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AFCRL-72-0179

MODAL ANALYSIS OF LOADED N-PORT SCATTERERS

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Contract No. F19628-68-C-0180
Project No. 5635
Task No. 563506
Work Unit No. 56350601

Scientific Report No. 16
March 1972

Contract Monitor: John F. McIlvenna
Microwave Physics Laboratory

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CONTENTS

PART ONE

GENERAL THEORY

I. INTRODUCTION

An N-port loaded scatterer is one having N ports (or terminal pairs) to which an N-port load network is connected [1,2]. The electromagnetic scattering characteristics of the system depend upon the load network as well as the scatterer itself. This report gives methods for determining the modes of a loaded scatterer and for using them to obtain a modal solution for the scattered field. Also included is the concept of modal resonance and synthesis. The theory uses N-port characteristic modes, which are analogous to those defined for material bodies [3,4,5]. The procedures used are similar to those developed for continuously loaded scatterers [6].

The theory can be formulated equally well in terms of either open-circuit or short-circuit network parameters. For a particular problem one representation may seem more natural than the other, but both lead to the same solution. Computationally, the short-circuit parameters are obtained more directly from a subsectional moment solution than the open-circuit parameters [7]. When approximations are made in the theory, the two representations may lead to different results.

II. FORMULATION OF THE PROBLEM

A loaded scatterer is basically two N-port networks connected together. The load network is passive, and its terminal characteristics can be represented by its N-port impedance matrix $[Z_L]$. The scatterer, illuminated by an impressed electric field $\underset{\sim}{E}^i$, is an active network and its terminal characteristics can be represented by a Thévenin equivalent circuit [8]. This equivalent consists of an N-port impedance matrix $[Z_S]$ obtained by removing the excitation (impressed field), plus series voltage sources at each port equal to the open circuit port voltages V_n^{oc} which exist when all ports are open circuited. Figure 1 shows this Thévenin equivalent connected to the load network. The terminal equation is

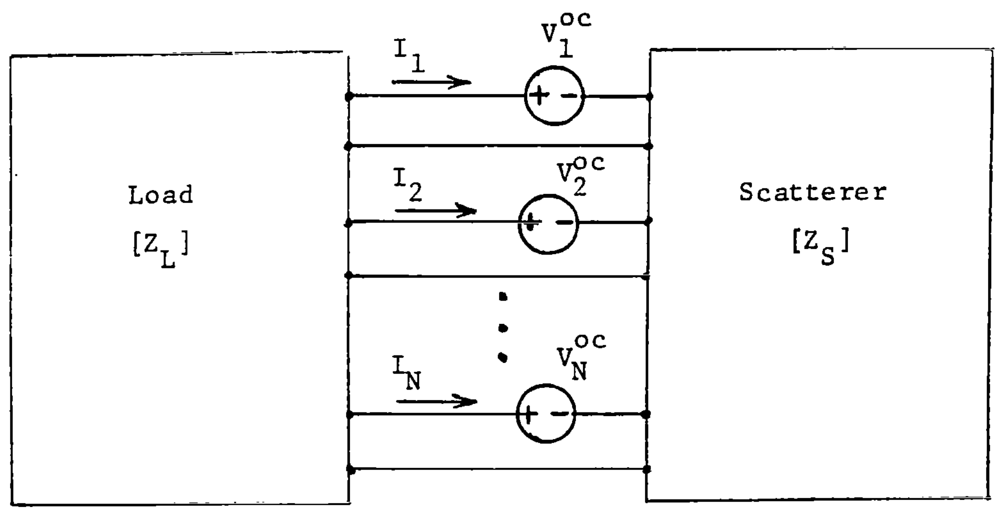


Fig. 1. Thévenin equivalent of the illuminated scatterer connected to the load network.

$$\vec{V}^{oc} = -\,[Z_S + Z_L]\vec{I} \tag{1}$$

where $\vec{V}^{oc}$ and $\vec{I}$ are the column matrices (vectors) of the voltage sources and port currents, respectively, with reference conditions as shown in Fig. 1. We shall consistently choose the reference condition for current to be into the positive voltage terminals for the scatterer and out of the positive voltage terminals for the load.

The scattered electric field $\mathbf{E}^s$ can be written as the superposition

$$\mathbf{E}^s = \mathbf{E}_0^{oc} + \sum_{n=1}^{N} I_n\, \mathbf{E}_n^{oc} \tag{2}$$

Here $\mathbf{E}_0^{oc}$ is the field scattered when all ports are open circuited, and $\mathbf{E}_n^{oc}$ is the field radiated when a unit current exists at port n and all other ports are open circuited. In matrix form we can write (2) as

$$\mathbf{E}^s = \mathbf{E}_0^{oc} + \tilde{\mathbf{E}}^{oc}\,\vec{I} \tag{3}$$

where $\tilde{\mathbf{E}}^{oc}$ is the row matrix of the $\mathbf{E}_n^{oc}$, $n=1,2,\ldots,N$. Equation (1) can be solved for the port currents as

$$\vec{I} = -\,[Z_S + Z_L]^{-1}\,\vec{V}^{oc} \tag{4}$$

Substituting this into (3), we obtain

$$\mathbf{E}^s = \mathbf{E}_0^{oc} - \tilde{\mathbf{E}}^{oc}\,[Z_S + Z_L]^{-1}\,\vec{V}^{oc} \tag{5}$$

The current $\mathbf{J}$ on the scatterer must also be a superposition of the form (5), or

$$\mathbf{J} = \mathbf{J}_0^{oc} - \tilde{\mathbf{J}}^{oc}[Z_S + Z_L]^{-1}\,\vec{V}^{oc} \tag{6}$$

Here $\mathbf{J}_0^{oc}$ is the current induced on the scatterer by $\mathbf{E}^i$ when all ports are open circuited, and $\tilde{\mathbf{J}}^{oc}$ is the row matrix with elements $\mathbf{J}_n^{oc}$, the current on the scatterer when a unit current exists at port n and all other ports are open circuited.

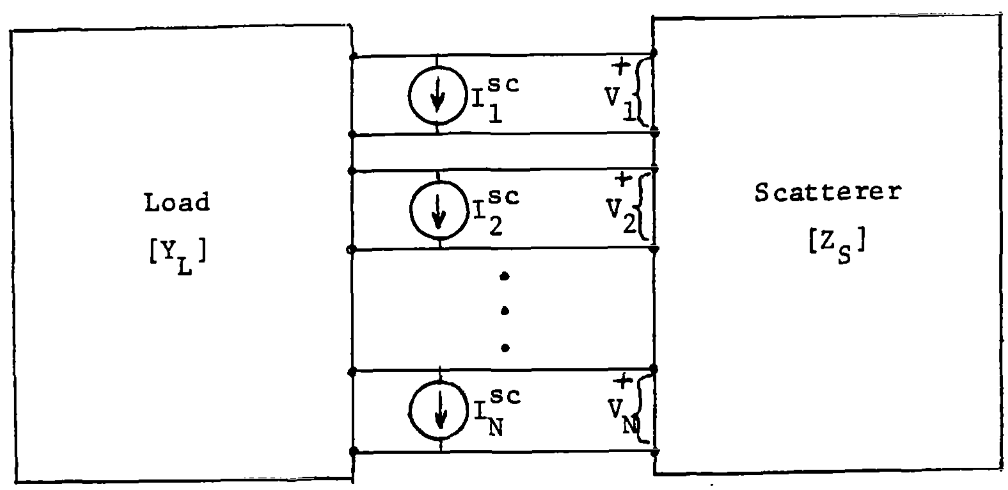


Fig. 2. Norton equivalent of the illuminated scatterer connected to the load network.

The dual short circuit formulation is obtained by representing the load by its N-port admittance matrix $[Y_L]$ and the illuminated scatterer by its Norton equivalent circuit [8]. This equivalent consists of the N-port admittance matrix $[Y_S]$ obtained by removing the excitation (impressed field), plus shunt current sources at each port equal to the short circuit port currents I_n^{sc} which exist when all ports are short circuited. Figure 2 shows this Norton equivalent connected to the load network. The terminal equation is

$$\vec{I}^{sc} = - [Y_S + Y_L]\vec{V} \tag{7}$$

where $\vec{I}^{sc}$ and $\vec{V}$ are the column matrices of the current sources and port voltages, respectively, with reference conditions as shown in Fig. 2.

The scattered electric field $\underset{\sim}{E}^s$ can be written as the superposition

$$\underset{\sim}{E}^s = \underset{\sim}{E}_0^{sc} + \sum_{n=1}^{N} V_n \underset{\sim}{E}_n^{sc} \tag{8}$$

where $\underset{\sim}{E}_0^{sc}$ is the field scattered when all ports are short circuited, and $\underset{\sim}{E}_n^{sc}$ is the field radiated when a unit voltage exists at port n and all other ports are short circuited. In matrix form (8) can be expressed as

$$\underset{\sim}{E}^s = \underset{\sim}{E}_0^{sc} + \underset{\sim}{\tilde{E}}^{sc} \vec{V} \tag{9}$$

where $\underset{\sim}{\tilde{E}}^{sc}$ is the row matrix of the $\underset{\sim}{E}_n^{sc}$, n=1,2,...,N. Equation (7) can be solved for the port voltages as

$$\vec{V} = - [Y_S + Y_L]^{-1} \vec{I}^{sc} \tag{10}$$

Substituting this into (9), we have

$$\underset{\sim}{E}^s = \underset{\sim}{E}_0^{sc} - \underset{\sim}{\tilde{E}}^{sc}[Y_S + Y_L]^{-1} \vec{I}^{sc} \tag{11}$$

Finally, the current $\underset{\sim}{J}$ on the scatterer must also be a superposition of the form (11), or

$$\underset{\sim}{J} = \underset{\sim}{J}_0^{sc} - \underset{\sim}{\tilde{J}}^{sc}[Y_S + Y_L]^{-1} \vec{I}^{sc} \tag{12}$$

Here $\underset{\sim}{J}_0^{sc}$ is the current induced on the scatterer when all ports are short circuited, and $\tilde{\underset{\sim}{J}}^{sc}$ is the row matrix with elements $\underset{\sim}{J}_n^{sc}$, the current when a unit voltage exists at port n and all other ports are short circuited.

III. N-PORT CHARACTERISTIC MODES

The characteristic modes for a loaded N-port system are defined in a manner analogous to those for continuously loaded bodies [6]. Both $[Z_S]$ and $[Z_L]$ are assumed symmetric. Hence their sum is symmetric and can be expressed in terms of real and imaginary parts as

$$[Z] = [Z_S + Z_L] = [R] + j[X] \tag{13}$$

where

$$[R] = \frac{1}{2}[Z + Z^*] \tag{14}$$

$$[X] = \frac{1}{2j}[Z - Z^*] \tag{15}$$

The characteristic modes of the N-port system are defined by the weighted eigenvalue equation

$$[Z]\vec{I}_n = (1+j\lambda_n)[R]\vec{I}_n \tag{16}$$

where $\vec{I}_n$ are the eigenvectors and $(1+j\lambda_n)$ are the eigenvalues. Substituting from (13) into (16), and cancelling the common $[R]\vec{I}_n$ terms, we have

$$[X]\vec{I}_n = \lambda_n[R]\vec{I}_n \tag{17}$$

This is a real symmetric eigenvalue equation. Hence, all eigenvalues λ_n are real and all eigenvectors $\vec{I}_n$ may be chosen real. More generally, the $\vec{I}_n$ are equiphasal, that is, a real vector times a complex constant. In (16), we note that $[R]\vec{I}_n$ is real when $\vec{I}_n$ is real, and $(1+j\lambda_n)$ is just a complex constant. Hence, $[Z]\vec{I}_n$, which can be viewed as the voltage sources of Fig. 1 which produce the mode currents, are also equiphasal.

The matrix [R] is normally positive definite, since $\tilde{\vec{I}}^*[R]\vec{I}$ is the time-average power radiated and/or dissipated by the system. For convenience, we normalize the mode currents so that they deliver unit power, that is,

$$\tilde{I}_n^*[R]\vec{I}_n = 1 \tag{18}$$

The usual equations of orthonormality of eigenvectors can now be written as

$$\tilde{I}_m^* [R]\vec{I}_n = \delta_{mn} \tag{19}$$

$$\tilde{I}_m^* [X]\vec{I}_n = \delta_{mn}\lambda_n \tag{20}$$

$$\tilde{I}_m^* [Z]\vec{I}_n = \delta_{mn}(1+j\lambda_n) \tag{21}$$

where δ_{mn} is the Kronecker delta. Because the $\vec{I}_n$ are real or equiphasal, the orthogonality relationships remain valid even without conjugation of the first vector. For loss-free systems, all power input is radiated, and (19) leads to an orthogonality relationship for radiation fields. In fact, the port modes have many properties in common with the modes of material bodies [4,5].

The dual formulation in terms of admittance matrices leads to a set of characteristic voltage modes. Again assume $[Y_S]$ and $[Y_L]$ symmetric and let

$$[Y] = [Y_S + Y_L] = [G] + j[B] \tag{22}$$

where [G] and [B] are given by equations analogous to (14) and (15). Define the characteristic voltages of the N-port system by the weighted eigenvalue equation

$$[Y]\vec{V}_n = (1+j\mu_n)[G]\vec{V}_n \tag{23}$$

Substituting from (22) into (23), and cancelling the common $[G]\vec{V}_n$ terms we have

$$[B]\vec{V}_n = \mu_n[G]\vec{V}_n \tag{24}$$

Again this is a real symmetric eigenvalue equation, hence all eigenvalues μ_n are real and all eigenvectors $\vec{V}_n$ are equiphasal. Analogous to the previous case, (23) implies that $[Y]\vec{V}_n$, which can be viewed as the current sources of Fig. 2 which produce the mode voltages, are also equiphasal.

The matrix [G] is normally positive definite, since $\tilde{\vec{V}}^*[G]\vec{V}$ is the time-average power radiated and/or dissipated by the system. We normalize the mode voltages so that they deliver unit power, that is

$$\tilde{\vec{V}}_n^* [G]\vec{V}_n = 1 \tag{25}$$

The usual equations of orthonormality of eigenvectors can now be written as

$$\tilde{\vec{V}}_m^*[G]\vec{V}_n = \delta_{mn} \tag{26}$$

$$\tilde{\vec{V}}_m^*[B]\vec{V}_n = \delta_{mn}\mu_n \tag{27}$$

$$\tilde{\vec{V}}_m^*[Y]\vec{V}_n = \delta_{mn}(1+j\mu_n) \tag{28}$$

Because the $\vec{V}_n$ are real or equiphasal, the orthogonality relationships remain valid even without conjugation of the first vector. For a given load, the μ_n and $\vec{V}_n$ are different from the λ_n and $\vec{I}_n$, since $[Y] \neq [Z]^{-1}$. However, the eigenvoltages $\vec{V}_n$ of the $[Y_L] = 0$ case are related to the eigencurrents $\vec{I}_n$ of the $[Z_L] = 0$ case by $\vec{V}_n = [R]\vec{I}_n$, and the eigenvalues by $\mu_n = -\lambda_n$. This can be deduced from Figs. 1 and 2 by observing that the cases $[Z_L] = 0$ and $[Y_L] = 0$, respectively, reduce to the same circuit.

IV. MODAL SOLUTIONS

Modal solutions for the port currents are obtained in the usual way by using the mode currents as a basis for the port currents. To be explicit, let the port current be represented as

$$\vec{I} = \sum_{n=1}^{N} \alpha_n \vec{I}_n \tag{29}$$

where the α_n are coefficients to be determined. Substitute (29) into (1), and obtain

$$\vec{V}^{oc} = - \sum_{n=1}^{N} \alpha_n [Z_S + Z_L]\vec{I}_n \tag{30}$$

Now take the scalar product of (30) with each $\tilde{\vec{I}}_m^*$ in turn to obtain the set of equations

$$\tilde{I}_m^* \vec{V}^{oc} = - \sum_{n=1}^{N} \alpha_n \tilde{I}_m^* [Z_S + Z_L] \vec{I}_n \tag{31}$$

m=1,2,...,N. Because of the orthogonality relationship (21), only the n=m term remains, and (31) reduces to

$$\tilde{I}_n^* \vec{V}^{oc} = - \alpha_n (1+j\lambda_n) \tag{32}$$

Substituting these values of α_n into (29), we have the modal solution for the port currents

$$\vec{I} = - \sum_{n=1}^{N} \frac{\tilde{I}_n^* \vec{V}^{oc}}{1+j\lambda_n} \vec{I}_n \tag{33}$$

If the mode currents are not normalized, the factors $1 + j\lambda_n$ should be replaced by $(1+j\lambda_n)\tilde{I}_n^*[R]\vec{I}_n$.

That part of the scattered field controlled by the load can also be expressed in spectral form. The total scattered field is obtained by substituting (33) into (3), giving

$$\underset{\sim}{E}^s = \underset{\sim}{E}_0^{oc} - \sum_{n=1}^{N} \frac{\tilde{I}_n^* \vec{V}^{oc}}{1+j\lambda_n} \underset{\sim}{E}(\vec{I}_n) \tag{34}$$

Here $\underset{\sim}{E}(\vec{I}_n)$ is the field radiated when $\vec{I}_n$ exists at the scatterer ports. A modal solution for the total current on the scatterer is obtained in the same way. The result is

$$\underset{\sim}{J} = \underset{\sim}{J}_0^{oc} - \sum_{n=1}^{N} \frac{\tilde{I}_n^* \vec{V}^{oc}}{1+j\lambda_n} \underset{\sim}{J}(\vec{I}_n) \tag{35}$$

Here $\underset{\sim}{J}(\vec{I}_n)$ is the current on the scatterer when $\vec{I}_n$ exists at its ports. Again, if unnormalized mode currents are used, the factors $1 + j\lambda_n$ in (34) and (35) should be replaced by $(1+j\lambda_n)\tilde{I}_n^*[R]\vec{I}_n$.

Similar modal solutions in terms of the port mode voltages can be found in terms of short circuit parameters. Hence, analogous to (33) we have for the port voltages

$$\vec{V} = - \sum_{n=1}^{N} \frac{\tilde{V}_n^* \vec{I}^{sc}}{1+j\mu_n} \vec{V}_n \tag{36}$$

Analogous to (34) we have a modal solution for the scattered field

$$\underset{\sim}{E}^{s} = \underset{\sim}{E}_0^{sc} - \sum_{n=1}^{N} \frac{\tilde{V}_n^{*} \vec{I}^{sc}}{1+j\mu_n} \underset{\sim}{E}(\vec{V}_n) \tag{37}$$

Here $\underset{\sim}{E}(\vec{V}_n)$ is the field radiated when $\vec{V}_n$ exists at the scatterer ports. Analogous to (35) we have a modal solution for the total current on the scatterer.

$$\underset{\sim}{J} = \underset{\sim}{J}_0^{sc} - \sum_{n=1}^{N} \frac{\tilde{V}_n^{*} \vec{I}^{sc}}{1+j\mu_n} \underset{\sim}{J}(\vec{V}_n) \tag{38}$$

Here $\underset{\sim}{J}(\vec{V}_n)$ is the current on the scatterer when V_n exists at its ports. If unnormalized mode voltages are used, the factors $1+j\mu_n$ in (36) to (38) should be replaced by $(1+j\mu_n)\tilde{V}_n^{*}[G]\vec{V}_n$.

V. MODAL RESONANCE

We can view the mode currents $\vec{I}_n$ as being excited by voltage sources $\vec{V}^{oc} = -[Z]\vec{I}_n$ in the circuit of Fig. 1. The power delivered by the sources is

$$P = \tilde{I}_n^{*}[Z]\vec{I}_n = 1+j\lambda_n \tag{39}$$

where the last equality follows from (21). A mode current is said to be in resonance when its eigenvalue λ_n is zero. Hence, at resonance, the reactive power λ_n is zero and the driving voltage is in phase with the current.

Similarly, we can view the mode voltages $\vec{V}_n$ as being excited by the current sources $\vec{I}^{sc} = -[Y]\vec{V}_n$ in the circuit of Fig. 2. The power delivered by the sources is

$$P = \tilde{V}_n[Y^{*}]\vec{V}_n^{*} = 1-j\mu_n \tag{40}$$

where the last equality follows from (28). A mode voltage is said to be in resonance when its eigenvalue μ_n is zero. Hence, at resonance, the reactive power $-\mu_n$ is zero and the driving current is in phase with the voltage.

Any equiphase current vector $\vec{I}$ can be resonated by choosing the proper load $[Z_L]$. This means that a desired $\vec{I}$ can be made an eigencurrent with eigenvalue $\lambda = 0$. We see from (17) that this requires

$$[X]\vec{I} = [X_S + X_L]\vec{I} = 0 \tag{41}$$

or

$$[X_L]\vec{I} = -[X_S]\vec{I} \tag{42}$$

This condition can always be satisfied by the diagonal load matrix

$$[X_L] = \begin{bmatrix} X_{L1} & 0 & 0 \ldots \\ 0 & X_{L2} & 0 \ldots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & X_{LN} \end{bmatrix} \tag{43}$$

Explicitly, the solution is then

$$X_{Li} = \frac{-1}{I_i}([X_S]\vec{I})_i \tag{44}$$

where $([X_S]\vec{I})_i$ denotes the i-th component of the column matrix $[X_S]\vec{I}$. Loads more complicated than (43) can also be used, and perhaps the nonuniqueness of the solution can be used for optimization purposes.

Similarly, any equiphase voltage vector $\vec{V}$ can be resonated by choosing the proper load $[Y_L]$. The desired $\vec{V}$ thereby becomes an eigenvoltage with eigenvalue $\mu = 0$. Analogous to (42), this requires

$$[B_L]\vec{V} = -[B_S]\vec{V} \tag{45}$$

which can always be satisfied by a diagonal load matrix with elements B_{Li}. The solution, analogous to (44), is

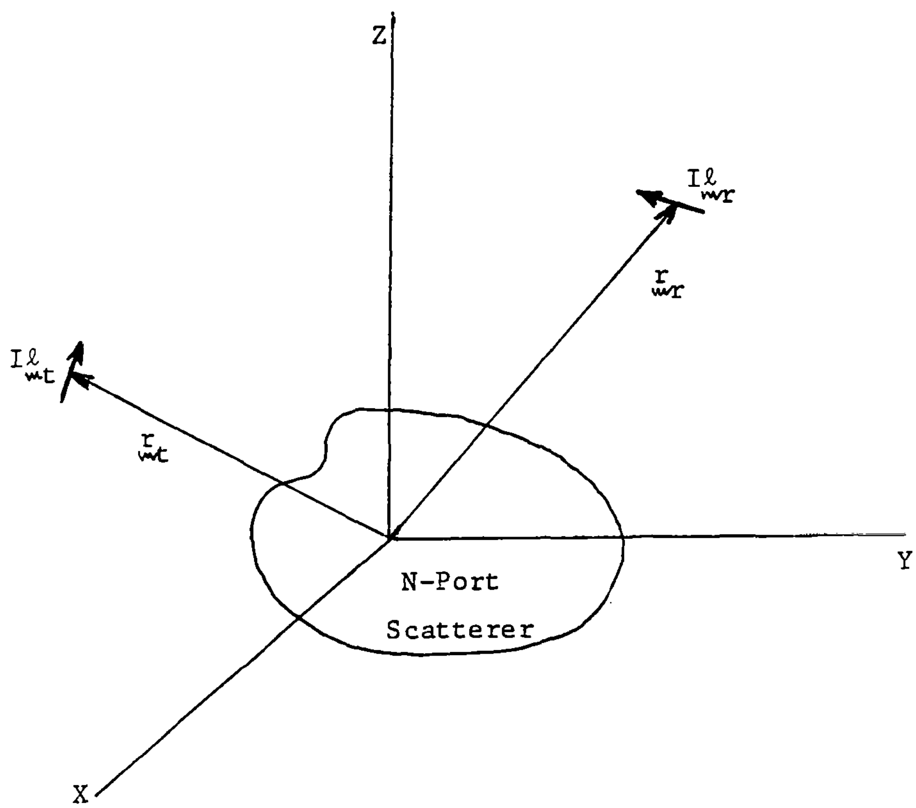


Fig. 3. The dipole scattering problem.

$$B_{Li} = \frac{-1}{V_i} ([B_S]\vec{V})_i \tag{46}$$

Again this solution is not unique, and nondiagonal loads $[B_L]$ can be used to resonate $\vec{V}$ if desired.

Resonating a desired equiphase $\vec{I}$ according to (42) or a desired equiphase $\vec{V}$ according to (45) will be called modal synthesis. In many cases when a current $\vec{I}$ is resonated all terms on the right-hand side of (34) are small except that involving $\underset{\sim}{E}(\vec{I})$. Then

$$\underset{\sim}{E}^s \approx -(I*\vec{V}^{oc})\underset{\sim}{E}(\vec{I}) \tag{47}$$

and we have an approximate procedure for synthesis of scattering patterns. This procedure usually gives good results if the scattered field from the open circuited scatterer is small and the scatterer is not electrically large. The analogous result for the dual formulation is

$$\underset{\sim}{E}^s \approx -(V*\vec{I}^{sc})\underset{\sim}{E}(\vec{V}) \tag{48}$$

where $\vec{V}$ is the equiphase resonated voltage. In this case the approximation is usually good if the scattered field from the short circuited scatterer is small and the scatterer is not electrically large. The remaining problem is to determine the desired $\vec{I}$ or $\vec{V}$ to resonate.

VI. DIPOLE SCATTERING

The basic radar problem consists of a transmitter, a receiver, and an N-port loaded scatterer. To remove the transmitting and receiving antennas from consideration, we take them to be terminals in empty space. A current impressed across either the transmitter or receiver terminals is then just an electric dipole. Figure 3 represents the general problem, where $I\underset{\sim}{\ell}_t$ is a current element across the transmitter terminals at position $\underset{\sim}{r}_t$, and $I\underset{\sim}{\ell}_r$ is a current element across the receiver terminals at position $\underset{\sim}{r}_r$. The loaded N-port scatterer is in the vicinity of the coordinate origin. The impressed field is now just the free-space field of $I\underset{\sim}{\ell}_t$, and the scattered field at the receiver terminals is proportional to $\underset{\sim}{E}^s \cdot I\underset{\sim}{\ell}_r$. From (5), we have

$$\underset{\sim}{E}^{s} \cdot \underset{\sim}{I\ell}_{r} = \underset{\sim}{E}_{0}^{oc} \cdot \underset{\sim}{I\ell}_{r} - \tilde{\underset{\sim}{E}}^{oc} \cdot \underset{\sim}{I\ell}_{r} \, [Z_{S} + Z_{L}]^{-1} \vec{V}_{t}^{oc} \qquad (49)$$

The subscript t has been added to V_t^{oc} to indicate that it is due to $\underset{\sim}{I\ell}_t$. We next use the reciprocity theorem [9]

$$\iiint \underset{\sim}{E}^{a} \cdot \underset{\sim}{J}^{b} d\tau = \iiint \underset{\sim}{E}^{b} \cdot \underset{\sim}{J}^{a} d\tau \qquad (50)$$

to obtain a relationship between $\tilde{\underset{\sim}{E}}^{oc} \cdot \underset{\sim}{I\ell}_r$ and $\vec{V}_r^{oc}$, the vector of port voltages due to excitation by $\underset{\sim}{I\ell}_r$. For this, let $\underset{\sim}{J}^a$ be $\underset{\sim}{I\ell}_r$ radiating in the presence of the scatterer with all ports open circuited, and let $\underset{\sim}{E}^a$ be the field produced. Let $\underset{\sim}{J}^b$ be a unit current applied at port i, also in the presence of the scatterer with all ports open circuited, and let $\underset{\sim}{E}^b$ be the field produced. Because currents induced on the scatterer do not enter into the reciprocity integrals, (50) reduces to

$$(V_r^{oc})_i = - \underset{\sim}{E}_i^{oc} \cdot \underset{\sim}{I\ell}_r \qquad (51)$$

where the subscripts i denote the i-th element of the vectors. The above procedure is applied to each port in turn, and the resulting set (51) written as the vector equation

$$\vec{V}_r^{oc} = - \vec{\underset{\sim}{E}}^{oc} \cdot \underset{\sim}{I\ell}_r \qquad (52)$$

This relationship also applies with r replaced by t, that is, for transmitter excitation. Using the transpose of (52) in (49), we have

$$\underset{\sim}{E}^{s} \cdot \underset{\sim}{I\ell}_{r} = \underset{\sim}{E}_{0}^{oc} \cdot \underset{\sim}{I\ell}_{r} + \tilde{V}_{r}^{oc} \, [Z_{S} + Z_{L}]^{-1} \vec{V}_{t}^{oc} \qquad (53)$$

If $\underset{\sim}{I\ell}_r = \underset{\sim}{u}$, a unit vector, then (53) is an equation for the $\underset{\sim}{u}$ component of $\underset{\sim}{E}^s$ at the point $\underset{\sim}{r}_r$.

The open circuit voltage vectors $\vec{V}_t^{oc}$ and $\vec{V}_r^{oc}$ are the port voltages when the excitation is $\underset{\sim}{I\ell}_t$ and $\underset{\sim}{I\ell}_r$, respectively. These voltage vectors can be expressed in terms of any basis. In particular, we choose $\{[R]\vec{I}_n\}$ as the basis, where $\vec{I}_n$ are real modal currents, and let

$$(I_r^{sc})_i = \underset{\sim}{E}_i^{sc} \cdot \underset{\sim}{I\ell}_r \tag{59}$$

where subscripts i denote the i-th element of the vectors. The above procedure is applied to each port in turn, and the resulting set (59) written as the vector equation

$$\vec{I}_r^{sc} = \vec{\underset{\sim}{E}}^{sc} \cdot \underset{\sim}{I\ell}_r \tag{60}$$

This relationship also applies with r replaced by t. Using the transpose of (60) in (57) we have

$$\underset{\sim}{E}^s \cdot \underset{\sim}{I\ell}_r = \underset{\sim}{E}_0^{sc} \cdot \underset{\sim}{I\ell}_r - \tilde{\vec{I}}_r^{sc}[Y_S + Y_L]^{-1}\,\vec{I}_t^{sc} \tag{61}$$

If $\underset{\sim}{I\ell}_r = \underset{\sim}{u}$, a unit vector, then (61) is an equation for the $\underset{\sim}{u}$ component of $\underset{\sim}{E}^s$ at the point $\underset{\sim}{r}_r$.

Finally, we can express the short circuit current vectors $\vec{I}_t^{sc}$ and $\vec{I}_r^{sc}$ in terms of any basis. In particular we choose $\{[G]\vec{V}_n\}$ as the basis, where $\{\vec{V}_n\}$ are real modal voltages, and let

$$\vec{I}^{sc} = \sum_{n=1}^{N} \beta_n [G]\vec{V}_n \tag{62}$$

The $\{\vec{V}_n\}$ is an orthonormal set with respect to the weight matrix [G]. Hence, the components β_n are given by

$$\beta_n = \tilde{\vec{V}}_n \vec{I}^{sc} \tag{63}$$

Equations (62) and (63) apply to both t and r quantities. In the basis $\{\vec{I}_n\}$ the matrix $[Y_S + Y_L]$ becomes diagonal with elements $1+j\mu_n$. Hence, (61) reduces to

$$\underset{\sim}{E}^s \cdot \underset{\sim}{I\ell}_r = \underset{\sim}{E}_0^{sc} \cdot \underset{\sim}{I\ell}_r - \sum_{n=1}^{N} \frac{\beta_n^r \beta_n^t}{1+j\mu_n} \tag{64}$$

Here β_n^r and β_n^t are the components (63) when the excitation is $\underset{\sim}{I\ell}_r$ and $\underset{\sim}{I\ell}_t$, respectively.

VII. PLANE-WAVE SCATTERING

The solution for plane-wave scattering is obtained by letting the transmitter and receiver recede to infinity. The amplitudes of the current dipoles are adjusted to produce unit plane waves in the vicinity of the scatterer. Hence, the incident wave from $\mathbf{I\ell}_t$ becomes [7]

$$\mathbf{E}_t^i = \mathbf{u}_t \, e^{-j\mathbf{k}_t \cdot \mathbf{r}} \tag{65}$$

where $\mathbf{k}_t$ is a vector of magnitude k and direction that of $-\mathbf{r}_t$. The incident wave from $\mathbf{I\ell}_r$ is given by (65) with subscripts t replaced by r. Figure 4 illustrates the general plane-wave scattering problem. To produce the unit plane wave (65), the amplitude of $\mathbf{I\ell}_t$ must be

$$\mathbf{I\ell}_t = -\frac{4\pi r_t}{j\omega\mu} e^{jkr_t} \mathbf{u}_t \tag{66}$$

and similarly for $\mathbf{I\ell}_r$. It is also convenient to normalize radiation fields according to

$$\mathbf{E}(r,\theta,\phi) = \frac{-j\omega\mu}{4\pi r} e^{-jkr} \mathbf{F}(\theta,\phi) \tag{67}$$

The function $\mathbf{F}$ is called the field pattern on the radiation sphere.

The results for dipole scattering can now be rewritten for plane-wave scattering as follows. Corresponding to (53), we have

$$\mathbf{F}^s \cdot \mathbf{u}_r = \mathbf{F}_0^{oc} \cdot \mathbf{u}_r + \tilde{V}_r^{oc}[Z_S + Z_L]^{-1}\vec{V}_t^{oc} \tag{68}$$

where $\tilde{V}_r^{oc}$ and $\vec{V}_t^{oc}$ now result from unit plane waves. Similarly, the modal solution (56) becomes

$$\mathbf{F}^s \cdot \mathbf{u}_r = \mathbf{F}_0^{oc} \cdot \mathbf{u}_r + \sum_{n=1}^{N} \frac{\alpha_n^r \alpha_n^t}{1+j\lambda_n} \tag{69}$$

where the α_n^r and α_n^t result from unit plane waves. An alternative form of the solution can be obtained by using reciprocity. From (52), using (66) and (67), we have

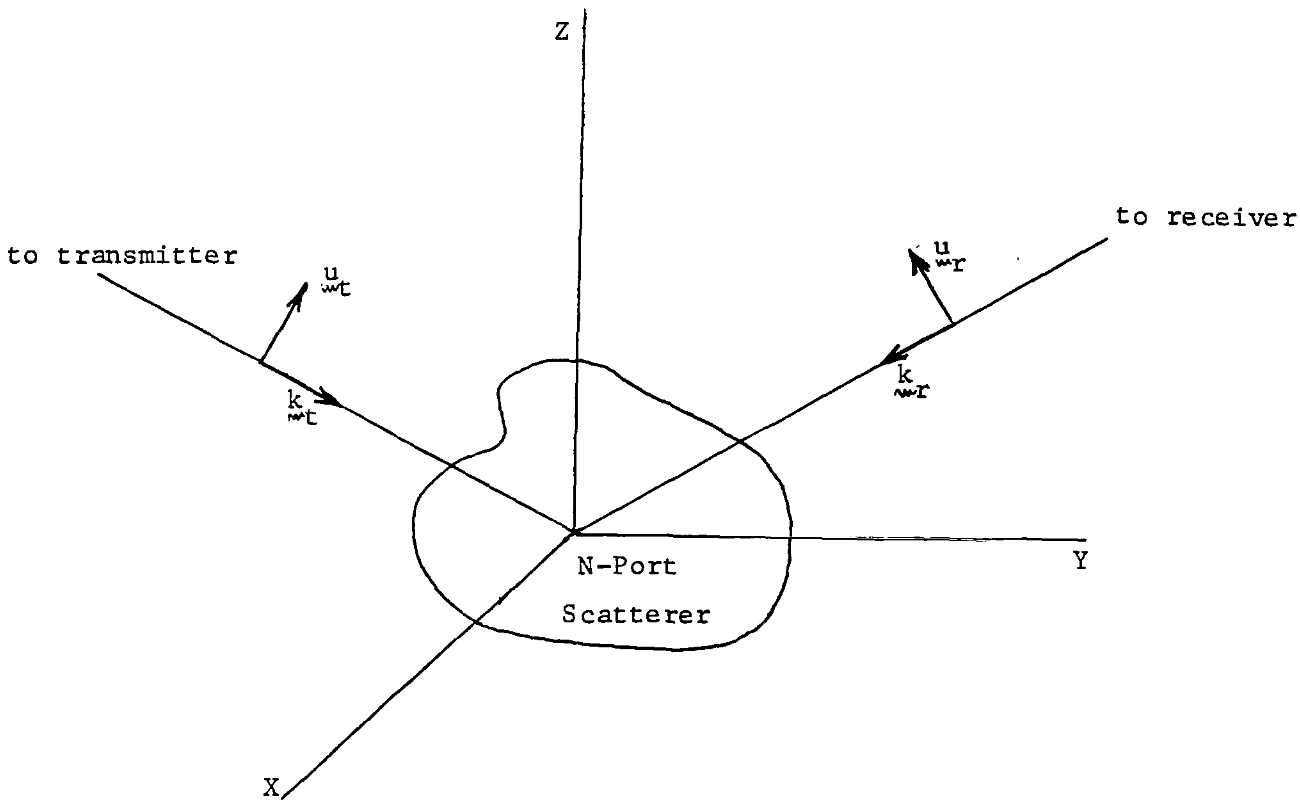


Fig. 4. The plane-wave scattering problem.

$$\vec{V}_r^{oc} = - \vec{F}_{\approx r}^{oc} \cdot \underset{\approx}{u}_r \tag{70}$$

and similarly for $\vec{V}_t^{oc}$. The $\underset{\approx}{F}$'s can be changed to a modal basis, with the result that

$$\alpha_n^r = - \underset{\approx}{F}(\vec{I}_n) \cdot \underset{\approx}{u}_r \tag{71}$$

Here $\underset{\approx}{F}(\vec{I}_n)$ is the mode pattern due to the real mode current $\vec{I}_n$. A similar result holds for α_n^t. We next define the tensor

$$\overset{\approx}{\mathcal{F}}^{oc} = \sum_{n=1}^{N} \frac{\underset{\approx}{F}(\vec{I}_n)\ \underset{\approx}{F}(\vec{I}_n)}{1+j\lambda_n} \tag{72}$$

which is analogous to Garbacz's bilinear scattering tensor [10]. Using (71) and (72) in (69), we have

$$\underset{\approx}{F}^s \cdot \underset{\approx}{u}_r = \underset{\approx}{F}_0^{oc} \cdot \underset{\approx}{u}_r + \underset{\approx}{u}_t \cdot \overset{\approx}{\mathcal{F}}^{oc} \cdot \underset{\approx}{u}_r \tag{73}$$

The radar cross section is given by

$$\sigma = \frac{\omega^2 \mu^2}{4\pi} \left| \underset{\approx}{F}^s \cdot \underset{\approx}{u}_r \right|^2 \tag{74}$$

where $\underset{\approx}{F}^s \cdot \underset{\approx}{u}_r$ can be computed from (68), (69), or (73).

Analogous results for the short circuit formulation can also be written. Dual to (68), we have from (61)

$$\underset{\approx}{F}^s \cdot \underset{\approx}{u}_r = \underset{\approx}{F}_0^{sc} \cdot \underset{\approx}{u}_r - \tilde{I}_r^{sc} [Y_S + Y_L]^{-1} \vec{I}_t^{sc} \tag{75}$$

where $\tilde{I}_r^{sc}$ and $\vec{I}_t^{sc}$ result from unit plane waves. Dual to (69), we have from (64)

$$\underset{\approx}{F}^s \cdot \underset{\approx}{u}_r = \underset{\approx}{F}_0^{sc} \cdot \underset{\approx}{u}_r - \sum_{n=1}^{N} \frac{\beta_n^r \beta_n^t}{1+j\mu_n} \tag{76}$$

where the β_n^r and β_n^t result from unit plane waves. Again we have an alternative form obtained by using reciprocity. From (60), using (66) and (67), we have

$$\vec{I}_r^{sc} = \underset{\sim}{F}_r^{sc} \cdot \underset{\sim}{u}_r \tag{77}$$

and similarly for $\vec{I}_t^{sc}$. In terms of the modal basis,

$$\beta_n^r = \underset{\sim}{F}(\vec{V}_n) \cdot \underset{\sim}{u}_r \tag{78}$$

where $\underset{\sim}{F}(\vec{V}_n)$ is the mode pattern due to the real mode voltage $\vec{V}_n$. A similar result holds for β_n^t. Next define the tensor

$$\overleftrightarrow{Y}^{sc} = \sum_{n=1}^{N} \frac{\underset{\sim}{F}(\vec{V}_n)\ \underset{\sim}{F}(\vec{V}_n)}{1+j\mu_n} \tag{79}$$

analogous to (72). Using (78) and (79) in (76), we have

$$\underset{\sim}{F}^s \cdot \underset{\sim}{u}_r = \underset{\sim}{F}_0^{sc} \cdot \underset{\sim}{u}_r - \underset{\sim}{u}_t \cdot \overleftrightarrow{Y}^{sc} \cdot \underset{\sim}{u}_r \tag{80}$$

Note that the $\overleftrightarrow{Y}^{sc}$ is not the same tensor as $\overleftrightarrow{Y}^{oc}$ of (73). If the radar cross section is desired, it is still given by (74), where $\underset{\sim}{F}^s \cdot \underset{\sim}{u}_r$ can be computed from (75), (76), or (80).

VIII. METHOD OF COMPUTATION

The parameters needed for the analysis can be calculated by the method of moments in a straight forward manner [7]. For a perfectly conducting scatterer over the surface S, the basic equation is

$$Z(\underset{\sim}{J}) = \underset{\sim}{E}_{tan}^i \tag{81}$$

where $\underset{\sim}{E}_{tan}^i$ is the tangential component of the incident field on S and -Z is the operator relating $\underset{\sim}{J}$ to the tangential $\underset{\sim}{E}$ field it produces on S. We approximate $\underset{\sim}{J}$ by the superposition

$$\underset{\sim}{J} = \sum_{n=1}^{M} I_n \underset{\sim}{J}_n \tag{82}$$

where $\underset{\sim}{J}_n$ are expansion functions and I_n are constants to be determined. The number of expansion functions M must be greater than or equal to the number of ports N, that is, $M \geq N$. Equation (81) is approximated by a matrix equation in the usual way [7]. The result is

$$[Z]\vec{I} = \vec{V} \tag{83}$$

where $\vec{I}$ is the current vector with elements I_n, $\vec{V}$ is the excitation vector with elements

$$V_n = \iint_S \underset{\sim}{J}_n \cdot \underset{\sim}{E}^i \, ds \tag{84}$$

and [Z] is the generalized impedance matrix with elements

$$Z_{mn} = \iint_S \underset{\sim}{J}_m \cdot Z(\underset{\sim}{J}_n)\, ds \tag{85}$$

The solution to (83) can be represented by

$$\vec{I} = [Z]^{-1}\vec{V} = [Y]\vec{V} \tag{86}$$

The approximate current on S is then given by (82), where I_n are the elements of $\vec{I}$.

To simplify the theory, we choose all $\underset{\sim}{J}_m = 0$ at port n except $\underset{\sim}{J}_n$, which is normalized so that I_n equals the current at port n. This implies that expansion functions n=1,2,...,N are port currents, and n=N+1, N+2,...,M are nonport currents. The matrices of (86) are then partitioned as

$$\begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} [Y_{11}] & [Y_{12}] \\ [Y_{21}] & [Y_{22}] \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix} \tag{87}$$

where subscripts 1 denote port elements and subscripts 2 denote nonport elements. Note that $\vec{I}_1$ is now simply the port current. When the excitation is concentrated at the ports only, $\vec{V}_1$ is the port voltage and the scatterer N-port matrices are simply

$$[Y_S] = [Y_{11}] \tag{88}$$

$$[Z_S] = [Y_{11}]^{-1} \tag{89}$$

When the excitation is due to an external impressed field, the short circuit port current is

$$\vec{I}_1 = [Y_{11}]\vec{V}_1 + [Y_{12}]\vec{V}_2 \tag{90}$$

Hence, when the excitation is due to $\underset{\sim}{I\ell}_r$, the short circuit port current is

$$\vec{I}_r^{sc} = [Y_{11}]\vec{V}_{1r} + [Y_{12}]\vec{V}_{2r} \tag{91}$$

and when the excitation is due to $\underset{\sim}{I\ell}_t$, the short circuit port current is

$$\vec{I}_t^{sc} = [Y_{11}]\vec{V}_{1t} + [Y_{12}]\vec{V}_{2t} \tag{92}$$

The subscripts r and t have been added to $\vec{V}_1$ and $\vec{V}_2$ to denote excitation by $\underset{\sim}{I\ell}_r$ and $\underset{\sim}{I\ell}_t$, respectively. Finally, the scattering due to the short-circuited scatterer is given by the formula

$$\underset{\sim}{E}_0^{sc} \cdot \underset{\sim}{I\ell}_r = \tilde{\vec{V}}_r[Y]\vec{V}_t \tag{93}$$

where [Y] is the complete matrix of (87).

To obtain the open circuit port voltages, we excite the scatterer simultaneously with an external impressed field plus port voltages sources such that $\vec{I}_1 = 0$. In this case (87) becomes

$$\begin{bmatrix} 0 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} [Y_{11}] & [Y_{12}] \\ [Y_{21}] & [Y_{22}] \end{bmatrix} \begin{bmatrix} \vec{V}^{oc} + \vec{V}_1 \\ \vec{V}_2 \end{bmatrix} \tag{94}$$

where $\vec{V}^{oc}$ is due to port voltage sources, and $\vec{V}_1$ and $\vec{V}_2$ are due to the external impressed field. Solving (94) for $\vec{V}^{oc}$, we have

$$\vec{V}^{oc} = -\vec{V}_1 - [Y_{11}]^{-1}[Y_{12}]\vec{V}_2 \tag{95}$$

Hence, when the excitation is due to $\underset{\sim}{I\ell}_r$, the open circuit port voltage is

$$\vec{V}_r^{oc} = -\vec{V}_{1r} - [Y_{11}]^{-1}[Y_{12}]\vec{V}_{2r} \tag{96}$$

and when the excitation is due to $I\underset{\sim}{\ell}_t$, the open circuit port voltage is

$$\vec{V}_t^{oc} = -\vec{V}_{1t} - [Y_{11}]^{-1}[Y_{12}]\vec{V}_{2t} \tag{97}$$

Again the subscripts r and t denote excitation by $I\underset{\sim}{\ell}_r$ and $I\underset{\sim}{\ell}_t$, respectively. Finally, the scattering due to the open-circuited scatterer is obtained as follows. Eliminate $\vec{V}^{oc}$ from (94) and solve for $\vec{I}_2$, obtaining

$$\vec{I}_2 = [Y_{22} - Y_{21}\, Y_{11}^{-1}\, Y_{12}]\vec{V}_2 \tag{98}$$

Let $\vec{V}_2$ be $\vec{V}_{2t}$ and form $\tilde{V}_{2r}\vec{I}_2$. By reciprocity the result is equal to $\underset{\sim}{E}_0^{oc} \cdot I\underset{\sim}{\ell}_r$. Hence,

$$\underset{\sim}{E}_0^{oc} \cdot I\underset{\sim}{\ell}_r = \tilde{V}_{2r}[Y_{22} - Y_{21}Y_{11}^{-1}\, Y_{12}]\vec{V}_{2t} \tag{99}$$

Note that the central matrix on the right hand side must be the generalized admittance matrix for the open-circuited scatterer. It should not be confused with the port admittance matrix $[Y_S]$. Equations (93) and (94) remain valid if the left hand sides are replaced by $\underset{\sim}{F}_0^{sc} \cdot \underset{\sim}{u}_r$ and $\underset{\sim}{F}_0^{oc} \cdot \underset{\sim}{u}_r$, respectively.

IX. EXAMPLES

Programs for computing the characteristic port modes of wire objects, and for using them in modal solutions, are given in Part II. We here consider examples of their application to a wire triangle with two cross wires, as shown in Fig. 5. The four points at which the wires cross the z axis are input ports, labeled (1), (2), (3) and (4) in Fig. 5. The tip angle (at z=0) is 30°, the wire diameter is a/100. All computations are made using 38 triangle functions in a Galerkin solution, as described in previous work [4,10].

The port impedance matrix $[Z_S] = [R] + j[X]$ is obtained from the generalized impedance matrix using available programs [10]. The eigencurrents $\vec{I}_n$ and eigenvalues λ_n are computed for the unloaded (short-circuited) wire triangle from (17)

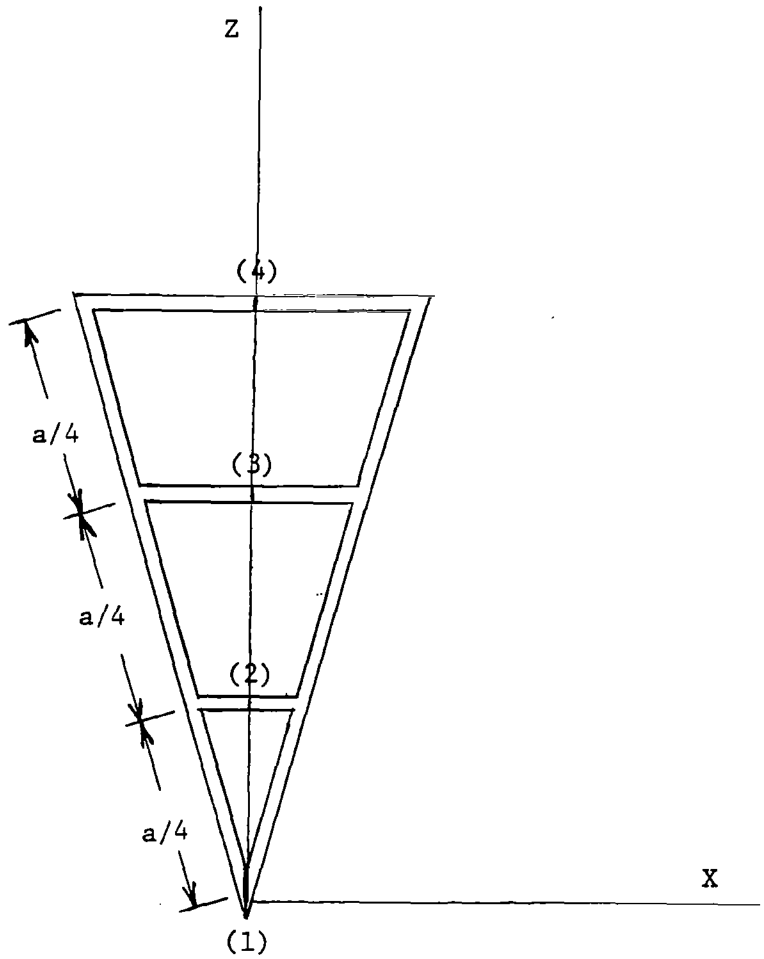


Fig. 5. Wire triangle with cross wires, tip angle = 30°, a = one wavelength, wire diameter = a/100.

using a Jacobi method [11]. The results are tabulated in Table 1, normalized so that the maximum port current is unity. The eigenvoltages $\vec{V}_n$ and eigenvalues λ_n are computed for the unloaded (open-circuited) wire triangle from (23). These results are tabulated in Table 2, normalized so that the maximum port voltage is unity. Note that the λ_n of Table 1 are related to the μ_n of Table 2 by $\lambda_n = -\mu_n$. It can also be shown that the $\vec{I}_n$ of Table 1 are related to the $\vec{V}_n$ of Table 2 by $\vec{I}_n = c[G]\vec{V}_n$, where c is a normalizing constant. It should be emphasized that the two unloaded cases, Tables 1 and 2, are really for two different loading conditions, $[Z_L] = 0$ and $[Y_L] = 0$, respectively.

Table 1. Port mode currents $\vec{I}_n$ for the unloaded wire triangle, $[Z_L] = 0$

n	λ_n	port (1)	port (2)	port (3)	port (4)
1	-0.1552	-0.1338	0.4326	0.8419	1.0000
2	-10.12	-0.6078	1.0000	0.8054	-0.6458
3	-50.54	1.0000	-0.5374	0.0137	0.0740
4	816.4	0.5441	1.0000	-0.2839	0.0778

Table 2. Port mode voltages $\vec{V}_n$ for the unloaded wire triangle, $[Y_L] = 0$.

n	μ_n	port (1)	port (2)	port (3)	port (4)
1	0.1552	-0.0205	0.1158	0.6425	1.0000
2	10.12	0.1904	0.2524	1.0000	-0.9256
3	50.54	1.0000	-0.2887	0.7884	-0.4050
4	-816.2	0.5313	1.0000	-0.6749	0.2066

Corresponding to each current vector $\vec{I}$ at the scatter ports there exists a unique current distribution over the entire scatterer. Figure 6 shows the current distributions on the wire triangle when mode currents $\vec{I}_n$, n=1,2,3,4, exist at the scatterer ports. The graphs are divided into segments

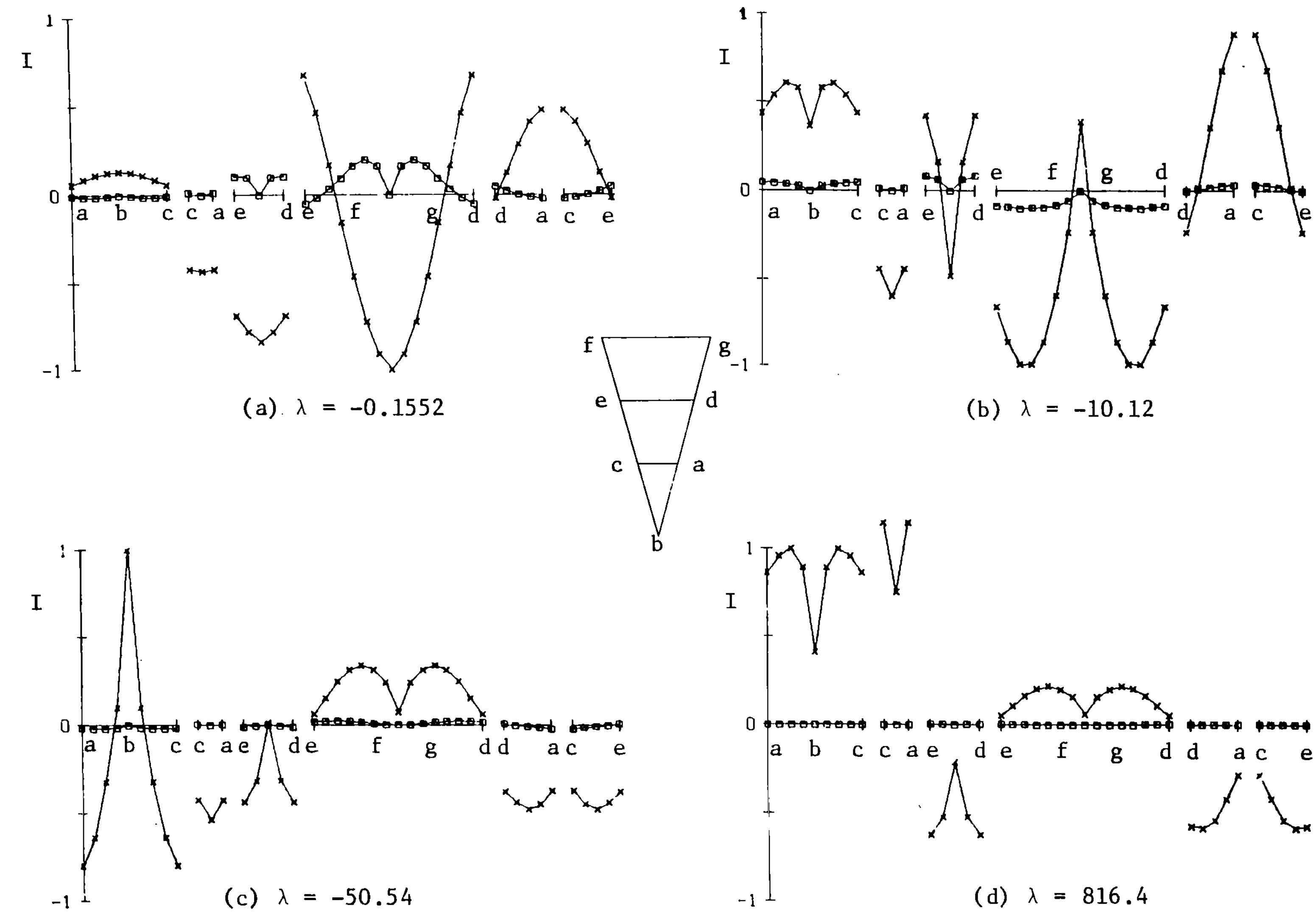


Fig. 6. Current distributions produced by the characteristic currents of the unloaded body, open circuit formulation, $[Z_L] = 0$. The x's denote real part, □'s imaginary part. Current distribution produced by the eigenvoltages of the unloaded body, short circuit formulation, $[Y_L] = 0$, are the same, but $\mu = -\lambda$.

corresponding to wire segments on the triangle, with points identified by letters a through g. The current graphs are normalized so that the magnitude of the maximum coefficient of the current expansion is unity. Since the current on one of the branches of a junction is the sum of two expansion coefficients, the maximum current plotted may be greater or less than unity.

Corresponding to each voltage vector $\vec{V}$ at the scatterer ports there exists a unique current distribution over the entire scatterer. As noted earlier, when a mode voltage $\vec{V}_n$ of the open-circuited scatterer exists at the scatterer ports, the resultant port current is $\vec{I}_n$, a mode current. Hence, the plots of Fig. 6 are also the current distributions on the wire triangle when mode voltages $\vec{V}_n$, n=1,2,3,4, exist at the scatterer ports.

Corresponding to each current vector $\vec{I}$ at the scatterer ports there exists a unique field radiated by the scatterer. Figure 7 shows the radiation gain patterns when the mode currents $\vec{I}_n$, n=1,2,3,4, exist at the scatterer ports. Plotted for each case are gain patterns in the two planes x=0 and y=0. These are labeled $g_{x\phi}$ and $g_{y\theta}$, respectively, where the second subscript denotes the polarization of the $\underset{\sim}{E}$ field.

Corresponding to each voltage vector $\vec{V}$ at the scatterer ports there exists a unique field radiated by the scatterer. Again, when a mode voltage $\vec{V}_n$ of the open-circuited scatterer exists at the scatterer ports, the resultant port current is $\vec{I}_n$, a mode current. Hence, the plots of Fig. 7 are also the radiation gain patterns when the mode voltages $\vec{V}_n$, n=1,2,3,4, exist at the scatterer ports.

When the scatterer is illuminated by an incident field, the scattered field can be computed either by matrix inversion [7] or by a modal solution of the form (34) or (37). We have investigated convergence of the modal solution as modes are added in the order of increasing $|\lambda_n|$ or $|\mu_n|$. Figure 8 shows the results using the open circuit formulation (34). The incident wave is an x-polarized z-traveling plane wave. The two solid curves on each plot are the matrix inversion solutions for radar cross section/wavelength squared in the planes x=0 and y=0 for the open-circuited scatterer, $[Z_L] = 0$. The

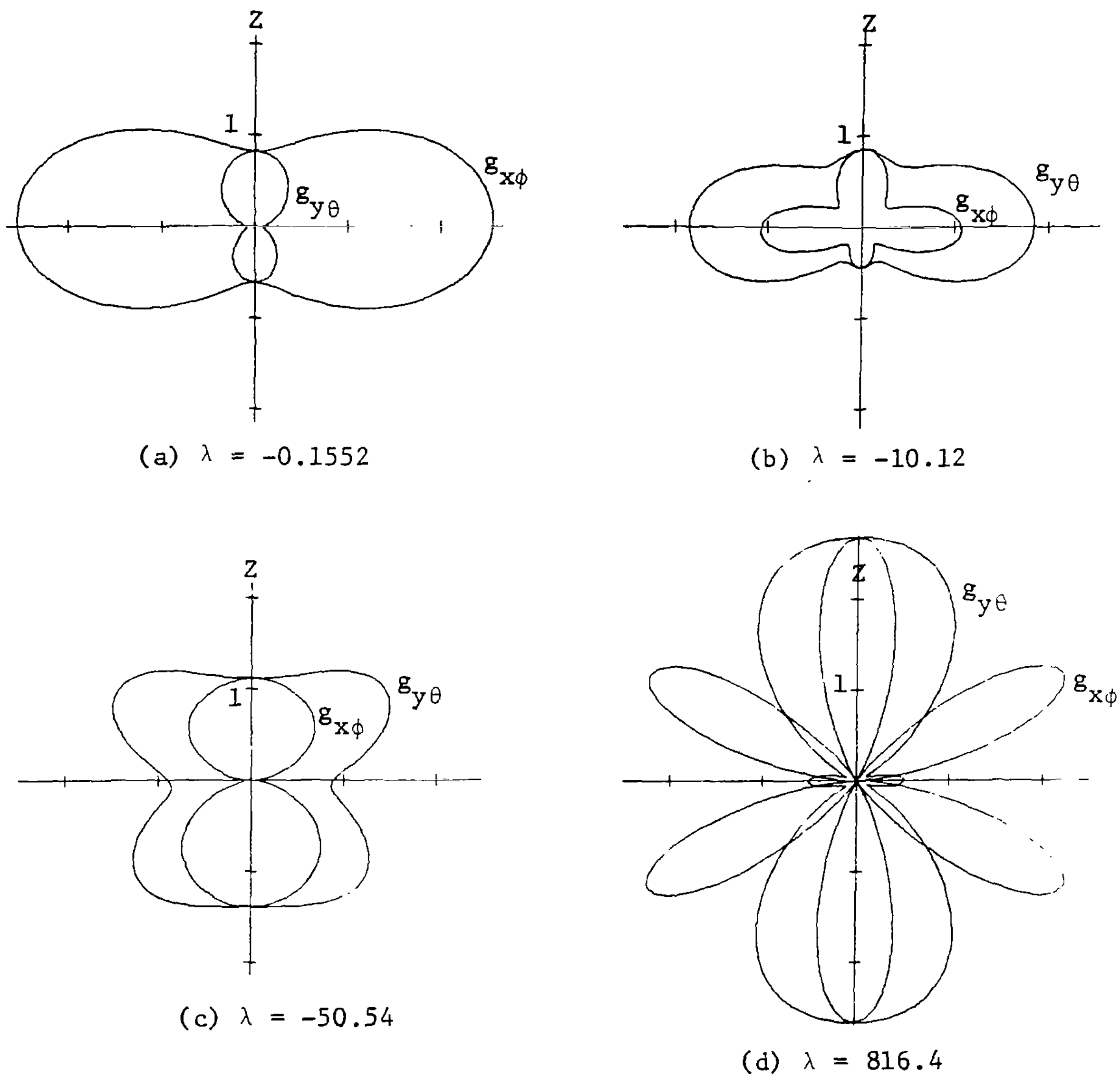


Fig. 7. Gain patterns for the characteristic currents of the unloaded body, open circuit formulation, $[Z_L] = 0$. Curves labeled $g_{x\phi}$ are in the x=0 plane and the field is ϕ polarized. Curves labeled $g_{y\theta}$ are in the y=0 plane and the field is θ polarized. Gain patterns for the characteristic voltages of the unloaded body, short circuit formulation, $[Y_L] = 0$, are the same, but $\mu = -\lambda$.

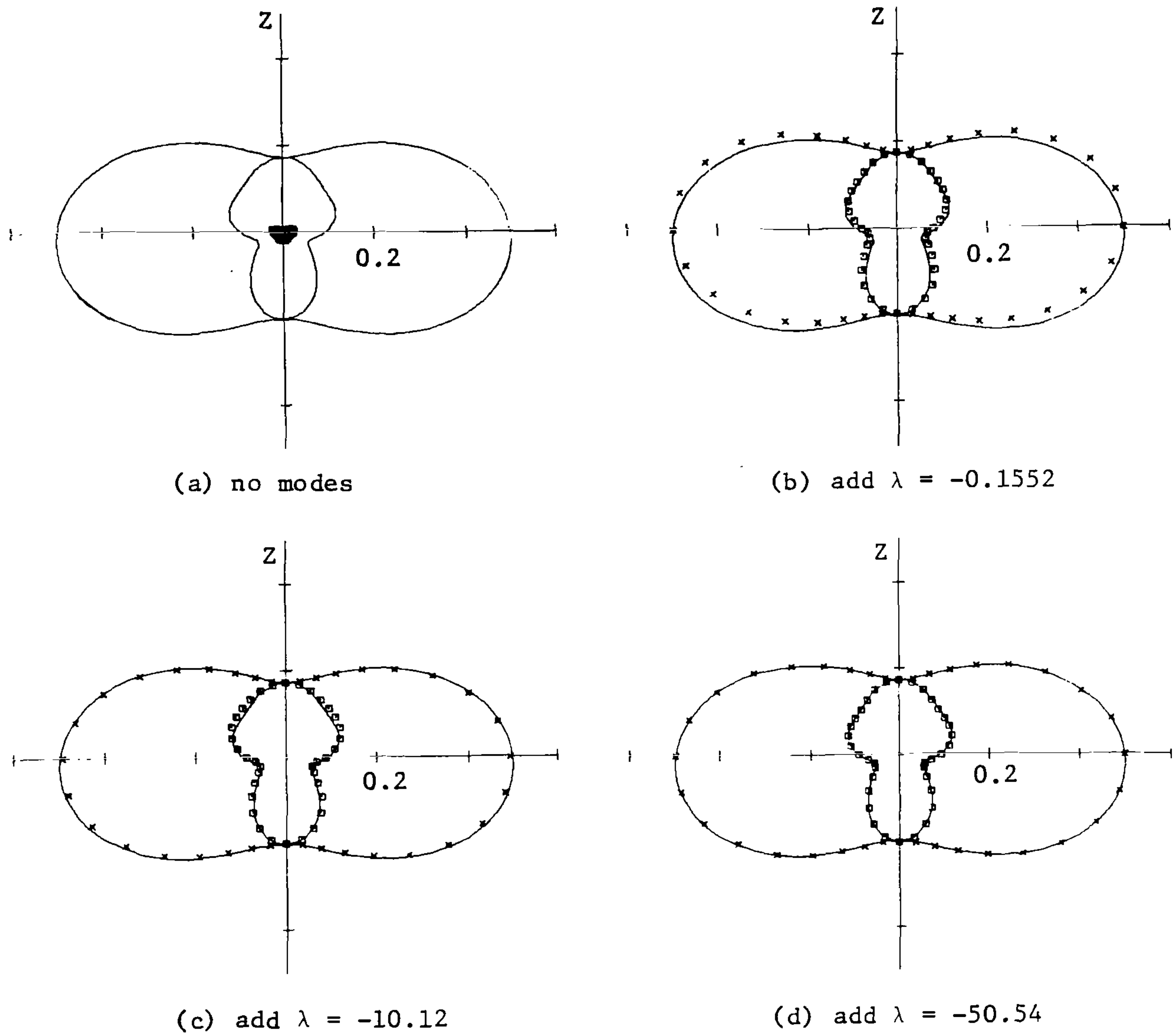


Fig. 8. Convergence of the open circuit modal formulation for bistatic radar cross section/wavelength squared for the case $[Z_L] = 0$. Excitation is an x-polarized plane wave traveling in the +z direction. The solid curves are the short circuit scattering patterns obtained by matrix inversion. The modal solutions are shown by x's in the x=0 plane and by □'s in the y=0 plane.

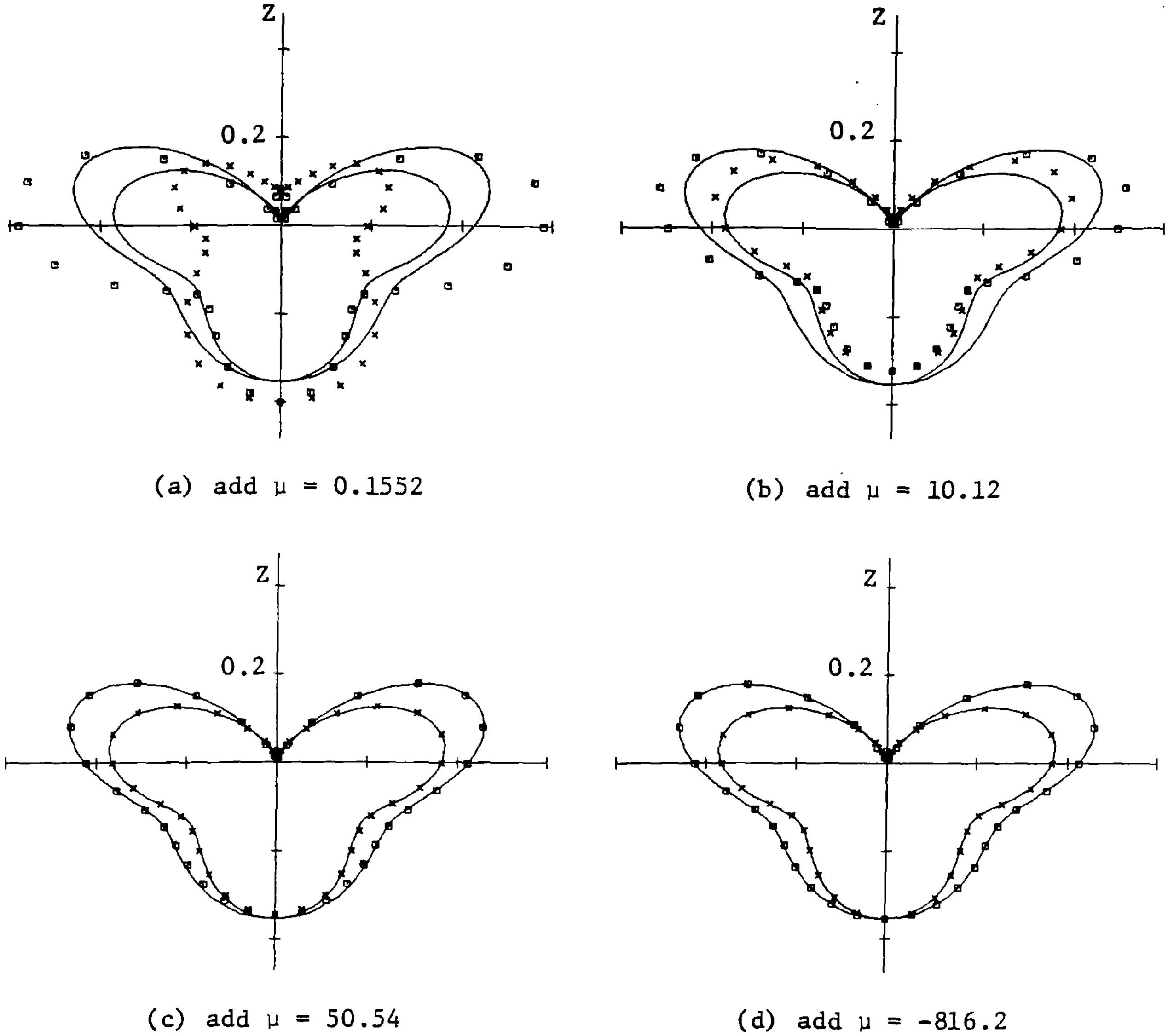


Fig. 9. Convergence of the short circuit modal formulation for bistatic radar cross section/wavelength squared for the case $[Y_L] = 0$. Excitation is an x-polarized plane wave traveling in the +z direction. The solid curves are the open circuit scattering patterns obtained by matrix inversion. The modal solutions are shown by x's in the x=0 plane and by □'s in the y=0 plane.

modal solutions are shown by x's in the x=0 plane and by □'s in the y=0 plane. Figure 8a includes no modes, and includes only the E_0^{oc} term of (34). This is actually the scattering from the short-circuited scatterer. Note that all x's and □'s fall into an almost solid spot at the origin. This is because the scattering from the open-circuited scatterer is an order of magnitude smaller than that from the short-circuited scatterer. Figure 8b includes the E_0^{oc} term plus the $\lambda = -0.1552$ mode in the modal solution. The pattern is now a close approximation to the matrix inversion solution. This is to be expected, since $\lambda = -0.1552$ is the only small eigenvalue and the E_0^{oc} term is small compared to E^s. In fact, the $\lambda = -0.1552$ mode alone, without the E_0^{oc} term, should also be a good approximation to the matrix inversion solution. Figure 8c shows the modal solution when two modes are included in (34), and Fig. 8d when three modes are included. The case of all four modes included is not shown, but is indistinguishable from Fig. 8d.

Figure 9 shows the convergence of the modal solution (37) when applied to the open-circuited scatterer. The incident wave is again an x-polarized z-traveling plane wave. The two solid curves on each plot are the matrix inversion solutions for radar cross section/wavelength squared in the planes x=0 and y=0 for the open circuited scatterer, $[Y_L] = 0$. The modal solutions are shown by x's in the x=0 plane and by □'s in the y=0 plane. The case of no modes, which would include only the E_0^{sc} term of (37), is not shown because the points all lie off scale. It is, of course, the scattering from the short-circuited scatterer, which is an order of magnitude larger than that from the open-circuited scatterer. (This is evident from Fig. 8a.) Figure 9a shows the modal solution when one mode is included in (37), Fig. 9b when two modes are included, Fig. 9c when three modes are included, and Fig. 9d when all four modes are included. Note that the modal solution converges to the matrix inversion solution more slowly in this case than in the previous case, Fig. 8. This is because the E_0^{sc} term of (37) is large compared to the final result E^s whereas in the previous case the E_0^{oc} term was small compared to the final result E^s.

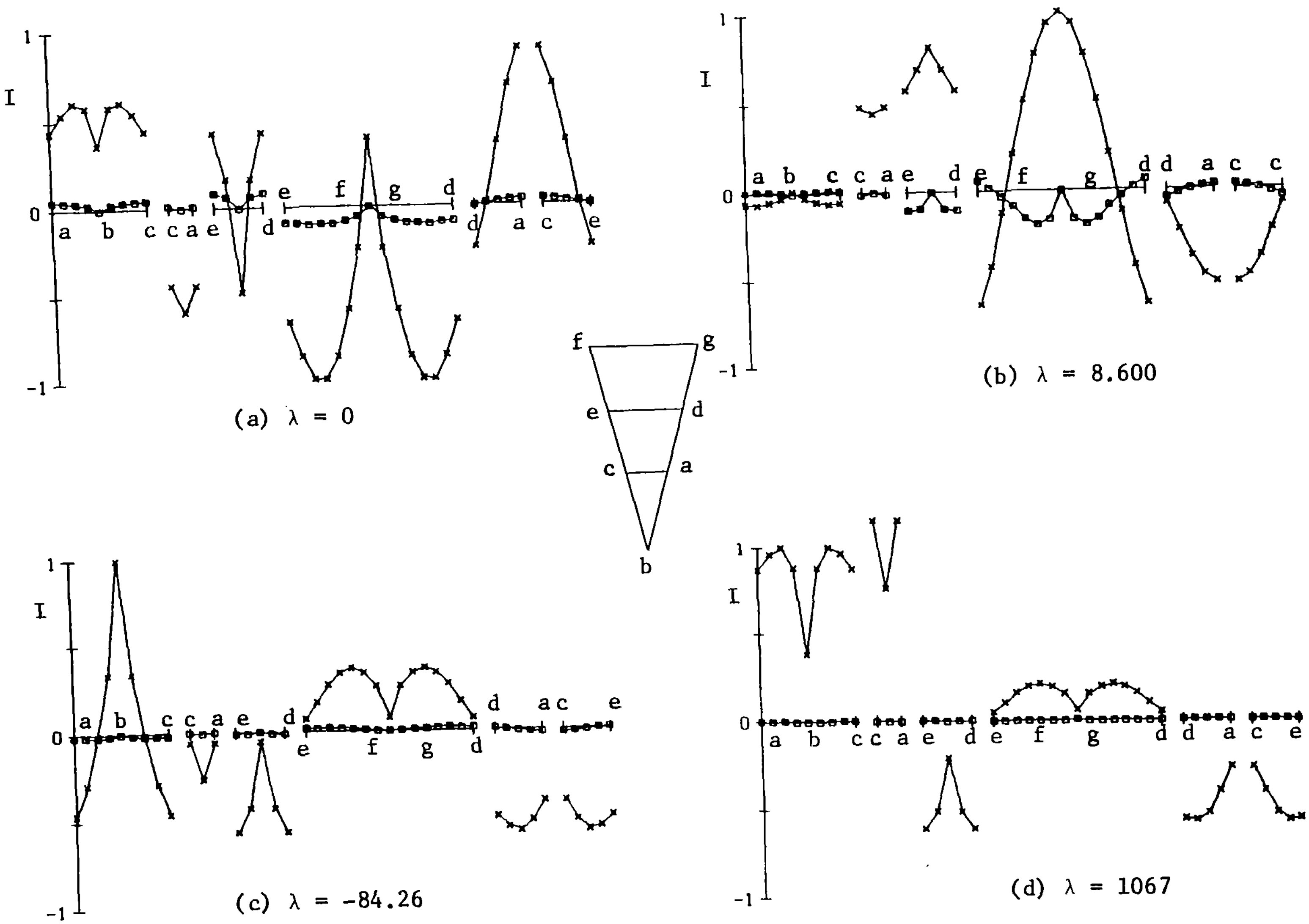


Fig. 10. Current distributions produced by the characteristic currents of the loaded body, open circuit formulation $[Z_L]$ = diagonal load to resonate the $\lambda = -10.12$ mode of the unloaded body. The x's denote real part, □'s imaginary part.

The remaining examples deal with the wire triangle reactively loaded to resonate a mode current. First, the diagonal load matrix $[X_L]$ is calculated according to (44) to resonate the mode current corresponding to $\lambda=-10.12$ of the unloaded wire triangle. The resultant loads, in ohms, are

$$X_{L1} = -215.6 \qquad X_{L3} = 854.7$$
$$X_{L2} = 173.7 \qquad X_{L4} = 986.7 \tag{100}$$

Next the loaded scatterer matrix $[Z] = [Z_S + jX_L]$ is formed, and the characteristic port currents $\vec{I}_n$ and eigenvalues λ_n are computed according to (17). These results are tabulated in Table 3.

Table 3. Port mode currents $\vec{I}_n$ for the loaded wire triangle, $[Z_L] = j[X_L]$ to resonate the $\lambda = -10.12$ mode of the unloaded wire triangle.

n	λ_n	port (1)	port (2)	port (3)	port (4)
1	0	-0.6078	1.0000	0.8054	-0.6458
2	8.600	0.0120	0.4430	0.8115	1.0000
3	-84.26	1.0000	-0.2654	-0.0506	0.0787
4	1067.	0.5069	1.0000	-0.2801	0.0743

The current distributions on the wire triangle when the characteristic currents $\vec{I}_n$, n=1,2,3,4, of the loaded wire triangle exist at the scatterer ports are shown in Fig. 10. The gain patterns of the wire triangle when the characteristic currents $\vec{I}_n$, n=1,2,3,4, of the loaded wire triangle exist at the scatterer ports are shown in Fig. 11. Convergence of the open-circuit modal formulation (34) for bistatic radar cross section/wavelength squared for the loaded wire triangle is shown in Fig. 12. Interpretation of these figures is analogous to that for Figs. 6, 7, and 8. Note that, even though the eigenvalues λ_n of the loaded case are different from those for the unloaded case, the current distributions and mode patterns (Figs. 6 and 7 and

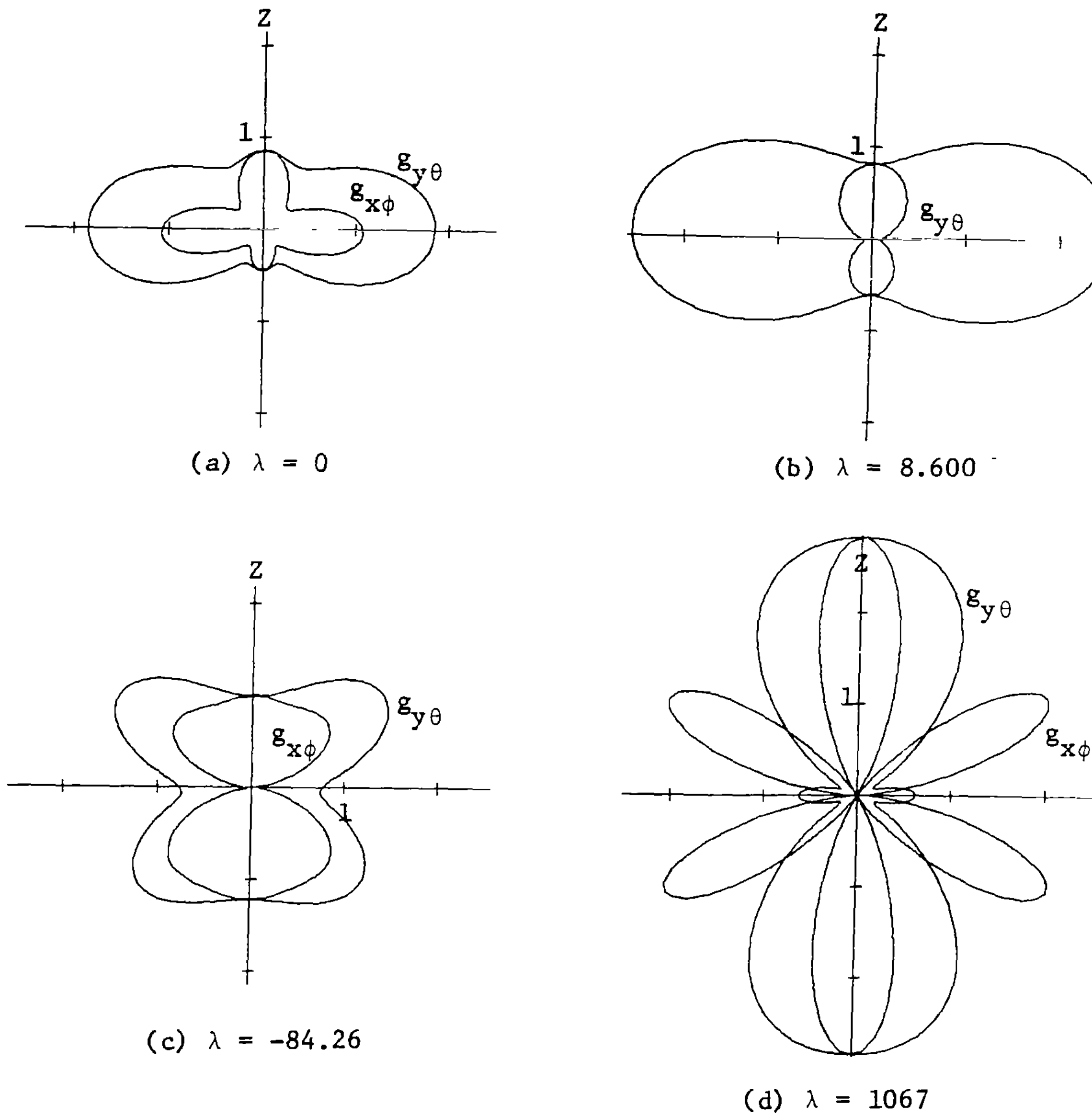


Fig. 11. Gain patterns for the characteristic currents of the loaded body, open circuit formulation, $[Z_L]$ = diagonal load to resonate the $\lambda = -10.12$ mode of the unloaded body. Curves labeled $g_{x\phi}$ are in the x=0 plane and the field is ϕ polarized. Curves labeled $g_{y\theta}$ are in the y=0 plane and the field is θ polarized.

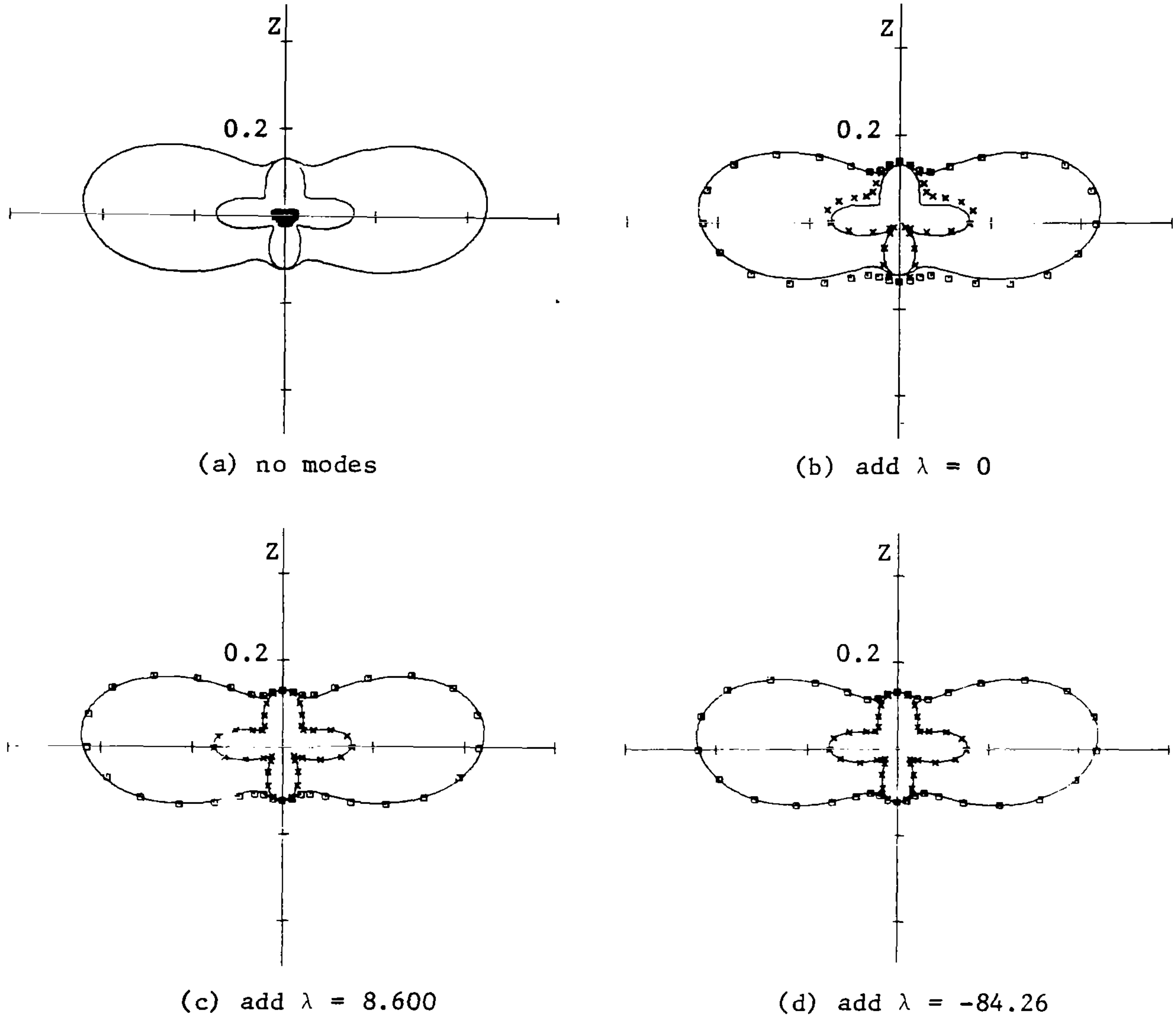


Fig. 12. Convergence of the open circuit modal formulation for bistatic radar cross section/wavelength squared for the case $[Z_L]$ = diagonal load to resonate the $\lambda = -10.12$ mode of the unloaded body. The modal solutions are shown by x's in the x=0 plane and by □'s in the y=0 plane. Solid curves are the matrix inversion solution.

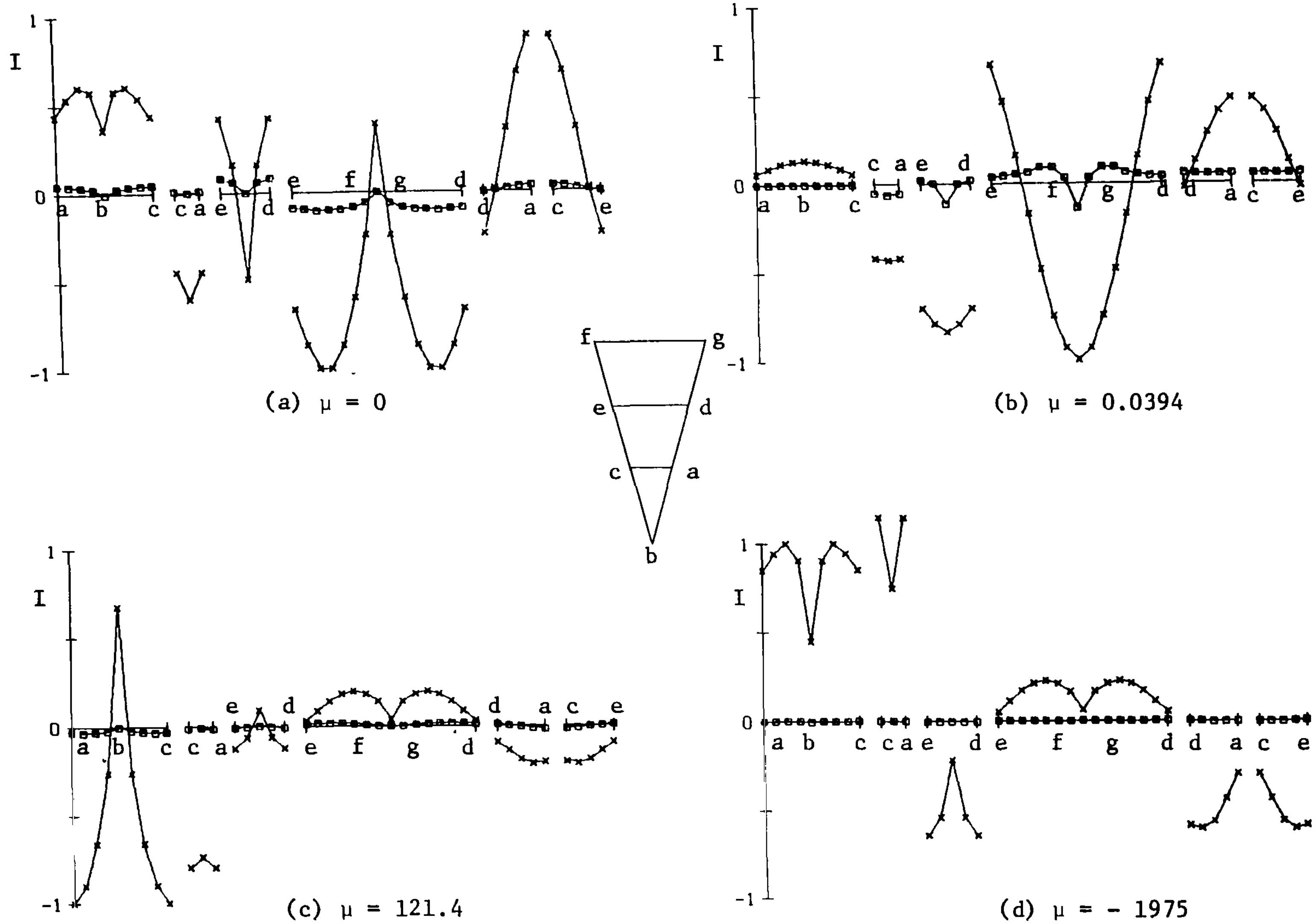


Fig. 13. Current distributions produced by the characteristic voltages of the loaded body, short circuit formulation, $[Y_L]$ = diagonal load to resonate the μ = 10.12 mode of the unloaded body. The x's denote real part, ◻'s imaginary part.

Figs. 10 and 11) are somewhat similar. Note also that the convergence of the modal solutions in the two cases, Figs. 8 and 12, are qualitatively similar.

For the final example, the diagonal load matrix $[B_L]$ is calculated according to (46) to resonate the mode voltage corresponding to the $\mu = 10.12$ mode of the unloaded wire triangle, $[Y_L] = 0$. The resultant loads, in mhos, are

$$B_{L1} = 0.004592 \qquad B_{L3} = -0.001159$$
$$B_{L2} = -0.005700 \qquad B_{L4} = -0.001004 \tag{101}$$

Next, the loaded scatterer matrix $[Y] = [Y_S + jB_L]$ is formed, and the characteristic port voltages $\vec{V}_n$ and eigenvalues μ_n are computed according to (24). These results are tabulated in Table 4.

Table 4. Port mode voltages $\vec{V}_n$ for the loaded wire triangle, $[Y_L] = j[B_L]$ to resonate the $\mu = 10.12$ mode of the unloaded wire triangle.

n	μ_n	port (1)	port (2)	port (3)	port (4)
1	0	0.1904	0.2524	1.0000	-0.9256
2	0.0394	-0.0054	0.0773	0.7014	1.0000
3	121.4	1.0000	0.1850	0.3390	-0.2319
4	-1975	0.4986	1.0000	-0.6917	0.2164

The current distributions on the wire triangle when the characteristic voltages $\vec{V}_n$, n=1,2,3,4, of the loaded wire triangle exist at the scatterer ports are shown in Fig. 13. The gain patterns of the wire triangle when the characteristic voltages $\vec{V}_n$, n=1,2,3,4, of the loaded wire triangle exist at the scatterer ports are shown in Fig. 14. Convergence of the short circuit modal formulation (37) for bistatic radar cross section/wavelength squared for the loaded wire triangle is shown in Fig. 15. Interpretation of these

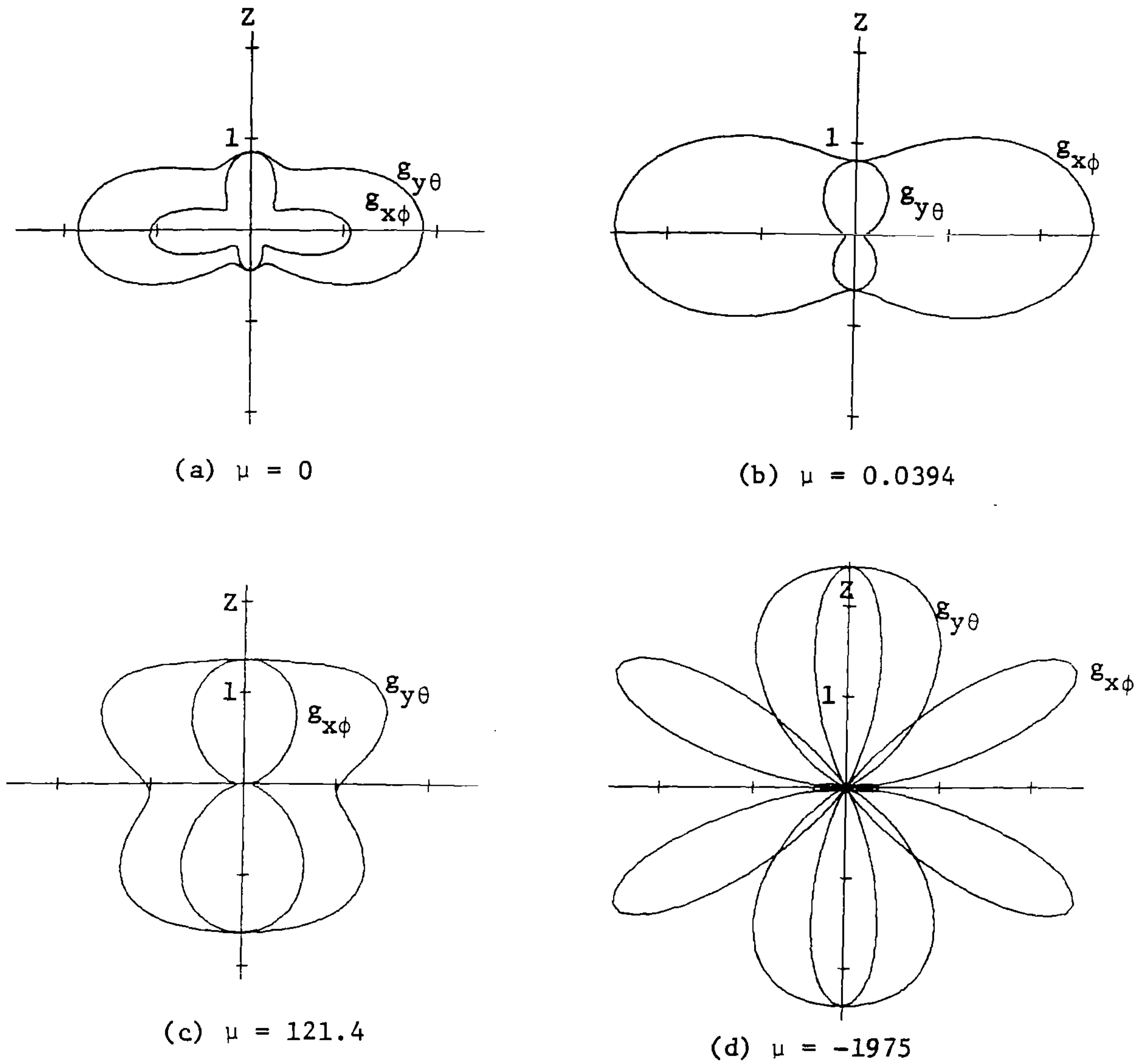


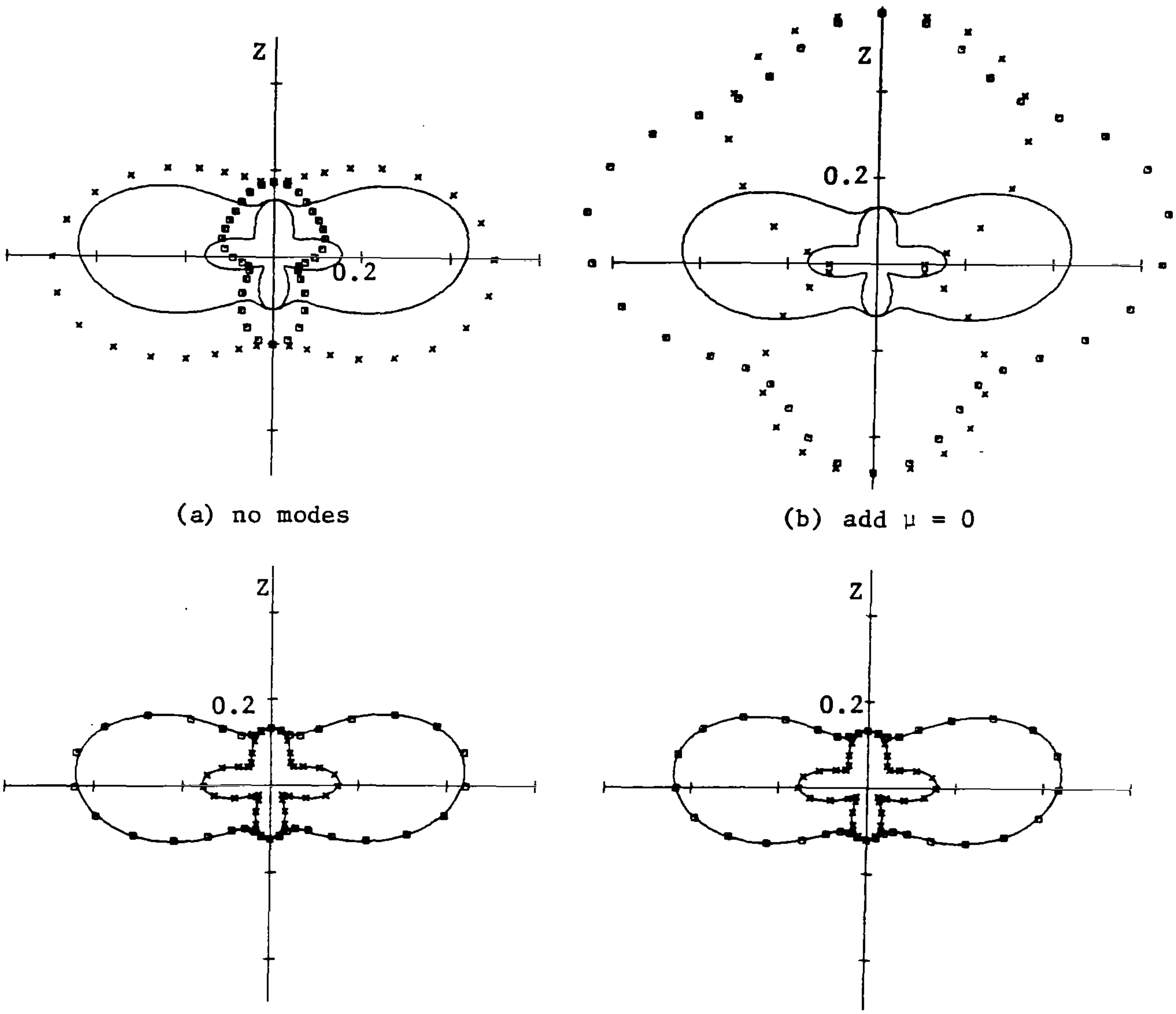
Fig. 14. Gain patterns for the characteristic voltages of the loaded body, short circuit formulation, $[Y_L]$ = diagonal load to resonate the $\mu = 10.12$ mode of the unloaded body. The x's denote real part, □'s imaginary part.

figures is analogous to that for Figs. 6, 7, and 9. Again note that, even though the port eigenvalues μ_n for the loaded case are different from those for the unloaded case, the current distributions and the mode patterns (Figs. 6 and 13 and Figs. 9 and 14) are somewhat similar. Also, convergence of the modal solutions in the two cases, Figs. 12 and 15, are qualitatively similar. However, in the loaded case, Fig. 15, we have two modes with small eigenvalues, namely, $\mu = 0$ and $\mu = 0.0394$. Hence, we should expect to require both of these modes in the modal solution before obtaining a good approximation to the matrix inversion solution. We also note that Fig. 15b, which includes one mode in the solution, gives a worse approximation to the matrix inversion solution than does Fig. 15a. This is not surprising because the modal solution gives a least squares approximation to $\underset{\sim}{E}^s - \underset{\sim}{E}_0^{sc}$ on the radiation sphere, not to $\underset{\sim}{E}^s$ alone. This is in contrast to the modal solution using modes of the complete body [4], in which case there is no term corresponding to $\underset{\sim}{E}_0^{sc}$.

Finally, note that the synthesized patterns of Figs. 12 and 15 are the same to within plotting accuracy. This is because the loads, $[X_L]$ in the first case and $[B_L]$ is the second case, were chosen to resonate the same mode of the unloaded wire triangle. The two results actually differ slightly, because of the $\underset{\sim}{E}_0^{oc}$ and $\underset{\sim}{E}_0^{sc}$ terms in (34) and (37), respectively. The computed loads $[X_L]$ and $[B_L]$ are actually not the same loads in the two cases. It can be shown that, when a mode of the unloaded body is resonated, the loads in the two cases are related by

$$[B_L] = - \left(\frac{\lambda_n^2}{1+\lambda_n^2}\right) [X_L]^{-1} \tag{102}$$

where $\lambda_n = -\mu_n$ is the eigenvalue of the mode resonated. The loads (100) and (101) satisfy (102) with $\lambda = -10.12$.



(a) no modes

(b) add $\mu = 0$

(c) add $\mu = 0.0394$

(d) add $\mu = 121.4$

Fig. 15. Convergence of the short circuit modal formulation for bistatic radar cross section/wavelength squared for the case $[Y_L]$ = diagonal load to resonate the $\mu = 10.12$ mode of the unloaded body. The modal solutions are shown by x's in the x=0 plane and by □'s in the y=0 plane. Solid curves are the matrix inversion solution.

X. DISCUSSION

The characteristic port modes of an N-port system form a convenient basis for expressing the field scattered or radiated by an N-port system. The mode currents (or voltages) diagonalize the loaded N-port impedance matrix (or admittance matrix). The mode fields form an orthonormal set on the radiation sphere. Because of these properties, problems of analysis, synthesis, and optimization become conceptually simpler. For example, the principle of modal resonance introduced in Section V can be used to synthesize and optimize scattering patterns from a loaded N-port scatterer. The procedure is similar to that used for continuously loaded scatterers [6]. The modes should find similar uses for the synthesis and optimization of N-port antenna patterns. While all the examples of this report have used computed parameters of N-port systems, measured parameters could be similarly used. This would, of course, bring many experimental problems into the picture, and require considerable further research.

PART TWO

COMPUTER PROGRAMS

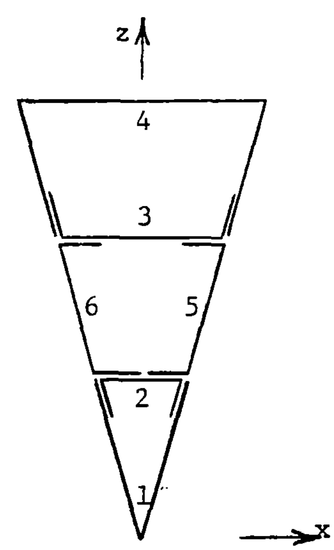
I. INTRODUCTION

The programs used to compute the examples of this report are described and listed here in Part two. Each program is accompanied by an explanation of the input data, a verbal flow chart, and sample input-output data. In general, the input data for the program of a given section depends upon the output of programs of previous sections. If each program is run with the input data listed in this report, the input data for any one of these programs can be verified in terms of the output of programs previously run. The Calcomp Plotter is used only in the two programs of Sections X and XI. The sole purpose of these two programs is to plot existing data.

II. DATA POINTS ON THE WIRE TRIANGLE

The sample data for all the programs of this report is that of the wire triangle with two cross wires, shown in Fig. 5. Distances are measured along the wire axis. The parameter a is one wavelength, and the wire diameter is a/100. Ports are defined at the 4 points at which the wire crosses the z axis.

The wire triangle is decomposed into six wires with some overlapping as shown below.



Six wire arrangement for the wire triangle.

The axis of the first wire is defined by 17 data points, including one at each end of the wire. These 17 data points divide the first wire into 16 segments of unit length. For convenience, we have chosen the length a to be 32 units in which case the propagation constant k is $\frac{\pi}{16}$. The next 9 data points divide the second wire into 8 segments of which the first two and the last two are one unit long and the remaining four are 4 sin (15°) long. The next 13 data points divide the third wire into 12 segments of which the first two and the last two are one unit long and the remaining eight are 4 sin (15°) long. The next 29 data points divide the fourth wire into 28 segments of which the first 8 and the last 8 are one unit long and the remaining 12 are 4 sin (15°) long. The next 13 data points divide the fifth wire into 12 setments of which the first two and the last two are 4 sin (15°) long and the remaining 8 are one unit long. The last 13 data points divide the sixth wire into 12 segments of which the first two and the last two are 4 sin (15°) long and the remaining 8 are one unit long. In general, a wire either terminates at a junction or extends two segments through the junction. If a wire terminates at a junction, its last two segments must be overlapped by one of the other wires. A junction with n branches should have overlapping on n-1 of its branches.

The vector $\vec{I}$ appearing in (83) is the current at every odd data point of each wire except the first and last data point of each wire. For example, only the currents at the 3rd, 5th, and 7th of the 9 data points of the second wire are included in $\vec{I}$. Only elements of $\vec{I}$ are eligible to become port currents. We have not explored the possibility of a port current at a junction, or rather on one of the branches of a junction.

A short program to obtain the x and z coordinates of the data points is listed next.

LISTING OF PROGRAM FOR X AND Z COORDINATES OF WIRE TRIANGLE

```
//                (0034,EE,1,1),'MAUTZ,JOE',REGION=140K
// EXEC WATFIV
//GO.SYSIN DD *
$JOB              MAUTZ
      DIMENSION X(50),Z(50),XX(25)
      PI=3.141593
      ANG=PI/12.
      SN=SIN(ANG)
      CS=COS(ANG)
      DO 11 J=1,25
      FJ=J-1
      X(J)=FJ*SN
      Z(J)=FJ*CS
   11 CONTINUE
      WRITE(3,13)(X(J),J=1,25)
   13 FORMAT('OX'/(1X,10F8.4))
      WRITE(3,14)(Z(J),J=1,25)
   14 FORMAT('OZ'/(1X,10F8.4))
      DO 12 J=1,7
      FJ=4*(J-1)
      XX(J)=FJ*SN
   12 CONTINUE
      WRITE(3,15)(XX(J),J=1,7)
   15 FORMAT('OXX'/(1X,10F8.4))
      STOP
      END
$DATA
$STOP
/*
//
```

PRINTED OUTPUT

```
X
  0.0000  0.2588  0.5176  0.7765  1.0353  1.2941  1.5529  1.8117  2.0706  2.3294
  2.5882  2.8470  3.1058  3.3646  3.6235  3.8823  4.1411  4.3999  4.6587  4.9176
  5.1764  5.4352  5.6940  5.9528  6.2117

Z
  0.0000  0.9659  1.9319  2.8978  3.8637  4.8296  5.7956  6.7615  7.7274  8.6933
  9.6593 10.6252 11.5911 12.5570 13.5230 14.4889 15.4548 16.4207 17.3867 18.3526
 19.3185 20.2844 21.2504 22.2163 23.1822

XX
  0.0000  1.0353  2.0706  3.1058  4.1411  5.1764  6.2117
```

In this program, DO loop 11 calculates the x and z coordinates of the right hand side of the triangle. DO loop 12 stores the x coordinates of the right half of the top side of the wire triangle in XX. Since the wire triangle is in the y=0 plane, all y coordinates are zero. The foregoing x, z, XX and appropriate zeros are arranged to obtain PX, PY, and PZ, the x, y, and z coordinates at the 94 data points. PX, PY, and PZ appear in the sample data of the next program which calculates the generalized impedance matrix.

III. GENERALIZED IMPEDANCE MATRIX

A program for the generalized impedance matrix Z of (85) appears on pages 12-15 of [10], but because a few minor errors have been corrected in the subroutine CALZ and because the main program has been changed considerably, the listing of the program, sample input-output data, and explanation of the punched card input will be included here.

The activity on data sets 1 (punched card input) and 6 (direct access input-output is as follows.

```
        REWIND 6
        READ (1,43) NF, N6, NP, NW, RAD
43      FORMAT (4I3, E14.7)
        READ (1,10)(PX(I), I = 1, NP)
        READ (1,10)(PY(I), I = 1, NP)
        READ (1,10)(PZ(I), I = 1, NP)
10      FORMAT (10F8.4)
        READ (1,15)(LL(I), I = 1, NW)
15      FORMAT (20I3)
        READ (1,44)(BKK(I), I = 1, NF)
44      FORMAT (5E14.7)
        SKIP N6 RECORDS ON DATA SET 6
        DO 14 K = 1, NF
        WRITE (6)(Z(I), I = 1, NZ)
14      CONTINUE
```

PX, PY, and PZ are the x, y, and z coordinates of the NP data points which describe the axes of NW wires. On each wire there are an odd number ≥ 5 of data points of which two occur at the ends of the wire. RAD is the radius of the wires. The LL(I)th data point marks the beginning of the Ith wire. LL(1) should be 1. DO loop 14 stores the impedance matrix for propagation constant k = BKK(K) columnwise in Z.

Minimum allocations are given in the main program by

```
COMPLEX Z(NZ)
COMMON LL(NW+1), RAD2(NP),PX(NP),PY(NP),PZ(NP)
DIMENSION BKK(NF), MD6(NF)
```

and in the subroutine CALZ by

```
COMPLEX PSI(4*N1), Z(NZ)
COMMON LL(NW+1),RAD2(NP),PX(NP),PY(NP),PZ(NP)
DIMENSION L(NW+1),XX(N1),XY(N1),XZ(N1),TX(N1),
          TY(N1),TZ(N1),AL(N1),T(4*N),
          TP(4*N),DC(4*N1)
```

where

$$N1 = NP - NW$$
$$N = \frac{N1}{2} - NW$$
$$NZ = N*N$$

Because J_j as an expansion function is approximated by 4 pulses while J_j as a testing function is approximated by 4 impulses, Z is not exactly symmetric. DO loop 41 in the main program makes Z symmetric by averaging corresponding off diagonal elements. A minor error has been found and corrected between statements 22 and 28 of the subroutine CALZ. Also, the old CALZ would not function properly if wires of different radii met at a junction. This difficulty has been avoided by replacing RAD, the radius of each wire by RAD2, the square of the wire radius on each segment. A segment is the portion of a wire between consecutive data points on that wire.

The sample data is such as to store the impedance matrix on record 1 of data set 6.

LISTING OF PROGRAM TO CALCULATE THE GENERALIZED IMPEDANCE MATRIX

```
//                (0034,EE,2,2),'MAUTZ,JOE',REGION=140K
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      SUBROUTINE CALZ
      COMPLEX U,U3,U4,U2,U5,U6,PSI(400),Z(1600)
      COMMON Z,KT,NP,N,LL(9),RAD2(100),BK,PX(100),PY(100),PZ(100)
      DIMENSION L(9),XX(100),XY(100),XZ(100),TX(100),TY(100),TZ(100)
      DIMENSION AL(100),T(200),TP(200),DC(400)
      IF(KT.NE.1) GO TO 9
      U=(0.,1.)
      PI=3.141593
      ETA=376.730
      C1=.125/PI
      C2=.25/PI
      J4=2
      N1=0
      J1=1
      DO 8 J=1,NP
      IF(LL(J1)-J) 7,6,7
    6 J4=J4-1
      L(J1)=J4
      J1=J1+1
      GO TO 8
    7 N1=N1+1
      J3=J-1
      IF((N1/2*2-N1).EQ.0) J4=J4+1
      XX(N1)=.5*(PX(J)+PX(J3))
      XY(N1)=.5*(PY(J)+PY(J3))
      XZ(N1)=.5*(PZ(J)+PZ(J3))
      S1=PX(J)-PX(J3)
      S2=PY(J)-PY(J3)
      S3=PZ(J)-PZ(J3)
      S4=SQRT(S1*S1+S2*S2+S3*S3)
      TX(N1)=S1/S4
      TY(N1)=S2/S4
      TZ(N1)=S3/S4
      AL(N1)=S4
    8 CONTINUE
      N=J4-2
      L(J1)=J4
      J1=1
      J2=-2
      DO 5 J=1,N
      IF(L(J1)-J) 3,4,3
    4 J2=J2+2
      J1=J1+1
    3 J3=(J-1)*4
      J4=J3+1
      J5=J4+1
      J6=J5+1
      J7=J6+1
      K4=J2+1
      K5=K4+1
      K6=K5+1
      K7=K6+1
      S1=AL(K4)+AL(K5)
      S2=AL(K6)+AL(K7)
      T(J4)=AL(K4)*.5*AL(K4)/S1
```

```
      T(J5)=AL(K5)*(AL(K4)+.5*AL(K5))/S1
      T(J6)=AL(K6)*(AL(K7)+.5*AL(K6))/S2
      T(J7)=AL(K7)*.5*AL(K7)/S2
      TP(J4)=AL(K4)/S1
      TP(J5)=AL(K5)/S1
      TP(J6)=-AL(K6)/S2
      TP(J7)=-AL(K7)/S2
      J2=J2+2
    5 CONTINUE
    9 U3=U*BK*ETA
      U4=-U/BK*ETA
      BK2=BK*BK/2.
      BK3=BK2*BK/3.
      N9=0
      N2=1
      N3=-2
      DO 10 NS=1,N
      IF(L(N2)-NS) 12,11,12
   11 KK=1
      N3=N3+2
      N2=N2+1
      GO TO 13
   12 KK=3
      DO 14 NF=1,N1
      N4=NF+N1
      N5=N4+N1
      N6=N5+N1
      DC(NF)=DC(N5)
      DC(N4)=DC(N6)
      PSI(NF)=PSI(N5)
      PSI(N4)=PSI(N6)
   14 CONTINUE
   13 DO 15 K=KK,4
      N7=N3+K
      K1=(K-1)*N1
      DO 16 NF=1,N1
      N8=NF+K1
      S1=XX(N7)-XX(NF)
      S2=XY(N7)-XY(NF)
      S3=XZ(N7)-XZ(NF)
      R2=S1*S1+S2*S2+S3*S3+RAD2(NF)
      R=SQRT(R2)
      RT=ABS(S1*TX(N7)+S2*TY(N7)+S3*TZ(N7))
      RT2=RT*RT
      RH=(R2-RT2)
      ALP=.5*AL(N7)
      AR=ALP/R
      S1=BK*R
      U2=COS(S1)-U*SIN(S1)
      IF(AR-.1) 22,22,21
   21 U2=U2*C1/ALP
      S1=RT-ALP
      S2=RT+ALP
      S3=SQRT(S1*S1+RH)
      S4=SQRT(S2*S2+RH)
      IF(S1) 18,18,19
   18 AI1=ALOG((S2+S4)*(-S1+S3)/RH)
      GO TO 20
   19 AI1=ALOG((S2+S4)/(S1+S3))
   20 AI2=AL(N7)
```

```
      AI3=(S2*S4-S1*S3+RH*AI1)/2.
      AI4=AI2*(RH+ALP*ALP/3.+RT2)
      S3=AI1*R
      S1=AI1-BK2*(AI3-R*(2.*AI2-S3))
      S2=-BK*(AI2-S3)+BK3*(AI4-3.*AI3*R+R2*(3.*AI2-S3))
      GO TO 28
   22 U2=U2*C2/R
      BA=BK*ALP
      BA2=BA*BA
      AR2=AR*AR
      AR3=AR2*AR
      ZR=RT/R
      ZR2=ZR*ZR
      ZR3=ZR2*ZR
      ZR4=ZR3*ZR
      A1=AR*(-1.+3.*ZR2)/6.+(3.-30.*ZR2+35.*ZR4)*AR3/40.
      A0=1.+AR*A1
      A2=-ZR2/6.-AR2*(1.-12.*ZR2+15.*ZR4)/40.
      A3=AR*(3.*ZR2-5.*ZR4)/60.
      A4=ZR4/120.
      S1=A0+BA2*(A2+BA2*A4)
      S2=BA*(A1+BA2*A3)
   28 PSI(N8)=U2*(S1+U*S2)
      DC(N8)=TX(NF)*TX(N7)+TY(NF)*TY(N7)+TZ(NF)*TZ(N7)
   16 CONTINUE
   15 CONTINUE
      N3=N3+2
      J3=(NS-1)*4
      J7=-2
      J9=1
      DO 25 NF=1,N
      J1=(NF-1)*4
      IF(L(J9)-NF) 26,27,26
   27 J9=J9+1
      J7=J7+2
   26 N9=N9+1
      U5=0.
      U6=0.
      J5=0
      DO 23 JS=1,4
      J4=J3+JS
      J8=J5+J7
      DO 24 JF=1,4
      J6=J8+JF
      J2=J1+JF
      U5=T(J2)*T(J4)*DC(J6)*PSI(J6)+U5
      U6=TP(J2)*TP(J4)*PSI(J6)+U6
   24 CONTINUE
      J5=J5+N1
   23 CONTINUE
      Z(N9)=U5*U3+U6*U4
      J7=J7+2
   25 CONTINUE
   10 CONTINUE
      RETURN
      END
      COMPLEX Z(1600)
      COMMON Z,KT,NP,N,LL(9),RAD2(100),BK,PX(100),PY(100),PZ(100)
      DIMENSION BKK(100),MD6(100)
      REWIND 6
```

```
      READ(1,43) NF,N6,NP,NW,RAD
   43 FORMAT(4I3,E14.7)
      WRITE(3,16) NF,N6,NP,NW,RAD
   16 FORMAT('0 NF N6 NP NW',6X,'RAD'/1X,4I3,E14.7)
      RAD=RAD*RAD
      DO 39 I=1,NP
      RAD2(I)=RAD
   39 CONTINUE
   40 READ(1,10)(PX(I),I=1,NP)
      READ(1,10)(PY(I),I=1,NP)
      READ(1,10)(PZ(I),I=1,NP)
   10 FORMAT(10F8.4)
      WRITE(3,29)(PX(I),I=1,NP)
      WRITE(3,30)(PY(I),I=1,NP)
      WRITE(3,31)(PZ(I),I=1,NP)
   29 FORMAT('0PX'/(1X,10F8.4))
   30 FORMAT('0PY'/(1X,10F8.4))
   31 FORMAT('0PZ'/(1X,10F8.4))
      READ(1,15)(LL(I),I=1,NW)
   15 FORMAT(20I3)
      WRITE(3,33)(LL(I),I=1,NW)
   33 FORMAT('0LL'/(1X,10I3))
      LL(NW+1)=10000
      DO 46 I=1,NF
      MD6(I)=0
   46 CONTINUE
      MD6(1)=1
      READ(1,44)(BKK(I),I=1,NF)
   44 FORMAT(5E14.7)
      WRITE(3,45)(BKK(I),I=1,NF)
   45 FORMAT('0BKK'/(1X,5E14.7))
      IF(N6) 23,23,24
   24 DO 26 J=1,N6
      READ(6)
   26 CONTINUE
   23 DO 14 K=1,NF
      KT=MD6(K)
      BK=BKK(K)
      CALL CALZ
      DO 41 J=1,N
      J1=(J-1)*N
      DO 42 I=1,J
      J2=J1+I
      J3=(I-1)*N+J
      Z(J2)=.5*(Z(J2)+Z(J3))
      Z(J3)=Z(J2)
   42 CONTINUE
   41 CONTINUE
      NZ=N*N
      WRITE(6)(Z(I),I=1,NZ)
      WRITE(3,38) N
   38 FORMAT('0IMPEDANCE MATRIX OF ORDER',I3)
      WRITE(3,37)(Z(I),I=1,5)
   37 FORMAT(1X,7E11.4)
   14 CONTINUE
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                X
//               VOLUME=SER=SU0004,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
```

```
//              BUFNO=1)
//GO.SYSIN DD *
  1  0 94   6 0.1600000E+00
  2.0706  1.8117  1.5529  1.2941  1.0353  0.7765  0.5176  0.2588  0.0000 -0.2588
 -0.5176 -0.7765 -1.0353 -1.2941 -1.5529 -1.8117 -2.0706 -1.5529 -1.8117 -2.0706
 -1.0353  0.0000  1.0353  2.0706  1.8117  1.5529 -4.6587 -4.3999 -4.1411 -3.1058
 -2.0706 -1.0353  0.0000  1.0353  2.0706  3.1058  4.1411  4.3999  4.6587 -4.1411
 -4.3999 -4.6587 -4.9176 -5.1764 -5.4352 -5.6940 -5.9528 -6.2117 -5.1764 -4.1411
 -3.1058 -2.0706 -1.0353  0.0000  1.0353  2.0706  3.1058  4.1411  5.1764  6.2117
  5.9528  5.6940  5.4352  5.1764  4.9176  4.6587  4.3999  4.1411  2.0706  3.1058
  4.1411  3.8823  3.6235  3.3646  3.1058  2.8470  2.5882  2.3294  2.0706  1.0353
  0.0000  0.0000 -1.0353 -2.0706 -2.3294 -2.5882 -2.8470 -3.1058 -3.3646 -3.6235
 -3.8823 -4.1411 -3.1058 -2.0706
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000
  7.7274  6.7615  5.7956  4.8296  3.8637  2.8978  1.9319  0.9659  0.0000  0.9659
  1.9319  2.8978  3.8637  4.8296  5.7956  6.7615  7.7274  5.7956  6.7615  7.7274
  7.7274  7.7274  7.7274  7.7274  6.7615  5.7956 17.3867 16.4207 15.4548 15.4548
 15.4548 15.4548 15.4548 15.4548 15.4548 15.4548 15.4548 16.4207 17.3867 15.4548
 16.4207 17.3867 18.3526 19.3185 20.2844 21.2504 22.2163 23.1822 23.1822 23.1822
 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822
 22.2163 21.2504 20.2844 19.3185 18.3526 17.3867 16.4207 15.4548 15.4548 15.4548
 15.4548 14.4889 13.5230 12.5570 11.5911 10.6252  9.6593  8.6933  7.7274  7.7274
  7.7274  7.7274  7.7274  7.7274  8.6933  9.6593 10.6252 11.5911 12.5570 13.5230
 14.4889 15.4548 15.4548 15.4548
  1 18 27 40 69 82
 0.1963495E+00
/*
//
```

PRINTED OUTPUT

```
 NF N6 NP NW         RAD
  1  0 94   6 0.1600000E+00
```

PX

```
  2.0706  1.8117  1.5529  1.2941  1.0353  0.7765  0.5176  0.2588  0.0000 -0.2588
 -0.5176 -0.7765 -1.0353 -1.2941 -1.5529 -1.8117 -2.0706 -1.5529 -1.8117 -2.0706
 -1.0353  0.0000  1.0353  2.0706  1.8117  1.5529 -4.6587 -4.3999 -4.1411 -3.1058
 -2.0706 -1.0353  0.0000  1.0353  2.0706  3.1058  4.1411  4.3999  4.6587 -4.1411
 -4.3999 -4.6587 -4.9176 -5.1764 -5.4352 -5.6940 -5.9528 -6.2117 -5.1764 -4.1411
 -3.1058 -2.0706 -1.0353  0.0000  1.0353  2.0706  3.1058  4.1411  5.1764  6.2117
  5.9528  5.6940  5.4352  5.1764  4.9176  4.6587  4.3999  4.1411  2.0706  3.1058
  4.1411  3.8823  3.6235  3.3646  3.1058  2.8470  2.5882  2.3294  2.0706  1.0353
  0.0000  0.0000 -1.0353 -2.0706 -2.3294 -2.5882 -2.8470 -3.1058 -3.3646 -3.6235
 -3.8823 -4.1411 -3.1058 -2.0706
```

PY

```
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
```

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000						

PZ

7.7274	6.7615	5.7956	4.8296	3.8637	2.8978	1.9319	0.9659	0.0000	0.9659
1.9319	2.8978	3.8637	4.8296	5.7956	6.7615	7.7274	5.7956	6.7615	7.7274
7.7274	7.7274	7.7274	7.7274	6.7615	5.7956	17.3867	16.4207	15.4548	15.4548
15.4548	15.4548	15.4548	15.4548	15.4548	15.4548	15.4548	16.4207	17.3867	15.4548
16.4207	17.3867	18.3526	19.3185	20.2844	21.2504	22.2163	23.1822	23.1822	23.1822
23.1822	23.1822	23.1822	23.1822	23.1822	23.1822	23.1822	23.1822	23.1822	23.1822
22.2163	21.2504	20.2844	19.3185	18.3526	17.3867	16.4207	15.4548	15.4548	15.4548
15.4548	14.4889	13.5230	12.5570	11.5911	10.6252	9.6593	8.6933	7.7274	7.7274
7.7274	7.7274	7.7274	7.7274	8.6933	9.6593	10.6252	11.5911	12.5570	13.5230
14.4889	15.4548	15.4548	15.4548						

LL

1 18 27 40 69 82

BKK

0.1963494E+00

IMPEDANCE MATRIX OF ORDER 38

0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03 0.2875E+01 0.4522E+02 0.1978E+00
0.1688E+01-0.2424E+01-0.1208E+02

IV. EXCITATION VECTOR FOR PLANE WAVE INCIDENCE

A subroutine VOLT to calculate $\vec{V}$ of (84) appears on pages 51-52 of [6]. Because $\vec{V}$ is desired for various frequencies, polarizations, and angles of incidence, it was necessary to rewrite the main program which calls VOLT. In the rewritten main program, the activity on data sets 1 (punched card input) and 6 (direct access input-output) is as follows.

```
      READ (1,9) NF, N6, NP, NW, NT, NPAT
  9   FORMAT (6I3)
      READ (1,18)(BK(I), I = 1, NF)
 18   FORMAT (5E14.7)
      READ (1,9)(NPA(I), I = 1, NPAT)
      READ (1, 10)(PX(I), I = 1, NP)
      READ (1, 10)(PY(I), I = 1, NP)
      READ (1, 10)(PZ(I), I = 1, NP)
 10   FORMAT (10F8.4)
      READ (1,15)(LL(I), I = 1, NW)
 15   FORMAT (20I3)
      REWIND 6
      SKIP N6 RECORDS ON DATA SET 6
      WRITE (6)(V(I), I = 1, KV1)
```

PX, PY, and PZ are the x, y, and z coordinates of the NP data points describing the axes of the NW wires. The LL(I)th data point is the first data point on the Ith wire. BK(I) is the propagation constant k at the Ith frequency. $\vec{V}$ of (84) is computed for NF frequencies, NPAT polarizations and NT angles and stored in V(I + (J-1)*N + (K-1)*N*NT + (L-1)*N*NT*NPAT) for the Ith expansion function $\underset{\sim}{J}_I$, the Jth angle $\frac{2\pi(J-1)}{NT-1}$ radians, the NPA(K)th polarization, and the Lth frequency. As in (17)-(22) on page 31 of [10], six different polarizations are defined for the incident field $\underset{\sim}{E}^i$

$$\begin{array}{llll} 1. & \underset{\sim}{u}_\theta\ ,\ x = 0 & 4. & \underset{\sim}{u}_\phi\ ,\ y = 0 \\ 2. & \underset{\sim}{u}_\phi\ ,\ x = 0 & 5. & \underset{\sim}{u}_\theta\ ,\ z = 0 \\ 3. & \underset{\sim}{u}_\theta\ ,\ y = 0 & 6. & \underset{\sim}{u}_\phi\ ,\ z = 0 \end{array} \qquad (103)$$

In the first polarization, $\underset{\sim}{E}^i$ is polarized in the $\underset{\sim}{u}_\theta$ direction and the propagation vector lies in the x=0 plane.

Minimum allocations are given by

```
COMPLEX VR(N*6), ZZ(N1*6)
COMMON L(NW+1), T(N*4), BX(N1), BY(N1), BZ(N1),
     TX(N1), TY(N1), TZ(N1)
```

in the subroutine VOLT and by

```
COMPLEX VR(N*6), V(N*NT*NPAT*NF), ZZ(N1*6)
COMMON L(NW+1), T(N*4), BX(N1), BY(N1), BZ(N1),
     TX(N1), TY(N1), TZ(N1)
DIMENSION PX(NP), PY(NP), PZ(NP), LL(NW+1),
     AL(N1), BK(NF), RX(N1), RY(N1), RZ(N1),
     SND(NT), CSD(NT)
```

in the main program where

```
N1 = NP - NW
 N = N1/2 - NW
```

The sample punched card input obtains only the frequency with propagation constant $\pi/16$ and the third polarization (u_θ, y=0) for $0° \leq \theta \leq 360°$ in steps of 2.5°. $\vec{V}$ is stored on record 2 of data set 6 immediately after the previously computed impedance matrice.

CALCULATION OF EXCITATION VECTOR FOR PLANE WAVE INCIDENCE

```
//                (0034,EE,2,2),'MAUTZ,JOE',REGION=200K
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      SUBROUTINE VOLT
      COMPLEX VR(228),U,ZZ(528),U3,U4,U5
      COMMON VR,U,ZZ,SN,CS,N1,L(9),N,NN6
      COMMON T(200),BX(100),BY(100),BZ(100),TX(100),TY(100),TZ(100)
      DO 30 I=1,N1
      S1=-BY(I)*SN
      S2=BZ(I)*CS
      S3=S1+S2
      S4=BX(I)*SN+S2
      S5=BX(I)*CS-S1
      U3=COS(S3)+U*SIN(S3)
      U4=COS(S4)+U*SIN(S4)
      U5=COS(S5)+U*SIN(S5)
      S1=TY(I)*CS
      S2=TZ(I)*SN
      J2=I+N1
      J3=J2+N1
      J4=J3+N1
      J5=J4+N1
      J6=J5+N1
      ZZ(I)=(-S2-S1)*U3
      ZZ(J2)=TX(I)*U3
      ZZ(J3)=(TX(I)*CS-S2)*U4
      ZZ(J4)=TY(I)*U4
      ZZ(J5)=-TZ(I)*U5
      ZZ(J6)=(-TX(I)*SN+S1)*U5
   30 CONTINUE
      J4=-2
      J5=1
      DO 49 I=1,N
      J2=(I-1)*4
      IF(L(J5)-I) 50,51,50
   51 J4=J4+2
      J5=J5+1
   50 J6=J4
      DO 53 J=I,NN6,N
      VR(J)=0.
      DO 52 K=1,4
      K3=J6+K
      K2=J2+K
      VR(J)=T(K2)*ZZ(K3)+VR(J)
   52 CONTINUE
      J6=J6+N1
   53 CONTINUE
      J4=J4+2
   49 CONTINUE
      RETURN
      END
      COMPLEX VR(228),V(11020),U,ZZ(528)
      COMMON VR,U,ZZ,SN,CS,N1,L(9),N,NN6
      COMMON T(200),BX(100),BY(100),BZ(100),TX(100),TY(100),TZ(100)
      DIMENSION PX(100),PY(100),PZ(100),LL(9),AL(100),BK(100)
      DIMENSION NPA(6),RX(100),RY(100),RZ(100),SND(145),CSD(145)
      U=(0.,1.)
```

```
      READ(1,9) NF,N6,NP,NW,NT,NPAT
    9 FORMAT(6I3)
      WRITE(3,14) NF,N6,NP,NW,NT,NPAT
   14 FORMAT('O NF N6 NP NW NT NPAT'/(1X,6I3))
      READ(1,18)(BK(I),I=1,NF)
   18 FORMAT(5E14.7)
      WRITE(3,19)(BK(I),I=1,NF)
   19 FORMAT('OBK'/(1X,5E14.7))
      READ(1,9)(NPA(I),I=1,NPAT)
      WRITE(3,26)(NPA(I),I=1,NPAT)
   26 FORMAT('ONPA'/(1X,6I3))
      READ(1,10)(PX(I),I=1,NP)
      READ(1,10)(PY(I),I=1,NP)
      READ(1,10)(PZ(I),I=1,NP)
   10 FORMAT(10F8.4)
      WRITE(3,11)(PX(I),I=1,NP)
      WRITE(3,12)(PY(I),I=1,NP)
      WRITE(3,13)(PZ(I),I=1,NP)
   11 FORMAT('OPX'/(1X,10F8.4))
   12 FORMAT('OPY'/(1X,10F8.4))
   13 FORMAT('OPZ'/(1X,10F8.4))
      READ(1,15)(LL(I),I=1,NW)
   15 FORMAT(20I3)
      WRITE(3,16)(LL(I),I=1,NW)
   16 FORMAT('OLL'/(1X,20I3))
      LL(NW+1)=10000
      REWIND 6
      IF(N6) 30,30,31
   31 DO 32 J=1,N6
      READ(6)(RX(I),I=1,2)
      WRITE(3,35)(RX(I),I=1,2)
   35 FORMAT(1X,2E14.7)
   32 CONTINUE
   30 N1=0
      J4=2
      J1=1
      DO 8 J=1,NP
      IF(LL(J1)-J) 7,6,7
    6 J4=J4-1
      L(J1)=J4
      J1=J1+1
      GO TO 8
    7 N1=N1+1
      J3=J-1
      IF((N1/2*2-N1).EQ.0) J4=J4+1
      S1=PX(J)-PX(J3)
      S2=PY(J)-PY(J3)
      S3=PZ(J)-PZ(J3)
      S4=SQRT(S1*S1+S2*S2+S3*S3)
      TX(N1)=S1/S4
      TY(N1)=S2/S4
      TZ(N1)=S3/S4
      RX(N1)=PX(J)+PX(J3)
      RY(N1)=PY(J)+PY(J3)
      RZ(N1)=PZ(J)+PZ(J3)
      AL(N1)=S4
    8 CONTINUE
      N=J4-2
      DO 34 I=1,NPAT
      NPA(I)=(NPA(I)-1)*N
```

```
34 CONTINUE
   L(J1)=J4
   J1=1
   J2=-2
   DO 5 J=1,N
   IF(L(J1)-J) 3,4,3
 4 J2=J2+2
   J1=J1+1
 3 J4=(J-1)*4+1
   J5=J4+1
   J6=J5+1
   J7=J6+1
   K4=J2+1
   K5=K4+1
   K6=K5+1
   K7=K6+1
   S1=AL(K4)+AL(K5)
   S2=AL(K6)+AL(K7)
   T(J4)=AL(K4)*.5*AL(K4)/S1
   T(J5)=AL(K5)*(AL(K4)+.5*AL(K5))/S1
   T(J6)=AL(K6)*(AL(K7)+.5*AL(K6))/S2
   T(J7)=AL(K7)*.5*AL(K7)/S2
   J2=J2+2
 5 CONTINUE
   NTN=NT*N
   NTP=NTN*NPAT
   NN6=N*6
   IF(NT-1) 23,23,24
23 DEL=0.
   GO TO 25
24 DEL=2.*3.141593/(NT-1)
25 DO 36 J=1,NT
   THET=(J-1)*DEL
   SND(J)=SIN(THET)
   CSD(J)=COS(THET)
36 CONTINUE
   DO 20 J=1,NF
   BK5=.5*BK(J)
   DO 21 I=1,N1
   BX(I)=BK5*RX(I)
   BY(I)=BK5*RY(I)
   BZ(I)=BK5*RZ(I)
21 CONTINUE
   KV=(J-1)*NTP
   DO 22 I=1,NT
   SN=SND(I)
   CS=CSD(I)
   KV3=(I-1)*N
   CALL VOLT
   DO 29 KK=1,NPAT
   KV1=KV+(KK-1)*NTN+KV3
   KV2=NPA(KK)
   DO 28 K=1,N
   KV1=KV1+1
   KV2=KV2+1
   V(KV1)=VR(KV2)
28 CONTINUE
29 CONTINUE
22 CONTINUE
20 CONTINUE
```

```
      WRITE(3,37)(V(I),I=1,5)
   37 FORMAT('OV'/(1X,7E11.4))
      WRITE(6)(V(I),I=1,KV1)
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                  X
//               VOLUME=SER=SU0004,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//               BUFNO=1)
//GO.SYSIN DD *
  1  1 94   6145   1
 0.1963495E+00
  3
  2.0706   1.8117   1.5529   1.2941   1.0353   0.7765   0.5176   0.2588   0.0000  -0.2588
 -0.5176  -0.7765  -1.0353  -1.2941  -1.5529  -1.8117  -2.0706  -1.5529  -1.8117  -2.0706
 -1.0353   0.0000   1.0353   2.0706   1.8117   1.5529  -4.6587  -4.3999  -4.1411  -3.1058
 -2.0706  -1.0353   0.0000   1.0353   2.0706   3.1058   4.1411   4.3999   4.6587  -4.1411
 -4.3999  -4.6587  -4.9176  -5.1764  -5.4352  -5.6940  -5.9528  -6.2117  -5.1764  -4.1411
 -3.1058  -2.0706  -1.0353   0.0000   1.0353   2.0706   3.1058   4.1411   5.1764   6.2117
  5.9528   5.6940   5.4352   5.1764   4.9176   4.6587   4.3999   4.1411   2.0706   3.1058
  4.1411   3.8823   3.6235   3.3646   3.1058   2.8470   2.5882   2.3294   2.0706   1.0353
  0.0000   0.0000  -1.0353  -2.0706  -2.3294  -2.5882  -2.8470  -3.1058  -3.3646  -3.6235
 -3.8823  -4.1411  -3.1058  -2.0706
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000
  7.7274   6.7615   5.7956   4.8296   3.8637   2.8978   1.9319   0.9659   0.0000   0.9659
  1.9319   2.8978   3.8637   4.8296   5.7956   6.7615   7.7274   5.7956   6.7615   7.7274
  7.7274   7.7274   7.7274   7.7274   6.7615   5.7956  17.3867  16.4207  15.4548  15.4548
 15.4548  15.4548  15.4548  15.4548  15.4548  15.4548  15.4548  16.4207  17.3867  15.4548
 16.4207  17.3867  18.3526  19.3185  20.2844  21.2504  22.2163  23.1822  23.1822  23.1822
 23.1822  23.1822  23.1822  23.1822  23.1822  23.1822  23.1822  23.1822  23.1822  23.1822
 22.2163  21.2504  20.2844  19.3185  18.3526  17.3867  16.4207  15.4548  15.4548  15.4548
 15.4548  14.4889  13.5230  12.5570  11.5911  10.6252   9.6593   8.6933   7.7274   7.7274
  7.7274   7.7274   7.7274   7.7274   8.6933   9.6593  10.6252  11.5911  12.5570  13.5230
 14.4889  15.4548  15.4548  15.4548
  1 18 27 40 69 82
/*
//
```

PRINTED OUTPUT

```
 NF  N6  NP  NW  NT  NPAT
  1   1 94   6145   1

BK
 0.1963494E+00

NPA
  3

PX
  2.0706   1.8117   1.5529   1.2941   1.0353   0.7765   0.5176   0.2588   0.0000  -0.2588
```

```
-0.5176 -0.7765 -1.0353 -1.2941 -1.5529 -1.8117 -2.0706 -1.5529 -1.8117 -2.0706
-1.0353  0.0000  1.0353  2.0706  1.8117  1.5529 -4.6587 -4.3999 -4.1411 -3.1058
-2.0706 -1.0353  0.0000  1.0353  2.0706  3.1058  4.1411  4.3999  4.6587 -4.1411
-4.3999 -4.6587 -4.9176 -5.1764 -5.4352 -5.6940 -5.9528 -6.2117 -5.1764 -4.1411
-3.1058 -2.0706 -1.0353  0.0000  1.0353  2.0706  3.1058  4.1411  5.1764  6.2117
 5.9528  5.6940  5.4352  5.1764  4.9176  4.6587  4.3999  4.1411  2.0706  3.1058
 4.1411  3.8823  3.6235  3.3646  3.1058  2.8470  2.5882  2.3294  2.0706  1.0353
 0.0000  0.0000 -1.0353 -2.0706 -2.3294 -2.5882 -2.8470 -3.1058 -3.3646 -3.6235
-3.8823 -4.1411 -3.1058 -2.0706

PY
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0. 000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000

PZ
  7.7274  6.7615  5.7956  4.8296  3.8637  2.8978  1.9319  0.9659  0.0000  0.9659
  1.9319  2.8978  3.8637  4.8296  5.7956  6.7615  7.7274  5.7956  6.7615  7.7274
  7.7274  7.7274  7.7274  7.7274  6.7615  5.7956 17.3867 16.4207 15.4548 15.4548
 15.4548 15.4548 15.4548 15.4548 15.4548 15.4548 15.4548 16.4207 17.3867 15.4548
 16.4207 17.3867 18.3526 19.3185 20.2844 21.2504 22.2163 23.1822 23.1822 23.1822
 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822
 22.2163 21.2504 20.2844 19.3185 18.3526 17.3867 16.4207 15.4548 15.4548 15.4548
 15.4548 14.4889 13.5230 12.5570 11.5911 [illegible]  9.6593  8.6933  7.7274  7.7274
  7.7274  7.7274  7.7274  7.7274  8.6933  9.6593 10.6252 [illegible] 12.5570 13.5230
 14.4889 15.4548 15.4548 15.4548

LL
  1 18 27 40 69 82
 0.3056983E+01-0.5124519E+02

V
 -0.2147E+00-0.4636E+00-0.3706E+00-0.3512E+00-0.4744E+00-0.1891E+00-0.5107E+00
-0.7308E-01-0.4744E+00-0.1191E+00
```

V. PORT PARAMETERS

A program has been written to calculate and store on direct access data set 6 the various port parameters appearing in Part One Section VIII. In this program, the activity on data sets 1 (punched card input) and 6 (direct access input-output) is as follows.

```
        READ (1,10) NF, NT, M, N, N6V, N6Z, N6P, NVT
10      FORMAT (20I3)
        READ (1,13)(BK(I), I = 1, NF)
13      FORMAT (5E14.7)
        READ (1, 10)(N4(I), I = 1, M)
        READ (1,10) NY, NZ, MIV, MII, MISC, NISC, KESC,
           MIθC, NVθC, KEθC
        KV = NT*NT*M
        REWIND 6
        IF(NT. EQ. 0) GO TO 16
        SKIP N6V RECORDS ON DATA SET 6
        READ (6) (V(I), I = 1, KV)
16      SKIP N6Z RECORDS ON DATA SET 6
        MM = M*M
        DO 27 K = 1, NF
        READ (6)(Z(I), I = 1, MM)
27      CONTINUE
        SKIP N6P RECORDS ON DATA SET 6
        WRITE (6)(PP(I), I = 1, NPP)
```

The impedance matrix previously computed at propagation constant BK(K) is read in through Z in DO loop 27. Note that the order of the impedance matrix is now M, N being henceforth reserved for the number of ports. The previously computed excitation vectors $\vec{V}$ are read in through V. The L^{th} excitation vector $\vec{V}$ at the K^{th} frequency resides in V((K-1)*NT*M+(L-1)*M+1) through V((K-1)*NT*M+L*M) where $1 \leq K \leq NF$, $1 \leq L \leq NT$.

The impedance matrix Z is stored in the present program exactly as it was previously computed, namely so that

$$Z \vec{I}^d = \vec{V}^d \tag{104}$$

where the vectors $\vec{I}^d$ and $\vec{V}^d$ are the same as those in (83) except that the elements are arranged in the order dictated by the NP data points instead of having the port elements come first. The present vector $\vec{I}^d$ is related to $\vec{I}$ of (83) and more especially (87) through N4.

$$(\vec{I})_J = (\vec{I}^d)_{N4(J)} \tag{105}$$

The subscript J indicates the J^{th} element of the vector.

The port parameters are stored in PP which is written on data set 6 at the end of the program. A fixed point variable is associated with each port parameter. If the fixed point variable MISC = 0, $\vec{I}_M^{SC}$ is not stored in PP. If MISC = 1, only the $\vec{I}_M^{SC}$ inside nested DO loops 27 and 42 is stored in PP. If MISC = 2, only the $\vec{I}_M^{SC}$ outside DO loop 27 is stored in PP. The latter $\vec{I}_M^{SC}$ is the same as the former $\vec{I}_M^{SC}$ evaluated at L = NVT. MIθC controls $\vec{I}_M^{OC}$ in the same way that MISC controls $\vec{I}_M^{SC}$. If a fixed point variable other than MIθC or MISC is one, the relevant port parameter is stored in PP but if this fixed point variable is zero, the port parameter is not stored in PP. The port parameters are stored in PP in the following order.

Looping	Port Parameter	Number of Elements	Fixed Point Variable	DO Loop
DO 27K = 1, NF				
	Y_S	N*N	NY	32
	Z_S	N*N	NZ	34
	I_V	M*N	MIV	35
	I_I	M*N	MII	38
DO 42L = 1, NT				
	$\vec{I}_M^{SC}$	M	MISC	60
	$\vec{I}^{SC}$	N	NISC	62
	$F_0^{SC} \cdot u_r$	1	KESC	64
	$\vec{I}_M^{OC}$	M	MIθC	66
	$\vec{V}^{OC}$	N	NVθC	69
	$F_0^{OC} \cdot u_r$	1	KEθC	71
42 CONTINUE				
27 CONTINUE				
	$\vec{I}_M^{SC}$	M	MISC	97
	$\vec{I}_M^{OC}$	M	MIθC	98

The column labeled "DO Loop" gives the number of the DO loop which stores the port parameter in PP. There are no vacancies internal to PP. For instance, if NY = MIV = 1 and NZ = 0 in which case Y_S and I_V are stored but Z_S is not, then I_V is stored immediately after Y_S. The index K of DO loop 27 obtains the K^{th} frequency. The port parameters inside DO loop 42 are computed from the L^{th} excitation vector $\vec{V}$ at the K^{th} frequency residing in V((K-1)*NT*M+(L-1)*M+1) through V((K-1)*NT*M+L*M). If NT = 0, DO loop 42 is omitted. The port parameters Y_S, Z_S, $\vec{I}^{SC}$, $F_0^{SC} \cdot u_r$, $\vec{V}^{OC}$, and $F_0^{OC} \cdot u_r$ are defined in Part One of this report. The normalized scattered fields $F_0^{SC} \cdot u_r$ and $F_0^{OC} \cdot u_r$ are computed from the transmitter excitation vector $\vec{V}$ labeled $\vec{V}_t$ in (93) which resides in

V((K-1)*NT*M+(NVT-1)*M+1) through V((K-1)*NT*M+NVT*M). The set of elements I_V((J-1)*M+1) through I_V(J*M) is $\vec{I}^d$ of (104) when the J^{th} port is driven by one volt and all other ports are short circuited. The set of elements I_I((J-1)*M+1) through I_I(J*M) is $\vec{I}^d$ of (104) when the J^{th} port is driven by one ampere and all other ports are open circuited. $\vec{I}_M^{SC}$ is $\vec{I}^d$ for the short circuited scatterer while $\vec{I}_M^{OC}$ is $\vec{I}^d$ for the open circuited scatterer.

Minimum allocations are given by

```
DIMENSION LR(M)
```

in the subroutine LINEQ and by

```
COMPLEX Z(M*M), V(NF*NT*M), YS(N*N), ZS(N*N),
    PP(2*NF*(N*N+M*N+NT*(M+N+1))), YV2(M),
    ZYV(N), FI2(M), YV1(M), YV(M), CR1(M), CR2(M)
DIMENSION N4(M), BK(NF)
```

in the main program. The above allocation for PP is based on the assumption that all ten of the fixed point variables NY, NZ, ... KEθC are one.

Statement 92 inverts the impedance matrix Z. DO loop 29 and statement 93 obtain the port matrices Y_S and Z_S. DO loops 32 and 34 store Y_S and Z_S in PP. DO loop 35 puts I_V in PP. DO loop 38 puts I_I in PP. I_I is calculated by noting that driving the J^{th} port with one ampere when all other ports are open circuited is equivalent to driving with port voltages given by the J^{th} column of Z_S. DO loop 46 stores $\begin{bmatrix} Y_{12} \\ Y_{22} \end{bmatrix}$ $[V_2]$ in YV2. DO loop 49 puts $Z_S Y_{12}$ in ZYV. DO loop 52 stores $\vec{I}_2$ of (98) in FI2. DO loop 58 stores $\vec{I}_M^{SC}$ in YV. The portion of DO loop 42 beyond statement 58 employs DO loops 60, 62, 64, 66, 69, and 71 to store respectively $\vec{I}_M^{SC}$, $\vec{I}^{SC}$, $\underset{\sim}{F}_O^{SC} \cdot \underset{\sim}{u}_r$, $\vec{I}_M^{OC}$, $\vec{V}^{OC}$, and $\underset{\sim}{F}_O^{OC} \cdot \underset{\sim}{u}_r$ in PP.

The sample data is such that all port parameters except $\vec{I}_M^{SC}$ and $\vec{I}_M^{OC}$ are stored on record 3 of data set 6. The first four port parameters are computed from the previously stored impedance matrix Z at propagation constant $\frac{\pi}{16}$. The remaining port parameters are computed from Z and the 145 excitation vectors $\vec{V}$ for $\underset{\sim}{u}_\theta$ polarized plane waves every 2.5° in the y=0 plane.

LISTING OF PROGRAM TO CALCULATE PORT PARAMETERS

```
//                (0034,EE,2,2),'MAUTZ,JOE',REGION=220K
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      SUBROUTINE LINEQ(LL,C)
      COMPLEX C(1),STOR,STO,ST,S
      DIMENSION LR(40)
      DO 20 I=1,LL
      LR(I)=I
   20 CONTINUE
      M1=0
      DO 18 M=1,LL
      K=M
      DO 2 I=M,LL
      K1=M1+I
      K2=M1+K
      IF(CABS(C(K1))-CABS(C(K2))) 2,2,6
    6 K=I
    2 CONTINUE
      LS=LR(M)
      LR(M)=LR(K)
      LR(K)=LS
      K2=M1+K
      STOR=C(K2)
      J1=0
      DO 7 J=1,LL
      K1=J1+K
      K2=J1+M
      STO=C(K1)
      C(K1)=C(K2)
      C(K2)=STO/STOR
      J1=J1+LL
    7 CONTINUE
      K1=M1+M
      C(K1)=1./STOR
      DO 11 I=1,LL
      IF(I-M) 12,11,12
   12 K1=M1+I
      ST=C(K1)
      C(K1)=0.
      J1=0
      DO 10 J=1,LL
      K1=J1+I
      K2=J1+M
      C(K1)=C(K1)-C(K2)*ST
      J1=J1+LL
   10 CONTINUE
   11 CONTINUE
      M1=M1+LL
   18 CONTINUE
      J1=0
      DO 9 J=1,LL
      IF(J-LR(J)) 14,8,14
   14 LRJ=LR(J)
      J2=(LRJ-1)*LL
   21 DO 13 I=1,LL
      K2=J2+I
      K1=J1+I
      S=C(K2)
```

```
      C(K2)=C(K1)
      C(K1)=S
   13 CONTINUE
      LR(J)=LR(LRJ)
      LR(LRJ)=LRJ
      IF(J-LR(J)) 14,8,14
    8 J1=J1+LL
    9 CONTINUE
      RETURN
      END
      COMPLEX Z(1444),V(11020),YS(16),ZS(16),PP(3312),YV2(38),ZYV(4)
      COMPLEX FI2(38),YV1(38),YV(38),CR1(38),CR2(38)
      DIMENSION N4(38),BK(50)
      READ(1,10) NF,NT,M,N,N6V,N6Z,N6P,NVT
   10 FORMAT(20I3)
      WRITE(3,11) NF,NT,M,N,N6V,N6Z,N6P,NVT
   11 FORMAT('0 NF NT  M  N N6V N6Z N6P NVT'/1X,4I3,4I4)
      READ(1,13)(BK(I),I=1,NF)
   13 FORMAT(5E14.7)
      WRITE(3,14)(BK(I),I=1,NF)
   14 FORMAT('0BK'/(1X,5E14.7))
      READ(1,10)(N4(I),I=1,M)
      WRITE(3,15)(N4(I),I=1,M)
   15 FORMAT('0N4'/(1X,20I3))
      READ(1,10) NY,NZ,MIV,MII,MISC,NISC,KESC,MIOC,NVOC,KEOC
      WRITE(3,12) NY,NZ,MIV,MII,MISC,NISC,KESC,MIOC,NVOC,KEOC
   12 FORMAT('0 NY NZ MIV MII MISC NISC KESC MIOC NVOC KEOC'/1X,2I3,2I4,
     16I5)
      KV=NF*NT*M
      REWIND 6
      IF(NT.EQ.0) GO TO 16
      IF(N6V) 17,17,18
   18 DO 19 J=1,N6V
      READ(6)(Z(I),I=1,2)
      WRITE(3,20)(Z(I),I=1,2)
   20 FORMAT(1X,4E11.4)
   19 CONTINUE
   17 READ(6)(V(I),I=1,KV)
      WRITE(3,21)(V(I),I=1,3)
   21 FORMAT('0V'/(1X,7E11.4))
   16 N6=IABS(N6Z)
      IF(N6Z) 22,23,24
   22 DO 25 J=1,N6
      BACKSPACE 6
   25 CONTINUE
      GO TO 23
   24 DO 26 J=1,N6
      READ(6)
   26 CONTINUE
   23 MM=M*M
      NN=N*N
      NPP=0
      NP1=N+1
      NZZ=NZ+MII+MIOC+NVOC+KEOC
      KYV=MISC+KESC+MIOC+KEOC
      NZYV=NIOC+NVOC+KEOC
      NFI2=MIOC+KEOC
      NYV1=MISC+NISC+KESC
      NYV2=MISC+KESC
      DO 27 K=1,NF
```

```
      NV1=(K-1)*NT*M
      NV2=NV1+(NV1-1)*M
      READ(6)(Z(I),I=1,MM)
      WRITE(3,28)(Z(I),I=1,3)
   28 FORMAT('0Z'/(1X,7E11.4))
   92 CALL LINEQ(M,Z)
      J4=0
      DO 29 J=1,N
      J1=(N4(J)-1)*M
      DO 30 I=1,N
      J2=J1+N4(I)
      J4=J4+1
      YS(J4)=Z(J2)
      ZS(J4)=YS(J4)
   30 CONTINUE
   29 CONTINUE
      IF(NZZ.EQ.0) GO TO 48
   93 CALL LINEQ(N,ZS)
   48 IF(NY.EQ.0) GO TO 31
      DO 32 J=1,NN
      NPP=NPP+1
      PP(NPP)=YS(J)
   32 CONTINUE
   31 IF(NZ.EQ.0) GO TO 33
      DO 34 J=1,NN
      NPP=NPP+1
      PP(NPP)=ZS(J)
   34 CONTINUE
   33 IF(MIV.EQ.0) GO TO 37
      DO 94 I=1,N
      J1=(N4(I)-1)*M
      DO 35 J=1,M
      NPP=NPP+1
      J2=J1+J
      PP(NPP)=Z(J2)
   35 CONTINUE
   94 CONTINUE
   37 IF(MII.EQ.0) GO TO 36
      DO 38 J=1,N
      J1=(J-1)*N
      DO 39 I=1,M
      J3=(I-1)*M
      NPP=NPP+1
      PP(NPP)=0.
      DO 40 L=1,N
      J2=L+J1
      J4=N4(L)+J3
      PP(NPP)=PP(NPP)+Z(J4)*ZS(J2)
   40 CONTINUE
   39 CONTINUE
   38 CONTINUE
   36 IF(NT.EQ.0) GO TO 27
      DO 42 L=1,NT
      NV=NV1+(L-1)*M
      NP2=M
      IF(KYV.EQ.0) NP2=NP1
      DO 46 J=1,NP2
      YV2(J)=0
      J1=(N4(J)-1)*M
      DO 47 I=NP1,M
```

```
      J3=NV+N4(I)
      J4=J1+N4(I)
      YV2(J)=YV2(J)+Z(J4)*V(J3)
   47 CONTINUE
   46 CONTINUE
   43 IF(NZYV.EQ.0) GO TO 45
      DO 49 J=1,N
      ZYV(J)=0.
      J1=(J-1)*N
      DO 50 I=1,N
      J2=J1+I
      ZYV(J)=ZYV(J)+ZS(J2)*YV2(I)
   50 CONTINUE
   49 CONTINUE
   45 IF(NFI2.EQ.0) GO TO 51
      DO 52 J=NP1,M
      FI2(J)=YV2(J)
      J1=(N4(J)-1)*M
      DO 53 I=1,N
      J2=J1+N4(I)
      FI2(J)=FI2(J)-Z(J2)*ZYV(I)
   53 CONTINUE
   52 CONTINUE
   51 NP2=1
      IF(NYV1.EQ.0) NP2=NP1
      NP3=M
      IF(NYV2.EQ.0) NP3=N
      IF(NP2.GT.NP3) GO TO 54
      DO 55 J=NP2,NP3
      YV1(J)=0.
      J1=(N4(J)-1)*M
      DO 56 I=1,N
      J3=NV+N4(I)
      J4=J1+N4(I)
      YV1(J)=YV1(J)+Z(J4)*V(J3)
   56 CONTINUE
   55 CONTINUE
   54 IF(NYV2.EQ.0) GO TO 57
      DO 58 J=1,M
      J1=N4(J)
      YV(J1)=YV1(J)+YV2(J)
   58 CONTINUE
   57 IF(MISC.EQ.0) GO TO 59
      GO TO (105,106),MISC
  105 DO 60 J=1,M
      NPP=NPP+1
      PP(NPP)=YV(J)
   60 CONTINUE
      GO TO 59
  106 IF(L.NE.NVT) GO TO 59
      DO 107 J=1,M
      CR1(J)=YV(J)
  107 CONTINUE
   59 IF(NISC.EQ.0) GO TO 61
      DO 62 J=1,N
      NPP=NPP+1
      J2=N4(J)
      PP(NPP)=YV(J2)
   62 CONTINUE
   61 IF(KESC.EQ.0) GO TO 63
```

```
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                  X
//               VOLUME=SER=SU0004,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//               BUFNO=1)
//GO.SYSIN DD *
  1145 38  4  1 -2  1 73
 0.1963495E+00
   4  9 13 22  1  2  3  5  6  7  8 10 11 12 14 15 16 17 18 19
 20 21 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38
  1  1  1  1  0  1  1  0  1  1
/*
//
```

PRINTED OUTPUT

```
 NF NT  M  N N6V N6Z N6P NVT
  1145 38  4   1  -2   1  73

BK
 0.1963494E+00

N4
   4  9 13 22  1  2  3  5  6  7  8 10 11 12 14 15 16 17 18 19
 20 21 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38

 NY NZ MIV MII MISC NISC KESC MIOC NVOC KEOC
  1  1   1   1    0    1    1    0    1    1
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03

V
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00-0.4744E+00-0.1891E+00

Z
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03 0.2875E+01 0.4522E+02
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00

PP
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02-0.6371E-03-0.2576E-03
```

VI. MODES OF A LOADED N PORT

The characteristic port voltages $\vec{V}_n$ of (23) and characteristic port currents $\vec{I}_n$ of (17) can be obtained from the eigencurrent program on pages 18-26 of [10]. Since equations (23) and (17) are of the same form, the eigencurrent program will automatically calculate either $\vec{V}_n$ of (23) or $\vec{I}_n$ of (17) depending upon whether the input is either $[Y_S + Y_L]$ or $[Z_S + Z_L]$. The loads (Y_L or Z_L) are read in via punched cards but the eigencurrent program expects each input matrix (Y_S or Z_S) to reside on a separate record on data set 6.

A short program was written to extract Y_S and Z_S from the previously stored port parameters and to store Y_S and Z_S on separate records on data set 6. In this program, the activity on data sets 1 (punched card input) and 6 (direct access input-output) is as follows.

```
      READ (1,10) N, N6, N7
10    FORMAT (20I3)
      REWIND 6
      SKIP N6 RECORDS ON DATA SET 6
      NN = N*N
      NN2 = NN*2
      READ (6)(PP(I), I = 1, NN2)
      SKIP N7 RECORDS ON DATA SET 6
      WRITE (6) (PP(I), I = NN)
      J1 = NN + 1
      WRITE (6)(PP(I), I = J1, NN2)
```

There are N ports. The sample data is such that Y_S and Z_S are read from record 3 and written on records 4 and 5 respectively.

STORE YS AND ZS ON SEPARATE RECORDS ON DATA SET 6

```
//                 (0034,EE,1,1),'MAUTZ,JOE',REGION=140K
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      COMPLEX PP(200)
      READ(1,10) N,N6,N7
   10 FORMAT(20I3)
      WRITE(3,11) N,N6,N7
   11 FORMAT('0   N N6 N7'/1X,3I3)
      REWIND 6
      IF(N6) 12,12,13
   13 DO 14 J=1,N6
      READ(6)(PP(I),I=1,2)
      WRITE(3,16)(PP(I),I=1,2)
   16 FORMAT(1X,7E11.4)
   14 CONTINUE
   12 NN=N*N
      NN2=NN*2
      READ(6)(PP(I),I=1,NN2)
      WRITE(3,17)(PP(I),I=1,3)
   17 FORMAT('OPP'/(1X,7E11.4))
      IF(N7) 18,18,19
   19 DO 20 I=1,N7
      READ(6)
   20 CONTINUE
   18 WRITE(6)(PP(I),I=1,NN)
      WRITE(3,17)(PP(I),I=1,3)
      J1=NN+1
      WRITE(6)(PP(I),I=J1,NN2)
      J2=NN+3
      WRITE(3,17)(PP(I),I=J1,J2)
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                 X
//                VOLUME=SER=SU0004,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//                BUFNO=1)
//GO.SYSIN DD *
  4  2  0
/*
//
```

PRINTED OUTPUT

```
  N N6 N7
  4  2  0
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00

PP
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02-0.6371E-03-0.2576E-03

PP
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02-0.6371E-03-0.2576E-03

PP
 0.6553E+01-0.1817E+03 0.7365E+00 0.1965E+03 0.1113E+02-0.3768E+03
```

```
EIGENCURRENT FOR WHICH LAMBDA =  0.5054E+02
  1.0000 -0.2887  0.7884 -0.4050

EIGENCURRENT FOR WHICH LAMBDA =  0.1012E+02
  0.1904  0.2524  1.0000 -0.9256

EIGENCURRENT FOR WHICH LAMBDA = 0.1552E+00
 -0.0205  0.1158  0.6425  1.0000

EIGENCURRENT FOR WHICH LAMBDA = -0.8162E+03
  0.5313  1.0000 -0.6749  0.2066

  N     EPS         EPL
  4 0.1000E-03 0.1000E+11

ZL
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00
 0.0000E+00

EIGENVALUES OF THE MATRIX R
 0.1133E+03 0.8199E+02 0.4562E+01 0.5509E+00

EIGENVALUES OF THE MATRIX B
 0.8164128E+03-0.1551809E+00-0.1011935E+02-0.50540[illegible]4E+02

EIGENCURRENT FOR WHICH LAMBDA =  0.8164E+03
  0.5441  1.0000 -0.2839  0.0778

EIGENCURRENT FOR WHICH LAMBDA = -0.1552E+00
 -0.1338  0.4326  0.8419  1.0000

EIGENCURRENT FOR WHICH LAMBDA = -0.1012E+02
 -0.6078  1.0000  0.8054 -0.6458

EIGENCURRENT FOR WHICH LAMBDA = -0.5054E+02
  1.0000 -0.5374  0.0137  0.0740
```

VII. MODE CURRENTS

The program to be described in this section uses the previously computed port parameter $\vec{I}_M^{SC}$ to calculate the currents (mode currents) excited by the port mode voltages $\vec{V}_n$ of (23). If presented with the port parameter $\vec{I}_M^{OC}$ and the port mode currents $\vec{I}_n$ of (17), this program will automatically calculate the currents excited by the port mode currents $\vec{I}_n$.

In the present program, the activity on data sets 1 (punched card input) and 6 (direct access input-output) is as follows.

```
        READ (1,10) NF
10      FORMAT (20I3)
        DO 13L = 1, NF
        READ (1,10), M,N,NE, N6P, N6V, N6C, NC, NθRM
        NEN = NE*N
        NPP = NC + N*M
        REWIND 6
        SKIP N6P RECORDS ON DATA SET 6
        READ (6)(PP(I), I = 1, NPP)
        SKIP N6V RECORDS ON DATA SET 6
        READ (6) (VN(I), I = 1, NEN)
        SKIP N6C RECORDS ON DATA SET 6
        J2 = NE*M
        WRITE (6) (CUR(I), I = 1, J2)
13      CONTINUE
```

There are M expansion functions associated with the method of moments, N ports and NE port modes. The port parameter I_V is in PP. More precisely, the current excited by one volt at the J^{th} port is in PP(NC+(J-1)*M+1) through PP(NC+J*M). The port mode voltages $\vec{V}_n$ of (23) are read in through VN. The current excited by the K^{th} port mode voltage is stored in CUR((K-1)*M+1) through CUR(K*M). The elements CUR((K-1)*M+1) through CUR(K*M) are in the same order as those of I^d of (104). The magnitude of the element largest in magnitude is normalized to one and the phase of the NC^{th} element CUR((K-1)*M+NORM) is normalized to zero.

Minimum allocations are given by

```
COMPLEX PP(NC+N*M), CUR(NE*M)
DIMENSION VN(NE*N)
```

The index K of DO loop 26 obtains the K^{th} current CUR((K-1)*M+1) through CUR(K*M). The index J of DO loop 24 obtains the J^{th} element of the K^{th} current. In DO loop 25, the current PP(J2) excited by one volt at the I^{th} port is multiplied by the port mode voltage VN(J6) at the I^{th} port to form a contribution to the current CUR(J4).

The sample data is such as to store the currents excited by the port mode voltages on record 8 of data set 6 and the currents excited by the port mode currents on record 9 of data set 6.

LISTING OF PROGRAM TO COMPUTE MODE CURRENTS

```
//                (0034,EE,1,1),'MAUTZ,JOE',REGION=140K
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      COMPLEX PP(336),CUR(152),U1
      DIMENSION VN(16)
      READ(1,10) NF
   10 FORMAT(20I3)
      WRITE(3,12) NF
   12 FORMAT('ONF'/1X,I3)
      DO 13 L=1,NF
      READ(1,10) M,N,NE,N6P,N6V,N6C,NC,NORM
      WRITE(3,11) M,N,NE,N6P,N6V,N6C,NC,NORM
   11 FORMAT('0  M  N NE N6P N6V N6C NC NORM'/1X,3I3,3I4,I3,I5)
      NEN=NE*N
      NPP=NC+N*M
      REWIND 6
      IF(N6P) 14,14,15
   15 DO 16 J=1,N6P
      READ(6)(PP(I),I=1,2)
      WRITE(3,8)(PP(I),I=1,2)
    8 FORMAT(1X,4E11.4)
   16 CONTINUE
   14 READ(6)(PP(I),I=1,NPP)
      WRITE(3,22)(PP(I),I=1,2)
   22 FORMAT('OPP'/(1X,7E11.4))
      N6=IABS(N6V)
      IF(N6V) 17,18,19
   17 DO 20 J=1,N6
      BACKSPACE 6
   20 CONTINUE
      GO TO 18
   19 DO 21 J=1,N6
      READ(6)(VN(I),I=1,2)
      WRITE(3,8)(VN(I),I=1,2)
   21 CONTINUE
   18 READ(6)(VN(I),I=1,NEN)
      WRITE(3,23)(VN(I),I=1,3)
   23 FORMAT('OVN'/(1X,7E11.4))
      DO 26 K=1,NE
      J5=(K-1)*N
      J3=(K-1)*M
      S1=0.
      DO 24 J=1,M
      J4=J3+J
      CUR(J4)=0.
      J2=J+NC
      DO 25 I=1,N
      J6=J5+I
      CUR(J4)=CUR(J4)+PP(J2)*VN(J6)
      J2=J2+M
   25 CONTINUE
      S2=CABS(CUR(J4))
      IF(S2.GT.S1) S1=S2
   24 CONTINUE
      J2=J3+NORM
      U1=CABS(CUR(J2))/(CUR(J2)*S1)
      DO 27 J=1,M
      J2=J+J3
```

```
      CUR(J2)=CUR(J2)*U1
   27 CONTINUE
      J1=J3+1
      WRITE(3,28) K,(CUR(I),I=J1,J2)
   28 FORMAT('0',I3,'TH MODE CURRENT'/(1X,10F8.4))
   26 CONTINUE
      N6=IABS(N6C)
      IF(N6C) 29,30,31
   29 DO 32 J=1,N6
      BACKSPACE 6
   32 CONTINUE
      GO TO 30
   31 DO 33 J=1,N6
      READ(6)(VN(I),I=1,2)
      WRITE(3,8)(VN(I),I=1,2)
   33 CONTINUE
   30 WRITE(6)(CUR(I),I=1,J2)
   13 CONTINUE
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                 X
//             VOLUME=SER=SU0004,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//             BUFNO=1)
//GO.SYSIN DD *
  2
 38   4   4   2   2   1 32   4
 38   4   4   2   3   1184   4
/*
//
```

PRINTED OUTPUT

```
NE
  2

  M   N  NE  N6P  N6V  N6C  NC  NORM
 38   4   4    2    2    1  32     4
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00

PP
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.1917E-03 0.3061E-02
 0.6553E+01-0.1817E+03

VN
 0.1000E+01-0.2887E+00 0.7884E+00

  1TH MODE CURRENT
 -0.6359 -0.0223 -0.3245 -0.0212  0.0964 -0.0163  1.0000  0.0000  0.0964 -0.0163
 -0.3245 -0.0212 -0.6359 -0.0223 -0.7989 -0.0197 -0.5374 -0.0000 -0.7989 -0.0198
 -0.0617 -0.0176 -0.3187 -0.0075  0.0137 -0.0000 -0.3187 -0.0075 -0.0617 -0.0176
  0.1547  0.0218  0.2494  0.0232  0.3124  0.0211  0.3392  0.0158  0.3143  0.0095
  0.2446  0.0046  0.0740 -0.0000  0.2446  0.0046  0.3143  0.0095  0.3392  0.0158
  0.3124  0.0212  0.2494  0.0232  0.1547  0.0218 -0.3766  0.0058 -0.4392 -0.0002
 -0.4766 -0.0079 -0.4498 -0.0149 -0.3734 -0.0202 -0.3734 -0.0202 -0.4498 -0.0149
 -0.4766 -0.0079 -0.4392 -0.0002 -0.3766  0.0058

2TH MODE CURRENT
```

```
 0.5398  0.0429  0.6039  0.0371  0.5789  0.0260  0.3635 -0.0000  0.5789  0.0260
 0.6039  0.0371  0.5398  0.0429  0.4360  0.0453 -0.5980  0.0000  0.4360  0.0453
 0.6599  0.0852  0.1644  0.0609 -0.4817  0.0000  0.1644  0.0609  0.6599  0.0852
-0.8641 -0.0948 -0.9949 -0.1004 -0.9908 -0.0998 -0.8637 -0.0964 -0.5946 -0.0846
-0.2359 -0.0583  0.3862  0.0000 -0.2359 -0.0583 -0.5946 -0.0845 -0.8637 -0.0964
-0.9908 -0.0997 -0.9949 -0.1004 -0.8641 -0.0947 -0.2358 -0.0047  0.0172  0.0059
 0.3588  0.0193  0.6738  0.0296  0.8793  0.0335  0.8792  0.0335  0.6738  0.0296
 0.3588  0.0193  0.0172  0.0059 -0.2358 -0.0047

3TH MODE CURRENT
 0.0869 -0.0143  0.1120 -0.0128  0.1272 -0.0087  0.1338  0.0000  0.1272 -0.0087
 0.1120 -0.0128  0.0869 -0.0143  0.0578 -0.0095 -0.4326 -0.0000  0.0578 -0.0095
-0.6716  0.0548 -0.7813  0.0983 -0.8419 -0.0000 -0.7813  0.0983 -0.6716  0.0548
 0.4626 -0.0167  0.1659  0.0332 -0.1554  0.0926 -0.4621  0.1629 -0.7244  0.1975
-0.9109  0.1634 -1.0000 -0.0000 -0.9109  0.1634 -0.7244  0.1975 -0.4621  0.1629
-0.1554  0.0926  0.1659  0.0332  0.4626 -0.0167 -0.0181  0.0499  0.1299  0.0208
 0.2877  0.0002  0.4119 -0.0134  0.4793 -0.0225  0.4793 -0.0225  0.4119 -0.0134
 0.2877  0.0002  0.1299  0.0208 -0.0181  0.0499

4TH MODE CURRENT
 0.9551 -0.0003  1.0000 -0.0005  0.8903 -0.0005  0.4091  0.0000  0.8904 -0.0005
 1.0000 -0.0005  0.9551 -0.0003  0.8622 -0.0002  0.7519  0.0000  0.8622 -0.0001
-0.0516 -0.0003 -0.5248  0.0004 -0.2135  0.0000 -0.5248  0.0004 -0.0516 -0.0003
 0.1060  0.0002  0.1640  0.0001  0.2016 -0.0000  0.2158 -0.0001  0.1991 -0.0001
 0.1574 -0.0001  0.0585 -0.0000  0.1574 -0.0001  0.1992 -0.0002  0.2158 -0.0002
 0.2017 -0.0001  0.1640  0.0001  0.1060  0.0002 -0.5701  0.0008 -0.5822  0.0008
-0.5399  0.0007 -0.4190  0.0005 -0.2817  0.0003 -0.2817  0.0003 -0.4190  0.0005
-0.5399  0.0007 -0.5822  0.0008 -0.5701  0.0008
 0.5441E+00 0.1000E+01

  N  M NE N6P N5V N6C NC NORM
 38  4  4   2   3  1184    4
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00

PP
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.1917E-03 0.3061E-02
 0.6553E+01-0.1817E+03
 0.1000E+01-0.2887E+00

VN
 0.5441E+00 0.1000E+01-0.2839E+00
```

PLUS A SECOND SET OF FOUR MODE CURRENTS

THE SECOND SET OF MODE CURRENTS WAS THE SAME AS THE FIRST SET IN REVERSE ORDER. FOR EXAMPLE, THE FIRST MODE CURRENT OF THE SECOND SET WAS THE SAME AS THE FOURTH MODE CURRENT OF THE FIRST SET.

VIII. MODE PATTERNS

The mode pattern $F(\vec{V}_n)$ and gain $G(\vec{V}_n)$ of the port voltage mode $\vec{V}_n$ are given by

$$\underset{\sim}{F}(\vec{V}_n) \cdot \underset{\sim}{u}_r = \tilde{V}_n \vec{I}_r^{SC} \tag{106}$$

$$G(\vec{V}_n) = \frac{k^2 \eta |\tilde{V}_n \vec{I}_r^{SC}|^2}{4\pi \ \mathrm{Real}(\tilde{V}_n^* Y_S \vec{V}_n)} \tag{107}$$

The mode pattern $\underset{\sim}{F}(\vec{I}_n)$ and gain $G(\vec{I}_n)$ of the port current mode $\vec{I}_n$ are given by

$$\underset{\sim}{F}(\vec{I}_n) \cdot \underset{\sim}{u}_r = - \tilde{I}_n \vec{V}_r^{OC} \tag{108}$$

$$G(\vec{I}_n) = \frac{k^2 \eta |\tilde{I}_n \vec{V}_r^{OC}|^2}{4\pi \ \mathrm{Real}(\tilde{I}_n^* Z_s \vec{I})} \tag{109}$$

The pattern program of this section computes

$$\left(\frac{k^2 \eta}{4\pi \mathrm{Real}(\tilde{V}_n^* \, Y_s \vec{V}_n)}\right)^{1/2} \tilde{V}_n \vec{I}_r^{SC}$$

and $G(\vec{V}_n)$ of (107) and stores them in E and GN respectively. If presented with $\vec{I}_n$, $\vec{V}_n^{OC}$ and Z_S instead of $\vec{V}_n$, $\vec{I}_r^{SC}$ and Y_S, the program will automatically store the dual quantities

$$- \left(\frac{k^2 \eta}{4\pi \mathrm{Real}(\tilde{I}_n^* Z_S \vec{I}_n)}\right)^{1/2} \tilde{I}_n \vec{V}_r^{OC}$$

and $G(\vec{I}_n)$ in E and GN.

In the pattern program of this section, the activity on data sets 1 (punched card input) and 6 (direct access input-output) is as follows.

```
      READ (1,10) NF
10    FORMAT (20I3)
      READ (1,42)(BK(I), I = 1, NF)
```

```
42      FORMAT (5E14.7)
        DO 28 L = 1, NF
        READ (1, 10) NT, N, NE, NPA, N6P, N6V, N6W, NIV, NY, I1, I2
        REWIND 6
        SKIP N6P RECORDS ON DATA SET 6
        NPP = I1 + (NPA*NT-1)*I2 + N
        READ (6)(PP(I), I = 1,NPP)
        IF(N6V - 900) 49, 49, 50
49      SKIP N6V RECORDS ON DATA SET 6
        NEN = NE*N
        READ (6)(VN(I), I = 1, NEN)
        GO TO 51
50      J3 = 0
        DO 52 LL = 1, NE
        J1 = J3 + 1
        J2 = J3 + N
        J3 = J2
        READ (1,42)(VN(I), I = J1, J2)
52      CONTINUE
51      J5 = NE*NT*NPA
        WRITE (6) (GN(I), I = 1, J5)
28      CONTINUE
```

The propagation constant k is read in through BK. There are N ports and NE port modes. NIV = 0 indicates port voltage mode patterns while NIV $\neq$ 0 indicates port current mode patterns. For NIV = 0, the port admittance matrix Y_S is in PP(NY+1) through PP(NY+N*N). The J^{th} point on the port voltage mode pattern is computed from $\vec{V}_n$ residing in VN and $\vec{I}_r^{SC}$ residing in PP(I1+(J-1)*I2+1) through PP(I1+(J-1)*I2+N) where $1 \leq J \leq$ NPA*NT obtains NPA different polarizations.

Minimum allocations are given by

```
COMPLEX PP(I1+(NT-1)*I2+N)
DIMENSION BK(NF), VN(NE*N), G(N*N),
          GN(NE*NT*NPA), ANG(NT)
```

The index K of DO loop 24 indicates the K^{th} port voltage mode. Because V_n is real and Y_S is symmetric,

$$\tilde{V}_n^{\,*} Y_s \vec{V}_n = \tilde{V}_n \, \mathrm{Real}(Y_s) \vec{V}_n \qquad (110)$$

DO loop 25 stores (110) in VGV. The index KK of DO loop 29 indicates the KK^{th} point on the pattern. DO loop 30 stores $\tilde{V}_n \vec{I}_r^{\,SC}$ in E.

The sample punched card input data is such that the patterns of the port mode voltages (first four patterns in the printed output) and the patterns of the port mode currents (last four patterns in the printed output) are stored on records 10 and 11 respectively of data set 6.

LISTING OF PROGRAM TO COMPUTE MODE PATTERNS

```
//              (0034,EE,2,3),'MAUTZ,JOE',REGION=140K
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      COMPLEX PP(3312),E
      DIMENSION BK(2),VN(80),G(16),GN(2900),ANG(145)
      READ(1,10) NF
   10 FORMAT(20I3)
      WRITE(3,38) NF
   38 FORMAT('0NF'/1X,I3)
      READ(1,42)(BK(I),I=1,NF)
   42 FORMAT(5E14.7)
      WRITE(3,43)(BK(I),I=1,NF)
   43 FORMAT('0BK'/(1X,5E14.7))
      PI=3.141593
      ETA=376.730
      C1=SQRT(ETA/(4.*PI))
      DO 28 L=1,NF
      READ(1,10) NT,N,NE,NPA,N6P,N6V,N6W,NIV,NY,I1,I2
      WRITE(3,11) NT,N,NE,NPA,N6P,N6V,N6W,NIV,NY,I1,I2
   11 FORMAT('0 NT  N NE NPA N6P N6V N6W NIV NY I1 I2'/1X,3I3,5I4,3I3)
      REWIND 6
      N6=IABS(N6P)
      IF(N6P) 12,12,13
   13 DO 14 J=1,N6
      READ(6)(PP(I),I=1,2)
      WRITE(3,16)(PP(I),I=1,2)
   16 FORMAT(1X,4E11.4)
   14 CONTINUE
   12 NPP=I1+(NPA*NT-1)*I2+N
      READ(6)(PP(I),I=1,NPP)
      WRITE(3,17)(PP(I),I=1,2)
   17 FORMAT('0PP'/(1X,4E11.4))
      N6=IABS(N6V)
      IF(N6V-900) 49,49,50
   49 IF(N6V) 18,19,20
   18 DO 21 J=1,N6
      BACKSPACE 6
   21 CONTINUE
      GO TO 19
   20 DO 22 J=1,N6
      READ(6)(VN(I),I=1,4)
      WRITE(3,16)(VN(I),I=1,4)
   22 CONTINUE
   19 NEN=NE*N
      READ(6)(VN(I),I=1,NEN)
      WRITE(3,23)(VN(I),I=1,4)
   23 FORMAT('0VN'/(1X,4E11.4))
      GO TO 51
   50 J3=0
      DO 52 LL=1,NE
      J1=J3+1
      J2=J3+N
      J3=J2
      READ(1,42)(VN(I),I=J1,J2)
      WRITE(3,53)(VN(I),I=J1,J2)
   53 FORMAT('0VN'/(1X,5E14.7))
   52 CONTINUE
```

```
51 NN=N*N
   DO 27 J=1,NN
   J1=J+NY
   G(J)=REAL(PP(J1))
27 CONTINUE
   DEL=360./(NT-1)
   DO 37 J=1,NT
   ANG(J)=(J-1)*DEL
37 CONTINUE
   J5=0
   DO 24 K=1,NE
   J1=(K-1)*N
   VGV=0.
   DO 25 J=1,N
   J3=(J-1)*N
   SG=0.
   DO 26 I=1,N
   J2=J1+I
   J4=J3+I
   SG=SG+G(J4)*VN(J2)
26 CONTINUE
   J4=J1+J
   VGV=VGV+VN(J4)*SG
25 CONTINUE
   C2=BK(L)*C1/SQRT(VGV)
   WRITE(3,33) K
33 FORMAT('0',I3,'TH MODE GAIN PATTERN')
   WRITE(3,39)
39 FORMAT('0 ANGLE    REAL(E)       IMAG(E)',7X,'GAIN')
   IF(NIV.NE.0) C2=-C2
   J4=I1
   DO 48 LL=1,NPA
   DO 29 KK=1,NT
   E=0.
   DO 30 J=1,N
   J2=J1+J
   J3=J4+J
   E=E+PP(J3)*VN(J2)
30 CONTINUE
   J4=J4+I2
   E=E*C2
   J5=J5+1
   S1=CABS(E)
   GN(J5)=S1*S1
   J6=KK-1
   IF(J6/4*4.NE.J6) GO TO 29
   WRITE(3,41) ANG(KK),E,GN(J5)
41 FORMAT(1X,F6.1,3E12.4)
29 CONTINUE
48 CONTINUE
24 CONTINUE
   N6=IABS(N6W)
   IF(N6W) 34,35,36
34 DO 31 J=1,N6
   BACKSPACE 6
31 CONTINUE
   GO TO 35
36 DO 32 J=1,N6
   READ(6)(VN(I),I=1,2)
   WRITE(3,16)(VN(I),I=1,2)
```

```
   32 CONTINUE
   35 WRITE(6)(GN(I),I=1,J5)
   28 CONTINUE
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                  X
//               VOLUME=SER=SU0004,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//               BUFNO=1)
//GO.SYSIN DD *
  2
 0.1963495E+00 0.1963495E+00
145   4   4   1   2   2   3   0   0336  10
145   4   4   1   2   3   3   1  16341  10
/*
//
```

PRINTED OUTPUT

```
NF
  2

BK
 0.1963494E+00 0.1963494E+00

 NT    N  NE  NPA  N6P  N6V  N6W  NIV  NY  I1  I2
145    4   4    1    2    2    3    0  0336  10
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00

PP
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.6553E+01-0.1817E+03 0.7365E+00 0.1965E+03

VN
 0.1000E+01-0.2887E+00 0.7884E+00-0.4050E+00
```

1TH MODE GAIN PATTERN

ANGLE	REAL(E)	IMAG(E)	GAIN
0.0	0.9612E+00	0.4412E+00	0.1119E+01
10.0	0.9695E+00	0.4490E+00	0.1142E+01
20.0	0.1001E+01	0.4666E+00	0.1220E+01
30.0	0.1069E+01	0.4740E+00	0.1367E+01
40.0	0.1172E+01	0.4350E+00	0.1562E+01
50.0	0.1273E+01	0.3156E+00	0.1720E+01
60.0	0.1304E+01	0.1250E+00	0.1716E+01
70.0	0.1219E+01	-0.5938E-01	0.1489E+01
80.0	0.1058E+01	-0.1376E+00	0.1139E+01
90.0	0.9407E+00	-0.8438E-01	0.8920E+00
100.0	0.9549E+00	0.1264E-01	0.9119E+00
110.0	0.1076E+01	0.3436E-01	0.1160E+01
120.0	0.1202E+01	-0.5975E-01	0.1447E+01
130.0	0.1252E+01	-0.2163E+00	0.1615E+01
140.0	0.1226E+01	-0.3589E+00	0.1631E+01
150.0	0.1164E+01	-0.4495E+00	0.1557E+01
160.0	0.1107E+01	-0.4923E+00	0.1468E+01
170.0	0.1072E+01	-0.5075E+00	0.1408E+01
180.0	0.1061E+01	-0.5108E+00	0.1387E+01

190.0	0.1072E+01	-0.5075E+00	0.1408E+01
200.0	0.1107E+01	-0.4923E+00	0.1468E+01
210.0	0.1164E+01	-0.4495E+00	0.1557E+01
220.0	0.1226E+01	-0.3589E+00	0.1631E+01
230.0	0.1252E+01	-0.2163E+00	0.1615E+01
240.0	0.1202E+01	-0.5977E-01	0.1447E+01
250.0	0.1076E+01	0.3437E-01	0.1160E+01
260.0	0.9548E+00	0.1266E-01	0.9119E+00
270.0	0.9406E+00	-0.8436E-01	0.8919E+00
280.0	0.1058E+01	-0.1376E+00	0.1139E+01
290.0	0.1219E+01	-0.5941E-01	0.1489E+01
300.0	0.1304E+01	0.1249E+00	0.1716E+01
310.0	0.1273E+01	0.3155E+00	0.1720E+01
320.0	0.1172E+01	0.4349E+00	0.1562E+01
330.0	0.1069E+01	0.4739E+00	0.1367E+01
340.0	0.1001E+01	0.4665E+00	0.1220E+01
350.0	0.9695E+00	0.4489E+00	0.1142E+01
360.0	0.9612E+00	0.4412E+00	0.1119E+01

PLUS SEVEN MORE PATTERNS

THE LOSS OF SYMMETRY ABOUT 180 DEGREES DUE TO NUMERICAL INACCURACIES WAS MOST PRONOUNCED IN THE PATTERN OF THE FOURTH PORT MODE VOLTAGE (FOURTH PATTERN IN THE PRINTED OUTPUT)

ANGLE	REAL(E)	IMAG(E)	GAIN
110.0	0.3728E+00	0.6030E-01	0.1426E+00
120.0	0.2686E+00	0.4018E+00	0.2336E+00
130.0	0.3990E+00	0.7718E+00	0.7549E+00
230.0	0.3982E+00	0.7714E+00	0.7536E+00
240.0	0.2680E+00	0.4012E+00	0.2327E+00
250.0	0.3724E+00	0.5964E-01	0.1422E+00

AND IN THE PATTERN OF THE FIRST PORT MODE CURRENT (FIFTH PATTERN IN THE PRINTED OUTPUT)

ANGLE	REAL(E)	IMAG(E)	GAIN
110.0	-0.5975E-01	0.3728E+00	0.1426E+00
120.0	-0.4014E+00	0.2690E+00	0.2335E+00
130.0	-0.7714E+00	0.3998E+00	0.7549E+00
230.0	-0.7709E+00	0.3990E+00	0.7536E+00
240.0	-0.4008E+00	0.2683E+00	0.2327E+00
250.0	-0.5908E-01	0.3724E+00	0.1422E+00

IX. SCATTERING PATTERNS

The program of this section computes the NE patterns

$$\underset{\sim}{F}^{s} \cdot \underset{\sim}{u}_r \approx \underset{\sim}{F}_o^{SC} \cdot \underset{\sim}{u}_r - \sum_{n=1}^{K} \frac{\beta_{n'}^{r}\, \beta_{n'}^{t}}{\tilde{V}_{n'} [Y_S + Y_L] V_{n'}} \qquad (111)$$

for K = 1,2,...NE where

$$\beta_n = \tilde{V}_n \vec{I}^{SC} \qquad (63)$$

and NE is the number of port modes. Also, n' = LV(n). The variable LV is read into the program. The pattern

$$\underset{\sim}{F}^{s} \cdot \underset{\sim}{u}_r = \underset{\sim}{F}_o^{SC} \cdot \underset{\sim}{u}_r - \tilde{I}_r^{SC} [Y_S + Y_L]^{-1} \vec{I}_t^{SC} \qquad (75)$$

is computed for reference. Equation (111) is essentially (76). Except for a sign difference in the load dependent terms, the quantities dual to (111), (63), and (75) are obtained by replacing the input Y_S, Y_L, $\vec{I}^{SC}$, $\underset{\sim}{F}_o^{SC} \cdot \underset{\sim}{u}_r$, and $\vec{V}_n$ by Z_S, Z_L, $\vec{V}^{OC}$, $\underset{\sim}{F}_o^{OC} \cdot \underset{\sim}{u}_r$, and $\vec{I}_n$ respectively.

The activity on data sets 1 (punched card input) and 6 (direct access input-output) is as follows.

```
      READ (1, 10) NF
10    FORMAT (20I3)
      READ (1,42)(BK(I), I = 1, NF)
42    FORMAT (5E14.7)
      DO 28 L = 1, NF
      READ (1,10) NT, N,NE,NS,NPA,N6P,N6V,N6W,NIV, NVT, NY, I1,I2,I3
      READ (1,10)(LV(I),I = 1, NE)
      READ (1,42)(YL(J), J = 1, N)
      REWIND 6
      SKIP N6P RECORDS ON DATA SET 6
      NPP = I1 + (NT*NPA-1)* I2 + N + 1
      READ (6)(PP(I), I = 1, NPP)
```

```
        SKIP N6V RECORDS ON DATA SET 6
        NEN = NE*N
        READ (6)(VN(I), I = 1, NEN)
        SKIP N6W RECORDS ON DATA SET 6
        J6 = ((NT-1)/NS+1)*(NE+ 1)*NPA + NT*NPA
        WRITE (6) (SIG(I), I = 1, J6)
28      CONTINUE
```

The propagation constant k is read in through BK. There are N ports and NE port modes. NIV = 0 indicates the short circuit formulation (111), (63) and (75) while NIV ≠ 0 indicates the dual open circuit formulation. For the short circuit formulation, the admittance Y_L is read in through the complex variable Y_L. The port admittance matrix Y_S is in PP(NY+1) through PP(NY+N*N). There are NT*NPA vectors $\vec{I}^{SC}$ of which the J^{th} resides in PP(I1+(J-1)*I2+1) through PP(I1+(J-1)*I2+N). The particular $\vec{I}^{SC}$ associated with the incident plane wave field is labeled $\vec{I}_t^{SC}$ in (75) and is in PP(I1+(NVT-1)*I2+1) through PP(I1+(NVT-1)*I2+N). The I^{th} of NPA polarizations of $\underset{\sim}{F}_o^{SC} \cdot \underset{\sim}{u}_r$ is in (PP((I-1)*NT*I2+(J-1)*I2+I3), J=1, NT). The port voltage modes V_n are stored in VN in the order of decreasing eigenvalues μ.

The short circuit radar cross section per square wavelength σ/λ^2 given by

$$\frac{\sigma}{\lambda^2} = \frac{k^4\eta^2}{16\pi^3}\left|\underset{\sim}{F}_o^{SC} \cdot \underset{\sim}{u}_r\right|^2 \qquad (112)$$

is in SIG((I-1)*NTS+1) through SIG(I*NTS) where NTS = (NT-1)/NS+1 and I denotes the I^{th} polarization. The Ith polarization of σ/λ^2 for the K^{th} pattern in (108) is in SIG(K*NTS*NPA+(I-1)*NTS+1) through SIG(K*NTS*NPA+I*NTS). The I^{th} polarization of σ/λ^2 for (75) is in SIG((NE+1)*NTS*NPA+(I-1)*NT+1) through SIG((NE+1)*NTS*NPA+I*NT). The effect of NS is to evaluate the pattern $\underset{\sim}{F}_o^{SC} \cdot \underset{\sim}{u}_r$ and the K patterns (111) not at all NT points but from one to NT in steps of NS.

Minimum allocations are given by

DIMENSION LR(N)

in the subroutine LINEQ and by

```
COMPLEX YL(N), PP(I1+NPA*NT*I2),BT(NE),E(NT*NP)
        YS(N*N),VV(N)
DIMENSION BK(NF), LV(NE), VN(NE*N), G(N*N),
        B(N*N), ANG(NT), SIG(J6)
```

in the main program where J6=((NT-1)/NS+1)*(NE+1)*NPA+NT*NPA.

DO loop 46 stores $Y_S + Y_L$ in YS. DO loop 24 stores

$$\frac{\beta^t_{n'} C2}{\tilde{V}_{n'}[Y_S + Y_L]\vec{V}_{n'}}$$

in BT(K) where

$$n' = LV(K)$$

$$C2 = \frac{k^2\eta}{4(\pi)^{3/2}}$$

The variable C3 appearing at this point in the program is C2 for the short circuit formulation (111), (63), and (75) but is -C2 for the dual open circuit formulation. DO loop 25 stores the real and imaginary parts of $\tilde{V}_{n'}[Y_S + Y_L]\vec{V}_{n'}$ in VGV and VBV. DO loops 33 and 56 put $(C2)(\underset{\sim}{F}^{SC}_o \cdot \underset{\sim}{u}_r)$ in E and (112) in SIG. DO loop 35 computes the corresponding quantities for (111). The integer K appearing in (111) is supplied by the index of outer DO loop 34. Statement 67 inverts the matrix $(Y_S + Y_L)$ stored in YS. DO loop 47 stores

$$(C2)\ [Y_S + Y_L]^{-1}\ \vec{I}^{SC}_t$$

in VV. DO loop 44 stores $C2(\underset{\sim}{F}^S \cdot \underset{\sim}{u}_r)$ in E and $|C2(\underset{\sim}{F}^S \cdot \underset{\sim}{u}_r)|^2$ in SIG where $\underset{\sim}{F}^S \cdot \underset{\sim}{u}_r$ is computed according to (75).

The sample data is such as to store six patterns for the short circuit formulation and six patterns for the open circuit formulation on records 12 and 13 respectively of data set 6. LV is such that the modal sum (111) is performed in the order of increasing magnitude of eigenvalues. Not all 12 patterns appearing in the computer output are listed here, but a few observations concerning them should be brought out. All 12 patterns are symmetric about 180 degrees. The scattering pattern obtained using four modes is the same as the pattern obtained using matrix inversion for the short circuit formulation as well as for the open circuit formulation. Because all the admittance loads are zero, the matrix inversion pattern for the short circuit formulation is just the open circuit pattern. Because all the impedance loads are zero, the matrix inversion pattern for the open circuit formulation is just the short circuit pattern.

LISTING OF PROGRAM TO COMPUTE SCATTERING PATTERNS

THIS PROGRAM CALLS THE MATRIX INVERSION SUBROUTINE LINEQ LISTED WITH THE PROGRAM OF SECTION V.

```
//                 (0034,EE,1,2),'MAUTZ,JOE',REGION=140K
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      COMPLEX U,YL(4),PP(3236),BT(4),E(290),BR,YS(16),VV(4)
      DIMENSION BK(6),LV(4),VN(16),G(16),B(16),ANG(145),SIG(660)
      U=(0.,1.)
      ETA=376.730
      PI=3.141593
      C1=.25*ETA/SQRT(PI*PI*PI)
      READ(1,10) NF
   10 FORMAT(20I3)
      WRITE(3,38) NF
   38 FORMAT('ONF'/1X,I3)
      READ(1,42)(BK(I),I=1,NF)
   42 FORMAT(5E14.7)
      WRITE(3,43)(BK(I),I=1,NF)
   43 FORMAT('OBK'/(1X,5E14.7))
      DO 28 L=1,NF
      READ(1,10) NT,N,NE,NS,NPA,N6P,N6V,N6W,NIV,NVT,NY,I1,I2,I3
      WRITE(3,11) NT,N,NE,NS,NPA,N6P,N6V,N6W,NIV,NVT,NY,I1,I2,I3
   11 FORMAT('O NT  N NE NS NPA N6P N6V N6W NIV NVT NY I1 I2 I3'/1X,4I3,
     16I4,4I3)
      READ(1,10)(LV(I),I=1,NE)
      WRITE(3,31)(LV(I),I=1,NE)
   31 FORMAT('OLV'/(1X,20I3))
      READ(1,42)(YL(J),J=1,N)
      WRITE(3,50)(YL(J),J=1,N)
   50 FORMAT('OYL'/(1X,5E14.7))
      C2=BK(L)*BK(L)*C1
      C3=C2
      IF(NIV.NE.0) C3=-C3
      REWIND 6
      N6=IABS(N6P)
      IF(N6P) 12,12,13
   13 DO 14 J=1,N6
      READ(6)(PP(I),I=1,2)
      WRITE(3,16)(PP(I),I=1,2)
   16 FORMAT(1X,4E11.4)
   14 CONTINUE
   12 NTP=NT*NPA
      NPP=I1+(NTP-1)*I2+N+1
      READ(6)(PP(I),I=1,NPP)
      WRITE(3,17)(PP(I),I=1,2)
   17 FORMAT('OPP'/(1X,4E11.4))
      N6=IABS(N6V)
      IF(N6V) 18,19,20
   18 DO 21 J=1,N6
      BACKSPACE 6
   21 CONTINUE
      GO TO 19
   20 DO 22 J=1,N6
      READ(6)(VN(I),I=1,4)
      WRITE(3,16)(VN(I),I=1,4)
   22 CONTINUE
```

```
19 NEN=NE*N
   READ(6)(VN(I),I=1,NEN)
   WRITE(3,23)(VN(I),I=1,4)
23 FORMAT('OVN'/(1X,4E11.4))
   NN=N*N
   DO 45 J=1,NN
   J1=NY+J
   YS(J)=PP(J1)
45 CONTINUE
   J2=N+1
   J1=1
   DO 46 J=1,N
   YS(J1)=YS(J1)+YL(J)
   J1=J1+J2
46 CONTINUE
   DO 27 J=1,NN
   G(J)=REAL(YS(J))
   B(J)=AIMAG(YS(J))
27 CONTINUE
   DEL=360./(NT-1)
   DO 37 J=1,NT
   ANG(J)=(J-1)*DEL
37 CONTINUE
   J5=I1+(NVT-1)*I2
   DO 24 K=1,NE
   J1=(LV(K)-1)*N
   VGV=0.
   VBV=0.
   DO 25 J=1,N
   J3=(J-1)*N
   SG=0.
   SB=0.
   DO 26 I=1,N
   J2=J1+I
   J4=J3+I
   SG=SG+G(J4)*VN(J2)
   SB=SB+B(J4)*VN(J2)
26 CONTINUE
   J4=J1+J
   VGV=VGV+VN(J4)*SG
   VBV=VBV+VN(J4)*SB
25 CONTINUE
   BT(K)=0.
   DO 66 J=1,N
   J2=J+J5
   J3=J+J1
   BT(K)=BT(K)+VN(J3)*PP(J2)
66 CONTINUE
   BT(K)=C3*BT(K)/(VGV+U*VBV)
   S1=VBV/VGV
   WRITE(3,30) VGV,S1
30 FORMAT('OVGV=',E11.4,' S1=',E11.4)
24 CONTINUE
   DO 33 KK=1,NTP
   J1=I3+(KK-1)*I2
   PP(J1)=PP(J1)*C2
   E(KK)=PP(J1)
33 CONTINUE
   IF(NIV) 52,51,52
51 WRITE(3,54)
```

```
      GO TO 53
   52 WRITE(3,55)
   54 FORMAT('OSHORT CIRCUIT SCATTERING PATTERN')
   55 FORMAT('OOPEN CIRCUIT SCATTERING PATTERN')
   53 J6=0
      WRITE(3,57)
   57 FORMAT('O ANGLE    REAL(E)       IMAG(E)',4X,'SIG/(LAM)**2')
      DO 68 LL=1,NPA
      J7=(LL-1)*NT
      DO 56 KK=1,NT,NS
      J8=J7+KK
      J6=J6+1
      SIG(J6)=E(J8)*CONJG(E(J8))
      WRITE(3,58) ANG(KK),E(J8),SIG(J6)
   58 FORMAT(1X,F6.1,3E12.4)
   56 CONTINUE
   68 CONTINUE
      DO 34 K=1,NE
      J1=(LV(K)-1)*N
      WRITE(3,59) K
   59 FORMAT('OSCATTERING PATTERN USING',I3,' MODES')
      WRITE(3,57)
      DO 69 LL=1,NPA
      J7=(LL-1)*NT
      DO 35 KK=1,NT,NS
      J8=J7+KK
      J3=I1+(J8-1)*I2
      BR=0.
      DO 36 J=1,N
      J2=J+J1
      J4=J+J3
      BR=BR+VN(J2)*PP(J4)
   36 CONTINUE
      E(J8)=E(J8)-BR*BT(K)
      J6=J6+1
      SIG(J6)=E(J8)*CONJG(E(J8))
      WRITE(3,58) ANG(KK),E(J8),SIG(J6)
   35 CONTINUE
   69 CONTINUE
   34 CONTINUE
   67 CALL LINEQ(N,YS)
      J1=I1+(NVT-1)*I2
      DO 47 J=1,N
      J3=(J-1)*N
      VV(J)=0.
      DO 48 I=1,N
      J2=I+J1
      J4=J3+I
      VV(J)=VV(J)+YS(J4)*PP(J2)
   48 CONTINUE
      VV(J)=VV(J)*C3
   47 CONTINUE
      WRITE(3,60)
   60 FORMAT('OSCATTERING PATTERN FROM MATRIX INVERSION')
      WRITE(3,57)
      DO 70 LL=1,NPA
      J7=(LL-1)*NT
      DO 44 KK=1,NT
      J8=J7+KK
      J2=(J8-1)*I2
```

```
      J1=J2+I1
      J2=J2+I3
      E(J8)=PP(J2)
      DO 29 J=1,N
      J3=J+J1
      E(J8)=E(J8)-VV(J)*PP(J3)
   29 CONTINUE
      J6=J6+1
      SIG(J6)=E(J8)*CONJG(E(J8))
      J3=KK-1
      IF(J3/NS*NS.NE.J3) GO TO 44
      WRITE(3,58) ANG(KK),E(J8),SIG(J6)
   44 CONTINUE
   70 CONTINUE
      N6=IABS(N6W)
      IF(N6W) 61,62,63
   61 DO 64 J=1,N6
      BACKSPACE 6
   64 CONTINUE
      GO TO 62
   63 DO 65 J=1,N6
      READ(6)(VN(I),I=1,2)
      WRITE(3,16)(VN(I),I=1,2)
   65 CONTINUE
   62 WRITE(6)(SIG(I),I=1,J6)
   28 CONTINUE
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT= 314,                    X
//               VOLUME=SER=SU0004,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//               BUFNO=1)
//GO.SYSIN DD *
  2
 0.1963495E+00 0.1963495E+00
145   4   4   4   1   2   2   5   0  73   0336  10341
  3   2   1   4
 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 0.0000000E+00 0.0000000E+00 0.0000000E+00
145   4   4   4   1   2   3   5   1  73  16341  10346
  2   3   4   1
 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 0.0000000E+00 0.0000000E+00 0.0000000E+00
/*
//
```

```
PRINTED OUTPUT

NF
  2

BK
 0.1963494E+00 0.1963494E+00

 NT   N  NE  NS  NPA  N6P  N6V  N6W  NIV  NVT  NY  I1  I2  I3
145   4   4   4    1    2    2    5    0   73   0336  10341

LV
  3   2   1   4
```

```
YL
 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 0.0000000E+00 0.0000000E+00 0.0000000E+00
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00

PP
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.6553E+01-0.1817E+03 0.7365E+00 0.1965E+03

VN
 0.1000E+01-0.2887E+00 0.7884E+00-0.4050E+00

VGV= 0.1336E-01 S1= 0.1552E+00

VGV= 0.2189E-03 S1= 0.1012E+02

VGV= 0.8111E-04 S1= 0.5054E+02

VGV= 0.4411E-05 S1=-0.8163E+03

SHORT CIRCUIT SCATTERING PATTERN

 ANGLE    REAL(E)       IMAG(E)      SIG/(LAM)**2
   0.0 -0.4158E+00 -0.8135E-02  0.1730E+00
  10.0 -0.4102E+00  0.2575E-01  0.1689E+00
  20.0 -0.3794E+00  0.1206E+00  0.1585E+00
  30.0 -0.2902E+00  0.2486E+00  0.1460E+00
  40.0 -0.1177E+00  0.3498E+00  0.1362E+00
  50.0  0.1136E+00  0.3440E+00  0.1312E+00
  60.0  0.3080E+00  0.1846E+00  0.1290E+00
  70.0  0.3442E+00 -0.7495E-01  0.1241E+00
  80.0  0.1800E+00 -0.2822E+00  0.1120E+00
  90.0 -0.7939E-01 -0.2943E+00  0.9293E-01
 100.0 -0.2472E+00 -0.1083E+00  0.7285E-01
 110.0 -0.2095E+00  0.1293E+00  0.6062E-01
 120.0 -0.1126E-01  0.2503E+00  0.6279E-01
 130.0  0.2060E+00  0.1949E+00  0.8045E-01
 140.0  0.3300E+00  0.2377E-01  0.1095E+00
 150.0  0.3407E+00 -0.1635E+00  0.1428E+00
 160.0  0.2857E+00 -0.3022E+00  0.1729E+00
 170.0  0.2257E+00 -0.3779E+00  0.1937E+00
 180.0  0.2013E+00 -0.4007E+00  0.2011E+00
PLUS THE REST OF THE PATTERN FROM 190.0 TO 360.0

PLUS 11 MORE PATTERNS OF WHICH ONLY A PORTION OF
THE OPEN CIRCUIT SCATTERING PATTERN IS SHOWN BELOW.

 ANGLE    REAL(E)       IMAG(E)      SIG/(LAM)**2
   0.0 -0.2532E-01 -0.1244E-01  0.7957E-03
  10.0 -0.2512E-01 -0.4726E-02  0.6534E-03
  20.0 -0.2044E-01  0.1693E-01  0.7043E-03
  30.0 -0.1379E-02  0.4556E-01  0.2078E-02
  40.0  0.3952E-01  0.6466E-01  0.5742E-02
  50.0  0.9443E-01  0.5138E-01  0.1156E-01
  60.0  0.1336E+00 -0.6376E-02  0.1788E-01
  70.0  0.1201E+00 -0.8900E-01  0.2234E-01
  80.0  0.4375E-01 -0.1464E+00  0.2335E-01
  90.0 -0.5813E-01 -0.1338E+00  0.2127E-01
```

X. MODE CURRENT PLOTS

A program to plot the mode currents is described in this section. In this program, the activity on data sets 1 (punched card input) and 6 (direct access input-output) is as follows.

```
        READ (1,20) NF
20      FORMAT (20I3)
        DO 8 LF = 1, NF
        READ (1,20) NP, NW, M, NE, NX, IRC, N6C
        REWIND 6
        SKIP N6C RECORDS ON DATA SET 6
        NEM = NE*M
        READ (6) (CUR(I), I = 1, NEM)
        IF(NX.EQ.0) GO TO 8
        READ (1,20)(N1(I), I = 1, NW)
        NW2 = NW2*2
        READ(1,20)(N2(I), I = 1, NW2)
        READ (1,20)(N3(I), I = 1, NW2)
        READ (1,27)(PX(I), I = 1, NP)
        READ (1,27)(PY(I), I = 1, NP)
        READ (1,27)(PZ(I), I = 1, NP)
27      FORMAT (10F8.4)
 8      CONTINUE
```

PX, PY, and PZ are the NP data points describing the axes of the NW wires. There are NE mode currents of which the J^{th} resides in CUR((J-1)*M+1) through CUR(J*M). If the data NP, NW, M, N1, N2, N3, PX, PY, and PZ is the same as for the previous LF, duplicate calculations may be avoided by setting NX equal to zero. IRC $\leq$ 0 indicates real mode currents while IRC > 0 indicates complex mode currents. Note the comment statement concerning IRC in the main program. N1(I) is the number of expansion functions on the I^{th} wire. More precisely,

$$N1(I) = (ND - 3)/2$$

where ND is the number of data points on the I^{th} wire.

The variable N2 accounts for each of the following 4 possibilities on the I^{th} wire.

N2(2*I-1) = 1 ; Junction of the third data point

N2(2*I-1) = 2 ; No junction at the third data point

N2(2*I) = 1 ; Junction at the third from the last data point

N2(2*I) = 2 ; No junction at the third from the last data point

A junction at the third from the last data point means that the end of the wire extends through the junction. No junction at the third from the last data point means that the end of the wire does not extend through a junction. Similar statements concerning the third data point and the beginning of the wire are true.

Any extremity of the I^{th} wire which extends through a junction is not plotted because this extremity appears on the plot of one of the other wires. Thus the first point to appear on the plot of the I^{th} wire is either the first or third data point on that wire. The branch mode current at the first point on the plot of the I^{th} wire is most generally the sum of the element of CUR which is the loop mode current proper to that point on the wire plus

N3(J3)/ABS(N3(J3))*CUR((J-1)*M+ABS(N3(J3)))

where J3 = 2*I-1 and J indicates the J^{th} mode current. N3(2*I) pertains to the last point on the plot of the I^{th} wire. The variable N3 is necessary to translate the loop currents (elements of CUR) into branch currents suitable for plotting.

Minimum allocations are given by

```
COMPLEX CUR(NE*M), Y(JX)
DIMENSION N1(NW), N2(NW*2), N3(NW*2),
      PX(NP), PY(NP), PZ(NP), X(JX), L(NW),
      RY(JY), AY(JY)
```

where

$$JX = NW*2 + \sum_{J=1}^{NW} N1(J)$$

$$JY = 2 + \underset{J=1,2,\ldots NW}{\text{MAXIMUM}} (N1(J))$$

The above JX and JY may be slightly larger than necessary because they are based on the over simplification that the first and last data points on each wire are plotted. If IRC $\leq$ 0, RY and AY are never used.

DO loops 34 and 85 store in X the x coordinates for the plot. In DO loops 34 and 85, the index J indicates the J^{th} wire. DO loop 35 calculates the next X by adding the appropriate segment length to the previous X. L(J) is the number of X's to be plotted for the J^{th} wire. DO loop 85 normalizes the sum of the lengths of the wires to 5 inches and leaves a .25 inch gap between wires.

DO loop 89 plots the K^{th} mode current. DO loop 38 puts tick marks on the y axis drawn by statement 91. Statements 92, 93, and 94 put the scale -1, 0, 1 on the y axis. DO loop 39 obtains the J^{th} wire. The y coordinates to be plotted are stored in Y. The variable U5 is necessary to center the plot at y=5 inches. DO loop 43 considers the Y contribution from the elements of CUR proper to the J^{th} wire. The branches 64 and 66 before and after DO loop 43 admit extra points on the ends of the wire if N2=2. The logic between statements 65 and 71 considers the Y contribution from the elements of CUR proper to the wires which overlap the Jth wire at the beginning and end of the J^{th} wire. Statement 96 draws the x axis for the J^{th} wire and statements 95 and 97 supply tick marks at the beginning and end. If IRC $\leq$ 0, statement 81 plots the branch mode current on the J^{th} wire. If IRC > 0, statements 98 and 99 plot the branch mode current on the J^{th} wire using the symbol x for real part and the symbol □ for imaginary part.

The sample data is such as to plot the currents previously stored on records 8 and 9 of data set 6.

LISTING OF PROGRAM TO PLOT MODE CURRENTS

```
//                (0034,EE,1,1,,15),'MAUTZ,JOE',REGION=140K
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      COMPLEX U5,CUR(152),Y(100)
C     IF(IRC.LE.0) THE ABOVE STATEMENT MUST BE REPLACED BY
C     DIMENSION CUR(152),Y(100)
      DIMENSION AREA(400),N1(6),N2(12),N3(12),PX(100),PY(100),PZ(100)
      DIMENSION X(100),L(6),X1(4),Y1(4),RY(100),AY(100)
      U5=(5.,5.)
      CALL PLOTID
      CALL PLOTS(AREA,400)
      READ(1,20) NF
   20 FORMAT(20I3)
      WRITE(3,9) NF
    9 FORMAT('0 NF'/1X,I3)
      DO 8 LF=1,NF
      READ(1,20) NP,NW,M,NE,NX,IRC,N6C
      WRITE(3,21) NP,NW,M,NE,NX,IRC,N6C
   21 FORMAT('0 NP NW  M NE NX IRC N6C'/1X,5I3,2I4)
      REWIND 6
      IF(N6C) 6,6,7
    7 DO 5 J=1,N6C
      READ(6)(CUR(I),I=1,2)
      WRITE(3,4)(CUR(I),I=1,2)
    4 FORMAT(1X,10E11.4)
    5 CONTINUE
    6 NEM=NE*M
      READ(6)(CUR(I),I=1,NEM)
      WRITE(3,11)(CUR(I),I=1,2)
   11 FORMAT('0CUR'/(1X,4E11.4))
      DO 80 J=1,NEM
      CUR(J)=2.*CUR(J)
   80 CONTINUE
      IF(NX.EQ.0) GO TO 83
      READ(1,20)(N1(I),I=1,NW)
      WRITE(3,12)(N1(I),I=1,NW)
   12 FORMAT('0N1'/(1X,20I3))
      NW2=NW*2
      READ(1,20)(N2(I),I=1,NW2)
      WRITE(3,87)(N2(I),I=1,NW2)
   87 FORMAT('0N2'/(1X,20I3))
      READ(1,20)(N3(I),I=1,NW2)
      WRITE(3,88)(N3(I),I=1,NW2)
   88 FORMAT('0N3'/(1X,20I3))
      READ(1,27)(PX(I),I=1,NP)
      READ(1,27)(PY(I),I=1,NP)
      READ(1,27)(PZ(I),I=1,NP)
   27 FORMAT(10F8.4)
      WRITE(3,28)(PX(I),I=1,NP)
   28 FORMAT('0PX'/(1X,10F8.4))
      WRITE(3,29)(PY(I),I=1,NP)
   29 FORMAT('0PY'/(1X,10F8.4))
      WRITE(3,30)(PZ(I),I=1,NP)
   30 FORMAT('0PZ'/(1X,10F8.4))
      J3=0
      J2=0
      JX=0
      X(1)=0.
```

```
      DO 34 J=1,NW
      JXM=JX
      JX=JX+1
      IF(J.EQ.1) GO TO 14
      X(JX)=X(JXM)
   14 J2=J2+1
      K2=N2(J2)
      N6=N1(J)+1
      GO TO (57,58),K2
   57 N6=N6-1
      J3=J3+2
   58 J2=J2+1
      K2=N2(J2)
      GO TO (59,60),K2
   59 N6=N6-1
   60 IF(N6.LE.0) GO TO 73
      DO 35 I=1,N6
      JXN=JX
      JX=JX+1
      X(JX)=X(JXN)
      DO 36 K=1,2
      J3=J3+1
      J4=J3+1
      S1=PX(J4)-PX(J3)
      S2=PY(J4)-PY(J3)
      S3=PZ(J4)-PZ(J3)
      X(JX)=X(JX)+SQRT(S1*S1+S2*S2+S3*S3)
   36 CONTINUE
   35 CONTINUE
   73 J3=J3+1
      L(J)=JX-JXM
      GO TO (61,34),K2
   61 J3=J3+2
   34 CONTINUE
      XN=5./X(JX)
      DX=1.
      JX=0.
      WRITE(3,100)
  100 FORMAT('0X')
      DO 85 J=1,NW
      LJ=L(J)
      DO 86 I=1,LJ
      JX=JX+1
      X(JX)=DX+XN*X(JX)
   86 CONTINUE
      JX1=JX-LJ+1
      WRITE(3,90)(X(I),I=JX1,JX)
   90 FORMAT(1X,7E11.4)
      DX=DX+.25
   85 CONTINUE
      Y1(1)=5.
      Y1(2)=5.
      X1(3)=1.
      X1(4)=1.
      Y1(3)=3.
      Y1(4)=7.
   83 WRITE(3,101)
  101 FORMAT('0Y')
      DO 89 K=1,NE
   91 CALL LINE(X1(3),Y1(3),2,1,0,0)
```

```
      DO 38 J=1,5
      S1=8-J
      CALL SYMBOL(1.,S1,.14,13,90.,-1)
   38 CONTINUE
   92 CALL NUMBER(.64,2.93,.14,-1.,0.,-1)
   93 CALL NUMBER(.76,4.93,.14,0.,0.,-1)
   94 CALL NUMBER(.76,6.93,.14,1.,0.,-1)
      KM=(K-1)*M
      NC=KM
      JY=0
      J2=0
      DO 39 J=1,NW
      J2=J2+1
      K2=N2(J2)
      JY1=JY+1
      GO TO (63,64),K2
   64 JY=JY1
      Y(JY)=U5
   63 N6=N1(J)
      DO 43 I=1,N6
      NC=NC+1
      JY=JY+1
      Y(JY)=U5+CUR(NC)
   43 CONTINUE
      J2=J2+1
      K2=N2(J2)
      GO TO (65,66),K2
   66 JY=JY+1
      Y(JY)=U5
   65 J3=J2-1
      K3=IABS(N3(J3))
      JC=KM+K3
      IF(N3(J3)) 67,69,68
   67 Y(JY1)=Y(JY1)-CUR(JC)
      GO TO 69
   68 Y(JY1)=Y(JY1)+CUR(JC)
   69 J3=J3+1
      K3=IABS(N3(J3))
      JC=KM+K3
      IF(N3(J3)) 70,72,71
   70 Y(JY)=Y(JY)-CUR(JC)
      GO TO 72
   71 Y(JY)=Y(JY)+CUR(JC)
   72 X1(1)=X(JY1)
      X1(2)=X(JY)
      IF(K.EQ.1) WRITE(3,90)(Y(I),I=JY1,JY)
   95 CALL SYMBOL(X1(1),Y1(1),.14,13,0.,-1)
   96 CALL LINE(X1(1),Y1(1),2,1,0,0)
   97 CALL SYMBOL(X1(2),Y1(2),.14,13,0.,-1)
      J9=JY-JY1+1
      IF(IRC) 81,81,82
   81 CALL LINE(X(JY1),Y(JY1),J9,1,4,1)
      GO TO 39
   82 JY2=JY1-1
      DO 84 I=1,J9
      J3=JY2+I
      RY(I)=REAL(Y(J3))
      AY(I)=AIMAG(Y(J3))
   84 CONTINUE
   98 CALL LINE(X(JY1),RY(1),J9,1,4,1)
```

```
   99 CALL LINE(X(JY1),AY(1),J9,1,0,1)
   39 CONTINUE
      CALL PLOT(8.,0.,-3)
   89 CONTINUE
    8 CONTINUE
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                X
//               VOLUME=SER=SU0004,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//               BUFNO=1)
//GO.SYSIN DD *
  2
 94   6 38   4   1   1   7
  7   3   5 13   5   5
  2   2   1   1   1   1   2   2   1   1   1   1
 10   8-34-33 38 29-11-15   0   0   0   0
  2.0706  1.8117  1.5529  1.2941  1.0353  0.7765  0.5176  0.2588  0.0000 -0.2588
 -0.5176 -0.7765 -1.0353 -1.2941 -1.5529 -1.8117 -2.0706 -1.5529 -1.8117 -2.0706
 -1.0353  0.0000  1.0353  2.0706  1.8117  1.5529 -4.6587 -4.3999 -4.1411 -3.1058
 -2.0706 -1.0353  0.0000  1.0353  2.0706  3.1058  4.1411  4.3999  4.6587 -4.1411
 -4.3999 -4.6587 -4.9176 -5.1764 -5.4352 -5.6940 -5.9528 -6.2117 -5.1764 -4.1411
 -3.1058 -2.0706 -1.0353  0.0000  1.0353  2.0706  3.1058  4.1411  5.1764  6.2117
  5.9528  5.6940  5.4352  5.1764  4.9176  4.6587  4.3999  4.1411  2.0706  3.1058
  4.1411  3.8823  3.6235  3.3646  3.1058  2.8470  2.5882  2.3294  2.0706  1.0353
  0.0000  0.0000 -1.0353 -2.0706 -2.3294 -2.5882 -2.8470 -3.1058 -3.3646 -3.6235
 -3.8823 -4.1411 -3.1058 -2.0706
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   .0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   .0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   .0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000   .0000  0.0000  0.0000
  7.7274  6.7615  5.7956  4.8296  3.8637  2.8978  1.9319  0.9659  0.0000  0.9659
  1.9319  2.8978  3.8637  4.8296  5.7956  6.7615  7.7274  5.7956  6.7615  7.7274
  7.7274  7.7274  7.7274  7.7274  6.7615  5.7956 17.3867 16.4207 15.4548 15.4548
 15.4548 15.4548 15.4548 15.4548 15.4548 15.4548 15.4548 16.4207 17.3867 15.4548
 16.4207 17.3867 18.3526 19.3185 20.2844 21.2504 22.2163 23.1822 23.1822 23.1822
 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822 23.1822
 22.2163  .2504 20.2844 19.3185 18.3526 17.3867 16.4207 15.4548 15.4548 15.4548
 15.4548 14.4889 13.5230 12.5570 11.5911 10.6252  9.6593  8.6933  7.7274  7.7274
  7.7274  7.7274  7.7274  7.7274  8.6933  9.6593 10.6252 11.5911 12.5570 13.5230
 14.4889 15.4548 15.4548 15.4548
 94   6 38   4   0   1   8
/*
//
```

PRINTED OUTPUT

```
 NF
  2

 NP NW  NM NE NX IRC N6C
 94   6 38   4   1   1   7
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00
```

```
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.6553E+01-0.1817E+03 0.7365E+00 0.1965E+03
 0.1000E+01-0.2887E+00 0.7884E+00-0.4050E+00
 0.5441E+00 0.1000E+01-0.2839E+00 0.7782E-01

CUR
-0.6359E+00-0.2229E-01-0.3245E+00-0.2125E-01

N1
  7   3   5 13   5   5

N2
  2   2   1   1   1   1   2   2   1   1   1   1

N3
 10   8-34-33 38 29-11-15   0   0   0   0

PX
  2.0706   1.8117   1.5529   1.2941   1.0353   0.7765   0.5176   0.2588   0.0000  -0.2588
 -0.5176  -0.7765  -1.0353  -1.2941  -1.5529  -1.8117  -2.0706  -1.5529  -1.8117  -2.0706
 -1.0353   0.0000   1.0353   2.0706   1.8117   1.5529  -4.6587  -4.3999  -4.1411  -3.1058
 -2.0706  -1.0353   0.0000   1.0353   2.0706   3.1058   4.1411   4.3999   4.6587  -4.1411
 -4.3999  -4.6587  -4.9176  -5.1764  -5.4352  -5.6940  -5.9528  -6.2117  -5.1764  -4.1411
 -3.1058  -2.0706  -1.0353   0.0000   1.0353   2.0706   3.1058   4.1411   5.1764   6.2117
  5.9528   5.6940   5.4352   5.1764   4.9176   4.6587   4.3999   4.1411   2.0706   3.1058
  4.1411   3.8823   3.6235   3.3646   3.1058   2.8470   2.5882   2.3294   2.0706   1.0353
  0.0000   0.0000  -1.0353  -2.0706  -2.3294  -2.5882  -2.8470  -3.1058  -3.3646  -3.6235
 -3.8823  -4.1411  -3.1058  -2.0706

PY
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  0.0000   0.0000   0.0000   0.0000

PZ
  7.7274   6.7615   5.7956   4.8296   3.8637   2.8978   1.9319   0.9659   0.0000   0.9659
  1.9319   2.8978   3.8637   4.8296   5.7956   6.7615   7.7274   5.7956   6.7615   7.7274
  7.7274   7.7274   7.7274   7.7274   6.7615   5.7956  17.3867  16.4207  15.4548  15.4548
 15.4548  15.4548  15.4548  15.4548  15.4548  15.4548  15.4548  16.4207  17.3867  15.4548
 16.4207  17.3867  18.3526  19.3185  20.2844  21.2504  22.2163  23.1822  23.1822  23.1822
 23.1822  23.1822  23.1822  23.1822  23.1822  23.1822  23.1822  23.1822  23.1822  23.1822
 22.2163  21.2504  20.2844  19.3185  18.3526  17.3867  16.4207  15.4548  15.4548  15.4548
 15.4548  14.4889  13.5230  12.5570  11.5911  10.6252   9.6593   8.6933   7.7274   7.7274
  7.7274   7.7274   7.7274   7.7274   8.6933   9.6593  10.6252  11.5911  12.5570  13.5230
 14.4889  15.4548  15.4548  15.4548

Y
 0.1000E+01 0.1137E+01 0.1275E+01 0.1412E+01 0.1549E+01 0.1686E+01 0.1824E+01
 0.1961E+01 0.2098E+01
 0.2348E+01 0.2490E+01 0.2632E+01
 0.2882E+01 0.3025E+01 0.3167E+01 0.3309E+01 0.3451E+01
 0.3701E+01 0.3838E+01 0.3975E+01 0.4113E+01 0.4250E+01 0.4392E+01 0.4534E+01
```

```
0.4676E+01 0.4818E+01 0.4961E+01 0.5103E+01 0.5240E+01 0.5377E+01 0.5515E+01
0.5652E+01
0.5902E+01 0.6039E+01 0.6176E+01 0.6314E+01 0.6451E+01
0.6701E+01 0.6838E+01 0.6975E+01 0.7113E+01 0.7250E+01

Y
0.3402E+01 0.4960E+01 0.3728E+01 0.4955E+01 0.4351E+01 0.4958E+01 0.5193E+01
0.4967E+01 0.7000E+01 0.5000E+01 0.5193E+01 0.4967E+01 0.4351E+01 0.4958E+01
0.3728E+01 0.4955E+01 0.3402E+01 0.4961E+01
0.4149E+01 0.5001E+01 0.3925E+01 0.5000E+01 0.4149E+01 0.5001E+01
0.4123E+01 0.4976E+01 0.4363E+01 0.4985E+01 0.5027E+01 0.5000E+01 0.4363E+01
0.4985E+01 0.4123E+01 0.4976E+01
0.5123E+01 0.5035E+01 0.5309E+01 0.5044E+01 0.5499E+01 0.5046E+01 0.5625E+01
0.5042E+01 0.5678E+01 0.5032E+01 0.5629E+01 0.5019E+01 0.5489E+01 0.5009E+01
0.5148E+01 0.5000E+01 0.5489E+01 0.5009E+01 0.5629E+01 0.5019E+01 0.5678E+01
0.5032E+01 0.5625E+01 0.5042E+01 0.5499E+01 0.5046E+01 0.5309E+01 0.5044E+01
0.5123E+01 0.5035E+01
0.4247E+01 0.5012E+01 0.4122E+01 0.5000E+01 0.4047E+01 0.4984E+01 0.4100E+01
0.4970E+01 0.4253E+01 0.4960E+01
0.4253E+01 0.4960E+01 0.4100E+01 0.4970E+01 0.4047E+01 0.4984E+01 0.4122E+01
0.5000E+01 0.4247E+01 0.5012E+01

 NP NW  M NE NX IRC N6C
 94  6 38  4  0   1   8
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.6553E+01-0.1817E+03 0.7365E+00 0.1965E+03
 0.1000E+01-0.2887E+00 0.7884E+00-0.4050E+00
 0.5441E+00 0.1000E+01-0.2839E+00 0.7782E-01
-0.6359E+00-0.2229E-01-0.3245E+00-0.2125E-01

CUR
 0.9551E+00-0.2918E-03 0.1000E+01-0.4569E-03

Y
0.6724E+01 0.5000E+01 0.6910E+01 0.4999E+01 0.7000E+01 0.4999E+01 0.6781E+01
0.4999E+01 0.5818E+01 0.5000E+01 0.6781E+01 0.4999E+01 0.7000E+01 0.4999E+01
0.6910E+01 0.4999E+01 0.6724E+01 0.5000E+01
0.7288E+01 0.4999E+01 0.6504E+01 0.5000E+01 0.7288E+01 0.4999E+01
0.3757E+01 0.5001E+01 0.3950E+01 0.5001E+01 0.4573E+01 0.5000E+01 0.3950E+01
0.5001E+01 0.3757E+01 0.5001E+01
0.5103E+01 0.5001E+01 0.5212E+01 0.5000E+01 0.5328E+01 0.5000E+01 0.5403E+01
0.5000E+01 0.5432E+01 0.5000E+01 0.5398E+01 0.5000E+01 0.5315E+01 0.5000E+01
0.5117E+01 0.5000E+01 0.5315E+01 0.5000E+01 0.5398E+01 0.5000E+01 0.5432E+01
0.5000E+01 0.5403E+01 0.5000E+01 0.5328E+01 0.5000E+01 0.5212E+01 0.5000E+01
0.5103E+01 0.50001E+01
0.3860E+01 0.5002E+01 0.3836E+01 0.5002E+01 0.3920E+01 0.5001E+01 0.4162E+01
0.5001E+01 0.4437E+01 0.5001E+01
0.4437E+01 0.5001E+01 0.4162E+01 0.5001E+01 0.3920E+01 0.5001E+01 0.3836E+01
0.5002E+01 0.3860E+01 0.5002E+01
```

XI. PATTERN PLOTS

The program of this section plots both the previously computed mode patterns and scattering patterns. The activity on data sets 1 (punched card input) and 6 (direct access input-output) is as follows

```
          READ (1,10) NF
    10    FORMAT (20I3)
          DO 19 LF = 1, NF
          READ (1,10) NT, NE, NEP, NS, N6
          READ (1, 10) (N1(I), I = 1, NE)
          READ (1,10) (N2(I), I = 1, NEP)
          READ (1, 10)(N3(I), I=1, NEP)
          READ (1,10)(NSBL(I), I = 1, NEP)
          READ (1,24)(SCAL(J), J=1, NEP)
    24    FORMAT (7E11.4)
          REWIND 6
          SKIP N6 RECORDS ON DATA SET 6
          READ (6) (SIG(I), I = 1, KS)
    19    CONTINUE
```

There are NE patterns in SIG. N1(I) = 0 indicates that the I^{th} pattern in SIG has (NT-1)/NS + 1 elements while N1(I) = 1 indicates that the I^{th} pattern in SIG has NT elements. NEP patterns are plotted. The K^{th} pattern to be plotted is SCAL(K) times the $N2(K)^{th}$ pattern in SIG. If N3(K) $\leq$ 0, the $(K+1)^{th}$ pattern to be plotted is put on the same frame as the K^{th} pattern, but if N3(K) > 0, the K^{th} and $(K+1)^{th}$ patterns to be plotted are put on different frames. NSBL(I) is the symbol number for the symbols on the I^{th} pattern to be plotted. If N1(I) = 1, NSBL(I) is not used.

Minimum allocations are given by

```
DIMENSION N1(NE), N2(NEP), N3(NEP), SCAL(NEP), N4(NE+1), CS(NT),
SN(NT), SIG(KS), X(NT), Y(NT), NSBL(NEP)
```

where

$$KS = ((NT-1)/NS + 1)*(NE - \sum_{J=1}^{NE} N1(J)) + NT*\sum_{J=1}^{NE} N1(J)$$

The N4(J) calculated by DO loop 15 tells that the J^{th} pattern in SIG is stored in SIG(N4(J) + 1) through SIG(N4(J + 1)). DO loop 18 calculates the sines and cosines needed for polar plots.

The index K of DO loop 34 indicates the K^{th} pattern to be plotted. If the K^{th} pattern to be plotted has (NT-1)/NS + 1 elements, DO loop 28 plots it with the symbol NSBL(K). If the K^{th} pattern to be plotted has NT elements, DO loop 30 stores the horizontal and vertical coordinates in X and Y which are then plotted by statement 35. Statement 35 draws straight lines between points. If no change in X and Y is expected, DO loop 30 is bypassed. The logic between statements 31 and 38 draws the horizontal and vertical axes with tick marks and advances the origin 9 inches.

The sample data is such as to plot the mode patterns previously stored on records 10 and 11 of data set 6 and the scattering patterns stored on records 12 and 13 of data set 6. The mode patterns on records 10 and 11 are plotted in order, one to a frame. Of the six scattering patterns on record 12, the sixth is paired with the second, third, fourth, and fifth in successive frames. The scattering patterns on record 13 are plotted in the same way as those on record 12.

LISTING OF PROGRAM TO PLOT PATTERNS

```
//                (0034,EE,2,1,,20),'MAUTZ,JOE',REGION=140K
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      DIMENSION X1(2),Y1(2),AREA(400),N1(40),N2(64),N3(64),SCAL(64)
      DIMENSION N4(41),CS(145),SN(145),SIG(4936),X(145),Y(145)
      DIMENSION NSBL(64)
      PI=3.141593
      X1(1)=5.
      X1(2)=5.
      Y1(1)=2.
      Y1(2)=8.
      CALL PLOTID
      CALL PLOTS(AREA,400)
      READ(1,10) NF
   10 FORMAT(20I3)
      WRITE(3,9) NF
    9 FORMAT('0 NF'/1X,I3)
      DO 19 LF=1,NF
      READ(1,10) NT,NE,NEP,NS,N6
      WRITE(3,11) NT,NE,NEP,NS,N6
   11 FORMAT('0 NT NE NEP NS N6'/1X,2I3,I4,2I3)
      READ(1,10)(N1(I),I=1,NE)
      WRITE(3,12)(N1(I),I=1,NE)
   12 FORMAT('0N1'/(1X,20I3))
      READ(1,10)(N2(I),I=1,NEP)
      WRITE(3,13)(N2(I),I=1,NEP)
   13 FORMAT('0N2'/(1X,20I3))
      READ(1,10)(N3(I),I=1,NEP)
      WRITE(3,14)(N3(I),I=1,NEP)
   14 FORMAT('0N3'/(1X,20I3))
      READ(1,10)(NSBL(I),I=1,NEP)
      WRITE(3,39)(NSBL(I),I=1,NEP)
   39 FORMAT('0NSBL'/(1X,20I3))
      READ(1,24)(SCAL(J),J=1,NEP)
   24 FORMAT(7E11.4)
      WRITE(3,25)(SCAL(J),J=1,NEP)
   25 FORMAT('0SCAL'/(1X,7E11.4))
      NTS=(NT-1)/NS+1
      N4(1)=0
      DO 15 J=1,NE
      J1=J+1
      IF(N1(J)) 16,16,17
   16 N4(J1)=N4(J)+NTS
      GO TO 15
   17 N4(J1)=N4(J)+NT
   15 CONTINUE
      KS=N4(J1)
      DEL=2.*PI/(NT-1)
      DO 18 J=1,NT
      ANG=(J-1)*DEL
      CS(J)=COS(ANG)
      SN(J)=SIN(ANG)
   18 CONTINUE
      REWIND 6
      IF(N6) 20,20,21
   21 DO 22 J=1,N6
      READ(6)(SIG(I),I=1,4)
```

```
      WRITE(3,23)(SIG(I),I=1,4)
   23 FORMAT(1X,7E11.4)
   22 CONTINUE
   20 READ(6)(SIG(I),I=1,KS)
      WRITE(3,37)(SIG(I),I=1,4)
   37 FORMAT('0SIG'/(1X,7E11.4))
      J5=-1
      DO 34 K=1,NEP
      SCL=SCAL(K)
      J2=N2(K)
      J1=N1(J2)
      J3=N3(J2)
      J2=N4(J2)
      IF(J1) 26,26,27
   26 J4=J2
      J6=NSBL(K)
      DO 28 I=1,NT,NS
      J4=J4+1
      S5=SIG(J4)*SCL
      IF(ABS(S5).GT.5.) STOP 28
      XX=5.+S5*SN(I)
      YY=5.+S5*CS(I)
      CALL SYMBOL(XX,YY,.07,J6,0.,-1)
   28 CONTINUE
      GO TO 29
   27 IF(J2.EQ.J5) GO TO 35
      J5=J2
      DO 30 I=1,NT
      J4=I+J2
      S5=SIG(J4)*SCL
      IF(ABS(S5).GT.5.) STOP 30
      X(I)=5.+S5*SN(I)
      Y(I)=5.+S5*CS(I)
   30 CONTINUE
   35 CALL LINE(X(1),Y(1),NT,1,0,0)
   29 J3=N3(K)
      IF(J3) 34,34,31
   31 CALL LINE(Y1(1),X1(1),2,1,0,0)
      DO 32 I=1,7
      S1=9-I
      CALL SYMBOL(S1,5.,.14,13,0.,-1)
   32 CONTINUE
      CALL LINE(X1(1),Y1(1),2,1,0,0)
      DO 33 I=1,7
      S1=9-I
      CALL SYMBOL(5.,S1,.14,13,90.,-1)
   33 CONTINUE
   38 CALL PLOT(9.,0.,-3)
   34 CONTINUE
   19 CONTINUE
      CALL PLOT(6.,0.,-3)
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                  X
//               VOLUME=SER=SU0004,DCB=(RECFM=VS,BLKSIZE=2596,LRECL=2592,X
//               BUFNO=1)
//GO.SYSIN DD *
  4
145  4  4  4  9
```

```
  1   1   1   1
  1   2   3   4
  1   1   1   1
  0   0   0   0
 0.1000E+01 0.1000E+01 0.1000E+01 0.1000E+01
145   4   4   4 10
  1   1   1   1
  1   2   3   4
  1   1   1   1
  0   0   0   0
 0.1000E+01 0.1000E+01 0.1000E+01 0.1000E+01
145   6   8   4 11
  0   0   0   0   0   1
  2   6   3   6   4   6   5   6
  0   1   0   1   0   1   0   1
  4   4   4   4   4   4   4   4
 0.1000E+03 0.1000E+03 0.1000E+03 0.1000E+03 0.1000E+03 0.1000E+03 0.1000E+03
 0.1000E+03
145   6   8   4 12
  0   0   0   0   0   1
  2   6   3   6   4   6   5   6
  0   1   0   1   0   1   0   1
  4   4   4   4   4   4   4   4
 0.1000E+02 0.1000E+02 0.1000E+02 0.1000E+02 0.1000E+02 0.1000E+02 0.1000E+02
 0.1000E+02
/*
//
```

PRINTED OUTPUT

```
 NF
  4

 NT NE NEP NS N6
145  4   4  4  9

N1
  1   1   1   1

N2
  1   2   3   4

N3
  1   1   1   1

NSBL
  0   0   0   0

SCAL
 0.1000E+01 0.1000E+01 0.1000E+01 0.1000E+01
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.6553E+01-0.1817E+03 0.7365E+00 0.1965E+03
 0.1000E+01-0.2887E+00 0.7884E+00-0.4050E+00
 0.5441E+00 0.1000E+01-0.2839E+00 0.7782E-01
-0.6359E+00-0.2229E-01-0.3245E+00-0.2125E-01
 0.9551E+00-0.2918E-03 0.1000E+01-0.4569E-03
```

```
SIG
 0.1119E+01 0.1120E+01 0.1124E+01 0.1131E+01

 NT NE NEP NS N6
145  4   4  4 10

N1
  1   1   1   1

N2
  1   2   3   4

N3
  1   1   1   1

NSBL
  0   0   0   0

SCAL
 0.1000E+01 0.1000E+01 0.1000E+01 0.1000E+01
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.6553E+01-0.1817E+03 0.7365E+00 0.1965E+03
 0.1000[illegible]+01-0.2[illegible]7E+00 0.7884E+00-0.4050E+00
 0.5441E+00 0.1000E+01-0.2839E+00 0.7782E-01
-0.6[illegible]9E+00-0.2229E-01-0.3245E+00-0.2125E-01
 0.9551E+00-0.2918E-03 0.1000E+01-0.4569E-03
 0.1119E+01 0.1120E+01 0.1124E+01 0.1131E+01

SIG
 0.2681E+01 0.2678E+01 0.2671E+01 0.2658E+01

 NT NE NEP NS N6
145  6   8  4 11

N1
  0   0   0   0   0   1

N2
  2   6   3   6   4   6   5   6

N3
  0   1   0   1   0   1   0   1

NSBL
  4   4   4   4   4   4   4   4

SCAL
 0.1000E+03 0.1000E+03 0.1000E+03 0.1000E+03 0.1000E+03 0.1000E+03 0.1000E+03
 0.1000E+03
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.19[illegible]7E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.65[illegible]3E+01-0.1817E+03 0.7365E+00 0.1965E+03
 0.10[illegible]0E+01-0.2887E+00 0.7884E+00-0.4050E+00
 0.54[illegible]1E+00 0.1000E+01-0.2839E+00 0.7782E-01
-0.6[illegible]9E+00-0.2229E-01-0.3245E+00-0.2125E-01
```

```
 0.9551E+00-0.2918E-03 0.1000E+01-0.4569E-03
 0.1119E+01 0.1120E+01 0.1124E+01 0.1131E+01
 0.2681E+01 0.2678E+01 0.2671E+01 0.2658E+01

SIG
 0.1730E+00 0.1689E+00 0.1585E+00 0.1460E+00

 NT NE NEP NS N6
145  6   8  4 12

N1
  0   0   0   0   0   1

N2
  2   6   3   6   4   6   5   6

N3
  0   1   0   1   0   1   0   1

NSBL
  4   4   4   4   4   4   4   4

SCAL
 0.1000E+02 0.1000E+02 0.1000E+02 0.1000E+02 0.1000E+02 0.1000E+02 0.1000E+02
 0.1000E+02
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02
 0.6553E+01-0.1817E+03 0.7365E+00 0.1965E+03
 0.1000E+01-0.2887E+00 0.7884E+00-0.4050E+00
 0.5441E+00 0.1000E+01-0.2839E+00 0.7782E-01
-0.6359E+00-0.2229E-01-0.3245E+00-0.2125E-01
 0.9551E+00-0.2918E-03 0.1000E+01-0.4569E-03
 0.1119E+01 0.1120E+01 0.1124E+01 0.1131E+01
 0.2681E+01 0.2678E+01 0.2671E+01 0.2658E+01
 0.1730E+00 0.1689E+00 0.1585E+00 0.1460E+00

SIG
 0.7957E-03 0.6534E-03 0.7043E-03 0.2078E-02
```

XII. LOADS FOR MODAL RESONANCE

The program on page 44 of [6] calculates loads according to (44) or (46). To obtain more accuracy, format statements 28, 29, 25, and 26 of this program were changed to

```
28      FORMAT (5E14.7)
29      FORMAT ('OFI'/(1X, 5E 14.7))
25      FORMAT ('OXL'/(1X, 5E 14.7))
26      FORMAT (5E 14.7)
```

The sample data is such to calculate the susceptive loads to resonate the port mode voltage with eigenvalue $\mu = 10.12$. The punched card input data and resulting print out are listed here. Since the input quantities are those of the right hand side of (46), the loads printed out under XL are susceptances. If the input quantities were those of the right hand side of (44), the loads printed out under XL would be reactances.

```
PUNCHED CARD INPUT DATA

  4  1  3
 0.1904000E+00 0.2524000E+00 0.1000000E+01-0.9256000E+00
/*
//

PRINTED OUTPUT

 N MD5 N6
  4  1  3
 0.3057E+01-0.5125E+03 0.3011E+01 0.2382E+03
-0.2142E+00-0.4636E+00-0.3706E+00-0.3513E+00
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02

Z
 0.1917E-03 0.3061E-02-0.3933E-03-0.3197E-02-0.6371E-03-0.2576E-03

FI
 0.1903999E+00 0.2524000E+00 0.1000000E+01-0.9256000E+00

XL
 0.4593194E-02-0.5699836E-02-0.1158788E-02-0.1003725E-02
```

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