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SCATTERING FROM LOADED WIRE OBJECTS NEAR A LOADED SURFACE OF REVOLUTION

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I. BASIC THEORY

The electric current $\underset{\sim}{J}$ induced on loaded surfaces by an incident electric field $\underset{\sim}{E}^i$ satisfies

$$-\underset{\sim}{E}^s_{tan} + Z_L \underset{\sim}{J} = \underset{\sim}{E}^i_{tan} \qquad (1)$$

on the loaded surfaces. In (1), Z_L is the surface impedance and $\underset{\sim}{E}^s$ is the electric field due to $\underset{\sim}{J}$. The subscript tan denotes components tangential to the loaded surface in question. Let L be the operator which relates $-\underset{\sim}{E}^s$ to $\underset{\sim}{J}$. Hence

$$-\underset{\sim}{E}^s = L\underset{\sim}{J} \qquad (2)$$

The operator L can be expressed in terms of a vector potential $\underset{\sim}{A}$ and a scalar potential ϕ as

$$L\underset{\sim}{J} = j\omega\underset{\sim}{A} + \underset{\sim}{\nabla}\phi \qquad (3)$$

where

$$\underset{\sim}{A} = \mu_o \iint_s \underset{\sim}{J} \frac{e^{-jkR}}{4\pi R} ds \qquad (4)$$

$$\phi = \frac{1}{\epsilon_o} \iint_s \sigma \frac{e^{-jkR}}{4\pi R} ds \qquad (5)$$

Here s denotes all the loaded surfaces, R is the distance from a source point to a field point, μ_o and ϵ_o are the permeability and permittivity of free space, k is the propagation constant $\omega\sqrt{\mu_o \epsilon_o}$ for angular frequency ω, and σ is the surface charge associated with $\underset{\sim}{J}$. The current and charge are related by the equation of continuity

$$\nabla \cdot \underset{\sim}{J} = -j\omega\sigma \tag{6}$$

The surface current $\underset{\sim}{J}$ consists of a current $\underset{\sim}{J}^s$ on the surface of revolution and a current $\underset{\sim}{J}^w$ on the surface of the wires.

$$\underset{\sim}{J} = \underset{\sim}{J}^s + \underset{\sim}{J}^w \tag{7}$$

Henceforth, the superscript s will denote the surface of revolution and the superscript w will denote the surface of the wires. Next, $\underset{\sim}{J}^s$ and $\underset{\sim}{J}^w$ are expanded in terms of tangential vector functions $\{\underset{\sim}{J}_i^s\}$ and $\{\underset{\sim}{J}_i^w\}$.

$$\underset{\sim}{J}^s = \sum_{i=1}^{N_s} I_i^s \underset{\sim}{J}_i^s \tag{8}$$

$$\underset{\sim}{J}^w = \sum_{i=1}^{N_w} I_i^w \underset{\sim}{J}_i^w \tag{9}$$

Expressions (2), (7), (8), and (9) are substituted into (1) and the inner products of the resulting equation with $\underset{\sim}{J}_i^{*s}$, i = 1, 2, ... N_s and $\underset{\sim}{J}_i^{*w}$, i = 1, 2, ... N_w are taken, where * denotes the complex conjugate. Thus,

$$\begin{bmatrix} Z^{ss} + Z_L^s & Z^{sw} \\ Z^{ws} & Z^{ww} + Z_L^w \end{bmatrix} \begin{bmatrix} \vec{I}^s \\ \vec{I}^w \end{bmatrix} = \begin{bmatrix} \vec{V}^s \\ \vec{V}^w \end{bmatrix} \tag{10}$$

where

$$(Z^{ab})_{ij} = \langle J_i^{*a}, L J_j^b \rangle \tag{11}$$

$$(Z_L^a)_{ij} = \langle J_i^{*a}, Z_L^a J_j^a \rangle \tag{12}$$

$$(V^a)_i = \langle J_i^{*a}, E^i \rangle \tag{13}$$

in which a and b may be either s or w. The inner product $<J_i^a, E^i>$ is defined by

$$<J_i^a, E^i> = \iint_s \underset{\sim}{J}_i^a \cdot \underset{\sim}{E}^i \, ds \tag{14}$$

On the radiation sphere,

$$\underset{\sim}{E}^s \cdot \underset{\sim}{u} = -\frac{j\omega\mu e^{-jkr}}{4\pi r} \begin{bmatrix} \tilde{R}^s & \tilde{R}^w \end{bmatrix} \begin{bmatrix} I^s \\ I^w \end{bmatrix} \tag{15}$$

where

$$(R^a)_i = <J_i^a, E^u> \tag{16}$$

in which a may be either s or w and $\underset{\sim}{E}^u$ is the electric field coming from a current element $I\ell\underset{\sim}{u}$ at the point of observation of $\underset{\sim}{E}^s$. Here, $\underset{\sim}{u}$ is a unit vector tangential to the radiation sphere. The strength $I\ell$ of the current element is adjusted so that $\underset{\sim}{E}^u$ is unity at an origin in the vicinity of the surface of revolution and wires. In (15), r is the distance from the origin to the point of observation of $\underset{\sim}{E}^s$. The scattering cross section per wavelength squared $\frac{\sigma}{\lambda^2}$ is defined by

$$\frac{\sigma}{\lambda^2} = \frac{4\pi r^2 \, |\underset{\sim}{E}^s \cdot \underset{\sim}{u}|^2}{\lambda^2 \, |\underset{\sim}{E}^i|^2} \tag{17}$$

where $\underset{\sim}{E}^i$ is the incident plane wave electric field in the vicinity of the body of revolution and wires. Assuming that $|E^i|^2 = 1$ and substituting (15) into (17), we obtain

$$\frac{\sigma}{\lambda^2} = \frac{k^4 \eta^2}{16\pi^3} \left| \begin{bmatrix} \tilde{R}^s & \tilde{R}^w \end{bmatrix} \begin{bmatrix} \vec{I}^s \\ \vec{I}^w \end{bmatrix} \right|^2 \tag{18}$$

where $\vec{I}^s$ and $\vec{I}^w$ are obtained by solving the matrix equation (10).

II. WIRE PARAMETERS E^{ss}, Z, $\vec{R}$, AND $\vec{V}$

The purpose of this section is to reduce (18) from an $(N_s + N_w)$ order matrix function of the wire load matrix Z_L^w to an N_w order matrix expression involving Z_L^w and new parameters E^{ss}, Z, $\vec{R}$, and $\vec{V}$. E^{ss} is proportional to the field scattered by the loaded surface of revolution when the wires are absent. Whereas Z^{ww}, $\vec{R}^w$, and $\vec{V}^w$ are respectively the impedance matrix, and receiver and transmitter excitations of the unloaded wires in free space, the new parameters Z, $\vec{R}$ and $\vec{V}$ will be respectively the impedance matrix, and receiver and transmitter excitations of the unloaded wires in the presence of the loaded surface of revolution.

Writing (10) as two separate matrix equations and eliminating $\vec{I}^s$ between them, we obtain

$$\vec{I}^s = [Z^{ss} + Z_L^s]^{-1}[\vec{V}^s - Z^{sw}\,\vec{I}^w] \qquad (19)$$

$$\vec{I}^w = [Z + Z_L^w]^{-1}\,\vec{V} \qquad (20)$$

where

$$Z = Z^{ww} - Z^{ws}\,[Z^{ss} + Z_L^s]^{-1}\,Z^{sw} \qquad (21)$$

$$\vec{V} = \vec{V}^w - Z^{ws}\,[Z^{ss} + Z_L^s]^{-1}\,\vec{V}^s \qquad (22)$$

Substituting (19) and (20) into (18) yields

$$\frac{\sigma}{\lambda^2} = \frac{k^4\eta^2}{16\pi^3}\left| E^{ss} + \tilde{R}\,[Z + Z_L^w]^{-1}\,\vec{V}\right|^2 \qquad (23)$$

where

$$E^{ss} = \tilde{R}^s\,[Z^{ss} + Z_L^s]^{-1}\,\vec{V}^s \qquad (24)$$

$$\tilde{R} = \tilde{R}^w - \tilde{R}^s\,[Z^{ss} + Z_L^s]^{-1}\,Z^{sw} \qquad (25)$$

III. EXPANSION FUNCTIONS $\underset{\sim}{J}_i^s$ AND $\underset{\sim}{J}_i^w$

The expansion functions $\{\underset{\sim}{J}_i^s\}$ are defined by

$$\underset{\sim}{J}_k^s = \underset{\sim}{u}_t \frac{T_i^s(t)}{\rho} e^{jm\phi} \tag{26}$$

$$\underset{\sim}{J}_{k+NM}^s = \underset{\sim}{u}_\phi \frac{T_i^s(t)}{\rho} e^{jm\phi} \tag{27}$$

for $m = -(M-1), -(M-2), \ldots 0, 1, 2, \ldots (M-1)$

$i = 1, 2, \ldots NM$

where

$$k = (M - 1 + m) * 2 * NM + i \tag{28}$$

Here, $\underset{\sim}{u}_t$ is a unit vector in the direction of the generating curve of the surface of revolution and $\underset{\sim}{u}_\phi$ is a tangential vector in the azimuthal direction. Also, ρ is the distance of the generating curve from the axis of the surface of revolution. In (26) and (27), $T_i^s(t)$ is a triangle function defined by

$$T_i^s(t) = \begin{cases} 0 & t < t_{2i-1} \\ \dfrac{t - t_{2i-1}}{\Delta_{2i-1} + \Delta_{2i}} & t_{2i-1} \le t < t_{2i+1} \\ \dfrac{t_{2i+3} - t}{\Delta_{2i+1} + \Delta_{2i+2}} & t_{2i+1} \le t < t_{2i+3} \\ 0 & t_{2i+3} \le t \end{cases} \tag{29}$$

The generating curve of the surface of revolution is defined by the $2*NM + 3 = NP$ points t_i which define $NP - 1$ intervals of lengths $\Delta_1, \Delta_2, \ldots \Delta_{NP-1}$. In (29), t is the arc length along the generating curve.

For $m \geq 0$, the expansion functions (26) and (27) and the testing functions implied by (11) are the same as those in [1]. Hence the computer program in Appendix A on page 44 of [1] may be used to compute Z^{ss} appearing in (19). Because an $e^{jm\phi}$ current produces only an $e^{jm\phi}$ field, Z^{ss} is the block diagonal arrangement of submatrices $(Z^{ss})^{-M+1}$, $(Z^{ss})^{-M+2}$ ... $(Z^{ss})^{o}$, $(Z^{ss})^{1}$, ... $(Z^{ss})^{M-1}$ where $(Z^{ss})^{m}$ is the submatrix of Z^{ss} obtained when both J_i^a and J_j^b appearing in (11) are proportional to $e^{jm\phi}$. More explicitly,

$$Z^{ss} = \begin{bmatrix} (Z^{ss})^{-M+1} & 0 & 0 & \cdots & 0 \\ 0 & (Z^{ss})^{-M+2} & 0 & \cdots & 0 \\ 0 & 0 & (Z^{ss})^{-M+3} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & (Z^{ss})^{M-1} \end{bmatrix} \qquad (30)$$

where

$$(Z^{ss})^{m} = \begin{bmatrix} (Z^{ss})^{mtt} & (Z^{ss})^{mt\phi} \\ (Z^{ss})^{m\phi t} & (Z^{ss})^{m\phi\phi} \end{bmatrix} \qquad (31)$$

for m = 0, 1, 2, ... M-1

On the right hand side of (31), $(Z^{ss})^{mt\phi}$ is the submatrix of $(Z^{ss})^{m}$ obtained by using $\underset{\sim}{u}_\phi$ directed expansion functions (27) and $\underset{\sim}{u}_t$ directed testing functions (26). Once $(Z^{ss})^{m}$ of (31) has been computed by the program in Appendix A on page 44 of [1], the rest of the submatrices $(Z^{ss})^{-m}$ of (30) are obtained from

$$(Z^{ss})^{-m} = \begin{bmatrix} (Z^{ss})^{mtt} & -(Z^{ss})^{mt\phi} \\ -(Z^{ss})^{m\phi t} & (Z^{ss})^{m\phi\phi} \end{bmatrix} \qquad (32)$$

where m = 1, 2, ... (M-1)

The expansion functions $\{\underset{\sim}{J}_i^w\}$ are defined by

$$\underset{\sim}{J}_i^w = \frac{\underset{\sim}{u}_\ell T_i^w(\ell)}{2\pi a} \tag{33}$$

where $T_i^w(\ell)$ is the triangle function given by expression (2) on page 5 of [2]. Also, ℓ is the length measured along the axis of the wire, $\underset{\sim}{u}_\ell$ is a unit vector in the direction of the axis of the wire, and a is the radius of the wire. For thin wires, it is reasonable to approximate the surface current (33) by an equivalent filament current $\underset{\sim}{u}_\ell T_i^w(\ell)$ and that is what will be done here. However, the field of the filament current simulates that of the surface current only on the surface of the wire and outside the wire. There can be no resemblance inside the wire because the field of the filament current is singular at the filament. For this reason the corresponding equivalent filament current testing function must be placed not on the wire axis but on a contour parallel to the wire axis on the surface of the wire. Now, the wire expansion and testing functions are the same as those in [2]. Thus, the computer program on page 12 of [2] can be used to obtain Z^{ww} appearing in (21).

IV. INTERACTION SUBMATRICES Z^{sw} AND Z^{ws}

For computation of Z^{ws}, it is assumed that the testing functions are filaments of current, not on the surface of the wire as in Section III, but on the axis of the wire. From (11),

$$(Z^{wsm})_{ij} = \langle J_i^{*w}, LJ_{j-p}^s \rangle \tag{34}$$

where Z^{wsm} is the submatrix of Z^{ws} corresponding to $e^{jm\phi}$ expansion functions and

$$p = (M - 1 + m) * 2 * NM \tag{35}$$

In (34), the complex conjugate operation is not necessary because J_i^w is real. Since the operator L is self-adjoint and because taking the complex conjugate of J_{j-p}^s is equivalent to replacing m by -m,

$$(Z^{wsm})_{ij} = (Z^{sw(-m)})_{ji} \tag{36}$$

Hence, it is sufficient to calculate Z^{swm} for positive and negative m.

For calculation of Z^{swm}, the expansion function (33) is lumped into a filament on the axis of the wire. The filament, in turn, is lumped into discrete current elements. The net result is an expansion function given by the first of the two expressions (5) on page 5 of [2]. Actually, to obtain the vector nature of the expansion function, each of the four terms on the right hand side of the first of the two expressions (5) on page 5 of [2] should be multiplied by a unit vector tangential to the axis of the wire. The expansion function $\underset{\sim}{J}_i^w$ is now the sum of four current elements.

$$\underset{\sim}{J}_i^w = \sum_{p=1}^{4} (\underset{\sim}{I\ell})_p \tag{37}$$

If $\underset{\sim}{J}_i^w$ were one current element $\underset{\sim}{I\ell} = \underset{\sim}{u}_x I\ell_x + \underset{\sim}{u}_y I\ell_y + \underset{\sim}{u}_z I\ell_z$, the elements of Z^{swm} would be given by the negative of the sum of (30), (31), and (32) of [3]. However, (30), (31) and (32) of [3] assume that $\underset{\sim}{I\ell}$ is located at $\phi = 0$. To generalize to $\phi \neq 0$, we must change $I\ell_x$ and $I\ell_y$ to $I\ell_\rho$ and $I\ell_\phi$ respectively and multiply by $e^{-jm\phi}$.

V. SAMPLE OUTPUT FROM PROGRAM TO COMPUTE Z^{ss}

In the program listed in Appendix A on page 44 of [1], all statements between and including statements 81 and 93 were replaced by

```
   WRITE (6) (Z(I), I = 1, NZ)
   WRITE (3,88) (Z(K), K =1,2)
88 FORMAT ('OZ'/(1X, 5E14.7))
```

and the subroutine LINEQ was removed. Also, the data was changed so as to store on the first three records of direct acess data set 6 $(Z^{ss})^0$, $(Z^{ss})^1$, and $(Z^{ss})^2$ of (31) for the flat back cone of half cone angle 45° and length $\frac{0.4}{\pi}$ wavelengths measured along its axis. With these changes, the program was run. The resulting printed output is listed next.

```
PRINTED OUTPUT FROM PROGRAM TO COMPUTE ZSS

NN=  0 NP= 15 NPHI= 20 BK= 0.1000000E+00

RH
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  6.6667
  5.3333  4.0000  2.6667  1.3333  0.0000

ZH
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  8.0000
  8.0000  8.0000  8.0000  8.0000  8.0000

TJ
  2.8284  5.6569  8.4853 11.3137 13.9804 16.6470

Z
 0.3137305E+02-0.8578586E+04 0.3044434E+02 0.8296912E+03

NN=  1 NP= 15 NPHI= 20 BK= 0.1000000E+00

RH
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  6.6667
  5.3333  4.0000  2.6667  1.3333  0.0000

ZH
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  8.0000
  8.0000  8.0000  8.0000  8.0000  8.0000

TJ
  2.8284  5.6569  8.4853 11.3137 13.9804 16.6470

Z
 0.1604865E+02-0.4318449E+04 0.1576461E+02 0.1259779E+04

NN=  2 NP= 15 NPHI= 20 BK= 0.1000000E+00

RH
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  6.6667
  5.3333  4.0000  2.6667  1.3333  0.0000

ZH
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  8.0000
  8.0000  8.0000  8.0000  8.0000  8.0000

TJ
  2.8284  5.6569  8.4853 11.3137 13.9804 16.6470

Z
 0.1211805E+00-0.2956441E+04 0.1059700E+00 0.1093889E+04
```

VI. SAMPLE OUTPUT FROM PROGRAM TO COMPUTE Z^{ww}

There is an error in the program listed on pages 12 - 16 of [2]. To correct this error, replace the group of three statements

```
H1 = (3. - 30. * ZR2 + 35.* ZR4)*AR3/40.
A1 = AR*(- 1. + 3. * ZR2)/6.
A0 = 1. + AR * (A1 + H1)
```

on page 14 of [2] by

```
A1 = AR*(-1. + 3. * ZR2)/6. + (3. - 30.*ZR2 + 35. * ZR4) * AR3/40.
A0 = 1. + AR * A1
```

On page 15 of [2], statement 37 was replaced by 37 FORMAT (1X,6E11.4) to shorten the printed line. If, in the input data to this program, RAD(I) ≠ RAD(J) and if the I^{th} and J^{th} wires overlap, then an inconsistency is apparent from the definition of RAD(I) on page 9 of [2]. Here, it was decided to let all wires have the same radius and not worry about this potential inconsistency. For the input data, there are two wires. One is in the $\phi = 0°$ plane on the surface of the cone and the other is symmetrically disposed in the $\phi = 180°$ plane. The basic theory in Section I and the computer program which calculates Z^{sw} are designed for wires not on the surface of revolution. Hence, numerical results obtained for wires on the surface of revolution will be questionable.

With the alteration of the three statements defining H1, A1, and A0, the modification of format statement 37, and changes in the input data, the program listed on pages 12 - 16 of [2] was run so as to store Z^{ww} of (21) on record 4 of direct access data set 6.

```
PRINTED OUTPUT FROM THE PROGRAM TO COMPUTE ZW

MD5 MD1
   1    3

MD6
  1

 NP NW       SK
 14  2 0.1000000E+00

PX
  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000 -1.0000 -2.0000 -3.0000
 -4.0000 -5.0000 -6.0000 -7.0000

PY
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000

PZ
  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  1.0000  2.0000  3.0000
  4.0000  5.0000  6.0000  7.0000

LL
  1  8

RAD
 0.3000000E+00 0.3000000E+00

IMPEDANCE MATRIX OF ORDER  4
 0.1592E+01-0.6232E+03 0.1579E+01 0.2619E+03-0.2788E-01-0.1954E+02
-0.4543E-01-0.6574E+01
 0.1579E+01 0.2619E+03 0.1592E+01-0.6232E+03-0.4542E-01-0.6706E+01
-0.7403E-01-0.4424E+01
```

VII. COMPUTATION OF THE INTERACTION SUBMATRIX Z^{sw}

The program to compute Z^{sw} accepts input on data sets 1 (punched cards) and 6 (direct access) in the following manner.

```
   READ (1, 10) NPW, NW, NP, M1, N1, N6, BK
10 FORMAT (6I3, E 14.7)
   READ (1, 12) (PX (I), I = 1, NPW)
   READ (1, 12) (PY (I), I = 1, NPW)
   READ (1, 12) (PZ (I), I = 1, NPW)
12 FORMAT (10F8.4)
   READ (1, 16) (LL (I), I = 1, NW)
16 FORMAT (20I3)
   READ (1, 12) (RH (I), I = 1, NP)
   READ (1, 12) (ZH (I), I = 1, NP)
   REWIND 6
   SKIP N6 RECORDS ON DATA SET 6
54 WRITE (6) (Z(J), J = 1, NZM)
```

PX(I), PY(I), and PZ(I) are the x, y, z coordinates of the I^{th} data point on the wires. There are NW wires. The $LL(I)^{th}$ data point is the first data point on the I^{th} wire. RH(I) and ZH(I) are the ρ (distance from the z axis of revolution) and z coordinates of the I^{th} data point on the generating curve of the surface of revolution. M1 is M appearing in (30). The equation at the bottom of page (21) of [3] should be labeled Equation (33). The $\sum_{n=m}^{\infty}$ in (33) is approximated by $\sum_{n=m}^{N1-1}$. BK is the propagation constant k appearing in (4).

The minimum allocation for BS in the subroutine BES is given by BS(XJ + 22) where XJ is the maximum value of kr' or kρ of (33) of [3]. Minimum allocations in the subroutine LEG are given by PC(M1 + 3), and PS(N1) where both M1 and N1 appear in the input data of the main program. In the main program, minimum allocations are given by

```
   COMPLEX EX(LW*(M1+1)), Z(NZM), HJ(N1), HJR(N1)
 DIMENSION PX(NPW), PY(NPW), PZ(NPW), LL(NW),
           RH(NP), ZH(NP), XR(LW), XP(LW), XZ(LW),
           UR(LW), UP(LW), UZ(LW), UL(LW), TW(4*N),
           DH(NP-1), R(NP-1), ZS(NP-1), SV(NP-1),
           CV(NP-1), T((NP-3)*2), TP((NP-3)*2),
           PS(M1+N1), P1(L5), P2(L5), P3(L5),
           BJ1(N1), BJ2(N1), BJ3(N1), BY1(N1),
           BY2(N1), BY3(N1)
```

where LW is the total number of intervals (each triangle function extends over four intervals) on the wires.

$$LW = NPW - NW \tag{38}$$

Also, N is the total number of triangle functions on the wires.

$$N = \sum_{J=1}^{NW} (LL(J + 1) - LL(J)-3)/2 \tag{39}$$

where LL(NW+1) = NPW+1

Furthermore,

$$NZM = N*(NP-3)*(2*M1 - 1) \tag{40}$$

$$L5 = N1*M2 - M2*(M2 - 1)/2 \tag{41}$$

DO loop 21 stores the ρ, ϕ, and z coordinates of the midpoint of the LW^{th} interval on the wires in XR(LW), XP(LW), and XZ(LW). UR, UP, and UZ are the ρ, ϕ, and z components of the vector length of the interval while UL is the length of the interval. Statement 56 changes LL(J) to the number of intervals on the J^{th} wire. In DO loop 39, TW(KL) for L = 6, 7, 8, 9 are the four sample values needed in (37) of the I^{th} triangle function on the J^{th} wire.

In DO loop 22, DH(I2) is the length of the I2th interval on the generating curve of the surface of revolution. R and ZS are the ρ and z coordinates of the midpoint of this interval. SV and CV are the sine and cosine of the angle between the z axis and the tangent to the generating curve.

Nested DO loops 43 and 46 store $e^{-jm\phi}$ (by which (30) - (32) of [3] must be multiplied) in EX((m+1)*LW + I) for m = -1, 0, 1, ... (M1-1) for the Ith interval on the wires.

For the surface of revolution, DO loop 23 stores the segment length times values of $T_i^s(t)$ of (29) and its derivative in T and TP. DO loop 24 puts n! in PS(n+1). Statement 57 stores $P_n^m(0)$ of (33) of [3] in P1(m*N1 - m*(m-1)/2 + n - m + 1) and $\frac{\partial P_n^m(0)}{\partial \Theta}$ in the corresponding place in P2. Nested DO loops 26 and (27) multiply $P_n^m(0)$ and $\frac{\partial P_n^m(0)}{\partial \Theta}$ by $\frac{(2n+1)(n-m)!}{(n+m)!}$.

As mentioned in Section IV, the elements of Z^{sw} are obtained by generalizing (30) - (32) of [3]. The functions ρf_i and $\frac{d}{dt}(\rho f_i)$ in (30) - (32) of [3] are approximated by four impulses. $I\ell_x$, $I\ell_y$ and $I\ell_z$ of (30) - (32) of [3] must be replaced by the sum (37). Now, an element of Z^{sw} is a triple sum consisting of four term sums over both the surface of revolution coordinates and the wire coordinates and a three term sum over m. This triple sum is obtained in the following indirect manner. DO loop 28 obtains the JSth coordinate on the surface of revolution. Nested DO loops 29 and 37 obtain the JWth coordinate on the wires. Inner DO loop 35 computes G_{JM-1} of (33) of [3]. For the above triplet (JS, JW, JM), inner DO loops 40, 41, and 42 add the contributions to the various elements of Z^{sw}. G_m contributes to $Z^{sw(-m+1)}$, $Z^{sw(-m)}$, $Z^{sw(-m-1)}$, $Z^{sw(m-1)}$, Z^{swm}, and $Z^{sw(m+1)}$ in the following manner.

$$Z^{sw(-m+1)t} = Z^{sw(-m+1)t} - EX*TW*\left[-T*(UR - jUP)k^2 G_m \sin v + TP*(UR - jUP)\left(\frac{\partial G_m}{\partial \rho} + \frac{m}{\rho} G_m\right)\right] \tag{42}$$

$$Z^{sw(-m+1)\phi} = Z^{sw(-m+1)\phi} - EX*TW*\left[-T*(UR - jUP)\; j\left(k^2 G_m - \frac{(m-1)}{\rho}\left(\frac{\partial G_m}{\partial \rho} + \frac{m}{\rho} G_m\right)\right)\right] \tag{43}$$

$$Z^{sw(-m)t} = Z^{sw(-m)t} - EX*TW*UZ*\left[-T*2k^2 G_m \cos v - TP*\frac{2}{\rho}\frac{\partial G_m}{\partial \theta}\right] \tag{44}$$

$$Z^{sw(-m)\phi} = Z^{sw(-m)\phi} - EX*TW*UZ*\left[\frac{-j2m}{\rho^2}\frac{\partial G_m}{\partial \theta}\right]* T \tag{45}$$

$$Z^{sw(-m-1)t} = Z^{sw(-m-1)t} - EX*TW*\left[-T*(UR + jUP)k^2 G_m \sin v + TP*(UR + jUP)\left(\frac{\partial G_m}{\partial \rho} - \frac{m}{\rho} G_m\right)\right] \tag{46}$$

$$Z^{sw(-m-1)\phi} = Z^{sw(-m-1)\phi} - EX*TW*T*(UR + jUP)\; j\left(k^2 G_m + \frac{m+1}{\rho}\left(\frac{\partial G_m}{\partial \rho} - \frac{m}{\rho} G_m\right)\right) \tag{47}$$

$$Z^{sw(m-1)t} = Z^{sw(m-1)t} - EX*TW* \left[-T*(UR + jUP)k^2 G_m \sin v + TP*(UR + jUP) \left(\frac{\partial G_m}{\partial \rho} + \frac{m}{\rho} G_m \right) \right] \quad (48)$$

$$Z^{sw(m-1)\phi} = Z^{sw(m-1)\phi} - EX*TW*T*(UR + jUP)j \left(k^2 G_m - \frac{m-1}{\rho} \left(\frac{\partial G_m}{\partial \rho} + \frac{m}{\rho} G_m \right) \right) \quad (49)$$

$$Z^{swmt} = Z^{swmt} - EX*TW*UZ* \left[-T*2k^2 G_m \cos v - TP* \frac{2}{\rho} \frac{\partial G_m}{\partial \theta} \right] \quad (50)$$

$$Z^{swm\phi} = Z^{swm\phi} - EX*TW*UZ* \left[\frac{j2m}{\rho^2} \frac{\partial G_m}{\partial \theta} \right] * T \quad (51)$$

$$Z^{sw(m+1)t} = Z^{sw(m+1)t} - EX*TW* \left[-T*(UR - jUP)k^2 G_m \sin v + TP*(UR - jUP) \left(\frac{\partial G_m}{\partial \rho} - \frac{m}{\rho} G_m \right) \right] \quad (52)$$

$$Z^{sw(m+1)\phi} = Z^{sw(m+1)\phi} - EX*TW* \left[-T*(UR - jUP)j \left(k^2 G_m + \frac{m+1}{\rho} \left(\frac{\partial G_m}{\partial \rho} - \frac{m}{\rho} G_m \right) \right) \right] \quad (53)$$

The additional superscript t or ϕ denotes $\underset{\sim}{u}_t$ or $\underset{\sim}{u}_\phi$ directed testing functions on the surface of revolution. The variables TW, UR, UP, UZ, T, TP, and EX have already been defined in the program. For the I^{th} surface of revolution triangle function and the J^{th} wire triangle function, Z^{swmt} will be stored in Z((M1-1 + m)*N*(NP-3) + (J-1)*(NP-3) + I) where N is the number of wire expansion functions. $Z^{swm\phi}$ is stored (NP-3)/2 locations later in Z.

Now that he has some idea of what is being done in DO loop 28, the reader should be able to digest the following details. Statement 58 puts the spherical Bessel functions and their derivatives $j_n(k\rho)$, $j_n'(k\rho)$, $y_n(k\rho)$, and $y_n'(k\rho)$ in BJ1, BJ2, BY1, and BY2. The subroutine BES is described on page 31 of [3]. Just before entering DO loop 29, the JS^{th} interval on the surface of revolution is in the domain of the JSM^{th} and $(JSM+1)^{th}$ triangle functions on the surface of revolution. These triangle functions are characterized on the JS^{th} interval by the previously stored T(JST+2) and T(JST+4). Just before statement 59, the JW^{th} interval on the wires is in the domain of the $\left(\frac{JWM+1}{NM2}\right)^{th}$ and $\left(\frac{JWM+2}{NM2}\right)^{th}$ triangle functions on the wires. These triangle functions are characterized on the JW^{th} interval by the previously stored TW(JWT+2) and TW(JWT+4). Statement 59 stores the spherical Bessel functions $j_n(kr')$ and $y_n(kr')$ in BJ3 and BY3. Statement 60 stores $P_n^m(\cos\Theta')$ in P3. If $r' > \rho$, DO loop 32 puts $j_n(k\rho)h_n^{(2)}(kr')$ and $kj_n'(k\rho)h_n^{(2)}(kr')$ in HJ and HJR. If $r' \le \rho$, DO loop 33 puts $j_n(kr')h_n^{(2)}(k\rho)$ and $kj_n(kr')h_n^{(2)'}(k\rho)$ in HJ and HJR.

DO loop 35 adds the contributions to Z^{sw} due to G_{JM-1} and $G_{-(JM-1)}$. DO loop 36 stores G_m, $\frac{\partial G_m}{\partial \rho}$, and $\frac{\partial G_m}{\partial \Theta}$ in G, GR, and GT for m = JM-1. In terms of TF and TD, expressions (40+2*M) for M = 1, 2, ... 6 become

$$Z^{sw} = Z^{sw} - EX*TW*[T*TF(M) + TP*TD(M)] \tag{54}$$

In terms of PF, expressions (41+2*M) for M = 1, 2, ... 6 become

$$Z^{sw} = Z^{sw} - EX*TW*T*PF(M)^{*} \qquad (55)$$

DO loop 40 contributes to Z^{swm} for $m = \pm$ (JM+M-3). The variable limit MB on DO loop 40 cuts off contributions to Z^{swm} for $|m| >$ M1-1. As mentioned before, for fixed JS and JW, two surface of revolution triangle functions and two wire expansion functions come into play. DO loop 41 obtains these two triangle functions on the surface of revolution. DO loop 42 obtains these two triangle functions on the wires. The variable limits KA, KB, KC, and KD are necessary for DO loops 41 and 42 because the first, second, second from the last and last intervals on either the surface of revolution or one of the wires are in the domain of only one triangle function. Inside DO loop 42, branch statement 61 is necessary because when m = 0, the $G_{\pm m}$ contributions to Z are one and the same.

LISTING ØF PRØGRAM TØ CØMPUTE THE INTERACTIØN SUBMATRIX ZSW

```
//              (0034,EE,30S,2),'MAUTZ,JØE',REGIØN=200K
// EXEC WATFIV
//GØ.FT06F001 DD DSNAME=EE0034.REV1,DISP=ØLD,UNIT=3330,                    X
//              VØLUME=SER=SU0009,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//              BUFNØ=1)
//GØ.SYSIN DD *
$JØB            MAUTZ,TIME=1,PAGES=40
      SUBRØUTINE RES(L,LD,ID,NJ,XJ,BJ,BJP,BY,BYP)
      DIMENSIØN BJ(25),BJP(25),BY(25),BYP(25),BS(40)
      L1=(L-1)*NJ
      L3=(LD-1)*NJ
    6 IF(XJ-1.E-3) 3,3,4
    3 J1=L1+1
      J2=L1+NJ
      DØ 5 J=J1,J2
      BJ(J)=0.
    5 CØNTINUE
      BJ(1)=1.
      RETURN
    4 SN=SIN(XJ)
      CS=CØS(XJ)
      IF(XJ-15.) 11,12,12
   12 BJ(L1+1)=SN/XJ
      BJ(L1+2)=(BJ(L1+1)-CS)/XJ
      DØ 14 I=3,NJ
      I3=L1+I
      I2=I3-1
      I1=I3-2
      BJ(I3)=FLØAT(2*I-3)/XJ*BJ(I2)-BJ(I1)
   14 CØNTINUE
      B3=FLØAT(2*NJ-1)/XJ*BJ(I3)-BJ(I2)
      GØ TØ 15
   11 NBJ=XJ+22.
      BS(NBJ)=0.
      BS(NBJ-1)=1.
      NBJ2=NBJ-2
      DØ 193 I=1,NBJ2
      I2=NBJ-I
      I3=I2+1
      I1=I2-1
      FI=FLØAT(2*I1+1)/XJ
      BS(I1)=BS(I2)*FI-BS(I3)
  193 CØNTINUE
      B1=SN/XJ
      B2=B1/XJ-CS/XJ
      IF(ABS(B1)-ABS(B2))1,2,2
    2 BB=B1/BS(1)
      GØ TØ 9
    1 BB=B2/BS(2)
    9 DØ 194 I=1,NJ
      I1=L1+I
      BJ(I1)=BS(I)*BB
  194 CØNTINUE
      B3=BS(NJ+1)*BB
```

```
 15 BY(L1+1)=-CS/XJ
    BY(L1+2)=(BY(L1+1)-SN)/XJ
    DO 64 I=3,NJ
    I3=L1+I
    I2=I3-1
    I1=I3-2
    BY(I3)=FLOAT(2*I-3)/XJ*BY(I2)-BY(I1)
 64 CONTINUE
    B4=FLOAT(2*NJ-1)/XJ*BY(I3)-BY(I2)
    IF(ID.EQ.2) RETURN
    NJ1=NJ-1
    J1=L3+1
    J2=L1+2
    BJP(J1)=-BJ(J2)
    BYP(J1)=-BY(J2)
    DO 65 J=2,NJ1
    J2=L3+J
    J1=L1+J-1
    J3=J1+2
    FJ=2*(2*J-1)
    BJP(J2)=.5*(BJ(J1)-BJ(J3))-(BJ(J1)+BJ(J3))/FJ
    BYP(J2)=.5*(BY(J1)-BY(J3))-(BY(J1)+BY(J3))/FJ
 65 CONTINUE
    FJ=FJ+4.
    J2=J2+1
    J1=J1+1
    BJP(J2)=.5*(BJ(J1)-B3)-(BJ(J1)+B3)/FJ
    BYP(J2)=.5*(BY(J1)-B4)-(BY(J1)+B4)/FJ
    RETURN
    END
    SUBROUTINE LEG(L,LD,ID,NJ,M,XP,P,PP)
    DIMENSION PC(15),P(70),PP(70),PS(25)
    PC(1)=1.
    M1=M+1
    DO 7 J=1,M1
    PC(J+1)=PC(J)*FLOAT(2*J-1)
  7 CONTINUE
    L5=M*NJ-M*(M-1)/2
    L2=(L-1)*L5
    L4=(LD-1)*L5
    X2=ABS(1.-XP*XP)
    X1=SQRT(X2)
    DO 3 J=1,M1
    M2=L2+(J-1)*NJ-(J-2)*(J-1)/2
    X3=1.
    IF(J.NE.1) X3=X1**(J-1)
    PS(1)=PC(J)*X3
    PS(2)=PC(J+1)*XP*X3
    IF(J.EQ.M1) GO TO 14
    P(M2+1)=PS(1)
    P(M2+2)=PS(2)
 14 NJ1=NJ-J+1
    DO 4 I=3,NJ1
    I1=I-2
    I2=I-1
    PS(I)=2.*XP*PS(I2)-PS(I1)+FLOAT(2*J-3)/FLOAT(I2)*(XP*PS(I2)-PS(I1)
   1)
    IF(J.EQ.M1) GO TO 4
    J2=M2+I
```

```
      P(J2)=PS(I)
    4 CØNTINUE
    3 CØNTINUE
      IF(ID.EQ.2) RETURN
      DØ 5 J=1,M
      M2=L4+(J-1)*NJ-(J-2)*(J-1)/2
      M3=M2+L2-L4
      NJ1=NJ-J+1
      DØ 6 I=2,NJ1
      J2=M2+I
      J1=M3+I-NJ+J-1
      J3=M3+I+NJ-J
      IF(J.NE.1.AND.J.NE.M) GØ TØ 8
      IF(J.NE.1) GØ TØ 12
      PP(J2)=-P(J3)
      GØ TØ 6
   12 PP(J2)=.5*(FLØAT(I*(2*J+I-3))*P(J1)-PS(I-1))
      GØ TØ 6
    8 PP(J2)=.5*(FLØAT(I*(2*J+I-3))*P(J1)-P(J3))
    6 CØNTINUE
      J2=M2+1
      J1=M3-NJ+J
      IF(J.NE.1) GØ TØ 13
      PP(J2)=0.
      GØ TØ 5
   13 PP(J2)=.5*FLØAT(2*J-2)*P(J1)
    5 CØNTINUE
      RETURN
      END
      CØMPLEX EX(180),U,Z(1500),U1,U2,U3,U4,U5,U6,U7,U8,U9,HJ(25)
      CØMPLEX HJR(25),G,GR,GT,TF(6),TD(6),PF(6),CØNJG
      DIMENSIØN PX(30),PY(30),PZ(30),LL(6),RH(50),ZH(50),XR(29),YR(29)
      DIMENSIØN XZ(29),UR(29),UP(29),UZ(29),UL(29),TW(60),DH(49),R(49)
      DIMENSIØN ZS(49),SV(49),CV(49),T(100),TP(100),PS(70),P1(70),P2(70)
      DIMENSIØN P3(70),BJ1(25),BJ2(25),BJ3(25),BY1(25),BY2(25),BY3(25)
      ETA=376.730
      U=(0.,1.)
      READ(1,10) NPW,NW,NP,M1,N1,N6,BK
   10 FØRMAT(6I3,E14.7)
      WRITE(3,11) NPW,NW,NP,M1,N1,N6,BK
   11 FØRMAT('0NPW NW NP M1 N1 N6',6X,'BK'/1X,6I3,E14.7)
      READ(1,12)(PX(I),I=1,NPW)
      READ(1,12)(PY(I),I=1,NPW)
      READ(1,12)(PZ(I),I=1,NPW)
   12 FØRMAT(10F8.4)
      WRITE(3,13)(PX(I),I=1,NPW)
   13 FØRMAT('0PX'/(1X,10F8.4))
      WRITE(3,14)(PY(I),I=1,NPW)
   14 FØRMAT('0PY'/(1X,10F8.4))
      WRITE(3,15)(PZ(I),I=1,NPW)
   15 FØRMAT('0PZ'/(1X,10F8.4))
      READ(1,16)(LL(I),I=1,NW)
   16 FØRMAT(20I3)
      LL(NW+1)=NPW+1
      WRITE(3,17)(LL(I),I=1,NW)
   17 FØRMAT('0LL'/(1X,20I3))
      READ(1,12)(RH(I),I=1,NP)
      READ(1,12)(ZH(I),I=1,NP)
      WRITE(3,18)(RH(I),I=1,NP)
```

```
 18 FORMAT('ORH'/(1X,10F8.4))
    WRITE(3,19)(ZH(I),I=1,NP)
 19 FORMAT('OZH'/(1X,10F8.4))
    LW=0
    K6=1
    DO 20 J=1,NW
    J1=LL(J)
    J2=LL(J+1)-2
    K2=LW+1
    DO 21 I=J1,J2
    LW=LW+1
    J4=I+1
    XM=.5*(PX(J4)+PX(I))
    YM=.5*(PY(J4)+PY(I))
    ZM=.5*(PZ(J4)+PZ(I))
    XD=PX(J4)-PX(I)
    YD=PY(J4)-PY(I)
    ZD=PZ(J4)-PZ(I)
    XR(LW)=SQRT(XM*XM+YM*YM)
    XP(LW)=ATAN2(YM,XM)
    XZ(LW)=ZM
    UR(LW)=(XM*XD+YM*YD)/XR(LW)
    UP(LW)=(-YM*XD+XM*YD)/XR(LW)
    UZ(LW)=ZD
    UL(LW)=SQRT(XD*XD+YD*YD+ZD*ZD)
 21 CONTINUE
 56 LL(J)=J2-J1+1
    J6=LL(J)/2-1
    DO 39 I=1,J6
    K7=K6+1
    K8=K6+2
    K9=K6+3
    K3=K2+1
    K4=K2+2
    K5=K2+3
    DEL1=UL(K2)+UL(K3)
    DEL2=UL(K4)+UL(K5)
    TW(K6)=.5*UL(K2)/DEL1
    TW(K7)=(UL(K2)+.5*UL(K3))/DEL1
    TW(K8)=(UL(K5)+.5*UL(K4))/DEL2
    TW(K9)=.5*UL(K5)/DEL2
    K6=K6+4
    K2=K2+2
 39 CONTINUE
 20 CONTINUE
    N=K9/4
    DO 22 I=2,NP
    I2=I-1
    RR1=RH(I)-RH(I2)
    RR2=ZH(I)-ZH(I2)
    DH(I2)=SQRT(RR1*RR1+RR2*RR2)
    R(I2)=.5*(RH(I)+RH(I2))
    ZS(I2)=.5*(ZH(I)+ZH(I2))
    SV(I2)=RR1/DH(I2)
    CV(I2)=RR2/DH(I2)
 22 CONTINUE
    M2=M1+1
    S1=.25*ETA
    DO 43 I=1,LW
```

```
      UR(I)=S1*UR(I)
      UP(I)=S1*UP(I)
      UZ(I)=S1*UZ(I)
      J4=I
      DO 46 J=1,M2
      S2=(J-2)*XP(I)
      EX(J4)=COS(S2)-J*SIN(S2)
      J4=J4+LN
46    CONTINUE
43    CONTINUE
      NM2=NP-3
      NM=NM2/2
      DO 23 J=1,NM
      J2=2*(J-1)+1
      J3=J2+1
      J4=J3+1
      J5=J4+1
      J6=4*(J-1)+1
      J7=J6+1
      J8=J7+1
      J9=J8+1
      DEL1=DH(J2)+DH(J3)
      DEL2=DH(J4)+DH(J5)
      T(J6)=DH(J2)*DH(J2)/2./DEL1
      T(J7)=DH(J3)*(DH(J2)+DH(J3)/2.)/DEL1
      T(J8)=DH(J4)*(DH(J5)+DH(J4)/2.)/DEL2
      T(J9)=DH(J5)*DH(J5)/2./DEL2
      TP(J6)=DH(J2)/DEL1
      TP(J7)=DH(J3)/DEL1
      TP(J8)=-DH(J4)/DEL2
      TP(J9)=-DH(J5)/DEL2
23    CONTINUE
      J1=M1+N1-1
      PS(1)=1.
      DO 24 J=1,J1
      J2=J+1
      PS(J2)=PS(J)*J
24    CONTINUE
57    CALL LEG(1,1,1,N1,M2,0.,P1,P2)
      J1=0
      DO 26 J=1,M2
      DO 27 I=J,N1
      J1=J1+1
      X6=(2*I-1)*PS(I-J+1)/PS(I+J-1)
      P1(J1)=P1(J1)*X6
      P2(J1)=P2(J1)*X6
27    CONTINUE
26    CONTINUE
      BK2=BK*BK
      BK3=2.*BK2
      NZ=NM2*4
      NZM=NZ*(2*M1-1)
      DO 52 J=1,NZM
      Z(J)=0.
52    CONTINUE
      JST=-6
      JSM=-1
      NPM=NP-1
      DO 28 JS=1,NPM
```

```
      KA=1
      KB=2
      IF(JS.LE.2) KA=2
      IF(JS.GT.NM2) KB=1
      XJ=BK*R(JS)
 58   CALL BES(1,1,1,N1,XJ,BJ1,BJ2,BY1,BY2)
      J1=JS/2
      J2=JS-2*J1
      JSM=JSM+J2
      JST=JST+1+2*J2
      JWT=-2
      JWM=-NM2
      JW=0
      DO 29 JJ=1,NW
      J5=LL(JJ)
      JWM=JWM-NM2
      JWT=JWT-4
      J5M=J5-1
      DO 37 JK=1,J5
      KC=1
      KD=2
      IF(JK.LE.2) KC=2
      IF(JK.GE.J5M) KD=1
      JW=JW+1
      J1=JK/2
      J2=JK-2*J1
      JWM=JWM+J2*NM2
      JWT=JWT+1+2*J2
      Z1=XZ(JW)-ZS(JS)
      XJ=SQRT(Z1*Z1+XR(JW)*XR(JW))
      XJ1=BK*XJ
 59   CALL BES(1,1,2,N1,XJ1,BJ3,BJ1,BY3,BY1)
      XJ1=Z1/XJ
 60   CALL LEG(1,1,2,N1,M2,XJ1,P3,P1)
      IF(XJ-R(JS)) 30,30,31
 31   DO 32 J=1,N1
      U1=BJ3(J)-U*BY3(J)
      HJ(J)=BJ1(J)*U1
      HJR(J)=BK*BJ2(J)*U1
 32   CONTINUE
      GO TO 34
 30   DO 33 J=1,N1
      HJ(J)=BJ3(J)*(BJ1(J)-U*BY1(J))
      HJR(J)=BK*BJ3(J)*(BJ2(J)-U*BY2(J))
 33   CONTINUE
 34   J1=0
      DO 35 JM=1,M2
      G=0.
      GR=0.
      GT=0.
      DO 36 JN=JM,N1
      J1=J1+1
      S1=P1(J1)*P3(J1)
      G=G+HJ(JN)*S1
      GR=GR+HJR(JN)*S1
      GT=GT+HJ(JN)*P2(J1)*P3(J1)
 36   CONTINUE
      U4=-BK2*SV(JS)*G
      U5=U*UP(JW)
```

```
      U6=UR(JW)+U5
      U7=UR(JW)-U5
      TF(4)=U4*U6
      TF(6)=U4*U7
      TF(3)=TF(4)
      TF(1)=TF(6)
      S1=JM-2
      S2=JM-1
      S3=JM
      U4=S2/R(JS)*G
      U8=GR+U4
      U9=GR-U4
      TD(4)=U6*U8
      TD(6)= U7*U9
      TD(3)=U6*U9
      TD(1)=U7*U8
      U9=(1./R(JS))*U
      U2=S1*U9
      U3=S3*U9
      U9=BK2*J*G
      U4=U6*U9
      U5=U7*U9
      PF(4)=-U2*TD(4)+U4
      PF(6)=-U3*TD(6)-U5
      PF(3)=U3*TD(3)+U4
      PF(1)=U2*TD(1)-U5
      TF(5)=-BK3*CV(JS)*UZ(JW)*G
      TF(2)=TF(5)
      TD(5)=-2.*UZ(JW)/R(JS)*GT
      TD(2)=TD(5)
      PF(5)=-S2/R(JS)*U*TD(5)
      PF(2)=-PF(5)
      K8=JWM-NM2+JSM-1
      K7=(M1+JM-4)*NZ+K8
      K6=(M1-JM+2)*NZ+K8
      J9=JW+(JM-1)*LW
      IF(JM-M1) 47,48,49
49    MB=1
      GO TO 50
48    MB=2
      GO TO 50
47    MB=3
50    DO 40 M=1,MB
      U1=EX(J9)
      U2=CONJG(U1)
      K7=K7+NZ
      K6=K6-NZ
      M3=M+3
      DO 41 KS=KA,KB
      M8=JST+KS+KS
      K8=K6+KS
      K9=K7+KS
      DO 42 KW=KC,KD
      M7=JWT+KW+KW
      S1=-T(M8)*TW(M7)
      S2=-TP(M8)*TW(M7)
      L6=KW*NM2
      L8=K8+L6
      L9=K9+L6
```

```
      Z(L8)=Z(L8)+(TF(M)*S1+TD(M)*S2)*U2
      K4=L8+NM
      Z(K4)=Z(K4)+PF(M)*S1*U2
   61 IF(JM.EQ.1) GO TO 42
      Z(L9)=Z(L9)+(TF(M3)*S1+TD(M3)*S2)*U1
      K5=L9+NM
      Z(K5)=Z(K5)+PF(M3)*S1*U1
   42 CONTINUE
   41 CONTINUE
      J9=J9+LW
   40 CONTINUE
   35 CONTINUE
   37 CONTINUE
   29 CONTINUE
   28 CONTINUE
      REWIND 6
      IF(N6.EQ.0) GO TO 54
      DO 55 J=1,N6
      READ(6)
   55 CONTINUE
   54 WRITE(6)(Z(J),J=1,NZM)
      WRITE(3,53)(Z(J),J=1,2)
   53 FORMAT('0Z'/(1X,6E14.7))
      STOP
      END
$DATA
 14  2 15  3 10  4 0.1000000E+00
  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000 -1.0000 -2.0000 -3.0000
 -4.0000 -5.0000 -6.0000 -7.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000
  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  1.0000  2.0000  3.0000
  4.0000  5.0000  6.0000  7.0000
  1  8
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  6.6667
  5.3333  4.0000  2.6667  1.3333  0.0000
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  8.0000
  8.0000  8.0000  8.0000  8.0000  8.0000
$STOP
/*
//
```

PRINTED OUTPUT

```
NPW NW NP M1 N1 N6        BK
 14  2 15  3 10  4 0.1000000E+00

PX
  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000 -1.0000 -2.0000 -3.0000
 -4.0000 -5.0000 -6.0000 -7.0000

PY
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000

PZ
  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  1.0000  2.0000  3.0000
  4.0000  5.0000  6.0000  7.0000
```

```
LL
  1   8

RH
  0.0000   1.0000   2.0000   3.0000   4.0000   5.0000   6.0000   7.0000   8.0000   6.6667
  5.3333   4.0000   2.6667   1.3333   0.0000

ZH
  0.0000   1.0000   2.0000   3.0000   4.0000   5.0000   6.0000   7.0000   8.0000   8.0000
  8.0000   8.0000   8.0000   8.0000   8.0000

Z
 0.2252771E-01-0.2685371E+03 0.4502131E-01 0.4381604E+03
```

VIII. COMPUTATION OF THE WIRE PARAMETERS OF SECTION II

The program of this section computes the wire parameters E^{ss}, Z, $\vec{R}$, and $\vec{V}$ and uses them to obtain the backscattering cross section per wavelength squared $\frac{\sigma}{\lambda^2}$ given by (23) for an axially incident plane wave traveling in the positive z direction.

The activity on data sets 1 (punched card input) and 6 (direct access input and output) is as follows.

```
      READ (1, 10) NPW, NW, NP, LW, N6, M1, BK
   10 FORMAT (6I3, E14.7)
      READ (1,12)(PX(I), I = 1, NPW)
      READ (1,12)(PY(I), I = 1, NPW)
      READ (1,12)(PZ(I), I = 1, NPW)
   12 FORMAT (10F8.4)
      READ (1,16)(LL(I), I = 1, NW)
   16 FORMAT (20I3)
      READ (1,12)(RH(I), I = 1, NP)
      READ (1,12)(ZH(I), I = 1, NP)
      READ (1,8)(ZWL(I), I = 1, LW)
    8 FORMAT (7E11.4)
      NPM = NP-1
      READ (1,8)(ZST(I), I = 1, NPM)
      READ (1,8)(ZSP(I), I = 1, NPM)
      REWIND 6
      SKIP N6 RECORDS ON DATA SET 6
      LWW = LW*LW
   30 READ (6) (ZWW(I), I = 1, LWW)
      LS = NP-3
      LSW = LW*LS
      MSW = (2*M1-1)*LSW
```

```
      READ (6)(ZSW(I), I = 1, MSW)
      REWIND 6
      LSS = LS*LS
      DO 36 M = 1, M1
      READ (6)(ZSS(I), I = 1, LSS)
   36 CONTINUE
```

The x, y, and z coordinates of the data points on the wires are read in through PX, PY, and PZ. There are NW wires. The LL(I)th data point is the first data point on the Ith wire. The ρ and z coordinates of the data points on the generating curve of the surface of revolution are read in through RH and ZH. There are LW triangle functions on the wires. The wires are loaded by placing lumped impedance loads at the peaks of the triangle functions. ZWL(I) is the impedance load at the peak of the Ith triangle function. More specifically, it is assumed that Z_L^w of (23) is a diagonal matrix whose Ith element is ZWL(I). This restricts the loads at a wire junction to two branches of the junction. The surface of revolution is not a perfect conductor but a rotationally symmetric impedance sheet. The $\underset{\sim}{u}_t$ directed electric current sees an average surface impedance of ZST(I) in the Ith interval on the generating curve. Likewise, the $\underset{\sim}{u}_\phi$ directed electric current sees the surface impedance ZSP(I) in the Ith interval. It must be pointed out that if ZST(I) $\neq$ ZSP(I), then the electric field is not parallel to the electric current when the electric current has both $\underset{\sim}{u}_t$ and $\underset{\sim}{u}_\phi$ components and hence the usual concept of surface impedance does not apply. However, it is thought that very narrow axially symmetric impedance bands should be characterized by ZST(I) $\neq$ ZSP(I). For instance, if the narrow band is a good conductor, only the $\underset{\sim}{u}_\phi$ directed current will be affected whereas if the band is a good insulator, only the $\underset{\sim}{u}_t$ directed current will be affected. BK is the propagation constant k appearing in (4). The matrices Z^{ww} and Z^{sw} appearing in (21) are read in through ZWW and ZSW. Z^{ww} is stored by columns in ZWW. Each submatrix Z^{swm} is stored by columns in ZSW. Here, m runs between -M1+1 and M1-1. All elements of Z^{swm} precede all those of $Z^{sw(m+1)}$. In DO loop 36, the submatrix $Z^{ss(M-1)}$ of (31) is read in through ZSS.

In the main program, minimum allocations are given by

```
COMPLEX ZWL(LW), ZST(NP-1), ZSP(NP-1), EX(NP-1),
EX(J3), ZT(NM), ZP(NM), ZZT(MN), ZZP(NM),
RS(NP-3), VS(NP-3), VW(LW), RW(LW), ZWW(LW*LW),
ZSW( (2*M1-1)*LW*LS), ZSS(LS*LS), MAXIMUM OF ZZ(LW*LS) OR
ZZ(2*LS)

DIMENSION PX(NPW), PY(NPW), PZ(NPW), LL(NW+1),
RH(NP), ZH(NP), DH(NP-1), RR(NP-1), SV(NP-1),
UL(J3)
```

where NM = (NP-3)/2 and J3 is the maximum number of intervals on one wire.

For the surface of revolution, DO loop 22 stores the interval length, ρ, sin v, and e^{-jkz} in DH, RR, SV, and EX. The surface of revolution load impedance matrix Z_L^s of (10) is of the same form (30) and (31) as Z^{ss}. Moreover, the mϕt and mtϕ load impedance submatrices are zero and the mtt and m$\phi\phi$ submatrices are tridiagonal and do not depend on m.

From (12), (14), (26), and (27),

$$(Z_L^s)_{ii}^{mtt} = 2\pi \int Z_L^t \frac{\left(T_i^s(t)\right)^2}{\rho} dt \tag{56}$$

$$(Z_L^s)_{i,i-1}^{mtt} = (Z_L^s)_{i-1,i}^{mtt} = 2\pi \int Z_L^t \frac{T_i^s(t) T_{i-1}^s(t)}{\rho} dt \tag{57}$$

$$(Z_L^s)_{ii}^{m\phi\phi} = 2\pi \int Z_L^\phi \frac{\left[T_i^s(t)\right]^2}{\rho} dt \qquad (58)$$

$$(Z_L^s)_{i,i-1}^{m\phi\phi} = (Z_L^s)_{i-1,i}^{m\phi\phi} = 2\pi \int Z_L^\phi \frac{T_i^s(t)T_{i-1}(t)}{\rho} dt \qquad (59)$$

where Z_L^t and Z_L^ϕ are surface impedances corresponding to ZST and ZSP in the program. The logic prior to statement 59 in DO loop 21 evaluates (56) to (59) by sampling the integrand four times. In DO loop 21, S1, S2, S3, and S4 are the four sample values of the triangle function $T_J^s(t)$. DO loop 21 stores (56) - (59) in ZT, ZZT, ZP, and ZZP respectively. The logic beginning with statement 59 in DO loop 21 stores $(R_1^{t\theta})_J$ and $(R_1^{\phi\theta})_J$ of (83) of [4] in RS(J) and RS(J+NM). Hence, for the $e^{j\phi}$ expansion functions, $\vec{R}^s$ of (24) is stored in RS. Next, for the $e^{-j\phi}$ testing functions $\vec{V}^s$ of (24) is stored in VS. For $e^{-j\phi}$ expansion and $e^{j\phi}$ testing functions, (82) of [4] states that $\vec{R}^s$ and $\vec{V}^s$ interchange roles.

For the J3th interval on the Jth wire, DO loop 27 stores the interval length and $-\Delta x e^{-jkz}$ in UL(J3) and EX(J3) where Δx is the extent of the interval in the $\underset{\sim}{u}_x$ direction. DO loop 28 stores the element of $\vec{V}^w$ of (22) for the Ith triangle function on the Jth wire in VW. Both $\vec{V}^s$ and $\vec{V}^w$ of (22) have been computed for an incident electric field $\underset{\sim}{E}^i$ given by

$$\underset{\sim}{E}^i = -\underset{\sim}{u}_x e^{-jkz} \qquad (60)$$

Because $[Z^{ss} + Z_L^s]$ has the block diagonal nature (30), its inverse is the block diagonal arrangement of the inverses of the submatrices $(Z^{ss} + Z_L^s)^m$. Because $[Z^{ss} + Z_L^s]^{-1}$ is block diagonal, all matrix products involving $[Z^{ss} + Z_L^s]^{-1}$ in (21), (22), (24), and (25) become sums over m of the m submatrix products. For instance,

$$Z^{ws}[Z^{ss} + Z_L^s]^{-1}Z^{sw} = \sum_{m=-M1+1}^{M1-1} Z^{wsm}\left[(Z^{ss} + Z_L^s)^m\right]^{-1} Z^{swm} \tag{61}$$

where the superscript m denotes the submatrix obtained by using only $e^{jm\phi}$ expansion functions or $e^{-jm\phi}$ testing functions on the surface of revolution. DO loop 36 adds the $m = \pm (M-1)$ contributions to (61). In DO loop 36, Z, $\vec{V}$, E^{ss}, and $\vec{R}$ of (21), (22), (24), and (25) are accumulated in ZWW, VW, ESS, and RW respectively. Because $\vec{R}^s$ and $\vec{V}^s$ have only $m = \pm 1$ submatrices, only the $m = \pm 1$ terms contribute to V, E^{ss} and $\vec{R}$ of (22), (24), and (25) respectively.

DO loop 55 adds the load matrix to $(Z^{ss})^m$ of (31) to form $(Z^{ss} + Z_L^s)^m$ of (61). In DO loop 55, ZSS(J1) is the J^{th} diagonal element of $(Z^{ss})^{mtt}$. Also, ZSS(J1-1) and ZSS(J1-LS) are nearby off diagonal elements. The corresponding elements of $(Z^{ss})^{m\phi\phi}$ are referenced by adding J3 to the subscript of ZSS. Statement 60 inverts the matrix $(Z^{ss} + Z_L^s)^m$. Nested DO loops (37) and (38) store $[(Z^{ss} + Z_L^s)^m]^{-1}Z^{swm}$ of (61) in ZZ by columns. Nested DO loops 41 and 42 subtract $Z^{wsm}[(Z^{ss} + Z_L^s)^m]^{-1}Z^{swm}$ from Z^{ww} stored in ZWW. The elements of Z^{wsm} needed in inner DO loop 43 have been extracted from Z^{swm} according to (36). Just as DO loops 37 and 41 have added the m = + (M-1) contribution to Z of (21) residing in ZWW, DO loops 45 and 46 add the m = -(M-1) contribution to Z of (21) residing in ZWW. In inner DO loop 47, the elements of $[(Z^{ss} + Z_L^s)^{-m}]^{-1}$ are, according to property (32) which survives matrix inversion, extracted from $[(Z^{ss} + Z_L^s)^m]^{-1}$ residing in ZSS.

For m = 1, DO loop 49 puts the J^{th} elements of $[(Z^{ss} + Z_L^s)^m]^{-1}\vec{V}^{sm}$ and $\tilde{R}^{sm}[(Z^{ss} + Z_L^S)^m]^{-1}$ in ZZ(J) and ZZ(J+LS). From (32) and the nature of $\vec{V}^{sm}$, the column vector defined by

$$\vec{I}^m = \left[(Z^{ss} + Z_L^s)^m\right]^{-1} \vec{V}^{sm} \tag{62}$$

satisfies

$$\vec{I}^{(-m)t} = \vec{I}^{mt}$$
$$\vec{I}^{(-m)\phi} = -\vec{I}^{m\phi} \qquad (63)$$

The row matrix $\tilde{R}^{sm}[(Z^{ss} + Z_L^s)^m]^{-1}$ satisfies a relationship similar to (63). DO loop 51 adds the m = ± 1 contributions to $\vec{V}$ and $\tilde{R}$ of (22) and (25) stored in VW and RW. DO loop 53 adds the m = ± 1 contributions to E^{ss} of (24) stored in ESS. The m = -1 contribution to E^{ss} is equal to the m = 1 contribution.

DO loop 61 adds the diagonal wire load matrix Z_L^w to Z to form $[Z + Z_L^w]$ of (20) in ZWW. Statement 62 inverts the matrix $[Z + Z_L^w]$ residing in ZWW. DO loop 57 stores $\vec{I}^w$ of (20) in ZZ and accumulates $\tilde{R}[Z + Z_L^w]^{-1}\vec{V}$ of (23) in U1. Finally, σ/λ^2 of (23) is stored in SIG and printed out.

LISTING OF PROGRAM TO COMPUTE WIRE PARAMETERS

```
//                 (0034,EE,20S,2),'MAUTZ,JOE',REGION=200K
// EXEC WATFIV
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=3330,                    X
//                 VOLUME=SER=SU0009,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//                 BUFNO=1)
//GO.SYSIN DD *
$JOB               MAUTZ,TIME=1,PAGES=40
      SUBROUTINE LINEQ(LL,C)
      COMPLEX C(200),STOR,STO,ST,S
      DIMENSION LR(58)
      DO 20 I=1,LL
      LR(I)=I
   20 CONTINUE
      M1=0
      DO 18 M=1,LL
      K=M
      K2=M1+K
      S1=ABS(REAL(C(K2)))+ABS(AIMAG(C(K2)))
      DO 2 I=M,LL
      K1=M1+I
      S2=ABS(REAL(C(K1)))+ABS(AIMAG(C(K1)))
      IF(S2-S1) 2,2,6
    6 K=I
      S1=S2
    2 CONTINUE
      LS=LR(M)
      LR(M)=LR(K)
      LR(K)=LS
      K2=M1+K
      STOR=C(K2)
      J1=0
      DO 7 J=1,LL
      K1=J1+K
      K2=J1+M
      STO=C(K1)
      C(K1)=C(K2)
      C(K2)=STO/STOR
      J1=J1+LL
    7 CONTINUE
      K1=M1+M
      C(K1)=1./STOR
      DO 11 I=1,LL
      IF(I-M) 12,11,12
   12 K1=M1+I
      ST=C(K1)
      C(K1)=0.
      J1=0
      DO 10 J=1,LL
      K1=J1+I
      K2=J1+M
      C(K1)=C(K1)-C(K2)*ST
      J1=J1+LL
   10 CONTINUE
   11 CONTINUE
```

```
      M1=M1+LL
   18 CONTINUE
      J1=0
      DO 9 J=1,LL
      IF(J-LR(J)) 14,8,14
   14 LRJ=LR(J)
      J2=(LRJ-1)*LL
   21 DO 13 I=1,LL
      K2=J2+I
      K1=J1+I
      S=C(K2)
      C(K2)=C(K1)
      C(K1)=S
   13 CONTINUE
      LR(J)=LR(LRJ)
      LR(LRJ)=LRJ
      IF(J-LR(J)) 14,8,14
    8 J1=J1+LL
    9 CONTINUE
      RETURN
      END
      COMPLEX U,U1,U2,U3,U4,CONJG,ZWL(20),ZST(49),ZSP(49),FX(49),ZT(26)
      COMPLEX ZP(26),ZZT(26),ZZP(26),RS(52),VS(52),VW(20),RW(20)
      COMPLEX ZWW(200),ZSW(500),ZSS(200),ZZ(100),FSS
      DIMENSION PX(30),PY(30),PZ(30),LL(6),RH(50),ZH(50),CH(49),RR(49)
      DIMENSION SV(49),UL(30)
      READ(1,10) NPW,NW,NP,LW,N6,M1,BK
   10 FORMAT(6I3,E14.7)
      WRITE(3,11) NPW,NW,NP,LW,N6,M1,BK
   11 FORMAT('ONPW NW NP LW N6 M1',6X,'BK'/1X,6I3,E14.7)
      READ(1,12)(PX(I),I=1,NPW)
      READ(1,12)(PY(I),I=1,NPW)
      READ(1,12)(PZ(I),I=1,NPW)
   12 FORMAT(10F8.4)
      WRITE(3,13)(PX(I),I=1,NPW)
   13 FORMAT('OPX'/(1X,10F8.4))
      WRITE(3,14)(PY(I),I=1,NPW)
   14 FORMAT('OPY'/(1X,10F8.4))
      WRITE(3,15)(PZ(I),I=1,NPW)
   15 FORMAT('OPZ'/(1X,10F8.4))
      READ(1,16)(LL(I),I=1,NW)
   16 FORMAT(20I3)
      LL(NW+1)=NPW+1
      WRITE(3,17)(LL(I),I=1,NW)
   17 FORMAT('OLL'/(1X,20I3))
      READ(1,12)(RH(I),I=1,NP)
      READ(1,12)(ZH(I),I=1,NP)
      WRITE(3,18)(RH(I),I=1,NP)
   18 FORMAT('ORH'/(1X,10F8.4))
      WRITE(3,19)(ZH(I),I=1,NP)
   19 FORMAT('OZH'/(1X,10F8.4))
      READ(1,8)(ZWL(I),I=1,LW)
    8 FORMAT(7E11.4)
      WRITE(3,9)(ZWL(I),I=1,LW)
    9 FORMAT('OZWL'/(1X,7E11.4))
      NPM=NP-1
      READ(1,8)(ZST(I),I=1,NPM)
      WRITE(3,23)(ZST(I),I=1,NPM)
   23 FORMAT('OZST'/(1X,7E11.4))
```

```
      READ(1,8)(ZSP(I),I=1,NPM)
      WRITE(3,24)(ZSP(I),I=1,NPM)
   24 FORMAT('0ZSP'/(1X,7E11.4))
      U=(0.,1.)
      PI=3.141593
      P2=2.*PI
      DO 22 I=2,NP
      I2=I-1
      RR1=RH(I)-RH(I2)
      RR2=ZH(I)-ZH(I2)
      DH(I2)=SQRT(RR1*RR1+RR2*RR2)
      RR(I2)=.5*(RH(I)+RH(I2))
      SV(I2)=RR1/DH(I2)
      S1=BK*.5*(ZH(I)+ZH(I2))
      EX(I2)=COS(S1)-U*SIN(S1)
      ZST(I2)=P2*ZST(I2)
      ZSP(I2)=P2*ZSP(I2)
   22 CONTINUE
      LS=NP-3
      LSW=LW*LS
      LWW=LW*LW
      LSS=LS*LS
      NM=LS/2
      LSN=LS*NM
      ETA=376.730
      C1=.25*BK*BK*ETA/PI
      C1=C1*C1/PI
      J1=1
      DO 21 J=1,NM
      J2=J1+1
      J3=J1+2
      J4=J1+3
      DEL1=DH(J1)+DH(J2)
      DEL2=DH(J3)+DH(J4)
      S1=.5*DH(J1)/DEL1
      S2=(DH(J1)+.5*DH(J2))/DEL1
      S3=(DH(J4)+.5*DH(J3))/DEL2
      S4=.5*DH(J4)/DEL2
      S5=DH(J1)*S1
      S6=DH(J2)*S2
      S7=DH(J3)*S3
      S8=DH(J4)*S4
      W1=S1*S5/RR(J1)
      W2=S2*S6/RR(J2)
      W3=S3*S7/RR(J3)
      W4=S4*S8/RR(J4)
      ZT(J)=W1*ZST(J1)+W2*ZST(J2)+W3*ZST(J3)+W4*ZST(J4)
      ZP(J)=W1*ZSP(J1)+W2*ZSP(J2)+W3*ZSP(J3)+W4*ZSP(J4)
      IF(J.EQ.1) GO TO 25
      W1=S1*W5/RR(J1)
      W2=S2*W6/RR(J2)
      ZZT(J)=W1*ZST(J1)+W2*ZST(J2)
      ZZP(J)=W1*ZSP(J1)+W2*ZSP(J2)
   25 W5=S7
      W6=S8
   59 U1=S5*EX(J1)
      U2=S6*EX(J2)
      U3=S7*EX(J3)
      U4=S8*EX(J4)
```

```
      RS(J)=(-SV(J1)*U1-SV(J2)*U2-SV(J3)*U3-SV(J4)*U4)*PI
      J5=J+NM
      RS(J5)=PI*U*(U1+U2+U3+U4)
      VS(J)=RS(J)
      VS(J5)=-RS(J5)
      J1=J1+2
   21 CONTINUE
      J5=0
      DO 26 J=1,NW
      J1=LL(J)
      J2=LL(J+1)-2
      J3=0
      DO 27 I=J1,J2
      J3=J3+1
      J4=I+1
      XD=PX(J4)-PX(I)
      YD=PY(J4)-PY(I)
      ZD=PZ(J4)-PZ(I)
      UL(J3)=SQRT(XD*XD+YD*YD+ZD*ZD)
      ZM=.5*BK*(PZ(J4)+PZ(I))
      EX(J3)=-XD*(COS(ZM)-U*SIN(ZM))
   27 CONTINUE
      J6=(J2-J1-1)/2
      K1=1
      DO 28 I=1,J6
      J5=J5+1
      K2=K1+1
      K3=K2+1
      K4=K3+1
      DEL1=UL(K1)+UL(K2)
      DEL2=UL(K3)+UL(K4)
      S1=.5*UL(K1)/DEL1
      S2=(UL(K1)+.5*UL(K2))/DEL1
      S3=(UL(K4)+.5*UL(K3))/DEL2
      S4=.5*UL(K4)/DEL2
      VW(J5)=S1*EX(K1)+S2*EX(K2)+S3*EX(K3)+S4*EX(K4)
      RW(J5)=VW(J5)
      K1=K1+2
   28 CONTINUE
   26 CONTINUE
      WRITE(3,64)(VW(I),I=1,LW)
   64 FORMAT('0VW'/(1X,4E14.7))
      WRITE(3,65)(RW(I),I=1,LW)
   65 FORMAT('0RW'/(1X,4E14.7))
      REWIND 6
      IF(N6) 30,30,31
   31 DO 32 J=1,N6
      READ(6)
   32 CONTINUE
   30 READ(6)(ZWW(I),I=1,LWW)
      WRITE(3,33)(ZWW(I),I=1,LWW)
   33 FORMAT('0ZWW'/(1X,4E14.7))
      MSW=(2*M1-1)*LSW
      READ(6)(ZSW(I),I=1,MSW)
      REWIND 6
      WRITE(3,34)(ZSW(I),I=1,2)
   34 FORMAT('0ZSW'/(1X,4E14.7))
      JP=(M1-2)*LSW
      JM=M1*LSW
```

```
      DO 36 M=1,M1
      READ(6)(ZSS(I),I=1,LSS)
      WRITE(3,40)(ZSS(I),I=1,2)
   40 FORMAT('0ZSS'/(1X,4E14.7))
      J3=LSM+NM
      J6=LS+1
      J1=1
      DO 55 J=1,NM
      ZSS(J1)=ZSS(J1)+ZT(J)
      J2=J1+J3
      ZSS(J2)=ZSS(J2)+ZP(J)
      IF(J.EQ.1) GO TO 56
      J4=J1-1
      J5=J1-LS
      ZSS(J4)=ZSS(J4)+ZZT(J)
      ZSS(J5)=ZSS(J5)+ZZT(J)
      J4=J4+J3
      J5=J5+J3
      ZSS(J4)=ZSS(J4)+ZZP(J)
      ZSS(J5)=ZSS(J5)+ZZP(J)
   56 J1=J1+J6
   55 CONTINUE
   60 CALL LINEQ(LS,ZSS)
      JP=JP+LSW
      JM=JM-LSW
      J1=0
      J3=JP
      DO 37 J=1,LW
      DO 38 I=1,LS
      J1=J1+1
      J2=I
      ZZ(J1)=0.
      DO 39 K=1,LS
      J4=J3+K
      ZZ(J1)=ZZ(J1)+ZSS(J2)*ZSW(J4)
      J2=J2+LS
   39 CONTINUE
   38 CONTINUE
      J3=J3+LS
   37 CONTINUE
      J1=0
      J5=0
      DO 41 J=1,LW
      J3=JM
      DO 42 I=1,LW
      J1=J1+1
      DO 43 K=1,LS
      J3=J3+1
      J6=J5+K
      ZWW(J1)=ZWW(J1)-ZSW(J3)*ZZ(J6)
   43 CONTINUE
   42 CONTINUE
      J5=J5+LS
   41 CONTINUE
      IF(M.EQ.1) GO TO 36
      J1=0
      J3=JM
      DO 45 J=1,LW
      DO 20 I=1,NM
```

```
      J1=J1+1
      ZZ(J1)=0.
      J5=J1+NM
      ZZ(J5)=0.
      J2=I
      DO 47 K=1,NM
      J4=J3+K
      J6=J2+LSN
      J7=J4+NM
      ZZ(J1)=ZZ(J1)+ZSS(J2)*ZSW(J4)-ZSS(J6)*ZSW(J7)
      J8=J2+NM
      J6=J6+NM
      ZZ(J5)=ZZ(J5)-ZSS(J8)*ZSW(J4)+ZSS(J6)*ZSW(J7)
      J2=J2+LS
   47 CONTINUE
   20 CONTINUE
      J1=J5
      J3=J3+LS
   45 CONTINUE
      J1=0
      J5=0
      DO 46 J=1,LW
      J3=JP
      DO 44 I=1,LW
      J1=J1+1
      DO 48 K=1,LS
      J3=J3+1
      J6=J5+K
      ZWW(J1)=ZWW(J1)-ZSW(J3)*ZZ(J6)
   48 CONTINUE
   44 CONTINUE
      J5=J5+LS
   46 CONTINUE
      IF(M.NE.2) GO TO 36
      J3=0
      DO 49 J=1,LS
      J1=J+LS
      ZZ(J)=0.
      ZZ(J1)=0.
      J2=J
      DO 50 K=1,LS
      ZZ(J)=ZZ(J)+ZSS(J2)*VS(K)
      J3=J3+1
      ZZ(J1)=ZZ(J1)+ZSS(J3)*RS(K)
      J2=J2+LS
   50 CONTINUE
   49 CONTINUE
      J1=JM
      J5=JP
      DO 51 J=1,LW
      DO 52 K=1,NM
      J2=J1+K
      J3=J2+NM
      J4=K+NM
      J6=J5+K
      J7=J6+NM
      VW(J)=VW(J)-ZSW(J2)*ZZ(K)-ZSW(J3)*ZZ(J4)
      YW(J)=VW(J)-ZSW(J6)*ZZ(K)+ZSW(J7)*ZZ(J4)
      J8=K+LS
```

```
      J9=J4+LS
      RW(J)=RW(J)-ZSW(J6)*ZZ(J8)-ZSW(J7)*ZZ(J9)
      RW(J)=RW(J)-ZSW(J2)*ZZ(J8)+ZSW(J3)*ZZ(J9)
   52 CONTINUE
      J1=J3
      J5=J7
   51 CONTINUE
      ESS=0.
      DO 53 K=1,LS
      ESS=ESS+RS(K)*ZZ(K)
   53 CONTINUE
      ESS=2.*ESS
   36 CONTINUE
      WRITE(3,33)(ZWW(I),I=1,LWW)
      WRITE(3,64)(VW(I),I=1,LW)
      WRITE(3,65)(RW(I),I=1,LW)
      LWP=LW+1
      J1=1
      DO 61 J=1,LW
      ZWW(J1)=ZWW(J1)+ZWL(J)
      J1=J1+LWP
   61 CONTINUF
   62 CALL LINEQ(LW,ZWW)
      U1=C.
      DO 57 J=1,LW
      ZZ(J)=0.
      J1=J
      DO 58 I=1,LW
      ZZ(J)=ZZ(J)+ZWW(J1)*VW(I)
      J1=J1+LW
   58 CONTINUE
      U1=U1+RW(J)*ZZ(J)
   57 CONTINUE
      WRITE(3,63)(ZZ(I),I=1,LW)
   63 FORMAT('OWIRE CURRENT'/(1X,4E14.7))
      U2=ESS+U1
      S1=U2*CONJG(U2)
      SIG=S1*C1
      WRITE(3,66) ESS,U2
   66 FORMAT('OESS=',2E14.7,' U2=',2E14.7)
      WRITE(3,67) SIG
   67 FORMAT('OSIG=',E14.7)
      STOP
      END
$DATA
 14  2 15   4   3   3 0.1000000E+00
  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000 -1.0000 -2.0000 -3.0000
 -4.0000 -5.0000 -6.0000 -7.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000
  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  1.0000  2.0000  3.0000
  4.0000  5.0000  6.0000  7.0000
  1   8
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  6.6667
  5.3333  4.0000  2.6667  1.3333  0.0000
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  8.0000
  8.0000  8.0000  8.0000  8.0000  8.0000
 0.1000E+01 0.1000E+03 0.2000E+01 0.2000E+03 0.1000E+01 0.1000E+03 0.2000E+01
 0.2000E+03
```

```
 0.1000E+00 0.1000E+03 0.2000E+00 0.2000E+03 0.3000E+00 0.3000E+03 0.4000E+00
 0.4000E+00 0.5000E+03 0.5000E+00 0.6000E+03 0.6000E+00 0.7000E+03 0.7000E+00
 0.8000E+00 0.8000E+03 0.9000E+00 0.9000E+03 0.1000E+01 0.1000E+04 0.1100E+01
 0.1100E+01 0.1200E+04 0.1200E+01 0.1300E+04 0.1300E+01 0.1400E+04 0.1400E+01
 0.1000E+00 0.1000E+03 0.2000E+00 0.2000E+03 0.3000E+00 0.3000E+03 0.4000E+00
 0.4000E+00 0.5000E+03 0.5000E+00 0.6000E+03 0.6000E+00 0.7000E+03 0.7000E+00
 0.8000E+00 0.8000E+03 0.9000E+00 0.9000E+03 0.1000E+01 0.1000E+04 0.1100E+01
 0.1100E+01 0.1200E+04 0.1200E+01 0.1300E+04 0.1300E+01 0.1400E+04 0.1400E+01
$STOP
/*
//
```

PRINTED OUTPUT

```
NPW NW NP LW N6 M1        BK
 14  2 15  4  3  3 0.1000000E+00

PX
  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000 -1.0000 -2.0000 -3.0000
 -4.0000 -5.0000 -6.0000 -7.0000

PY
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000

PZ
  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  1.0000  2.0000  3.0000
  4.0000  5.0000  6.0000  7.0000

LL
  1  8

RH
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  6.6667
  5.3333  4.0000  2.6667  1.3333  0.0000

ZH
  0.0000  1.0000  2.0000  3.0000  4.0000  5.0000  6.0000  7.0000  8.0000  8.0000
  8.0000  8.0000  8.0000  8.0000  8.0000

ZWL
 0.1000E+01 0.1000E+03 0.2000E+01 0.2000E+03 0.1000E+01 0.1000E+03 0.2000E+01
 0.2000E+03

ZST
 0.1000E+00 0.1000E+03 0.2000E+00 0.2000E+03 0.3000E+00 0.3000E+03 0.4000E+00
 0.4000E+00 0.5000E+03 0.5000E+00 0.6000E+03 0.6000E+00 0.7000E+03 0.7000E+00
 0.8000E+00 0.8000E+03 0.9000E+00 0.9000E+03 0.1000E+01 0.1000E+04 0.1100E+01
 0.1100E+01 0.1200E+04 0.1200E+01 0.1300E+04 0.1300E+01 0.1400E+04 0.1400E+01

ZSP
 0.1000E+00 0.1000E+03 0.2000E+00 0.2000E+03 0.3000E+00 0.3000E+03 0.4000E+00
 0.4000E+00 0.5000E+03 0.5000E+00 0.6000E+03 0.6000E+00 0.7000E+03 0.7000E+00
 0.8000E+00 0.8000E+03 0.9000E+00 0.9000E+03 0.1000E+01 0.1000E+04 0.1100E+01
 0.1100E+01 0.1200E+04 0.1200E+01 0.1300E+04 0.1300E+01 0.1400E+04 0.1400E+01

VW
-0.1903517E+01 0.5888273E+00-0.1748591E+01 0.9552607E+00
 0.1903517E+01-0.5888273E+00 0.1748591E+01-0.9552607E+00
```

```
Rw
-0.1903517E+01 0.5888273E+00-0.1748591E+01 0.9552607E+00
 0.1903517E+01-0.5888273E+00 0.1748591E+01-0.9552607E+00

Zww
 0.1591979E+01-0.6232256E+03 0.1579350E+01 0.2619270E+03
-0.2787559E-01-0.1953976E+02-0.4543247E-01-0.6573979E+01
 0.1579350E+01 0.2619263E+03 0.1591979E+01-0.6232256E+03
-0.4541844E-01-0.6706268E+01-0.7403433E-01-0.4424469E+01
-0.2787559E-01-0.1953976E+02-0.4543247E-01-0.6573979E+01
 0.1591979E+01-0.6232256E+03 0.1579350E+01 0.2619270E+03
-0.4541844E-01-0.6706269E+01-0.7403433E-01-0.4424469E+01
 0.1579350E+01 0.2619263E+03 0.1591979E+01-0.6232256E+03

ZSw
 0.2252771E-01-0.2685371E+03 0.4502131E-01 0.4381604E+03

ZSS
 0.3137305E+02-0.8578586E+04 0.3044434E+02 0.8296912E+03

ZSS
 0.1604865E+02-0.4318449E+04 0.1576461E+02 0.1259779E+04

ZSS
 0.1211805E+00-0.2956441E+04 0.1059700E+00 0.1093889E+04

Zww
 0.1654465E+03 0.1905650E+03 0.1978964E+02 0.1650481E+03
-0.4175437E+01 0.2873213E+03-0.8640443E+01-0.3880681E+00
 0.1953658E+02 0.1637389E+03 0.5427119E+02-0.5051479E+03
-0.8529103E+01-0.1212996E+01 0.1820891E+02 0.4641051E+02
-0.4175326E+01 0.2873210E+03-0.8640484E+01-0.3880547E+00
 0.1654462E+03 0.1905645E+03 0.1978963E+02 0.1650484E+03
-0.8529051E+01-0.1212955E+01 0.1820892E+02 0.4641049E+02
 0.1953654E+02 0.1637389E+03 0.5427118E+02-0.5051479E+03

Vw
-0.1160264E+01-0.4321427E+01-0.2834294E+01-0.1397821E+01
 0.1160261E+01 0.4321425E+01 0.2834295E+01 0.1397821E+01

Rw
-0.1141512E+01-0.4348800E+01-0.2829595E+01-0.1392907E+01
 0.1141514E+01 0.4348797E+01 0.2829597E+01 0.1392907E+01

WIRE CURRENT
-0.1854982E-01-0.1562880E-01-0.7436503E-02-0.1311144E-01
 0.1854983E-01 0.1562880E-01 0.7436544E-02 0.1311148E-01

ESS
ESS= 0.3589605E+00-0.5239022E+00 U2= 0.2709360E+00-0.2319655E+00
SIG= 0.3639371E-02
```

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