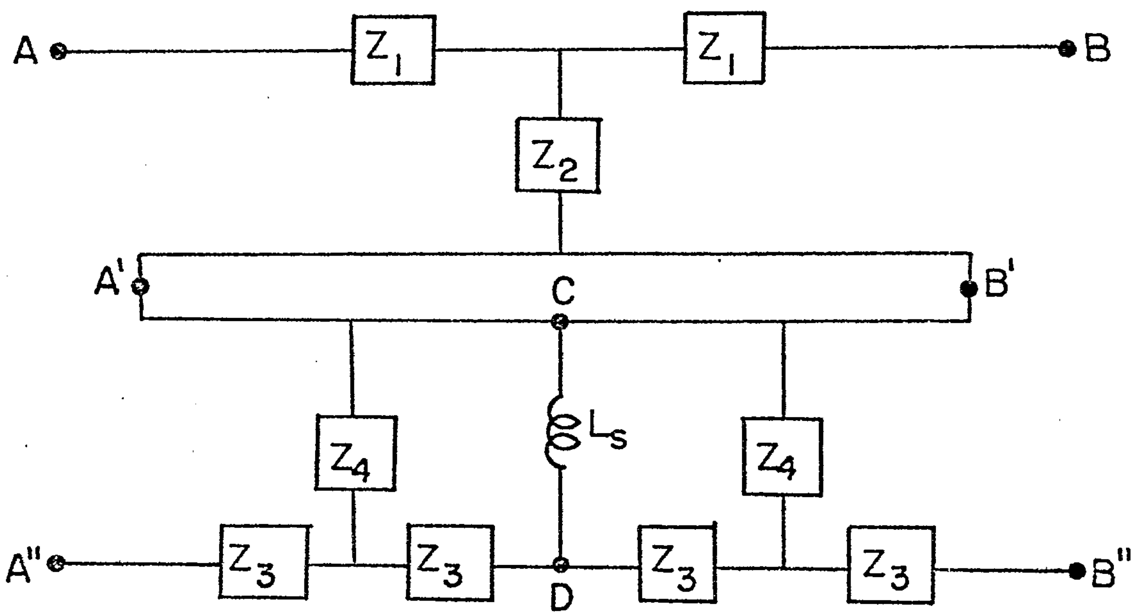

Figure 12. General Equivalent Circuit of Cable Clamp between Terminals A-A" and B-B".

SECTION III

SIMPLIFICATION OF ELEMENTS FOR ELECTRICALLY SMALL CLAMPS

The general equivalent circuit for the cable clamp shown in Figure 12 and the corresponding expression for the impedance elements in equations (1, 2, 6 and 7) can be made simpler by using the approximation that $\gamma\ell$ and $\gamma'\ell$ are small, i.e., the clamp length is small compared with a wavelength. For these cases, the impedance elements become:

$$Z_1 = Z_c \tanh(\gamma\ell/2) \cong j \frac{\omega}{c} Z_c \frac{\sqrt{\varepsilon_r}\,\ell}{2} \tag{16a}$$

$$Z_2 = Z_c/\sinh(\gamma\ell) \cong \frac{1}{j\omega} \frac{Z_c c}{\ell\sqrt{\varepsilon_r}} \tag{16b}$$

$$Z_3 = Z_c' \tan(\gamma'\ell/4) \cong j \frac{\omega}{c} \frac{Z_c'\,\ell}{4} \tag{16c}$$

$$Z_4 = Z_c'/\sinh(\gamma'\ell/2) \cong \frac{1}{j\omega} \frac{Z_c'\,2c}{\ell} \tag{16d}$$

Noting that these impedances can be represented by simple inductances or capacitances,

$$L_1 = \frac{Z_c\,\ell\sqrt{\varepsilon_r}}{2c} \tag{17a}$$

$$C_2 = \frac{\ell\sqrt{\varepsilon_r}}{Z_c \times c} \tag{17b}$$

$$L_3 = \frac{Z_c'\,\ell}{4c} \tag{17c}$$

$$C_4 = \frac{\ell}{2Z_c'\,c} \tag{17d}$$

The entire equivalent circuit for the clamp can thus be described in terms of single circuit elements. Figure 13a shows the resulting circuit after some simplification

For many clamp geometries typically found in aircraft, the impedances formed by L_3 and L_s will be much less than the impedance formed by C_4 at the frequencies found within an EMP waveform. Thus, it is possible to further simplify the cable clamp equivalent circuit by neglecting the capacitance C4. Figure 13b shows the resulting network.

This network can be put in the canonical tee form by a trivial rearrangement of the inductive elements, as shown in Figure 13c. Notice that the shunt inductance due to the strap, L_s, still occurs in series with the capacitance element. Under some circumstances, the inductance reactance of L_s is negligible compared with the capacitive reactance of C_2, and L_s can be eliminated from the model. For this case, the circuit of Figure 13d results.

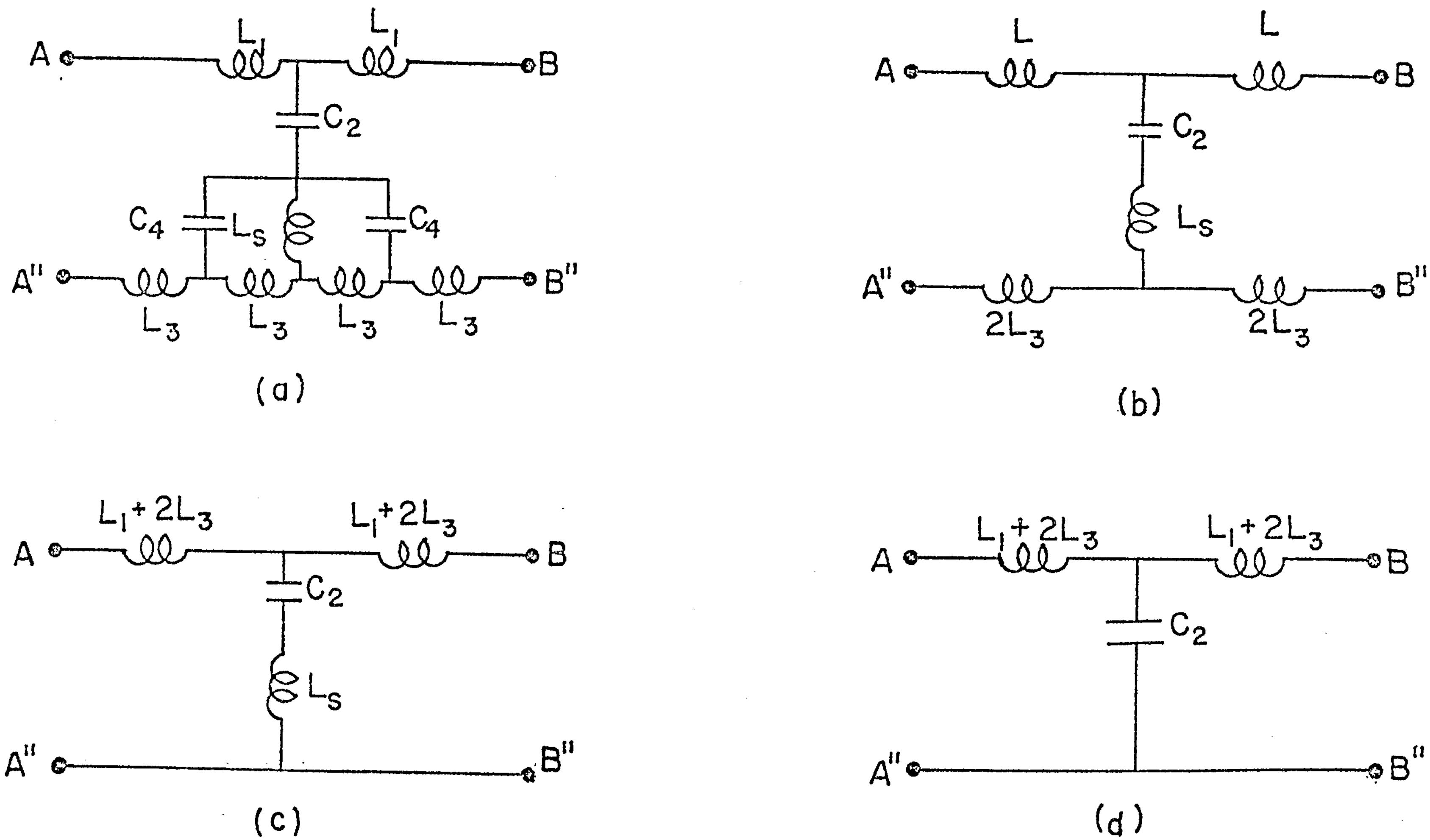

Figure 13. Various Simplifications for the Equivalent Circuit for the Cable Clamp

SECTION IV

CIRCUIT ELEMENTS FOR A SAMPLE CLAMP

To obtain an order of magnitude estimation of the parameters of a typical clamp, the dimensions of a clamp similar to one shown in Figure 4 were measured and yielded the following values for the dimensions indicated in Figure 5:

ℓ = 1.4 cm

b = 2.15 cm

h = 3 cm

a = 1.5 cm

t = 1.4 cm

ε_r = 2.25 (polyethylene assumed)

From these values, the characteristic impedance Z_c of equation (3) becomes

$$Z_c = \frac{120\pi \ \ln(2.15/1.5)}{2\pi\sqrt{2.25}} = 14.4 \ \Omega$$

Similarly, the characteristic impedance Z_c' of equation (8) is given by

$$Z_c' = \frac{120\pi}{2\pi} \ \ln\left(\frac{3.0}{2.15} + \sqrt{\left(\frac{3.0}{2.15}\right)^2 - 1}\right) = 51.7 \ \Omega$$

Since the cable length, ℓ , is much smaller than typical wavelengths found in an EMP waveform, it is sufficient to use the lumped capacitive and inductive elements given in Section III. Using equation (17), the following element values can be calculated:

$$L_1 = \frac{14.4\times 1.4\times 10^{-2}\times\sqrt{2.25}}{2\times 3\times 10^{8}} = .5\times 10^{-9} \text{ henry}$$

$$C_2 = \frac{1.4\times 10^{-2}\times\sqrt{2.25}}{14.4\times 3\times 10^{8}} = 4.8\times 10^{-12} \text{ farad}$$

$$L_3 = \frac{51.7\times 1.4\times 10^{-2}}{4\times 3\times 10^{8}} = 6.0\times 10^{-10} \text{ henry}$$

$$C_4 = \frac{1.4\times 10^{-2}}{2\times 51.7\times 3\times 10^{8}} = 4.5\times 10^{-13} \text{ farad}$$

and from equation (15), the strap inductance is

$$L_S = \frac{4\pi\times 10^{-7}}{4\pi}\left[\frac{1.4\times 10^{-2}}{4} - \sqrt{\left(\frac{1.4\times 10^{-2}}{4}\right)^2 + (3-2.15)\times 10^{-2}} + (3-2.15)\times 10^{-2}\right.$$

$$\left.\times\, \ell n\left(\frac{(3-2.15)\times 10^{-2} + \sqrt{\left[(3-2.15)\times 10^{-2}\right]^2 + \left(\frac{1.4\times 10^{-2}}{4}\right)^2}}{\frac{1.4\times 10^{-2}}{4}}\right)\right]$$

or

$$L_S = 8.1\times10^{-10} \text{ henry}$$

With these values of inductance and capacitance, the validity of the approximations made in obtaining Figures 13c and 13d can be examined. The shunting effect of the capacitance C_4 across the two inductors L_S and L_3 can be ignored for frequencies f such that

$$f \ll f_o = \frac{1}{2\pi \sqrt{(L_S + L_3)\times C_4}} = 6.32 \text{ gigahertz}$$

where f_o is the resonant frequency of the parallel L-C circuit formed by L_3, L_S, and C_4. For frequencies contained in nominal EMP waveforms, this inequality is upheld, so the effects of capacitor C_4 may be neglected.

Similarly, the series L-C branch of Figure 13c can be simplified for frequencies f such that

$$f \ll f_1 = \frac{1}{2\pi \sqrt{L_S\ C_2}} = 2.55 \text{ gigahertz}$$

where f_1 is the resonant frequency of the series L-C circuit. For typical EMP frequencies, it is thus possible to neglect the element L_S for this particular clamp and employ the equivalent circuit of Figure 13d. For other frequencies or for cable clamps having different dimensions, the more complicated forms of the equivalent circuit may be required.

SECTION V

CONCLUSION

A rather simple set of equivalent circuits which describe the behavior of a cable clamp on a single transmission line has been developed. Since the clamp is assumed to be electrically small, the equivalent circuits involve only lumped inductances and capacitances which are simple functions of the clamp geometry.

The precise effect of the clamp on energy propagation on the transmission line will depend upon the specific geometry of the cable and clamp being considered. The analysis of a specific transmission line, with and without the clamp, will be detailed in a separate report, to assess the degree of improvement in the single line model for internal EMP interaction calculations.

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1. King, R.W.P., Transmission Line Theory, Dover 1965.
2. Plonsey, R. and R.E. Collin, Principles and Applications of Electromagnetic Fields, McGraw-Hill, 1961.
3. Tai, C.T., "Cylindrical Dipoles," Ch. 3 of Antenna Engineering Handbook, ed. by H. Jasik, McGraw-Hill, 1961.