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ELECTROMAGNETIC TRANSMISSION THROUGH A RECTANGULAR APERTURE IN A PERFECTLY CONDUCTING PLANE

by

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ABSTRACT

A computer program is developed for calculating the transmission characteristics of a rectangular aperture in a perfectly conducting plane excited by an incident plane wave. The solution is obtained from the integral equation for the equivalent magnetic current using the method of moments. The expansion functions and testing functions are pulses in the direction transverse to current flow, and triangles in the direction of current flow. Quantities computed are the equivalent magnetic current and the transmission cross section patterns. To illustrate the solution, computations are given for narrow slots and for square apertures. The computer program is described and listed with sample input-output data.

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PART ONE

THEORY AND EXAMPLES

I. INTRODUCTION

Formulas for the computation of plane wave transmission through a rectangular aperture in a perfectly conducting plane are derived in Part One. The computer programs which use these formulas are given in Part Two. The general theory of solution is derived in a previous report [1]. Basically, the procedure is an application of the method of moments to an integral equation formulation of the problem. The unknown to be determined is the equivalent magnetic current over the aperture region, which is proportional to the tangential electric field in the aperture. The solution is expressed in terms of an aperture admittance matrix, which is dual to the impedance matrix for the complementary conducting plate. Once the equivalent magnetic current is obtained, the electromagnetic field can be computed via potential integrals. The notation used in this report is the same as that used in [1]. We abstract equations from this previous work as we need them, referring to them by equation number. We do not attempt to summarize the theory here.

Previous studies of aperture problems include those for small apertures [2,3], and those for circular apertures [4,5]. Some results for

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[1] R. F. Harrington and J. R. Mautz, "A Generalized Network Formulation for Aperture Problems," Scientific Report No. 8 on Contract F19628-73-C-0047 with A.F. Cambridge Research Laboratories, Report AFCRL-TR-75-0589, November 1975.

[2] H. A. Bethe, "Theory of Diffraction by Small Holes," Phys. Rev., vol. 66, pp. 163-182, October 1944.

[3] C. J. Bouwkamp, "Diffraction Theory," Repts. Progr. in Phys., vol. 17, pp. 35-100, 1954.

[4] G. Bekefi, "Diffraction of Electromagnetic Waves by an Aperture in a Large Screen," Journ. Appl. Phys., vol. 24, No. 9, pp. 1123-1130, September 1953.

[5] C. J. Bouwkamp, "Theoretical and Numerical Treatment of Diffraction Through a Circular Aperture," IEEE Trans. on Antennas and Propagation, vol. AP-18, No. 2, pp. 152-176, March 1970.

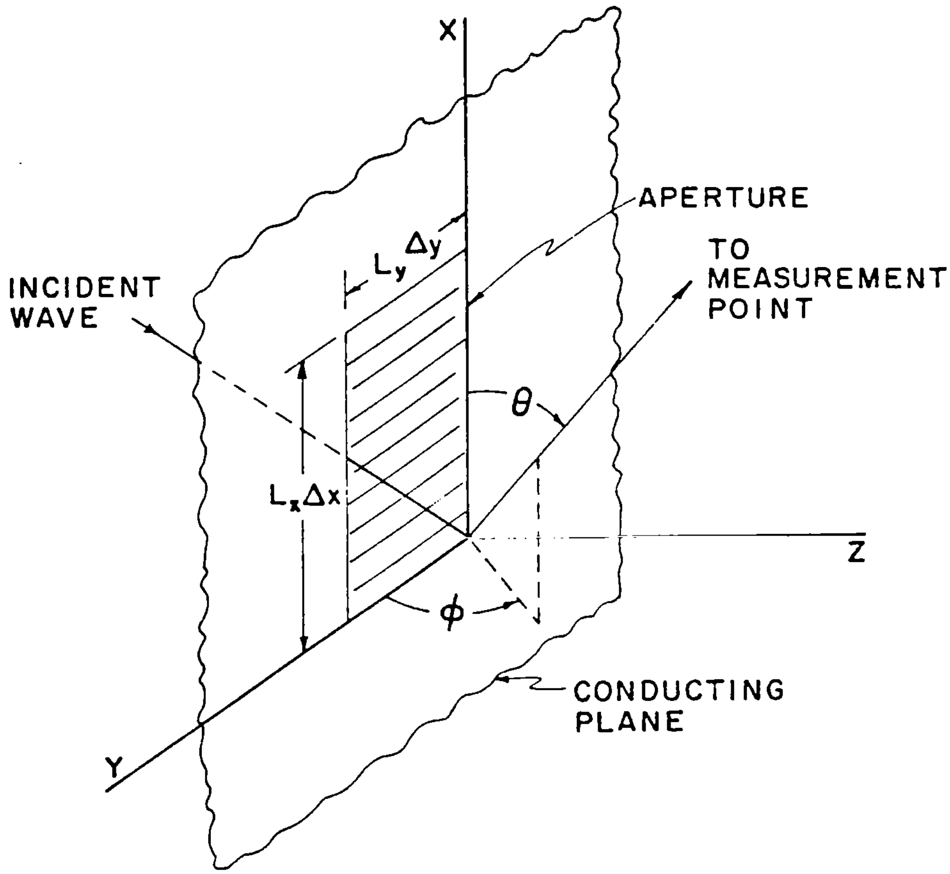

Fig. 1. Rectangular aperture in a conducting plane.

apertures of arbitrary shape have been obtained using Babinet's principle plus a wire grid approximation to the complementary conducting plate [6,7]. The reader may consult these papers for other references.

II. STATEMENT OF THE PROBLEM

Figure 1 shows the problem to be considered and defines the coordinates and parameters to be used. The infinitely conducting plate covers the entire z=0 plane except for the aperture, which is rectangular in shape with side lengths $L_x \Delta x$ and $L_y \Delta y$ in the x and y directions, respectively. The excitation of the aperture is a uniform plane wave incident from the region $z < 0$. The field to be computed is the far zone magnetic field in the region $z > 0$, at the angles θ, ϕ.

The solution is expressed in terms of the equivalent magnetic current $\underset{\sim}{M} = \hat{\underset{\sim}{z}} \times \underset{\sim}{E}$, where $\hat{\underset{\sim}{z}}$ is the unit z-directed vector and $\underset{\sim}{E}$ is the electric field in the aperture. To compute $\underset{\sim}{M}$, we use a linear expansion in terms of basis functions $\underset{\sim}{M}_i$ and evaluate the coefficients by the method of moments. This involves determining a generalized admittance matrix, evaluated in Section III, and an excitation vector. To determine the field produced by $\underset{\sim}{M}$, we need a measurement vector. The excitation and measurement vectors for the present problem are of the same form, and are evaluated in Section IV.

III. ADMITTANCE MATRIX

According to [1, Eq. (28)] and [1, Eq. (10)], the admittance matrix [Y] is given by

$$Y_{ij} = (Y^a + Y^b)_{ij} = -4 \langle W_i, H(M_j) \rangle \tag{1}$$

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[6] A. T. Adams, C. B. Varnado, D. E. Warren, "Aperture Coupling by Matrix Methods," 1973 IEEE EMC Symposium Record, New York City, June 1973, pp. 226-240.

[7] J-L Lin, W. L. Curtis, M. C. Vincent, "On the Field Distribution of an Aperture," IEEE Trans. on Antennas and Propagation, vol. AP-22, No. 3, pp. 467-471, May 1974.

where $\mathbf{H}(\mathbf{M}_j)$ is the magnetic field produced by $\mathbf{M}_j$ radiating in free space. The magnetic field $\mathbf{H}(\mathbf{M}_j)$ can be expressed in terms of an electric vector potential $\mathbf{F}$ and magnetic scalar potential ϕ as [8]

$$\mathbf{H}(\mathbf{M}_j) = -j\omega\mathbf{F}_j - \nabla\phi_j \tag{2}$$

where

$$\mathbf{F}_j = \frac{\varepsilon}{4\pi} \iint_{\text{apert.}} \mathbf{M}_j \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} ds \tag{3}$$

$$\phi_j = \frac{1}{4\pi\mu} \iint_{\text{apert.}} \rho_j \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} ds \tag{4}$$

$$\rho_j = \frac{\nabla \cdot \mathbf{M}_j}{-j\omega} \tag{5}$$

where $\mathbf{r}$ and $\mathbf{r}'$ are respectively the vectors to the field and source points in the aperture, ω is the angular frequency, ε is the capacitivity of free space, μ is the permeability of free space, and $k = \omega\sqrt{\mu\varepsilon}$ is the propagation constant in free space. Substituting [1, Eq. (7)] and (2) into (1), we obtain

$$Y_{ij} = 4 \iint_{\text{apert.}} \mathbf{W}_i \cdot (j\omega\mathbf{F}_j + \nabla\phi_j) ds \, . \tag{6}$$

Because of the identity

$$0 = \iint_{\text{apert.}} \nabla \cdot (\phi_j \mathbf{W}_i) ds = \iint_{\text{apert.}} \mathbf{W}_i \cdot \nabla\phi_j ds + \iint_{\text{apert.}} \phi_j \nabla \cdot \mathbf{W}_i ds \tag{7}$$

(6) becomes

$$Y_{ij} = 4j\omega \iint_{\text{apert.}} (\mathbf{F}_j \cdot \mathbf{W}_i + \phi_j \rho_i) ds \tag{8}$$

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[8] C. H. Papas, Theory of Electromagnetic Wave Propagation, McGraw-Hill Book Co., New York, 1965, p. 23.

where

$$\rho_i = \frac{\nabla \cdot W_i}{-j\omega} \tag{9}$$

We choose the set of testing functions W_i equal to the set of expansion functions M_i. The rectangular aperture $0 \leq x \leq L_x \Delta x$, $0 \leq y \leq L_y \Delta y$ where L_x and L_y are integers is divided into rectangular subareas of length Δx in x and Δy in y. The set M_i of expansion functions is split into a set M_i^x of x directed magnetic currents and a set M_i^y of y directed magnetic currents defined by

$$M^x_{p+(q-1)(L_x-1)} = \hat{x}\, T^x_p(x)\, P^y_q(y) \,, \qquad \begin{array}{l} p = 1,2,\ldots L_x-1 \\ q = 1,2,\ldots L_y \end{array} \tag{10}$$

$$M^y_{p+(q-1)L_x} = \hat{y}\, T^y_q(y)\, P^x_p(x) \,, \qquad \begin{array}{l} p = 1,2,\ldots L_x \\ q = 1,2,\ldots L_y-1 \end{array} \tag{11}$$

where $T^x_p(x)$ and $T^y_q(y)$ are triangle functions defined by

$$T^x_p(x) = \begin{cases} \dfrac{x - (p-1)\Delta x}{\Delta x} & (p-1)\,\Delta x \leq x \leq p\Delta x \\[2ex] \dfrac{(p+1)\Delta x - x}{\Delta x} & p\Delta x \leq x \leq (p+1)\Delta x \\[2ex] 0 & |x - p\Delta x| \geq \Delta x \end{cases} \tag{12}$$

$$T^y_q(y) = \begin{cases} \dfrac{y - (q-1)\Delta y}{\Delta y} & (q-1)\Delta y \leq y \leq q\Delta y \\[2ex] \dfrac{(q+1)\Delta y - y}{\Delta y} & q\Delta y \leq y \leq (q+1)\Delta y \\[2ex] 0 & |y - q\Delta y| \geq \Delta y \end{cases} \tag{13}$$

and $P^x_p(x)$ and $P^y_q(y)$ are pulse functions defined by

$$P_p^x(x) = \begin{cases} 1 & (p-1)\Delta x \le x < p\Delta x \\ 0 & \text{all other } x \end{cases} \tag{14}$$

$$P_q^y(y) = \begin{cases} 1 & (q-1)\Delta y \le y < q\Delta y \\ 0 & \text{all other } y \end{cases} \tag{15}$$

The magnetic charge sheets, say ρ^x and ρ^y associated with $\underset{\sim}{M}^x$ and $\underset{\sim}{M}^y$ are obtained from (5) as

$$\rho^x_{p+(q-1)(L_x-1)} = \frac{(P_p^x(x) - P_{p+1}^x(x))P_q^y(y)}{-j\omega\Delta x} \tag{16}$$

$$\rho^y_{p+(q-1)L_x} = \frac{(P_q^y(y) - P_{q+1}^y(y))P_p^x(x)}{-j\omega\Delta y} \tag{17}$$

Introduction of the two types of expansion functions $\underset{\sim}{M}_j^x$ and $\underset{\sim}{M}_j^y$ and the two types of testing functions $\underset{\sim}{M}_i^x$ and $\underset{\sim}{M}_i^y$ into (8) gives rise to four Y submatrices defined by

$$Y_{ij}^{uv} = 4j\omega \iint_{\text{apert.}} (\underset{\sim}{F}_j^v \cdot \underset{\sim}{M}_i^u + \phi_j^v \rho_i^u) ds \tag{18}$$

where u is either x or y and v is either x or y. In (18), $\underset{\sim}{F}_j^v$ and ϕ_j^v are the electric vector and magnetic scalar potentials due to $\underset{\sim}{M}_j^v$.

The integrations over the "field" magnetic current and charge $\underset{\sim}{M}_i^u$ and ρ_i^u explicit in (18) are approximated by sampling the integrands at two points. Hence,

$$Y_{ij}^{xv} = 4j\omega\Delta x\Delta y \, [\tfrac{1}{2} (\underset{\sim}{F}_j^v \cdot \hat{\underset{\sim}{x}})_{x_p,y_q} + \tfrac{1}{2} (\underset{\sim}{F}_j^v \cdot \hat{\underset{\sim}{x}})_{x_{p+1},y_q}$$

$$- \frac{1}{j\omega\Delta x} (\phi_j^v)_{x_p,y_q} + \frac{1}{j\omega\Delta x} (\phi_j^v)_{x_{p+1},y_q}] \tag{19}$$

$$Y_{ij}^{yv} = 4j\omega\Delta x\Delta y \; [\frac{1}{2} (\underset{\sim}{F}_j^v \cdot \underset{\sim}{\hat{y}})_{x_p, y_q} + \frac{1}{2} (\underset{\sim}{F}_j^v \cdot \underset{\sim}{\hat{y}})_{x_p, y_{q+1}}$$

$$- \frac{1}{j\omega\Delta y} (\phi_j^v)_{x_p, y_q} + \frac{1}{j\omega\Delta y} (\phi_j^v)_{x_p, y_{q+1}}] \tag{20}$$

where v is either x or y and

$$x_p = (p - .5)\Delta x \tag{21}$$

$$y_q = (q - .5)\Delta y \tag{22}$$

To determine p and q in terms of i in (19) or (20), refer to (10) or (11).

Substitution of (10), (11), (16), (17), (3) and (4) into (19) and (20) yields

$$Y_{ij}^{xx} = \frac{j\Delta x\Delta y}{\pi\eta} [\frac{1}{2} I_c(s-p,\ t-q) - \frac{1}{2} I_x(s-p+1,\ t-q)$$

$$+ \frac{(s-p + 3/2)}{2} I_c(s-p+1,\ t-q) + \frac{1}{2} I_x(s-p-1,\ t-q) - \frac{(s-p-3/2)}{2} I_c(s-p-1, t-q)$$

$$+ \frac{1}{k^2\Delta x^2} (I_c(s-p+1,\ t-q) - 2I_c(s-p,\ t-q) + I_c(s-p-1,\ t-q))] \tag{23}$$

$$Y_{ij}^{yx} = \frac{j}{\pi\eta k^2} [- I_c(s-p,\ t-q) + I_c(s-p+1,\ t-q)$$

$$+ I_c(s-p,\ t-q-1) - I_c(s-p+1,\ t-q-1)] \tag{24}$$

$$Y_{ij}^{xy} = \frac{j}{\pi\eta k^2} [- I_c(s-p,\ t-q) + I_c(s-p,\ t-q+1)$$

$$+ I_c(s-p-1,\ t-q) - I_c(s-p-1,\ t-q+1)] \tag{25}$$

$$Y_{ij}^{yy} = \frac{j\Delta x\Delta y}{\pi\eta}\Big[\frac{1}{2} I_c(s-p,\ t-q) - \frac{1}{2} I_y(s-p,\ t-q+1)$$

$$+ \frac{(t-q+3/2)}{2} I_c(s-p,\ t-q+1) + \frac{1}{2} I_y(s-p,\ t-q-1) - \frac{(t-q-3/2)}{2} I_c(s-p,t-q-1)$$

$$+ \frac{1}{k^2\Delta y^2}\left(I_c(s-p,\ t-q+1) - 2I_c(s-p,\ t-q) + I_c(s-p,\ t-q-1)\right)\Big] \qquad (26)$$

where $\eta = \sqrt{\frac{\mu}{\varepsilon}} = 376.730$ ohms is the intrinsic impedance for empty space and where

$$I_c(s,t) = k \int_{y=(t-1/2)\Delta y}^{(t+1/2)\Delta y} dy \int_{x=(s-1/2)\Delta x}^{(s+1/2)\Delta x} dx\ \frac{e^{-jk\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} \qquad (27)$$

$$I_x(s,t) = \frac{k}{\Delta x} \int_{y=(t-1/2)\Delta y}^{(t+1/2)\Delta y} dy \int_{x=(s-1/2)\Delta x}^{(s+1/2)\Delta x} x dx\ \frac{e^{-jk\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} \qquad (28)$$

$$I_y(s,t) = \frac{k}{\Delta y} \int_{y=(t-1/2)\Delta y}^{(t+1/2)\Delta y} y dy \int_{x=(s-1/2)\Delta x}^{(s+1/2)\Delta x} dx\ \frac{e^{-jk\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} \qquad (29)$$

$I_c(s,t)$ is even in both s and t, $I_x(s,t)$ is odd in s and even in t, and $I_y(s,t)$ is even in s and odd in t. In (23) to (26), Y_{ij}^{uv} is the interaction of the expansion function $\underset{\sim}{M}_j^v$ with the testing function $\underset{\sim}{M}_i^u$. For the testing function $\underset{\sim}{M}_i^x$,

$$i = p + (q-1)(L_x-1), \begin{cases} p = 1,2,\ldots L_x-1 \\ q = 1,2\ldots L_y \end{cases} \qquad (30)$$

whereas for the testing function $\underset{\sim}{M}_i^y$,

$$i = p + (q-1)L_x, \begin{cases} p = 1,2,\ldots L_x \\ q = 1,2,\ldots L_y-1 \end{cases} \qquad (31)$$

For the expansion function $\underset{\sim}{M}_j^x$,

$$j = s + (t-1)(L_x-1), \begin{cases} s = 1,2,\dots L_x-1 \\ t = 1,2,\dots L_y \end{cases} \tag{32}$$

whereas for the expansion function $\underset{\sim}{M}_j^y$,

$$j = s + (t-1)L_x, \begin{cases} s = 1,2,\dots L_x \\ t = 1,2,\dots L_y-1 \end{cases} \tag{33}$$

The four dimensional array Y_{ij}^{xx} of (23) in which s, p, t, and q vary separately can be constructed from the two dimensional array obtained by varying the two integers (s-p) and (t-q) in (23). In (23),

$$2 - L_x \leq (s-p) \leq L_x-2$$

$$1 - L_y \leq (t-q) \leq L_y-1$$

but because (23) is even in both (s-p) and (t-q),

$$\begin{aligned} (s-p) &= 0,1,2,\dots L_x-2 \\ (t-q) &= 0,1,2,\dots L_y-1 \end{aligned} \tag{34}$$

is sufficient. In (24),

$$1 - L_x \leq (s-p) \leq L_x-2$$

$$2 - L_y \leq (t-q) \leq L_y-1$$

but because (24) is odd about (s-p) = -1/2 and odd about (t-q) = 1/2,

$$\begin{aligned} (s-p) &= 0,1,2,\dots L_x-2 \\ (t-q) &= 1,2,3,\dots L_y-1 \end{aligned} \tag{35}$$

is sufficient. In (25),

$$2 - L_x \leq (s-p) \leq L_x - 1$$

$$1 - L_y \leq (t-q) \leq L_y - 2$$

but because (25) is odd about $(s-p) = 1/2$ and odd about $(t-q) = -1/2$,

$$(s-p) = 1,2,3,\ldots L_x - 1$$
$$(t-q) = 0,1,2,\ldots L_y - 2 \qquad (36)$$

is sufficient. Finally, in (26),

$$1 - L_x \leq (s-p) \leq L_x - 1$$

$$2 - L_y \leq (t-q) \leq L_y - 2$$

but because (26) is even in both $(s-p)$ and $(t-q)$,

$$(s-p) = 0,1,2,\ldots L_x - 1$$
$$(t-q) = 0,1,2,\ldots L_y - 2 \qquad (37)$$

is sufficient. From inspection of (23) to (26) and (34) to (37),

$$s = -1,0,1,\ldots L_x - 1$$
$$t = -1,0,1,\ldots L_y - 1 \qquad (38)$$

is adequate in (27) to (29).

The integrals (27) to (29) are evaluated by using the following four term approximation

$$e^{-jkr} \approx e^{-jkr_o}\left[1 - jk(r-r_o) - \frac{k^2}{2}(r-r_o)^2 + \frac{jk^3}{6}(r-r_o)^3\right] \qquad (39)$$

where

$$r = \sqrt{x^2 + y^2} \tag{40}$$

$$r_o = \sqrt{(s\Delta x)^2 + (t\Delta y)^2} \ . \tag{41}$$

Substitution of (39) into (27) yields

$$I_c(s,t) = [k(1 + jkr_o - \frac{k^2r_o^2}{2} - \frac{jk^3r_o^3}{6})\iint \frac{dxdy}{r}$$

$$+ k^2(-j + kr_o + \frac{jk^2r_o^2}{2})\iint dxdy + k^3(-\frac{1}{2} - \frac{jkr_o}{2}) \iint rdxdy$$

$$+ \frac{jk^4}{6} \iint r^2dxdy]e^{-jkr_o} \tag{42}$$

where the limits on all the integrals in (42) are the same as those in (27). The approximations to (28) or (29) are given by (42) with an additional factor of either $\frac{x}{\Delta x}$ or $\frac{y}{\Delta y}$ in the integrands. Three of the required integrals are

$$\iint dxdy = \Delta x\Delta y \tag{43}$$

$$\iint xdxdy = s\Delta x^2\Delta y \tag{44}$$

$$\iint ydxdy = t\Delta x\Delta y^2 \tag{45}$$

The indefinite integrals associated with the rest of the required integrals are

$$\iint \frac{dxdy}{r} = x \log(y + r) + y \log (x + r) \tag{46}$$

$$\iint rdxdy = \frac{xyr}{3} + \frac{x^3}{6} \log(y + r) + \frac{y^3}{6} \log (x + r) \tag{47}$$

$$\iint r^2 dxdy = \frac{xyr^2}{3} \tag{48}$$

$$\iint \frac{xdxdy}{r} = \frac{yr}{2} + \frac{x^2}{2} \log (y + r) \tag{49}$$

$$\iint xrdxdy = yr(\frac{r^2}{12} + \frac{x^2}{8}) + \frac{x^4}{8} \log (y + r) \tag{50}$$

$$\iint xr^2 dxdy = x^2 y(\frac{x^2}{4} + \frac{y^2}{6}) \tag{51}$$

$$\iint \frac{ydxdy}{r} = \frac{xr}{2} + \frac{y^2}{2} \log (x + r) \tag{52}$$

$$\iint yrdxdy = xr(\frac{r^2}{12} + \frac{y^2}{8}) + \frac{y^4}{8} \log (x + r) \tag{53}$$

$$\iint yr^2 dxdy = y^2 x \ (\frac{y^2}{4} + \frac{x^2}{6}) \tag{54}$$

The reader can verify (46) to (54) by showing that, in each case, the mixed second partial derivative $\frac{\partial^2}{\partial x \partial y}$ of the right hand side is equal to the integrand on the left hand side. The definite integral is obtained from the indefinite integral by adding the indefinite integral evaluated at both upper limits to that at both lower limits and subtracting both evaluations of the indefinite integral at the mixed (one upper, one lower) limits.

IV. PLANE WAVE EXCITATION AND MEASUREMENT VECTORS

The plane wave excitation vector $\vec{P}^i$ of [1, Eq. (32)] and the plane wave measurement vector $\vec{P}^m$ of [1, Eq. (37)] are of the same form except for a minus sign. We therefore need to evaluate only one of them, say the measurement vector $\vec{P}^m$. We specialize it to four principal plane patterns as

$$(P_i^{mu})_{\theta y} = -2 \iint_{\text{apert.}} \underset{\sim}{M}_i^u \cdot \hat{\underset{\sim}{\theta}} \, e^{jkx \cos\theta} \, dxdy \tag{55}$$

$$(P_i^{mu})_{yy} = -2 \iint_{\text{apert.}} \underset{\sim}{M}_i^u \cdot \hat{\underset{\sim}{y}} \, e^{jkx \cos\theta} \, dxdy \tag{56}$$

$$(P_i^{mu})_{\phi x} = -2 \iint_{\text{apert.}} \underset{\sim}{M}_i^u \cdot \hat{\underset{\sim}{\phi}} \, e^{jky \cos\phi} \, dxdy \tag{57}$$

$$(P_i^{mu})_{xx} = -2 \iint_{\text{apert.}} \underset{\sim}{M}_i^u \cdot \hat{\underset{\sim}{x}} \, e^{jky \cos\phi} \, dxdy \tag{58}$$

The superscript u is necessary because $\underset{\sim}{M}_i$ has been split up into $\underset{\sim}{M}_i^x$ and $\underset{\sim}{M}_i^y$ of (10) and (11). In (55) to (58), $\hat{\underset{\sim}{\theta}}$, $\hat{\underset{\sim}{y}}$, $\hat{\underset{\sim}{\phi}}$, and $\hat{\underset{\sim}{x}}$ are unit vectors in the θ, y, ϕ, and x directions respectively where, as shown in Fig. 1, θ is measured from the positive x axis in the y = 0 plane and ϕ is measured from the positive y axis in the x = 0 plane. For measurement vectors, $0° \leq \theta \leq 180°$, $0° \leq \phi \leq 180°$. $(P_i^{mu})_{\theta y}$ is for a $\hat{\underset{\sim}{\theta}}$ polarized measurement in the y = 0 plane, $(P_i^{mu})_{yy}$ is for a $\hat{\underset{\sim}{y}}$ polarized measurement in the y = 0 plane, $(P_i^{mu})_{\phi x}$ is for a $\hat{\underset{\sim}{\phi}}$ polarized measurement in the x = 0 plane, and $(P_i^{mu})_{xx}$ is for a $\hat{\underset{\sim}{x}}$ polarized measurement in the x = 0 plane. Because our set of testing functions $\underset{\sim}{W}_m$ is the same as the set of expansion functions $\underset{\sim}{M}_n$, the plane wave excitation vector $\vec{P}^i$ of [1, Eq. (32)] is obtained by putting $180° \leq \theta \leq 360°$, $180° \leq \phi \leq 360°$ in the negative of one of the equations (55) to (58).

Substituting (10) and (11) into (55) to (58) we obtain, with the help of [9]

[9] H. B. Dwight, Tables of Integrals and Other Mathematical Data, fourth edition, Macmillan Co., New York, 1961, Eq. 567.1.

$$(P^{mx}_{p+(q-1)(L_x-1)})_{\theta y} = 2\Delta x\Delta y \sin\theta \left(\frac{\sin\frac{k\Delta x\cos\theta}{2}}{\frac{k\Delta x\cos\theta}{2}}\right)^2 e^{jkp\Delta x\cos\theta}, \begin{cases} p=1,2,\ldots L_x-1 \\ q=1,2,\ldots L_y \end{cases} \tag{59}$$

$$(P^{my}_i)_{\theta y} = 0, \; i = 1,2,\ldots\; L_x(L_y-1) \tag{60}$$

$$(P^{mx}_i)_{yy} = 0, \; i = 1,2,\ldots\; (L_x-1)L_y \tag{61}$$

$$(P^{my}_{p+(q-1)L_x})_{yy} = -2\Delta x\Delta y \left(\frac{\sin\frac{k\Delta x\cos\theta}{2}}{\frac{k\Delta x\cos\theta}{2}}\right) e^{jk(p-1/2)\Delta x\cos\theta}, \begin{cases} p = 1,2,\ldots L_x \\ q = 1,2,\ldots L_y-1 \end{cases} \tag{62}$$

$$(P^{mx}_i)_{\phi x} = 0, \; i = 1,2,\ldots\; (L_x-1)L_y \tag{63}$$

$$(P^{my}_{p+(q-1)L_x})_{\phi x} = 2\Delta x\Delta y \sin\phi \left(\frac{\sin\frac{k\Delta y\cos\phi}{2}}{\frac{k\Delta y\cos\phi}{2}}\right)^2 e^{jkq\Delta y\cos\phi}, \begin{cases} p = 1,2,\ldots L_x \\ q = 1,2,\ldots L_y-1 \end{cases} \tag{64}$$

$$(P^{mx}_{p+(q-1)(L_x-1)})_{xx} = -2\Delta x\Delta y \left(\frac{\sin\frac{k\Delta y\cos\phi}{2}}{\frac{k\Delta y\cos\phi}{2}}\right) e^{jk(q-1/2)\Delta y\cos\phi}, \begin{cases} p=1,2,\ldots L_x-1 \\ q=1,2,\ldots L_y \end{cases} \tag{65}$$

$$(P^{my}_i)_{xx} = 0, \; i = 1,2,\ldots\; L_x(L_y-1) \tag{66}$$

V. REPRESENTATIVE COMPUTATIONS

A versatile computer program has been developed using the preceding formulas. This program is described and listed in Part Two of this report. Some representative computations obtained with this program are given in this section.

The first computations were made for a narrow slot, of width $\lambda/20$ and of variable length L. The far-zone quantity plotted was the transmission cross section, defined as [1, Eq. (39)]

$$\tau = 2\pi r^2 |H_m|^2 \tag{67}$$

where H_m is the component of magnetic field being considered. We use the notation:

$$\begin{aligned} \tau_{\theta y} &= 2\pi r^2 |H_\theta|^2 \quad \text{in the } y = 0 \text{ plane,} \\ \tau_{xx} &= 2\pi r^2 |H_x|^2 \quad \text{in the } x = 0 \text{ plane.} \end{aligned} \tag{68}$$

For the case being considered, the orthogonal components of $\underset{\sim}{H}$ in these two planes were zero. Figure 2 shows plots of $\tau_{\theta y}$ and τ_{xx} for x-directed slots of width $\lambda/20$ and length (a) $L = \lambda/4$, (b) $L = \lambda/2$, (c) $L = 3\lambda/4$, and (d) $L = \lambda$. In all cases the excitation was due to a plane wave normally incident on the conducting plane with the magnetic field in the x direction. Note the large transmission cross section for $L = \lambda/2$, case (b), due to the slot being near resonance. The plots of τ are of the same form as scattering cross section from the complementary conducting strips, as known from Babinet's principle.

Figure 3 shows plots of the equivalent magnetic current in the aperture region for the same slots. Since $\underset{\sim}{M} = \hat{\underset{\sim}{z}} \times \underset{\sim}{E}$, they are also plots of the tangential component of $\underset{\sim}{E}$ in the slots. Again note the large value of $\underset{\sim}{M}$ for the case $L = \lambda/2$, which is near resonance. Note also that, for short slots ($L \leq 3\lambda/4$), the $\underset{\sim}{M}$ is almost equiphasal and closely approximated by a half sine wave.

Next, computations were made to test the rate of convergence of the solution as the number of subsections was increased. A slot of width $\lambda/10$ and length 2λ was chosen for the study. Again the excitation is a plane-wave normally incident on the conducting plane with the magnetic field in the x direction. Figure 4 shows plots of $\tau_{\theta y}$ and τ_{xx} for the cases (a) 39, (b) 19, (c) 9, and (d) 4 triangular expansion

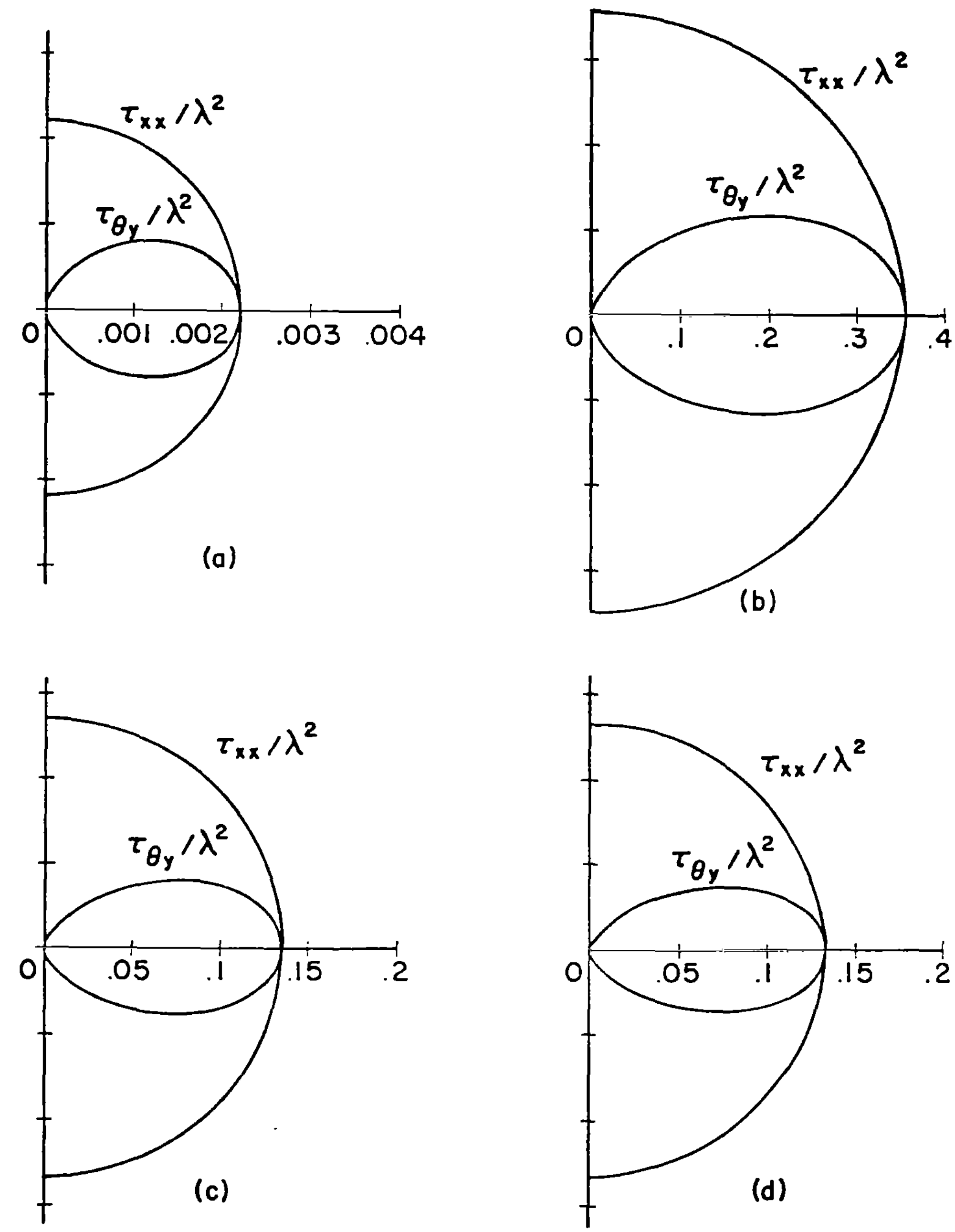

Fig. 2. Transmission cross section for slots of length L in the x direction and width $\lambda/20$ in the y direction. (a) $L = \lambda/4$, (b) $L = \lambda/2$, (c) $L = 3\lambda/4$, (d) $L = \lambda$. Excitation is by a plane wave normally incident on the conducting plane with magnetic field in the x direction.

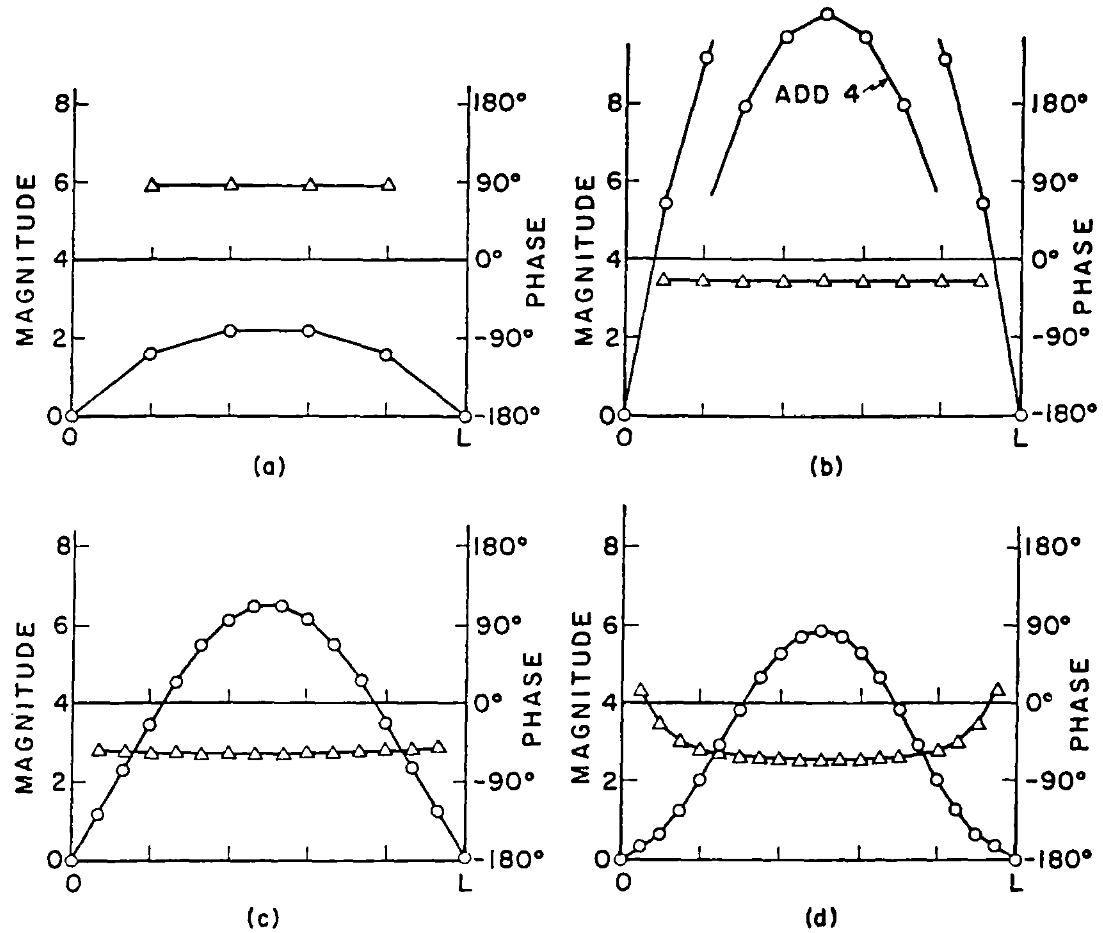

Fig. 3. Magnitude and phase of $|M/E^i|$, where M is the x-directed magnetic current and E^i is the incident electric field, for the same slots as for Fig. 2. (a) $L = \lambda/4$, (b) $L = \lambda/2$, (c) $L = 3\lambda/4$, (d) $L = \lambda$. Circles denote magnitude, triangles denote phase.

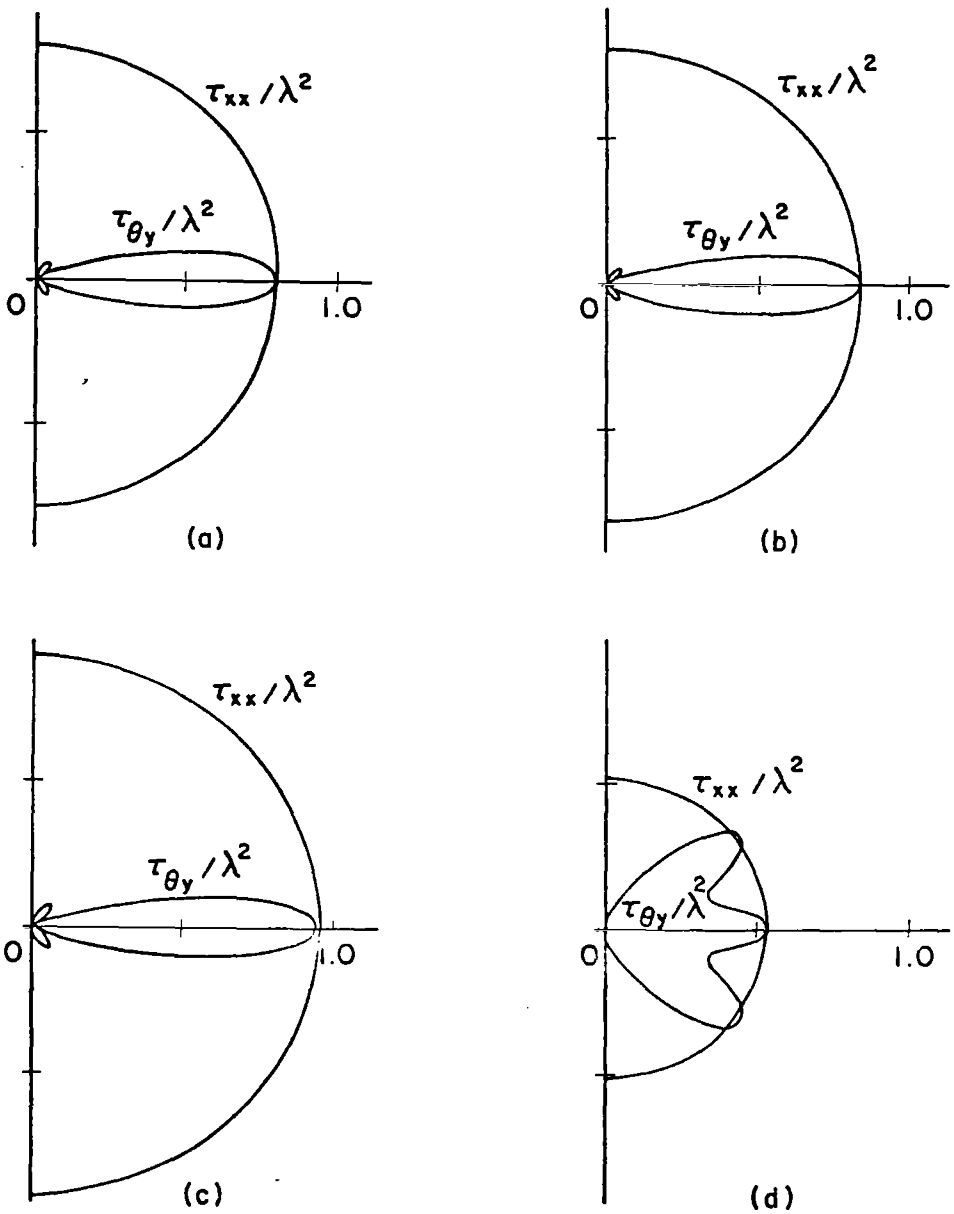

Fig. 4. Transmission cross section when the number of expansion functions is (a) 39, (b) 19, (c) 9, and (d) 4. Computations are for a slot of length 2λ in the x direction and width $\lambda/10$ in the y direction. Excitation is by a plane wave normally incident on the conducting plane with magnetic field in the x direction.

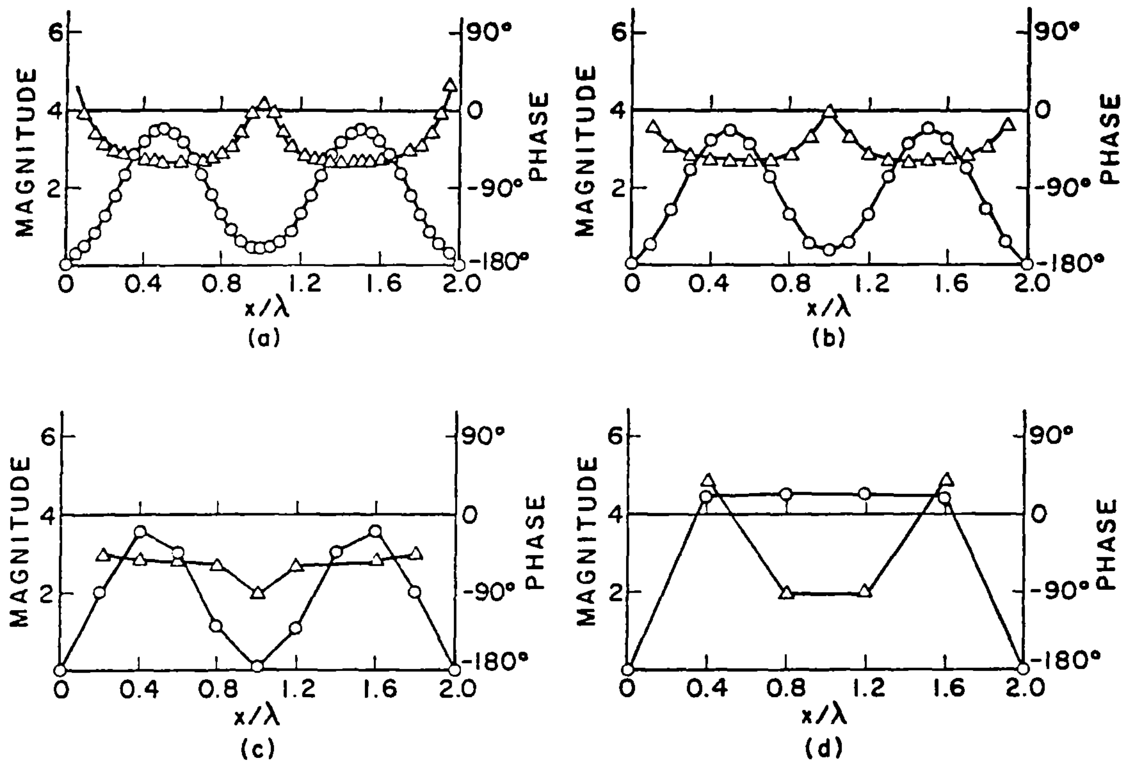

Fig. 5. Magnitude and phase of $|M/E^i|$, where M is the x-directed magnetic current and E^i is the incident electric field, when the number of expansion functions is (a) 39, (b) 19, (c) 9, and (d) 4. Circles denote magnitude, triangles denote phase. Computations are for the same slot as for Fig. 4.

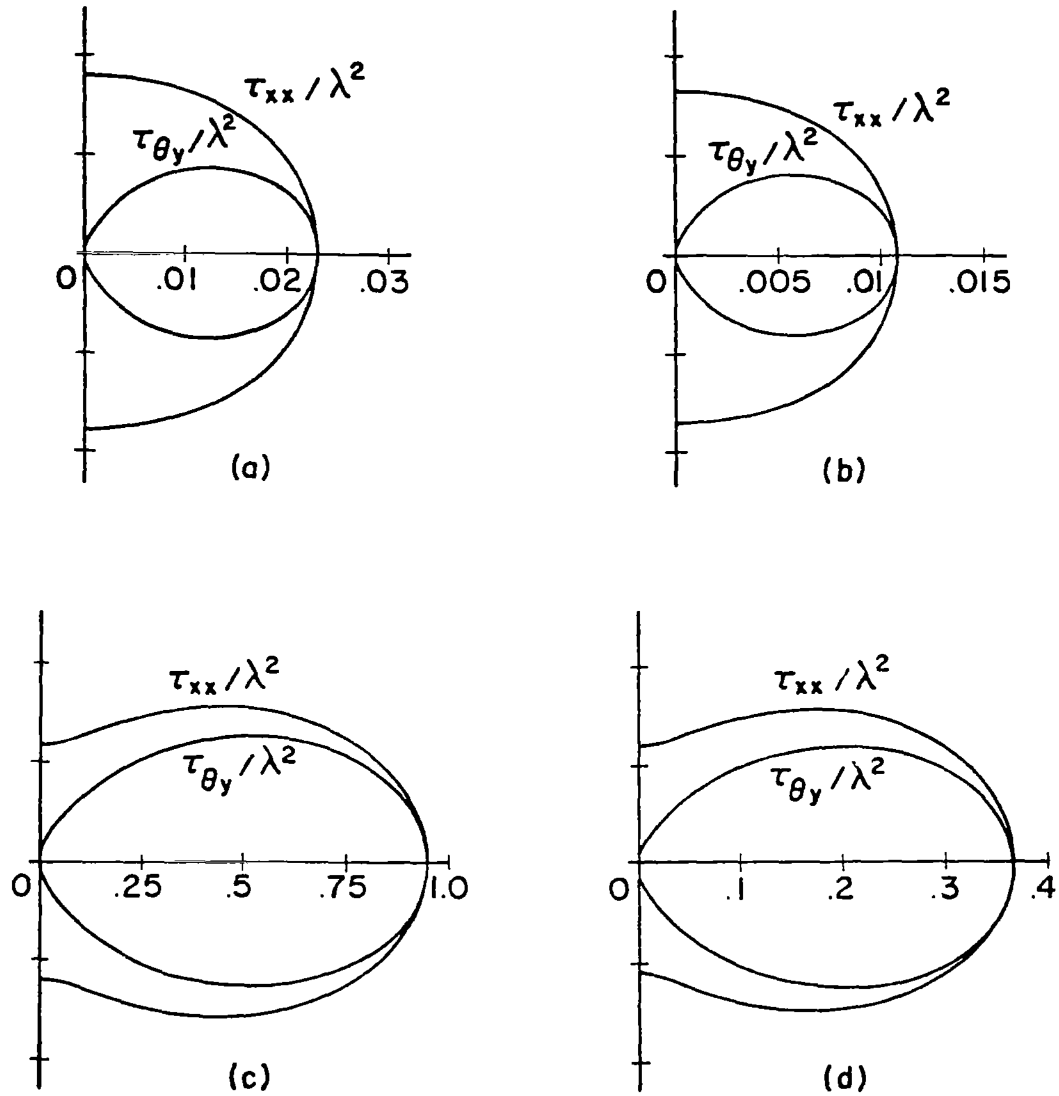

Fig. 6. Transmission cross sections for a square aperture of side length L, excited by a plane wave with H in the xz plane and incident at an angle θ from the normal direction in the H plane. (a) $L = \lambda/4$, $\theta = 0$. (b) $L = \lambda/4$, $\theta = 45°$. (c) $L = \lambda/2$, $\theta = 0$. (d) $L = \lambda/2$, $\theta = 45°$.

functions respectively. Note that the patterns (a) and (b) are essentially the same, and pattern (c) is only slightly different. They differ appreciably from (d), which results from only 4 expansion functions. The difference in the solutions as the number of expansion functions is decreased is better illustrated by plots of $\underset{\sim}{M}$, as shown in Fig. 5. These are for the same cases as the corresponding cases of Fig. 4. It can be seen clearly how the computed equivalent current in the slot region changes as the number of subsections is reduced. As a rule of thumb, for near-field quantities (such as $\underset{\sim}{M}$) one should use subareas of length $\lambda/10$ or less and for far-field quantities (such as τ) length $\lambda/5$ or less.

Finally, Fig. 6 shows some computations for wider apertures and excitations by waves not normally incident on the conducting plane. All cases shown are for square apertures, of side length L. Figures 6(a) and (b) are for $L = \lambda/4$, with the plane wave normally incident for (a) and incident 45° from the normal direction in the $\underset{\sim}{H}$ plane for (b). Figures 6(c) and (d) are for $L = \lambda/2$, with the plane wave normally incident for (c) and 45° from the normal direction in the $\underset{\sim}{H}$ plane for (d). Note that, for the relatively small slots chosen, there is little difference in the shapes of the patterns as the incident wave direction is changed from the normal direction. There is, however, an appreciable difference in the amplitudes of the patterns.

VI. DISCUSSION

The computer program, Part Two, is written explicitly for rectangular apertures, but the formulas are valid for any aperture composed of rectangular subsections. Other apertures, such as L-shaped, T-shaped, square O-shaped, etc., could be treated by appropriately changing the computer program. Apertures of arbitrary shape could be treated by approximating them by rectangular subsections. As with all moment solutions, the size of the apertures which can be treated depends upon the size of the matrix which can be computed and inverted. The examples indicate that the rectangular subsections should have side lengths not greater than 0.2 wavelengths for reasonable accuracy.

The aperture admittance matrix has application to any problem in which one region is bounded by a plane conductor, as shown in reference [1]. Hence, it can be used for waveguide-fed apertures in a ground plane, and for cavity-backed apertures in a ground plane. It is planned to treat these latter two problems in future reports.

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PART TWO

COMPUTER PROGRAMS

I. DESCRIPTION OF THE MAIN PROGRAM

The main program computes the complex coefficients V_n which determine the magnetic current $\underset{\sim}{M}$ according to [1, Eq. (5)], the transmission coefficient T of [1, Eq. (44)], and four patterns of the transmission cross section [1, Eq. (40)] per square wavelength τ/λ^2. The four patterns of τ/λ^2 are written on the first record of direct access data set 6. The main program calls the subroutines LINEQ, YMAT, and PLANE which are listed later on in this report.

One data card is read early in the main program according to

```
      READ (1,11) LX, LY, LI, NTH, DX, DY, TH
   11 FORMAT (4I3, 3E14.7)
```

The L_x and L_y appearing in (10) are read in through LX and LY respectively. Here, LX $\geq$ 2 and LY $\geq$ 1 which means that the long dimension of a rectangular aperture only one subsection wide must lie along the x axis. DX is $\Delta x/\lambda$ and DY is $\Delta y/\lambda$ where Δx and Δy appear in (12) and (13) and λ is the wavelength. The plane wave excitation vector [1, Eq. (32)] is the negative of expression (54 + LI) where LI is either 1,2,3 or 4 and where the angle (either θ or ϕ) in degrees appearing in equation (54 + LI) is TH. The four patterns of the transmission cross section [1, Eq. (40)] per square wavelength τ/λ^2 are generated by evaluating the plane wave measurement vectors (55) to (58) at angles (θ or ϕ) equal to (J-1)*180./(NTH-1) degrees, J = 1,2,...NTH.

Minimum allocations are given by

```
      COMPLEX Y(N*N), P(4*N), V(N)
      DIMENSION TA(4*NTH)
```

where

$$N = (LX-1)*LY + LX*(LY-1) \tag{69}$$

Statement 27 uses LX, LY, DX, and DY to store $\frac{\pi\eta}{j\Delta x\Delta y}$ $[Y^a + Y^b]$ where $[Y^a + Y^b]$ is the admittance matrix appearing in [1, Eq. (14)] by columns in Y. Here, $\eta = \sqrt{\frac{\mu}{\varepsilon}} = 376.730$ ohms is the intrinsic impedance for empty space. At the time statement 27 is executed, LX and LY are still the original input data, namely the numbers of subdivisions in x and y but DX and DY are $k\Delta x$ and $k\Delta y$ instead of the original input data $\Delta x/\lambda$ and $\Delta y/\lambda$.

Statement 28 inverts the N by N matrix stored in Y.

Statement 29 uses TH, LX, LY, DX, and DY to store $\frac{-1}{2\Delta x\Delta y} P_m^i$ in P(m + (K-1)*N), K = 1,2,3,4, m = 1,2,...N, where P_m^i is the plane wave excitation [1, Eq. (32)] and, in particular, the negative of expression (54 + K). At the time statement 29 is executed, TH is the angle θ or ϕ (see (55) to (58) and Fig. 1) in radians which specifies the direction from which the exciting plane wave comes, LX and LY are the numbers of subdivisions in x and y, DX is $k\Delta x$ and DY is $k\Delta y$.

Nested DO loops 16 and 17 multiply the matrix stored in Y by the column vector stored in P(1 + (LI-1)*N) through P(LI*N) and use the constant UV = $j2\pi\eta$ in order to store $\vec{V}$ of [1, Eq. (14)] in V. Substituting [1, Eq. (28)] into [1, Eq. (13)] we obtain

$$[Y^{hs}]\vec{V} = \frac{1}{2}\vec{I}^i \tag{70}$$

which simplifies [1, Eq. (44)] to

$$T = \frac{1}{2\eta S \cos\theta_{inc.}} \mathrm{Re}(\tilde{V}\,\vec{I}^{i*}) \tag{71}$$

DO loop 16 accumulates $\frac{-\tilde{V}\,\vec{I}^{i*}}{2\Delta x\Delta y}$ in U2. Statement 31 stores the transmission coefficient T of (69) in T.

[1, Eq. (40)] simplifies to

$$\tau/\lambda^2 = \frac{k^4}{32\pi^3\eta^2}\,|\tilde{P}^m\vec{V}|^2 \tag{72}$$

DO loop 19 stores the transmission cross section per square wavelength τ/λ^2 of (72) in TAU(K). Statement 30 uses TH = (J-1)*π/(NTH-1) radians, LX, LY, DX, and DY to store $\frac{1}{2\Delta x\Delta y} P_n^m$ in P(n + (K-1)*N). For the $\frac{1}{2\Delta x\Delta y} \vec{P}^m$ stored in P(1+(K-1)*N) through P(K*N), DO loop 21 accumulates $\frac{\tilde{P}^m \vec{V}}{2\Delta x\Delta y}$ in U1. Next, τ/λ^2 of (72) is stored in both TAU(K) and TA(J+(K-1)*NTH).

Statement 32 writes TA on the first record of data set 6 for possible input to the plot program listed later on in this report.

```
C      LISTING OF THE MAIN PROGRAM AND SAMPLE DATA
C
// EXEC WATFIV
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=3330,                         X
//               DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592)
//GO.SYSIN DD *
$JOB             MAUTZ,TIME=1,PAGES=40
C
C      MAIN PROGRAM
C      THIS PROGRAM CALLS THE SUBROUTINES LINEQ,YMAT,PLANE
       COMPLEX U,UV,Y(2500),P(200),U1,U2,V(50),CONJG
       DIMENSION TAU(4),TA(1168)
       PI=3.141593
       ETA=376.730
       U=(0.,1.)
       UV=2.*PI*ETA*U
       READ(1,11) LX,LY,LI,NTH,DX,DY,TH
    11 FORMAT(4I3,3E14.7)
       WRITE(3,12) LX,LY,LI,NTH,DX,DY,TH
    12 FORMAT('  LX LY LI NTH',5X,'DX',12X,'DY',12X,'TH'/1X,4I3,3E14.7)
       BK=2.*PI
       DX=DX*BK
       DY=DY*BK
       P8=180./PI
       TH=TH/P8
    27 CALL YMAT(LX,LY,DX,DY,Y)
       WRITE(3,13)(Y(I),I=1,3)
    13 FORMAT(' Y'/(1X,6E11.4))
       N=(LX-1)*LY+LX*(LY-1)
    28 CALL LINEQ(N,Y)
       WRITE(3,13)(Y(I),I=1,3)
    29 CALL PLANE(TH,LX,LY,DX,DY,P)
       WRITE(3,14)(P(I),I=1,3)
    14 FORMAT(' P'/1X,6E11.4)
       IA=1+(LI-1)*N
       IB=IA+N-1
       U2=0.
       DO 16 J=1,N
       U1=0.
       J1=J
       DO 17 I=IA,IB
       U1=U1+Y(J1)*P(I)
       J1=J1+N
    17 CONTINUE
       V(J)=U1*UV
       J1=IA+J-1
       U2=U2+V(J)*CONJG(P(J1))
    16 CONTINUE
       WRITE(3,24)(V(I),I=1,N)
    24 FORMAT(' COEFFICIENTS V OF MAGNETIC CURRENT EXPANSION FUNCTIONS'
      1/(1X,6E11.4))
    31 T=REAL(U2)/(LX*LY*ETA*SIN(TH))
       WRITE(3,18) T
    18 FORMAT(' TRANSMISSION COEFFICIENT T=',E14.7)
       CT=DX*DY/(PI*ETA)
       CT=CT*CT/(8.*PI)
       DTH=PI/(NTH-1)
       WRITE(3,23)
    23 FORMAT('0 ANGLE',4X,'TAU1',7X,'TAU2',7X,'TAU3',7X,'TAU4')
       DO 19 J=1,NTH
```

```
      TH=(J-1)*DTH
   30 CALL PLANE(TH,LX,LY,DX,DY,P)
      TH=TH*P8
      J1=0
      J2=J
      DO 20 K=1,4
      U1=0.
      DO 21 I=1,N
      J1=J1+1
      U1=U1+P(J1)*V(I)
   21 CONTINUE
      H=U1*CONJG(U1)
      TAU(K)=CT*H
      TA(J2)=TAU(K)
      J2=J2+NTH
   20 CONTINUE
      WRITE(3,22) TH,(TAU(I),I=1,4)
   22 FORMAT(1X,F7.2,4E11.4)
   19 CONTINUE
      KA=J2-NTH
      REWIND 6
   32 WRITE(6)(TA(J),J=1,KA)
      STOP
      END
$DATA
  5  1   1 19 0.5000000E-01 0.5000000E-01 0.2700000E+03
$STOP
/*
//
```

```
PRINTED OUTPUT
 LX LY LI NTH       DX                DY                TH
  5  1   1 19 0.5000000E-01 0.5000000E-01 0.2700000E+03
Y
-0.1531E+02-0.6525E-01 0.6646E+01-0.6463E-01 0.1312E+01-0.6275E-01
Y
-0.9999E-01 0.4015E-02-0.6822E-01 0.5537E-02-0.5131E-01 0.5523E-02
P
-0.1000E+01-0.3034E-06-0.1000E+01-0.6067E-06-0.1000E+01-0.9101E-06
COEFFICIENTS V OF MAGNETIC CURRENT EXPANSION FUNCTIONS
 0.4511E+02 0.5916E+03 0.6238E+02 0.8153E+03 0.6238E+02 0.8153E+03
 0.4511E+02 0.5916E+03
TRANSMISSION COEFFICIENT T= 0.1141254E+00

 ANGLE     TAU1        TAU2        TAU3        TAU4
   0.00 0.0000E+00 0.0000E+00 0.0000E+00 0.2186E-02
  10.00 0.5885E-04 0.0000E+00 0.0000E+00 0.2186E-02
  20.00 0.2308E-03 0.0000E+00 0.0000E+00 0.2188E-02
  30.00 0.5016E-03 0.0000E+00 0.0000E+00 0.2190E-02
  40.00 0.8462E-03 0.0000E+00 0.0000E+00 0.2193E-02
  50.00 0.1228E-02 0.0000E+00 0.0000E+00 0.2196E-02
  60.00 0.1602E-02 0.0000E+00 0.0000E+00 0.2199E-02
  70.00 0.1918E-02 0.0000E+00 0.0000E+00 0.2202E-02
  80.00 0.2129E-02 0.0000E+00 0.0000E+00 0.2203E-02
  90.00 0.2204E-02 0.0000E+00 0.0000E+00 0.2204E-02
 100.00 0.2129E-02 0.0000E+00 0.0000E+00 0.2203E-02
 110.00 0.1918E-02 0.0000E+00 0.0000E+00 0.2202E-02
 120.00 0.1602E-02 0.0000E+00 0.0000E+00 0.2199E-02
 130.00 0.1228E-02 0.0000E+00 0.0000E+00 0.2196E-02
 140.00 0.8462E-03 0.0000E+00 0.0000E+00 0.2193E-02
```

150.00	0.5016E-03	0.0000E+00	0.0000E+00	0.2190E-02
160.00	0.2308E-03	0.0000E+00	0.0000E+00	0.2188E-02
170.00	0.5885E-04	0.0000E+00	0.0000E+00	0.2186E-02
180.00	0.7663E-15	0.0000E+00	0.0000E+00	0.2186E-02

```
C     LISTING OF THE SUBROUTINE LINEQ
C
      SUBROUTINE LINEQ(LL,C)
      COMPLEX C(2500),STOR,STO,ST,S
      DIMENSION LR(50)
      DO 20 I=1,LL
      LR(I)=I
   20 CONTINUE
      M1=0
      DO 18 M=1,LL
      K=M
      K2=M1+K
      S1=ABS(REAL(C(K2)))+ABS(AIMAG(C(K2)))
      DO 2 I=M,LL
      K1=M1+I
      S2=ABS(REAL(C(K1)))+ABS(AIMAG(C(K1)))
      IF(S2-S1) 2,2,6
    6 K=I
      S1=S2
    2 CONTINUE
      LS=LR(M)
      LR(M)=LR(K)
      LR(K)=LS
      K2=M1+K
      STOR=C(K2)
      J1=0
      DO 7 J=1,LL
      K1=J1+K
      K2=J1+M
      STO=C(K1)
      C(K1)=C(K2)
      C(K2)=STO/STOR
      J1=J1+LL
    7 CONTINUE
      K1=M1+M
      C(K1)=1./STOR
      DO 11 I=1,LL
      IF(I-M) 12,11,12
   12 K1=M1+I
      ST=C(K1)
      C(K1)=0.
      J1=0
      DO 10 J=1,LL
      K1=J1+I
      K2=J1+M
      C(K1)=C(K1)-C(K2)*ST
      J1=J1+LL
```

```
10 CONTINUE
11 CONTINUE
   MI=MI+LL
18 CONTINUE
   JI=0
   DO 9 J=1,LL
   IF(J-LR(J)) 14,8,14
14 LRJ=LR(J)
   J2=(LRJ-1)*LL
21 DO 13 I=1,LL
   K2=J2+I
   K1=J1+I
   S=C(K2)
   C(K2)=C(K1)
   C(K1)=S
13 CONTINUE
   LR(J)=LR(LRJ)
   LR(LRJ)=LRJ
   IF(J-LR(J)) 14,8,14
 8 J1=J1+LL
 9 CONTINUE
   RETURN
   END
```

II. DESCRIPTION OF THE SUBROUTINE YMAT

The subroutine YMAT(LX, LY, DX, DY, Y) uses the input variables LX, LY, DX, and DY to calculate and to store $\frac{\pi\eta[Y]}{j\Delta x\Delta y}$ by columns in Y where [Y] is the admittance matrix (1) dealt with in Part One. With regard to the input variables, LX and LY are the numbers L_x and L_y (see (10)) of subdivisions in x and y, DX is $k\Delta x$ and DY is $k\Delta y$ where Δx and Δy (see (12) and (13)) are the lengths of the subdivisions in x and y. We require that LX $\geq$ 2, and LY $\geq$ 1.

Minimum allocations are given by

```
COMPLEX TC(J1), TX(J1), TY(J1), YXX(J2), Y(N*N)
```

where

```
J1 = (LX + 1) * (LY + 1)

J2 = MAX ((LX-1) * LY, LX * (LY-1))

N  = (LX -1) * LY + LX * (LY-1)
```

Here, MAX denotes the maximum value.

Nested DO loops 15 and 16 put $I_c(s,t)$ of (42) in TC(JST), $I_x(s,t)$ in TX(JST), and $I_y(s,t)$ in TY(JST) where

$$s = JS - 1$$

$$t = JT - 1$$

$$JST = s + 2 + (t+1)(L_x + 1)$$

As mentioned in Part One, the expressions for $I_x(s,t)$ and $I_y(s,t)$ are similar to (42). The variables x and y of integration in (43) to (54) are changed to kx and ky with the result that x and y is replaced by kx and ky everywhere on the right hand sides of (43) to (54) and the dangling factors of k, k^2, k^3, and k^4 in (42) disappear. The logic inside DO loop 16 is best understood by building up a table of variables in YMAT versus expressions in terms of variables appearing in Part One, Section III.

Variables in YMAT	Expressions in Part One, Section III
YL	$(t - .5)k\Delta y$
YU	$(t + .5)k\Delta y$
XL	$(s - .5)k\Delta x$
XU	$(s + .5)k\Delta x$
R1	kr_o
U1	$1 + jkr_o - \frac{k^2r_o^2}{2} - j\,\frac{k^3r_o^3}{6}$
U2	$-j + kr_o + j\,\frac{k^2r_o^2}{2}$
U3	$-\frac{1}{2} - j\,\frac{kr_o}{2}$
EX	e^{-jkr_o}
S1	$kx \log(y + r) + ky \log(x+r)$ evaluated at x,y limits
S3	$k^3x^3 \log(y+r) + k^3y^3 \log(x+r)$ evaluated at x,y limits
S5	$\frac{k^3xyr}{3} + \frac{k^3x^3}{6}\log(y+r) + \frac{k^3y^3}{6}\log(x+r)$ evaluated at x,y limits
TC(JST)	$I_c(s,t)$
S5	$k^4(\frac{yr^3}{12} + \frac{x^2yr}{8} + \frac{x^4}{8}\log(y+r))$ evaluated at x,y limits
S6	$k^5x^2y\,(\frac{x^2}{4} + \frac{y^2}{6})$ evaluated at x,y limits
TX(JST)	$I_x(s,t)$
S5	$k^4(\frac{xr^3}{12} + \frac{y^2xr}{8} + \frac{y^4}{8}\log(x+r))$ evaluated at x,y limits
S6	$k^5y^2x\,(\frac{y^2}{4} + \frac{x^2}{6})$ evaluated at x,y limits
TY(JST)	$I_y(s,t)$

In the preceding table, the first, second, and third S5 and the first and second S6 correspond respectively to the first, second, and third uses of S5 and the first and second uses of S6 in DO loop 16.

If $L_y = 1$, the logic between the statements 44 and 18 uses the fact that (27) is even in s to store $I_c(-1,0)$ in $TC(L_x+2)$. Similarly, $I_x(-1,0)$ is stored in $TX(L_x+2)$ and $I_y(-1,0)$ in $TY(L_x+2)$. If $L_y \neq 1$, the logic between statements 44 and 18 stores $I_c(s,-1)$ in TC(s+2), $s = -1,0,1,\ldots L_x-1$ and $I_c(-1,t)$ in $TC(1 + (t+1)(L_x + 1))$, $t = 0,1,\ldots L_y-1$ and similarly for I_x and I_y.

Nested DO loops 19 and 20 store $\frac{\pi\eta}{j\Delta x\Delta y} Y_{ij}^{xx}$ where Y_{ij}^{xx} is given by (23) with

$$(s - p) = JS - 2$$

$$(t - q) = JT - 2$$

in $YXX(s-p+1 + (t-q)(L_x-1))$. See (34) for bounds on (s-p) and (t-q).

Nested DO loops 24, 23, 22, and 21 store $\frac{\pi\eta}{j\Delta x\Delta y} Y_{ij}^{xx}$ where i and j are given by (30) and (32) where

$$p = JP$$
$$q = JQ$$
$$s = JS$$
$$t = JT$$

in Y(i + (j-1)*N) where N is given by (69). If

$$s - p \geq 0$$
$$t - q \geq 0$$

the subscript for YXX inside nested DO loops 24, 23, 22, and 21 is

$$s - p + 1 + (t-q)(L_x - 1)$$

The more general subscript

$$|s - p| + 1 + |t-q|(L_x - 1)$$

is a consequence of the fact that Y_{ij}^{xx} of (23) is even in both (s-p) and (t-q).

Nested DO loops 25 and 26 store $\frac{\pi\eta}{j\Delta x\Delta y} Y_{ij}^{yx}$ where Y_{ij}^{yx} is given by (24) with

$$s - p = JS - 2$$

$$t - q = JT - 2$$

in YXX$(s\text{-}p + 1 + (t\text{-}q\text{-}1)(L_x - 1))$. See (35) for bounds on (s-p) and (t-q).

Nested DO loops 30, 29, 28, and 27 store $\frac{\pi\eta}{j\Delta x\Delta y} Y_{ij}^{yx}$ where i and j are given by (31) and (32) where

$$p = JP$$

$$q = JQ$$

$$s = JS$$

$$t = JT$$

in $Y((L_x - 1)\ L_y + i + (j-1)*N)$. If

$$s - p \geq 0$$

$$t - q \geq 1$$

the subscript for YXX inside nested DO loops 30, 29, 28, and 27 is

$$s - p + 1 + (t-q-1)(L_x - 1)$$

The more general subscript

$$\left|s - p + \frac{1}{2}\right| + \frac{1}{2} + \left(\left|t - q - \frac{1}{2}\right| - \frac{1}{2}\right)(L_x - 1)$$

for YXX is a consequence of the fact that Y_{ij}^{yx} of (24) is odd about $s\text{-}p = -\frac{1}{2}$ and odd about $t\text{-}q = \frac{1}{2}$.

Nested DO loops 31 and 32 store $\frac{\pi\eta}{j\Delta x\Delta y} Y_{ij}^{xy}$ where Y_{ij}^{xy} is given by (25) with

$$s - p = JS - 2$$

$$t - q = JT - 2$$

in YXX $(s\text{-}p + (t\text{-}q)(L_x - 1))$. See (36) for bounds on (s-p) and (t-q).

Nested DO loops 36, 35, 34, and 33 store $\frac{\pi\eta}{j\Delta x\Delta y} Y_{ij}^{xy}$ where i and j are given by (30) and (33) where

$$p = JP$$

$$q = JQ$$

$$s = JS$$

$$t = JT$$

in $Y(N*(L_x - 1) * L_y + i + (j-1)*N)$. If

$$s - p \geq 1$$

$$t - q \geq 0$$

the subscript for YXX in nested DO loops 36, 35, 34, and 33 is

$$s - p + (t-q)\ (L_x - 1).$$

The more general subscript

$$|s - p - \tfrac{1}{2}| + \tfrac{1}{2} + (|t - q + \tfrac{1}{2}| - \tfrac{1}{2})(L_x - 1)$$

for YXX is a consequence of the fact that Y_{ij}^{xy} of (25) is odd about $s-p = \frac{1}{2}$ and odd about $t-q = -\frac{1}{2}$.

Nested DO loops 37 and 38 store $\frac{\pi\eta}{j\Delta x\Delta y} Y_{ij}^{yy}$ where Y_{ij}^{yy} is given by (26) with

$$s - p = JS - 2$$

$$t - q = JT - 2$$

in YXX $(s-p + 1 + (t-q)\ L_x)$. See (37) for bounds on (s-p) and (t-q).

Nested DO loops 42, 41, 40, and 39 store $\frac{\pi\eta}{j\Delta x\Delta y} Y_{ij}^{yy}$ where i and j are given by (31) and (33) where

$$p = JP$$

$$q = JQ$$

$$s = JS$$

$$t = JT$$

in $Y((N+1) * (L_x - 1) * L_y + i + (j-1) * N)$. If

$$s - p \geq 0$$

$$t - q \geq 0$$

the subscript for YXX in nested DO loops 42, 41, 40, and 39 is

$$s - p + 1 + (t-q)L_x$$

The more general subscript

$$|s - p| + 1 + |t - q|L_x$$

for YXX is a consequence of the fact that Y_{ij}^{yy} of (26) is even in both (s-p) and (t-q).

```
C     LISTING OF THE SUBROUTINE YMAT
C
      SUBROUTINE YMAT(LX,LY,DX,DY,Y)
      COMPLEX U,U1,U2,U3,U4,EX,TC(100),TX(100),TY(100),YXX(100),Y(2500)
      DX2=1./(DX*DX)
      DY2=1./(DY*DY)
      DXDY=DX*DY
      NX=(LX-1)*LY
      NY=(LY-1)*LX
      N=NX+NY
      LXP=LX+1
      LYP=LY+1
      LXM=LX-1
      LYM=LY-1
      U=(0.,1.)
      U4=.1666667*U
      JST=LX+1
      DO 15 JT=1,LY
      JST=JST+1
      YL=(JT-1.5)*DY
      YU=YL+DY
      YL2=YL*YL
      YU2=YU*YU
      Y1=(JT-1)*DY
      Y2=Y1*Y1
      DO 16 JS=1,LX
      XL=(JS-1.5)*DX
      XU=XL+DX
      XL2=XL*XL
      XU2=XU*XU
      X1=(JS-1)*DX
      X2=X1*X1
      R2=X2+Y2
      R1=SQRT(R2)
      RU1=1.-.5*R2
      U1=RU1+R1*(1.-.1666667*R2)*U
      U2=R1-RU1*U
      U3=-.5-.5*R1*U
      EX=COS(R1)-U*SIN(R1)
      JST=JST+1
      R5=XL2+YL2
      R6=XU2+YL2
      R7=XL2+YU2
      R8=XU2+YU2
      R1=SQRT(R5)
      R2=SQRT(R6)
      R3=SQRT(R7)
      R4=SQRT(R8)
      AYL=YL*ALOG((XU+R2)/(XL+R1))
      AYU=YU*ALOG((XU+R4)/(XL+R3))
      AXL=XL*ALOG((YU+R3)/(YL+R1))
      AXU=XU*ALOG((YU+R4)/(YL+R2))
      S1=AXU-AXL+AYU-AYL
      AYL=YL*AYL
      AYU=YU*AYU
      AXL=XL*AXL
      AXU=XU*AXU
      S3=XU*AXU-XL*AXL+YU*AYU-YL*AYL
      XY1=XL*YL
      XY2=XU*YL
```

```
      XY3=XL*YU
      XY4=XU*YU
      S5=.3333333*(XY4*R4-XY3*R3-XY2*R2+XY1*R1)+.1666667*S3
      TC(JST)=(S1*U1+DXDY*U2+S5*U3+.3333333*(XY4*R8-XY3*R7-XY2*R6+XY1*R5
     1)*U4)*EX
      YR1=YL*R1
      YR2=YL*R2
      YR3=YU*R3
      YR4=YU*R4
      S5=.8333333E-1*(YR4*R8-YR3*R7-YR2*R6+YR1*R5)+.125*(XU2*(YR4-YR2)-X
     1L2*(YR3-YR1)+XU2*AXU-XL2*AXL)
      S6=.25*DY*(XU2*XU2-XL2*XL2)+.3333333*X1*DX*(YU2*YU-YL2*YL)
      TX(JST)=.5*(YR4-YR3-YR2+YR1+AXU-AXL)*U1+X1*DXDY*U2+S5*U3+S6*U4
      TX(JST)=TX(JST)*EX/DX
      XR1=XL*R1
      XR2=XU*R2
      XR3=XL*R3
      XR4=XU*R4
      S5=.8333333E-1*(XR4*R8-XR3*R7-XR2*R6+XR1*R5)+.125*(YU2*(XR4-XR3)-Y
     1L2*(XR2-XR1)+YU2*AYU-YL2*AYL)
      S6=.25*DX*(YU2*YU2-YL2*YL2)+.3333333*Y1*DY*(XU2*XU-XL2*XL)
      TY(JST)=.5*(XR4-XR3-XR2+XR1+AYU-AYL)*U1+Y1*DXDY*U2+S5*U3+S6*U4
      TY(JST)=TY(JST)*EX/DY
   16 CONTINUE
   15 CONTINUE
      IF(LYM) 44,44,45
   44 J1=LXP+1
      J2=J1+2
      TC(J1)=TC(J2)
      TX(J1)=-TX(J2)
      TY(J1)=TY(J2)
      GO TO 46
   45 J1=2*LXP+1
      DO 17 JS=2,LXP
      J1=J1+1
      TC(JS)=TC(J1)
      TX(JS)=TX(J1)
      TY(JS)=-TY(J1)
   17 CONTINUE
      J1=1
      DO 18 JT=1,LYP
      J2=J1+2
      TC(J1)=TC(J2)
      TX(J1)=-TX(J2)
      TY(J1)=TY(J2)
      J1=J1+LXP
   18 CONTINUE
   46 J4=LX+2
      JY=0
      DO 19 JT=2,LYP
      DO 20 JS=2,LX
      J3=J4
      J4=J4+1
      J5=J4+1
      JY=JY+1
      YXX(JY)=.5*(TC(J4)+(JS-.5)*TC(J5)-(JS-3.5)*TC(J3)-TX(J5)+TX(J3))+D
     1X2*(TC(J5)-2.*TC(J4)+TC(J3))
   20 CONTINUE
      J4=J4+2
   19 CONTINUE
```

```
      JY=0
      DO 24 JT=1,LY
      DO 23 JS=1,LXM
      DO 22 JQ=1,LY
      JTQ=LXM*IABS(JT-JQ)+1
      DO 21 JP=1,LXM
      J1=JTQ+IABS(JS-JP)
      JY=JY+1
      Y(JY)=YXX(J1)
   21 CONTINUE
   22 CONTINUE
      JY=JY+NY
   23 CONTINUE
   24 CONTINUE
      IF(LYM.EQ.0) RETURN
      J4=2*LXP+1
      JY=0
      DO 25 JT=3,LYP
      DO 26 JS=2,LX
      JY=JY+1
      J4=J4+1
      J3=J4-LXP
      YXX(JY)=(-TC(J4)+TC(J3)+TC(J4+1)-TC(J3+1))/DXDY
   26 CONTINUE
      J4=J4+2
   25 CONTINUE
      JY=NX
      DO 30 JT=1,LY
      DO 29 JS=1,LXM
      DO 28 JQ=1,LYM
      JTQ=2*(JT-JQ)-1
      J2=LXM*(IABS(JTQ)-1)/2
      DO 27 JP=1,LX
      JSP=2*(JS-JP)+1
      J1=J2+(IABS(JSP)+1)/2
      JY=JY+1
      Y(JY)=YXX(J1)
      IF(JTQ*JSP.LT.0) Y(JY)=-Y(JY)
   27 CONTINUE
   28 CONTINUE
      JY=JY+NX
   29 CONTINUE
   30 CONTINUE
      JY=0
      J4=LXP+2
      DO 31 JT=2,LY
      DO 32 JS=3,LXP
      J3=J4
      J4=J4+1
      J5=J4+LXP
      JY=JY+1
      YXX(JY)=(-TC(J4)+TC(J3)+TC(J5)-TC(J5-1))/DXDY
   32 CONTINUE
      J4=J4+2
   31 CONTINUE
      JY=N*NX
      DO 36 JT=1,LYM
      DO 35 JS=1,LX
      DO 34 JQ=1,LY
      JTQ=2*(JT-JQ)+1
```

```
      J2=LXM*(IABS(JTQ)-1)/2
      DO 33 JP=1,LXM
      JY=JY+1
      JSP=2*(JS-JP)-1
      J1=J2+(IABS(JSP)+1)/2
      Y(JY)=YXX(J1)
      IF(JTQ*JSP.LT.0) Y(JY)=-Y(JY)
   33 CONTINUE
   34 CONTINUE
      JY=JY+NY
   35 CONTINUE
   36 CONTINUE
      JY=0
      J4=LX+2
      DO 37 JT=2,LY
      DO 38 JS=2,LXP
      JY=JY+1
      J4=J4+1
      J5=J4+LXP
      J3=J4-LXP
      YXX(JY)=.5*(TC(J4)+(JT-.5)*TC(J5)-(JT-3.5)*TC(J3)-TY(J5)+TY(J3))+D
     1Y2*(TC(J5)-2.*TC(J4)+TC(J3))
   38 CONTINUE
      J4=J4+1
   37 CONTINUE
      JY=(N+1)*NX
      DO 42 JT=1,LYM
      DO 41 JS=1,LX
      DO 40 JQ=1,LYM
      JTQ=LX*IABS(JT-JQ)+1
      DO 39 JP=1,LX
      J1=JTQ+IABS(JS-JP)
      JY=JY+1
      Y(JY)=YXX(J1)
   39 CONTINUE
   40 CONTINUE
      JY=JY+NX
   41 CONTINUE
   42 CONTINUE
      RETURN
      END
```

III. DESCRIPTION OF THE SUBROUTINE PLANE

The subroutine PLANE(TH, LX, LY, DX, DY, P) uses input variables TH, LX, LY, DX, and DY to store $\frac{1}{2\Delta x\Delta y}$ times the plane wave measurements (59) to (66) in $P(i)$, $P(N_x + i)$, $P(N + i)$, $P(N + N_x + i)$, $P(2N + i)$, $P(2N + N_x + i)$, $P(3N + i)$, and $P(3N + N_x + i)$ respectively where

$$i = p + (q-1)(L_x - 1) \qquad \text{in (59) and (65)}$$

$$i = p + (q-1)\,L_x \qquad \text{in (62) and (64)}$$

$$N_x = (L_x - 1)\,L_y$$

$$N = N_x + L_x(L_y - 1)$$

Both angles θ and ϕ appearing in (59) to (66) are equal to TH radians. The arguments LX and LY of PLANE are the numbers L_x and L_y of subdivisions in the x and y directions and DX and DY are the electrical lengths $k\Delta x$ and $k\Delta y$ of the x and y subdivisions. We require that LX $\geq$ 2, and LY $\geq$ 1.

Nested DO loops 81 and 87 store $\frac{1}{2\Delta x\Delta y}$ times (59) with

$$p = JP$$

$$q = JQ$$

in $P(p + (q-1)(L_x - 1))$. Nested DO loops 82 and 88 store $\frac{1}{2\Delta x\Delta y}$ times (62) with

$$p = JP$$

$$q = JQ$$

in $P(N + N_x + p + (q-1)L_x)$. Nested DO loops 83 and 84 store $\frac{1}{2\Delta x\Delta y}$ times (64) with

$$p = JP$$

$$q = JQ$$

in $P(2N + N_x + p + (q-1)L_x)$. Nested DO loops 85 and 86 store $\frac{1}{2\Delta x\Delta y}$ times (65) with

$$p = JP$$

$$q = JQ$$

in $P(3N + p + (q-1)(L_x - 1))$.

```
C     LISTING OF THE SUBROUTINE PLANE
C
      SUBROUTINE PLANE(TH,LX,LY,DX,DY,P)
      COMPLEX U,U1,P(200)
      U=(0.,1.)
      LXM=LX-1
      LYM=LY-1
      NX=LXM*LY
      N=NX+LYM*LX
      N4=N*4
      DO 89 J=1,N4
      P(J)=0.
   89 CONTINUE
      SN=SIN(TH)
      CS=COS(TH)
      X2=DX*CS
      X3=.5*X2
      S1=-SIN(X3)/X3
      S2=S1*S1*SN
      DO 81 JP=1,LXM
      S5=JP*X2
      U1=S2*(COS(S5)+U*SIN(S5))
      J1=JP
      DO 87 JQ=1,LY
      P(J1)=U1
      J1=J1+LXM
   87 CONTINUE
   81 CONTINUE
      IF(LYM.EQ.0) GO TO 90
      DO 82 JP=1,LX
      S5=(JP-.5)*X2
      U1=S1*(COS(S5)+U*SIN(S5))
      J1=N+NX+JP
      DO 88 JQ=1,LYM
      P(J1)=U1
      J1=J1+LX
   88 CONTINUE
   82 CONTINUE
   90 Y2=DY*CS
      Y3=.5*Y2
      S1=-SIN(Y3)/Y3
      S2=S1*S1*SN
      J1=2*N+NX
      IF(LYM.EQ.0) GO TO 91
      DO 83 JQ=1,LYM
      S5=JQ*Y2
      U1=S2*(COS(S5)+U*SIN(S5))
      DO 84 JP=1,LX
      J1=J1+1
      P(J1)=U1
   84 CONTINUE
   83 CONTINUE
   91 DO 85 JQ=1,LY
      S5=(JQ-.5)*Y2
      U1=S1*(COS(S5)+U*SIN(S5))
      DO 86 JP=1,LXM
      J1=J1+1
      P(J1)=U1
   86 CONTINUE
   85 CONTINUE
      RETURN
      END
```

IV. DESCRIPTION OF THE PROGRAM TO PLOT PATTERNS

This program plots patterns of the transmission cross section per square wavelength τ/λ^2 read from direct access data set 6.

Punched card data is read according to

```
      READ(1,22) NTH, NP
22    FORMAT(20I3)
      READ(1,22)(LP(K), K = 1, NP)
      READ(1,10)(SCL(K), K = 1, NP)
10    FORMAT(6E11.4)
```

The patterns of τ/λ^2 are read from direct access data set 6 according to

```
      REWIND 6
      J1 = NTH*NP
      READ(6)(TA(I), I = 1, J1)
```

TA(J + (K-1)*NTH) is the value of τ/λ^2 at angle (J-1)*π/(NTH-1) radians on the Kth pattern. Here, J = 1,2,...NTH and K = 1,2...NP. If LP(K) = 0, the Kth pattern is not plotted. If LP(K) ≠ 0, the Kth pattern is multiplied by SCL(K) and then plotted in inches.

Minimum allocations are given by

```
      DIMENSION LP(NP), SCL(NP), SN(NTH), CS(NTH),
         TA(NP*NTH), X(NTH), Y(NTH)
```

DO loop 15 plots the Kth pattern if LP(K) ≠ 0. DO loop 16 puts tick marks on the vertical axis drawn by statement 25. DO loop 18 puts tick marks on the horizontal axis drawn by statement 26. Statement 27 plots the pattern whose horizontal and vertical coordinates have been stored in X and Y by DO loop 20.

```
C     LISTING OF THE PROGRAM TO PLOT PATTERNS
C
// EXEC FORTGCLG
//FORT.SYSIN DD *
      DIMENSION LP(25),SCL(25),XX(4),YY(4),SN(73),CS(73),TA(1168)
      DIMENSION X(73),Y(73)
      READ(1,22) NTH,NP
   22 FORMAT(20I3)
      WRITE(3,11) NTH,NP
   11 FORMAT(' NTH NP'/1X,2I3)
      READ(1,22)(LP(K),K=1,NP)
      WRITE(3,23)(LP(K),K=1,NP)
   23 FORMAT(' LP'/(1X,20I3))
      READ(1,10)(SCL(K),K=1,NP)
   10 FORMAT(6E11.4)
      WRITE(3,24)(SCL(K),K=1,NP)
   24 FORMAT(' SCL'/(1X,6E11.4))
      XX(1)=1.
      YY(1)=1.
      XX(2)=1.
      YY(2)=9.
      XX(3)=1.
      YY(3)=5.
      XX(4)=5.
      YY(4)=5.
      PI=3.141593
      DTH=PI/(NTH-1)
      DO 19 J=1,NTH
      ANG=(J-1)*DTH
      SN(J)=SIN(ANG)
      CS(J)=COS(ANG)
   19 CONTINUE
      CALL PLOTID
      REWIND 6
   12 J1=NTH*NP
      READ(6)(TA(I),I=1,J1)
      WRITE(3,14) TA(1)
   14 FORMAT(' TA=',E11.4)
      J1=0
      DO 15 K=1,NP
      IF(LP(K).EQ.0) GO TO 21
   25 CALL LINE(XX(1),YY(1),2,1,0,0)
      S3=9.
      DO 16 J=1,9
   17 CALL SYMBOL(1.,S3,.14,13,90.,-1)
      S3=S3-1.
   16 CONTINUE
   26 CALL LINE(XX(3),YY(3),2,1,0,0)
      S1=5.
      DO 18 J=1,4
      CALL SYMBOL(S1,5.,.14,13,0.,-1)
      S1=S1-1.
   18 CONTINUE
      DO 20 J=1,NTH
      J2=J1+J
      S1=TA(J2)*SCL(K)
      X(J)=1.+S1*SN(J)
      Y(J)=5.+S1*CS(J)
   20 CONTINUE
   27 CALL LINE(X(1),Y(1),NTH,1,0,0)
```

```
      CALL PLOT(6.,0.,-3)
   21 J1=J1+NTH
   15 CONTINUE
      CALL PLOT(5.,0.,-3)
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EF0034.REV1,DISP=OLD,UNIT=3330,
//               DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592)
//GO.SYSIN DD *
 19   1
  1
 0.1000E+04
/*
//
```