INTERACTION NOTES

Note 265

Electromagnetic Field Analysis for a Coaxial Cable with Periodic Slots

23 February 1976

James R. Wait
C.I.R.E.S., University of Colorado/NOAA
Boulder, Colorado

ABSTRACT

The analysis deals with the excitation of coaxial structures with periodic apertures in the outer cylindrical shield. These apertures are taker to be finite circumferential slots of thin width. A quasi-static method is employed that reduces the problem to a field matching of angular wave functions over the angular extent of the slots. An approximate averaging technique is then used to obtain expressions for the effective admittance of the cable. It is then argued that this can be used to characterize the cable in more complicated environments.

INTRODUCTION

There is a need to understand how electromagnetic waves interact with leaky coaxial cables [1-4]. Whether such leakage through the cable shield is intentional or not, it is desirable to have a theoretical basis for specifying the cable's characteristics when it is located in a realistic environment. With this motivation, we have chosen an idealized model of a cable that has a perfectly conducting sheath or shield with periodic apertures. Such a structure is non-uniform in both the axial and azimuthal directions and, in this sense, the analysis is more general than most previous attempts to deal with such problems.

* Consultant to the Institute for Telecommunication Sciences, Office of Telecommunications, Boulder, CO

FORMULATION

The geometry of the posed problem is shown in Fig. 1. The coaxial cable consists of a cylindrical structure of infinite length with a solid perfectly conducting core of radius a with a shield of radius b. The cylindrical shield is perfectly conducting and with negligible thickness; it is continuous except for a periodic array of narrow circumferential slots of arc length $b\phi_o$ and with axial spacing ℓ. The concentric region between the inner core conductor and the shield is a homogeneous dielectric with permittivity ϵ. The external region is free space with permittivity ϵ_o. As indicated in Fig. 1, a cylindrical coordinate system (ρ,ϕ,z) is chosen to be coaxial with the cylindrical structure.

One convenient artifice for dealing with a field analysis of this type of periodic structure is to imagine it to be excited by an external plane wave whose wave normal, for example, subtends an angle θ_o with the negative z axis. Now if this induces a voltage V_o at the center of the slot at $z = 0$, it follows that the voltage at the center of the adjacent slot at $z = \ell$ is $V_o \exp(-i\beta_o \ell)$ where $\beta_o = k_o \cos\theta_o$. In fact, the voltage at the center of the slot at $z = p\ell$, where $p = \pm 1, \pm 2, \pm 3 \ldots.$, is $V_o \exp(-i\beta_o p\ell)$. Our first task will then be to deduce expressions for the fields internal to and external to the shield due to the array of circumferential slots with such an excitation. Because of the periodic nature of the solution, we need only work with the reference "cell" that extends from $z = -\ell/2$ to $z = +\ell/2$.

Of course, if we wish to obtain the natural propagation constants for the structure we let the excitation vanish after the problem has been formulated in the manner described above.

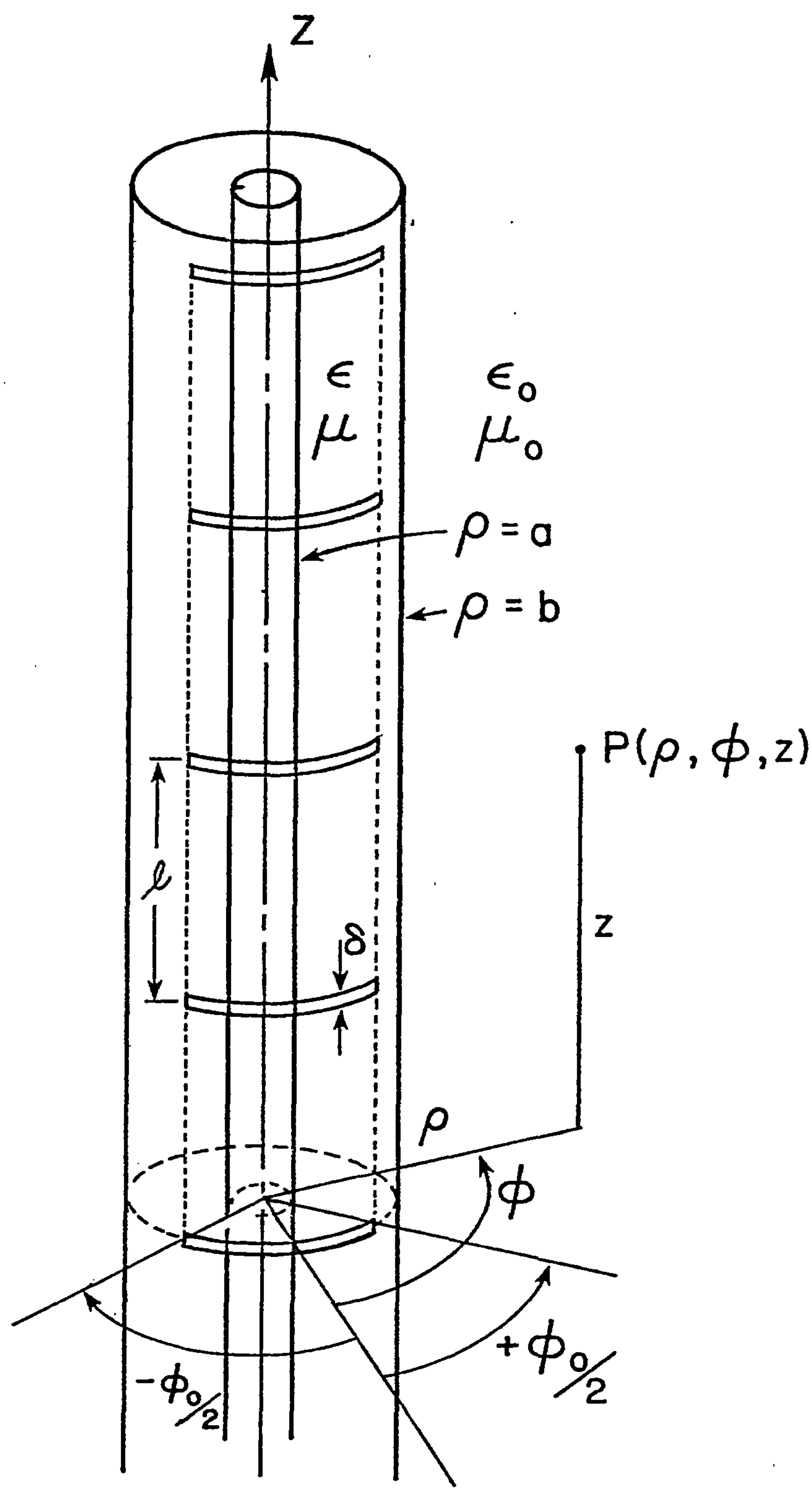

Fig. 1 A coaxial cable with a periodic array of circumferential slots cut in the shield.

THE HERTZ POTENTIALS

As usual in problems of this type, we express the fields in terms of electric and magnetic Hertz vectors that have only z components, Π and Π^*, respectively. Thus, for the region $a < \rho < b$,

$$E_\rho = \frac{\partial^2\Pi}{\partial\rho\partial z} - \frac{i\mu\omega}{\rho}\frac{\partial\Pi^*}{\partial\phi} \qquad (1) \qquad H_\rho = \frac{\partial^2\Pi^*}{\partial\rho\partial z} + \frac{i\varepsilon\omega}{\rho}\frac{\partial\Pi}{\partial\phi} \qquad (4)$$

$$E_\phi = \frac{1}{\rho}\frac{\partial^2\Pi}{\partial\phi\partial z} + i\mu\omega\frac{\partial\Pi^*}{\partial\rho} \qquad (2) \qquad H_\phi = \frac{1}{\rho}\frac{\partial^2\Pi^*}{\partial\phi\partial z} - i\varepsilon\omega\frac{\partial\Pi}{\partial\rho} \qquad (5)$$

$$E_z = \left(k^2 + \frac{\partial^2}{\partial z^2}\right)\Pi \qquad (3) \qquad H_z = \left(k^2 + \frac{\partial^2}{\partial z^2}\right)\Pi^* \qquad (6)$$

where $k^2 = \varepsilon\mu\omega^2$ and μ is the magnetic permeability. In order to satisfy the boundary conditions that $E_\phi = 0$ and $E_z = 0$ at $\rho = a$ suitable expansions for this concentric dielectric region are [5]

$$\Pi = \sum_n \sum_m A_{m,n} Z_n(u_m\rho)e^{in\phi}e^{-i\beta_m z} \qquad (7)$$

and

$$\Pi^* = \sum_n \sum_m A^*_{m,n} Z^*_n(u_m\rho)e^{in\phi}e^{-i\beta_m z} \qquad (8)$$

where $\beta_m = \beta_o + 2\pi m/\ell$, $u_m = \left(\beta_m^2 - k^2\right)^{1/2} = i\left(k^2 - \beta_m^2\right)^{1/2}$. The summations here over m and n extend from $-\infty$ to $+\infty$ through all integers including zero. The radial functions are expressed in terms of conventional modified Bessel functions and their derivatives as follows:

$$Z_n(u\rho) = I_n(u\rho) - \frac{I_n(ua)}{K_n(ua)}K_n(u\rho) , \qquad (9)$$

$$Z^*_n(u\rho) = I_n(u\rho) - \frac{I'_n(ua)}{K'_n(ua)}K_n(u\rho) . \qquad (10)$$

These forms permit the satisfaction of the boundary condition $E_\phi = E_z = 0$ at $\rho = a$.

The coefficients $A_{m,n}$ and $A^*_{m,n}$ are as yet undetermined.

APPLICATION OF FURTHER BOUNDARY CONDITIONS

To simplify the analysis we now argue that the circumferential slots are sufficiently thin that $E_\phi = 0$ at all points on the inside of the shield at $\rho = b$. This means that

$$A^*_{m,n} = -A_{m,n} \frac{n\beta_m}{i\mu\omega u_m b} \frac{Z_n(u_m b)}{Z^{*\prime}_n(u_m b)} \tag{11}$$

where $Z^{*\prime}_n(u_m b) = dZ^*_n(x)/dx$ evaluated at $x = u_m b$.

On the other hand, the axial electric field in the section $-\frac{\ell}{2} < z < \frac{\ell}{2}$ at $\rho = b$ is given by

$$E_z(b) = \frac{V_o}{\delta} f(\phi) g(z) e^{-i\beta_o z} \tag{12}$$

but vanishes outside the region $-\phi_o/2 < \phi < +\phi_o/2$ and $-\delta/2 < z < \delta/2$. Here we normalize the dimensionless functions $f(\phi)$ and $g(z)$ such that $f(0) = 1$ and $(1/\delta) \int_{-\delta/2}^{\delta/2} g(z)dz = 1$. Now, according to (3) and (7),

$$E_z(b) = -\sum_m \sum_n u_m^2 A_{m,n} Z_n(u_m b) e^{in\phi} e^{-i\beta_m z} \tag{13}$$

Then, on utilizing the usual orthogonality properties, we find that, on equating (12) and (13), that

$$A_{m,n} = -\frac{V_o}{2\pi u^2 \ell\delta} \frac{1}{Z_n(u_m b)} \int_{-\delta/2}^{\delta/2} g(z) e^{i(2\pi m/\ell)z} dz \int_{-\phi_o/2}^{\phi_o/2} f(\phi) e^{-in\phi} d\phi \tag{14}$$

Thus, the fields in the dielectric region $(a < \rho < b)$ can be expressed in terms of the slot voltage V_o via (1) to (8). In particular, the resulting field on the inside of the shield is to be obtained from

$$H_\phi(b,\phi,z) = \sum_m \sum_n \left[A^*_{m,n} \frac{n\beta_m}{b} Z^*_n(u_m b) - A_{m,n} i\epsilon\omega u_m Z'_n(u_m b) \right] \times e^{in\phi} e^{-i\beta_m z} \tag{15}$$

GREEN FUNCTION TYPE REPRESENTATION

For later application, (15) is conveniently written in the equivalent form

$$H_\phi(b,\phi,z) = \frac{V_o}{2\pi\delta} \int_{-\delta/2}^{\delta/2} g(z')G(b,\phi;z,z')dz' \; e^{-i\beta_o z} \tag{16}$$

where the "Green's function" is

$$G(b,\phi;z,z') = \frac{i\epsilon\omega}{\ell} \sum_m \sum_n \frac{F_n}{u_m} \frac{Z'_n(u_m b)}{Z_n(u_m b)} \left(1 - \Delta_{m,n}\right) e^{-i(2\pi m/\ell)(z-z')} e^{in\phi} \tag{17}$$

where

$$F_n = \int_{-\phi_o/2}^{\phi_o/2} f(\phi) e^{-in\phi} d\phi \tag{18}$$

and

$$\Delta_{m,n} = \frac{(n\beta_m)^2}{k^2(u_m b)^2} \frac{Z_n(u_m b)}{Z^{*\prime}_n(u_m b)} \frac{Z^*_n(u_m b)}{Z'_n(u_m b)} \tag{19}$$

Since we will need to use (17) in the region where $z-z'$ is small (i.e. within the slot) it is desirable to convert the m summation to a different form. This is simply accomplished by noting, as $|m| \to \infty$, that

$$Z'_n(u_m b)/Z_n(u_m b) \to 1$$

and

$$\Delta_{m,n} \to n^2/(kb)^2$$

where n is regarded as a finite integer. This immediately suggests writing (17) in the form

$$G(b,\phi;z,z') = \frac{i\varepsilon\omega}{\ell} \sum_n F_n e^{in\phi} \frac{Z_n'(u_o b)}{Z_n(u_o b)}$$

$$+ i\varepsilon\omega \sum_n \sum_m{}' F_n e^{in\phi} \left[\frac{1}{u_m \ell} \frac{Z_n'(u_m b)}{Z_n(u_m b)} \left(1 - \Delta_{m,n}\right) - \frac{1}{2\pi|m|} \left(1 - \frac{n^2}{k^2 b^2}\right)\right] e^{-i(2\pi/\ell)m(z-z')}$$

$$+ i\varepsilon\omega \sum_n \sum_m{}' F_n e^{in\phi} \frac{1}{2\pi|m|} \left(1 - \frac{n^2}{k^2 b^2}\right) e^{-i(2\pi/\ell)m(z-z')} \tag{20}$$

where the prime over the summation sign indicates that the $m = 0$ term is to be excluded. The latter summation over m can be expressed in closed form by noting that

$$\sum_1^\infty \frac{\cos mx}{m} = -\frac{1}{2} \ell n[2(1 - \cos x)] \quad \text{where} \quad 0 < x < 2\pi$$

$$\simeq -\ell n\, x \quad \text{where} \quad 0 < x << 1 \tag{21}$$

Thus, if $|z-z'|/\ell$ is a sufficiently small parameter, we can write (20) in the very convenient form

$$G(b,\phi;z,z') \cong i\varepsilon\omega \sum_n F_n e^{in\phi} \left[\frac{1}{u_o \ell} \frac{Z_n'(u_o b)}{Z_n(u_o b)} - \left(1 - \frac{n^2}{k^2 b^2}\right)\frac{1}{\pi} \ell n \left(\frac{2\pi}{\ell} |z-z'|\right) + \Delta_n\right] \tag{22}$$

where

$$\Delta_n = \sum_m{}' \left[\frac{1}{u_m \ell} \frac{Z_n'(u_m b)}{Z_n(u_m b)} \left(1 - \Delta_{m,n}\right) - \frac{1}{2\pi|m|} \left(1 - \frac{n^2}{k^2 b^2}\right)\right] \tag{23}$$

REPRESENTATIONS FOR THE EXTERNAL REGION

In dealing with the fields produced in the external region (i.e. $\rho > b$) by the same array of circumferential slots, the procedure is almost identical. Now, however, the appropriate Hertz potentials are

$$\Pi_o = \sum_n \sum_m B_{m,n} K_n(v_m \rho) e^{in\phi} e^{-i\beta_m z} \qquad (24)$$

and

$$\Pi_o^* = \sum_n \sum_m B_{m,n}^* K_n(v_m \rho) e^{in\phi} e^{-i\beta_m z} \qquad (25)$$

where

$$v_m = \left(\beta_m^2 - k_o^2\right)^{1/2} = i\left(k_o^2 - \beta_m^2\right)^{1/2}$$

These have the required physical behavior as $\rho \to \infty$ provided we always choose $\operatorname{Re} v_m > 0$. The task again amounts to determining the coefficients $B_{m,n}$ and $B_{m,n}^*$ in terms of the slot voltage V_o.

Assuming that $E_{o\phi} = 0$ for the entire surface $\rho = b$, we easily find that

$$\frac{B_{m,n}^*}{B_{m,n}} = - \frac{n\beta_m}{i\mu_o \omega v_m b} \frac{K_n(v_m b)}{K_n'(v_m b)} \qquad (26)$$

Again we require that the axial field E_{oz} at $\rho = b$ should have the form given by (12). Thus, it is found that

$$B_{m,n} = - \frac{V_o}{2\pi v_m^2 \ell} \frac{F_n}{K_n(v_m b)} \int_{-\delta/2}^{\delta/2} g(z') e^{i(2\pi/\ell)mz'} dz' \qquad (27)$$

where F_n is defined by (18). The corresponding field components $E_{o\rho}$, $E_{o\phi}$, etc. in the region $\rho > b$ can now be expressed in terms of V_o by operating on (24) and (25); here we use (1) to (6) with μ and ε replaced by μ_o and ε_o, respectively. In particular, we find that

$$H_{o\phi}(b,\phi,z) = \frac{V_o}{2\pi\delta}\int_{-\delta/2}^{\delta/2} g(z')G_o(b,\phi;z,z')dz' e^{-i\beta_o z} \tag{28}$$

where

$$G_o(b,\phi;z,z') = \frac{i\varepsilon_o\omega}{\ell}\sum_m\sum_n \frac{F_n}{v_m}\frac{K_n'(v_m b)}{K_n(v_m b)}\left(1 - \Omega_{m,n}\right)e^{-i(2\pi m/\ell)(z-z')}e^{in\phi} \tag{29}$$

where

$$\Omega_{m,n} = \frac{(n\beta_m)^2}{k_o^2(v_m b)^2}\left[\frac{K_n(v_m b)}{K_n'(v_m b)}\right]^2 \tag{30}$$

For $|m| \to \infty$ we now note that

$$K_n'(v_m b)/K_m(v_m b) \to -1$$

and

$$\Omega_{m,n} \to n^2/(k_o b)^2$$

Again, for the case where $|z-z'|/\ell$ is sufficiently small, we are led to the representation of the form

$$G_o(b,\phi;z,z') = i\varepsilon_o\omega \sum_n F_n e^{in\phi}\left[\frac{1}{v_o\ell}\frac{K_n'(v_o b)}{K_n(v_o b)} + \left(1 - \frac{n^2}{k_o^2 b^2}\right)\frac{1}{\pi}\ln\left(\frac{2\pi}{\ell}|z-z'|\right) + \Omega_n\right] \tag{31}$$

where

$$\Omega_n = \sum_m{}' \left[\frac{1}{v_m\ell}\frac{K_n'(v_m b)}{K_n(v_m b)}\left(1 - \Omega_{m,n}\right) + \frac{1}{2\pi|m|}\left(1 - \frac{n^2}{k_o^2 b^2}\right)\right] \tag{32}$$

FIELD MATCHING IN THE APERTURES

We are now in the position to match the tangential magnetic fields in the aperture of the reference slot at $z = 0$. Thus

$$H_\phi(b,\phi,z) = H_{o\phi}(b,\phi,z) + H_{a\phi}(b,\phi,z) \tag{33}$$

for $|z| < \delta/2$ and $|\phi| < \phi_o$ where $H_{a\phi}$ is the applied magnetic field due to the external excitation. By definition $H_{a\phi}$ is the value of the field on the outer surface of the shield if there were no slots present (i.e. in the limit $\delta \to 0$). We can always write

$$H_{a\phi}(b,\phi,z) = \sum_p H_p e^{ip\phi} e^{-i\beta_o z} \tag{34}$$

where H_p is specified by the form of the excitation and $p = 0,\pm1,\pm2,\ldots$ For present purposes, we do not need to give the explicit form of H_p. Using (16), (28) and (34), it follows that (33) is equivalent to

$$\frac{1}{\delta}\int_{-\delta/2}^{\delta/2} g(z)[G(b,\phi;z,z') - G_o(b,\phi;z,z')]dz = 2\pi \sum_p H_p e^{ip\phi} \tag{35}$$

that is to be satisfied for $|\phi| < \phi_o/2$ and $|z| < \delta/2$. By using (22) and (32) the following integral equation is obtained from (35):

$$V_o \sum_n F_n e^{in\phi} J_n = 2\pi \sum_p H_p e^{ip\phi} \tag{36}$$

where

$$J_n = \frac{i\varepsilon\omega}{u_o\ell}\frac{Z_n'(u_ob)}{Z_n(u_ob)} - \frac{i\varepsilon_o\omega}{v_o\ell}\frac{K_n'(v_ob)}{K_n(v_ob)} + i\varepsilon\omega\Delta_n - i\varepsilon_o\omega\Omega_n$$
$$- \frac{1}{\pi}\left[i\varepsilon\omega\left(1 - \frac{n^2}{k^2b^2}\right) + i\varepsilon_o\omega\left(1 - \frac{n^2}{k_o^2b^2}\right)\right]\frac{1}{\delta}\int_{-\delta/2}^{\delta/2} \ln\left(\frac{2\pi}{\ell}|z-z'|\right) g(z')dz' \tag{37}$$

Now the right-hand side of (36) does not depend on z; thus

$$\int_{-\delta/2}^{\delta/2} \ln\left(\frac{2\pi}{\ell}|z-z'|\right) g(z')dz' = \text{constant} \tag{38}$$

But

$$\frac{1}{\delta}\int_{-\delta/2}^{\delta/2} g(z')dz' = 1 \tag{39}$$

and therefore we deduce that

$$g(z') = \frac{\delta/2}{\pi[(\delta/2)^2 - (z')^2]^{1/2}} \quad \text{for} \quad |z'| < \delta/2 \tag{40}$$

This, of course, has the correct edge behavior as $|z'| \to \delta/2$. Furthermore, we can confirm that the "constant" on the right-hand side of (38) is $\ln(\pi\delta/2\ell)$ by making use of the integral formula [6]

$$\int_0^1 \frac{\ln(1 + x) + \ln(1 - x)}{(1 - x^2)^{1/2}}\, dx = - \pi \ln 2 \tag{41}$$

Thus,

$$J_n = i\omega \left\{ \frac{\varepsilon}{u_o \ell} \frac{Z_n'(u_o b)}{Z_n(u_o b)} - \frac{\varepsilon_o}{v_o \ell} \frac{K_n'(v_o b)}{K_n(v_o b)} + \varepsilon\Delta_n - \varepsilon_o \Omega_n \right. \\ \left. + \frac{1}{\pi} \left[\varepsilon\left(1 - \frac{n^2}{k^2 b^2}\right) + \varepsilon_o \left(1 - \frac{n^2}{k_o^2 b^2}\right) \right] \ln \frac{2\ell}{\pi\delta} \right\} \tag{42}$$

Using this formula for J_n, the next step is to solve (36) by requiring that it be satisfied for all $|\phi| < \phi_o/2$. In principle, this process determines the functional form of $f(\phi)$ which is the azimuthal distribution of the vertical electric field in the slot. Various numerical procedures such as point matching could be used to perform this operation. For the present discussion, we assume that the field distribution in the slot is given by

$$f(\phi) = \cos(\pi\phi/\phi_o) \quad \text{for} \quad |\phi| \leq \phi_o/2 \tag{43}$$

which, of course, means that $f(0) = 1$ and $f(\pm\phi_o/2) = 0$. [Actually, any other smooth distribution could be assumed without changing our conclusions]. Thus

$$F_n = \int_{-\phi_o/2}^{\phi_o/2} f(\phi) e^{-in\phi} d\phi = \frac{2\pi}{\phi_o} \frac{\cos(n\phi_o/2)}{(\pi/\phi_o)^2 - n^2} \tag{44}$$

and, in particular, $F_o = 2\phi_o/\pi$. To obtain an explicit, albeit approximate, expression for the slot voltage V_o, we now equate the average values of the two sides of (36) over the range $-\phi_o/2 < \phi < \phi_o/2$. Thus, we easily find that

$$V_o \simeq \left[2\pi \sum_p H_p \frac{\sin(p\phi_o/2)}{p\phi_o/2}\right] \Big/ \left[\sum_n F_n J_n \frac{\sin(n\phi_o/2)}{n\phi_o/2}\right] \tag{45}$$

where J_n is given by (42) and F_n is given by (44). In the usual situation where the cable diameter is electrically small (i.e. $k_o b << 1$), we only need retain the $p = 0$ term in the above summation. In the discussion below, this is assumed to be the case.

EFFECTIVE AXIAL ADMITTANCE

To interpret the present results, we define an effective admittance Y_{eff} as follows

$$Y_{eff} = \overline{H}_{o\phi}/\overline{E}_{oz} \tag{46}$$

where $\overline{E}_{oz}$ and $\overline{H}_{o\phi}$ are the average fields evaluated at the outer surface $\rho = b$ of the cable shield. For example

$$\overline{E}_{oz} = \frac{1}{2\pi\ell} \int_{-\ell/2}^{\ell/2} dz \int_{-\pi}^{\pi} d\phi \, E_{oz}(b,\phi,z) e^{i\beta_o z} \tag{47}$$

which reduces to

$$\overline{E}_{oz} = V_o \phi_o / \pi^2 \ell \tag{48}$$

Also, it easily follows that

$$\overline{H}_{o\phi} = \frac{i\varepsilon_o \omega V_o K_o'(v_o b) F_o}{2\pi v_o \ell K_o(v_o b)} + H_o \tag{49}$$

Here H_o is the average value of the azimuthal component of the "applied" magnetic field at $z = 0$; it corresponds to the value computed for a cable with no slots in the shield.

We now complete the calculation by noting that

$$Y_{eff} = -Y_e + (H_o/V_o)\pi^2\ell/\phi_o \tag{50}$$

where

$$Y_e = -\frac{i\varepsilon_o\omega}{v_o}\frac{K_o'(v_o b)}{K_o(v_o b)} = \frac{i\varepsilon_o\omega}{v_o}\frac{K_1(v_o b)}{K_o(v_o b)} \simeq \frac{i\varepsilon_o\omega}{v_o^2 b}/K_o(v_o b) \tag{51}$$

is the usual external wave admittance for cylindrical waves of order zero. Then it follows that

$$Y_{eff} = Y_i + Y_L \tag{52}$$

where

$$Y_i = \frac{i\varepsilon\omega}{u_o}\frac{Z_o'(u_o b)}{Z_o(u_o b)} \simeq \frac{i\varepsilon\omega}{u_o^2 b\,\ell n(b/a)} \tag{53}$$

is the corresponding internal wave admittance and

$$Y_L = -(Y_e + Y_i) + \sum f_n \frac{\sin(n\phi_o/2)}{(n\phi_o/2)}(J_n\ell) \tag{54}$$

where

$$f_n = F_n/F_o = \frac{\pi^2}{\phi_o^2}\left(\frac{\pi^2}{\phi_o^2} - n^2\right)^{-1}\cos\frac{n\phi_o}{2} \tag{55}$$

Here $(2\pi bY_L)^{-1}$ can be interpreted as the effective transfer impedance of the sheath per unit length as usually defined [4]. Using (54), with some rearrangement of the terms, we can write

$$Y_L = \frac{1}{i\omega L} + i\omega C + \Delta Y \tag{56}$$

where

$$\frac{1}{L} = \frac{\ell}{\pi b^2}\left(\frac{1}{\mu} + \frac{1}{\mu_o}\right)\ell n\frac{2\ell}{\pi\delta} \Bigg/ \sum_n f_n \frac{\sin(n\phi_o/2)}{(n\phi_o/2)} n^2 \tag{57}$$

$$C = \frac{\ell}{\pi}(\varepsilon + \varepsilon_o)\ \ell n \frac{2\ell}{\pi\delta} \sum_n f_n \frac{\sin(n\phi_o/2)}{(n\phi_o/2)} \tag{58}$$

and

$$\Delta Y = \sum_n f_n \frac{\sin(n\phi_o/2)}{(n\phi_o/2)} \left[Y_{i,n} + Y_{e,n} + i\omega\ell(\varepsilon\Delta_n - \varepsilon_o\Omega_n)\right] - (Y_e + Y_i) \tag{59}$$

where

$$Y_{i,n} = \frac{i\varepsilon\omega}{u_o} \frac{Z_n'(u_o b)}{Z_n(u_o b)} \tag{60}$$

and

$$Y_{e,n} = -\frac{i\varepsilon_o\omega}{v_o} \frac{K_n'(v_o b)}{K_n(v_o b)} \tag{61}$$

Here we note that $Y_{i,o} = Y_i$ and $Y_{e,o} = Y_e$.

The summations indicated in (57) and (58) are effected by using [7]

$$\sum_{n=1}^{\infty} \frac{n \sin ny}{(n^2 - a^2)} = \frac{\pi}{2} \frac{\sin a(\pi - y)}{\sin a\pi} \tag{62}$$

that is valid for $0 \leq y \leq \pi$ where a is not an integer, and the derived form

$$\sum_{n=1}^{\infty} \frac{\sin ny}{n(n^2 - a^2)} = \frac{1}{a^2}\left[\frac{\pi}{2} \frac{\sin a(\pi - y)}{\sin a\pi} - \frac{\pi - y}{2}\right] \tag{63}$$

valid under the same conditions. Thus

$$\frac{1}{L} = \frac{\ell}{b^2}\left(\frac{1}{\mu} + \frac{1}{\mu_o}\right)\frac{\pi^2}{\phi_o^3}\ \ell n \frac{2\ell}{\pi\delta} \tag{64}$$

and

$$C = \frac{2\ell}{\phi_o}(\varepsilon + \varepsilon_o)\ \ell n \frac{2\ell}{\pi\delta} \tag{65}$$

Evidently, if $(k_c b)^2 << 1$ and $(kb)^2 << 1$ the contribution from $i\omega C$ is negligible compared with $(i\omega L)^{-1}$. Of course, there may be a non-neglible contribution from the ΔY. In general, it depends on the axial wave number β_o.

FINAL REMARKS AND CONCLUSIONS

An interesting special case of the preceding development is to let $\phi_o = \pi$ corresponding to a slot that extends one-half way around the circumference of the cable shield. In this case, we note that

$$f_n \frac{\sin n\phi_o/2}{n\phi_o/2} = \frac{1}{2} \quad \text{for} \quad n = 1 \tag{66}$$

$$= 0 \quad \text{for} \quad n = 2,3,4,\ldots$$

Then (59) reduces to

$$\Delta Y \simeq Y_{i,1} + Y_{e,1} + i\omega\ell[\varepsilon(\Delta_o + \Delta_1) - \varepsilon_o(\Omega_o + \Omega_1)] \tag{67}$$

If we now consider $|u_o b|$ as a small parameter and, at the same time, we neglect the contribution from the latter term in (67), we see that

$$\Delta Y \simeq \frac{i\varepsilon\omega}{u_o^2 b}\left[\frac{b+a}{b-a}\right] + i\varepsilon_o\omega b \tag{68}$$

where, for $\mu = \mu_o$,

$$\frac{1}{i\omega L} \simeq -\frac{2i\varepsilon_o\omega}{\pi k_o^2 b}\left(\frac{\ell}{b}\right)\ell n\,\frac{2\ell}{\pi\delta} \tag{69}$$

Thus it appears that ΔY is small compared with $(i\omega L)^{-1}$ when

$$\frac{b+a}{b-a} \quad \frac{k^2}{k^2 - \beta_o^2} \quad \frac{(b/\ell)}{\ell n(2\ell/\pi\delta)} << 1 \,. \tag{70}$$

This obviously appears to be *not* the case when the axial propagation constant $i\beta_o$ is comparable with the propagation constant ik of the dielectric insulator in the cable. This condition is less stringent, however, in the case where ℓ/b is a large parameter (i.e. slot spacing is much greater than cable radius).

In general, it appears that the effective transfer impedance Y_L of the cable will indeed depend on the axial propagation constant. This is particularly the case when we are dealing with modes that are similar to the conventional TEM-like mode in the cable. In spite of this fact, the transfer impedance concept is useful when we wish to characterize the cable in relation to its environment. In principle, there is no reason why the β_o dependence of the effective transfer impedance could not be incorporated in such analyses.

ACKNOWLEDGEMENTS

I wish to thank Dr. K.F. Casey of Kansas State University for much useful insight that he has provided on these problems. I would also like to thank Dr. David A. Hill for his useful comments.

REFERENCES

[1] R.M. Whitmer, "Cable shielding performance and C.W. response", *IEEE Trans. Electromagn. Compat.*, Vol. EMC-15, No. 4, pp. 180-187, Nov. 1973.

[2] E.F. Vance, "Shielding effectiveness of braided wire shields", Interaction Notes, Note 172, April 1974.

[3] K.S.H. Lee and C.E. Baum, "Application of modal analysis to braided wire shields", Interaction Notes, Note 132, January 1973.

[4] J.R. Wait and D.A. Hill, "Propagation along a braided coaxial cable in a circular tunnel", *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-23, No. 5, pp. 401-405, May 1975.

[5] J.R. Wait, *Electromagnetic Radiation from Cylindrical Structures*, Oxford: Pergamon, 1959.

[6] I.S. Gradshteyn and I.M. Ryzhik, *Tables of Integrals, Series and Products*, Academic Press, New York, 1965, pg. 557.

[7] I.S. Gradshteyn and I.M. Ryzhik, *Tables of Integrals, Series and Products*, Academic Press, New York, 1965, pg. 40 (No. 5 with m = 0).

ADDITIONAL REFERENCES

D.C. Chang, "Equivalent-circuit representation and characteristics of a radiating cylinder driven through a radiating slot", *IEEE Trans. Antennas Propagat.*, Vol. AP-21, No. 6, pp. 792-796, Nov. 1973.

J.R. Wait and D.A. Hill, "Electromagnetic fields of a dielectric coated coaxial cable with an interrupted shield—Quasi-static approach", *IEEE Trans. Antennas Propagat.*, Vol. AP-23, No. 4, pp. 679-682, Sept. 1975.