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A SPECTRAL DOMAIN APPROACH FOR CONSTRUCTING ASYMPTOTIC SOLUTIONS TO HIGH FREQUENCY SCATTERING PROBLEMS

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ABSTRACT

In this report we present a new interpretation of Keller's diffraction coefficient in terms of a plane wave spectrum of the surface current distribution on the scatterer. We show that the scattered fields, expressed in terms of spectral diffraction coefficients, are well behaved in the entire range of observation angles, including the shadow boundaries and caustic regions, where the use of Keller's coefficients give rise to infinite fields. The application of the Spectral Domain approach is illustrated by considering a number of geometries, including the half-plane with planar and non-planar illuminations, apertures, and semi-infinite cylindrical structures. A brief comparsion with Ufimtsev's approach is also included.

I. Introduction

In this paper we introduce the concepts of Spectral Theory of Diffraction (STD), an approach for solving high frequency diffraction problems. The solution is constructed in the spectral, or equivalently the Fourier transform domain using a spectral diffraction coefficient which resembles the Keller's GTD coefficient. However, both the interpretation and the use of the STD coefficient is significantly different from that of the Keller's coefficient. Whereas the diffracted field computed using the Keller's coefficient diverges to infinity at the shadow boundaries and caustic directions, the result derived from STD has the correct behavior for all of these observation angles.

Recently, two uniform theories [1]-[2] have been developed for circumventing the difficulties associated with Keller's theory in the neighborhood of shadow boundaries. Each one of these theories is based on its own ansatz and they do yield different form for the expressions for the field. Since the present approach is based on an exact representation for the field it provides a convenient means for verifying and testing the different theories for some special cases. In addition, it may be conveniently applied to non-planar illuminating waves, shadow boundary-shadow boundary interaction, multiple edge diffraction, etc., where one or both of uniform theories may require significant modification. An added feature of STD is that it can even be applied at caustic directions, even when there is a confluence of shadow boundary and the caustic direction. In contrast the uniform theories mentioned above break down at the caustics.

The paper begins with an introduction of the basic concept of STD. This is done by constructing the half-plane solution for plane wave illumination in the spectral domain. The scattered field is represented in terms of a superposition integral of a continuous spectrum of plane waves with the spectral diffraction coefficient playing the role of the wighting function of the spectral plane waves. It is shown that even when this diffraction coefficient goes to infinity, the scattered field remains bounded, as of course it should. The next section treats the non-planar illumination of a half-plane and compares the results with other uniform theories. The last two sections demonstrate the usefulness of the STD concept by considering two geometries with either shadow boundary or caustic difficulties when treated with conventional GTD. These include an aperture in a plane illuminated by a normally incident plane wave and a semi-infinite cylinder with axial incidence. The paper finally concludes with a brief summary and suggestions for future work.

II. DIFFRACTION OF A PLANE WAVE BY A HALF-PLANE---A NEW INSIGHT

The problem of diffraction by a half-plane has been analyzed extensively in the existing literature on the theory of electromagnetism. Since Sommerfeld's well-known solution in 1896, many other workers have devoted their efforts toward solving this problem using a multitude of techniques. The interested reader may refer to the standard texts of Noble [3], Born and Wolf [4], Mittra and Lee [5], and others where comprehensive reviews of these techniques may be found. The principal reason why the half-plane solution plays such an important role in diffraction theory is that it forms an integral part of the solution of a large class of high frequency diffraction problems dealing with more complex bodies. This line of thought, in which the canonical solution of the half-plane problem is used to construct the solution of other complicated geometries such as apertures, was originally developed by MacDonald in 1905, Braunbek in 1950, Millar [6], Ufimtsev [7], and others, and later integrated with the ray optical viewpoint by Keller [8-9], Deschamps [10], and others. A good review is given in [11] where wedge diffraction is also studied.

In this section we will re-examine this classical problem of diffraction by a half-plane from a new angle in which the solution to the problem is constructed in the spectral domain after the concept of the spectral diffraction coefficient is introduced. Only a brief discussion of the solution is presented here, mainly with the objective of laying the foundation for more complex problems to be dealt with in the following sections.

A. Construction of the Solution

The geometry of an ideally conducting half plane illuminated by a plane wave is shown in Fig. 1. The Cartesian coordinates x, y, z and the cylindrical coordinates ρ, ϕ, z of the observation point are also indicated in the figure.

We let the direction of propagation of the incident plane wave be normal to the edge, i.e., $\vec{k}^i \cdot \hat{z} = 0$. This assumption changes the vector nature of the three-dimensional problem to a two-dimensional scalar diffraction problem (see for instance, the book by Born and Wolf [4]). Furthermore, the problem may be classified as E-wave or H-wave types by simply letting the incident E-field or H-field be directed alternatively, along the edge of the half plane. In this work, only the E-wave-type incident field will be considered; however, the H-wave-type solution can be constructed similarly. In scalar diffraction problems, the corresponding boundary condition, which is similar to the E-wave diffracting case, is called the soft body boundary condition.

Let the incident plane wave with unit amplitude take the following form at point $\vec{\rho}$

$$u^i = \exp\,(i\vec{k}^i \cdot \vec{\rho}) = e^{-ik\rho\cos\,(\phi - \phi^i)} = e^{-ik\,(x\cos\phi^i + y\sin\phi^i)}, \tag{1}$$

where ϕ^i and $\vec{k}^i$ denote the incident angle and wave vector, respectively. The problem at hand is to determine the total field diffracted by the half-plane due to the incident field (1). Let us resolve the total field u^t as

$$u^t = u^s + u^i, \tag{2}$$

where u^s designates the scattered field due to the induced discontinuity (or induced current) in the half plane. The scattered field u^s satisfies the reduced wave equation

$$(\partial_x^2 + \partial_y^2 + k^2)\, u^s = 0\ , \quad \text{for } y \neq 0, \tag{3}$$

and the soft body boundary condition

$$u^s = -u^i\ , \quad \text{for } x \leq 0. \tag{4}$$

In addition, for a unique solution it will be necessary to impose two other conditions, viz., the radiation condition and the edge condition. The boundary value problem described in (3) and (4) can be solved exactly by many different techniques. Here, we use the transform technique and solve a Wiener-Hopf type equation. The discussion is not new and may be found in standard texts, e.g., Noble [3], Jones [12], or Mittra and Lee [5]. The main purpose of this presentation is to introduce our notations and develop the fundamentals for subsequent sections.

Let us define the Fourier transform pair as follows:

$$U(\alpha) = \int_{-\infty}^{\infty} u(x)\, e^{i\alpha x}\, dx = \mathcal{F}[u(x)] \tag{5}$$

and

$$u(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U(\alpha)\, e^{-i\alpha x}\, d\alpha = \mathcal{F}^{-1}[U(\alpha)] \quad . \tag{6}$$

Upon transforming (3) and imposing the radiation condition, the general solution of this equation may be written as:

$$U^s(\alpha,y) = \begin{cases} A(\alpha)\, e^{-\gamma y} & y > 0 \\ B(\alpha)\, e^{\gamma y} & y < 0 \end{cases} \tag{7}$$

where $\gamma = \sqrt{\alpha^2 - k^2}$, with the requirement that Re $\gamma > 0$ and Im $\gamma < 0$ (this is discussed in [5]). Since U^s is a continuous function, it is easily found that

$$A(\alpha) = B(\alpha) \quad . \tag{8}$$

We use notations U_- and U_+ to denote a regular function in the lower- and upper-half α-plane, respectively. Using (1) and imposing the boundary condition (4) in the transform domain, one derives

$$U_-^s(\alpha,0) = \frac{-1}{i(\alpha - k\cos\phi^i)} \quad . \tag{9}$$

The crucial step in solving the Wiener-Hopf equation for the problem at hand is to realize that

$$\partial_y U^s_-(\alpha,0^+) - \partial_y U^s_-(\alpha,0-) = -2\gamma A(\alpha) = -X(\alpha) \tag{10}$$

where $X(\alpha)$ is a regular function in the lower-half α-plane. It is further noticed that

$$U^s_-(\alpha,0) + U^s_+(\alpha,0) = A(\alpha) = \frac{1}{2\gamma} X(\alpha) \quad . \tag{11}$$

$X(\alpha)$ may clearly be interpreted as the transform of the induced discontinuity (induced current) in the half-plane, i.e.,

$$X(\alpha) = \int_{-\infty}^{0} - \partial_y u^t \Big|_{0^+}^{0^-} e^{i\alpha x} dx \quad . \tag{12}$$

Upon application of the standard factorization and decomposition procedures one can determine $X(\alpha)$ from (9), (10) and (11) to obtain the following equation:

$$X(\alpha) = \frac{2i(k + k\cos\phi^i)^{1/2}\sqrt{\alpha - k}}{\alpha - k\cos\phi^i} \quad . \tag{13}$$

Substituting (13) into (11) and using (7) and (6), one can finally construct the scattered field

$$u^s(x,y) = \frac{1}{2\pi} \int_{-\infty+i\tau}^{\infty+i\tau} X(\alpha) \frac{e^{-i\alpha x-\gamma|y|}}{2\gamma} d\alpha \quad , \tag{14}$$

where τ is a small real number. We may notice that

$$\frac{e^{-\gamma|y|}}{2\gamma} = F\left[\frac{i}{4} H_0^{(1)}(k\rho)\right] \quad , \tag{15}$$

in which $H_0^{(1)}$ is the zero-order Hankel function of the first kind and $\frac{i}{4} H_0^{(1)}(k\rho)$ is the Green's function of the two-dimensional Helmholtz operator.

Introducing the change of variables $x = \rho\cos\phi$, $y = \rho\sin\phi$, $\alpha = -k\cos\omega$ and $\gamma = -ik\sin\omega$ into (14), one obtains

$$\underset{\sim}{u}^s(\rho,\phi) = u^s(\rho\cos\phi, \rho\sin\phi) = \frac{i}{4\pi} \int_{i\infty}^{\pi-i\infty} \underset{\sim}{X}(\phi^i,\omega)\, e^{ik\rho\cos(\omega-|\phi|)} d\omega \tag{16}$$

where the path of integration is the one used in [5], and

$$\underset{\sim}{X}(\phi^i,\omega) = X(-k\cos\omega) = \frac{-4\cos\frac{\phi^i}{2}\cos\frac{\omega}{2}}{\cos\phi^i + \cos\omega} \quad . \tag{17}$$

Upon introducing

$$\underset{\sim}{X}^i(\phi^i,\omega) = -\text{Sec}\,\frac{\omega - \phi^i}{2} \tag{18a}$$

and

$$\underset{\sim}{X}^r(\phi^i,\omega) = \text{Sec}\,\frac{\omega + \phi^i}{2} \tag{18b}$$

we arrive at

$$\underset{\sim}{X}(\phi^i,\omega) = \underset{\sim}{X}^i(\phi^i,\omega) - \underset{\sim}{X}^r(\phi^i,\omega) \tag{19}$$

where superscripts i and r are used to denote the incident and reflected diffraction coefficients, respectively. We may notice that $\underset{\sim}{X}^i(\cdot)$ and $\underset{\sim}{X}^r(\cdot)$ have the same functional form, i.e., Sec $(\cdot)$. This definition of $\underset{\sim}{X}$ is closely related to the definition used by Deschamps in [10].

Clearly, $\underset{\sim}{X}^i$ and $\underset{\sim}{X}^r$ are infinite at $\omega = -(\pi - \phi^i)$ and $\omega = \pi - \phi^i$, respectively. These two values of ω correspond to the incident and reflected shadow boundaries appearing in the GTD technique. As a matter of fact, $\underset{\sim}{X}(\phi^i,\omega)$ is precisely Keller's diffraction coefficient, when ω is replaced by the observation angle ϕ. Although $\underset{\sim}{X}(\phi^i,\omega)$ tends to infinity at the shadow boundaries, it does not mean that the field itself is also infinite as Keller's GTD theory predicts. Instead, the correct value of the field is obtained from (16), which is always bounded. To distinguish it from Keller's coefficient, which is associated with the diffracted field, we will refer to $\underset{\sim}{X}(\phi^i,\omega)$ as the "Spectral Diffraction Coefficient" for half planes. The terminology is chosen since $\underset{\sim}{X}(\phi^i,\omega)$ is associated with the spectrum, or equivalently the Fourier transform, of the induced current and appears only inside the kernel of the plane wave spectrum representation for the field and not directly in the form a factor multiplying the incident field as in the case of Keller's representation.

We may further use the defintion (12) and introduce the spectral diffraction coefficient of the physical optics field X_{PO} as the Fourier transform of the physical optics induced discontinuity in the half plane, i.e.,

$$X_{PO}(\alpha) = \int_{-\infty}^{0} -2\, \partial_y \, u^i \Big|_{y=0} e^{i\alpha x}\, dx, \tag{20}$$

where $-2\, \partial_y\, u^i|_{y=0}$ is the induced current of the physical optics contribution in the half plane. Since $\partial_y\, u^i\Big|_{y=0} = -ik \sin\phi^i\, e^{-ikx\cos\phi^i}$, one obtains

$$X_{PO}(\alpha) = \frac{2k\sin\phi^i}{\alpha - k\cos\phi^i} \quad . \tag{21}$$

Replacing α by $\alpha = -k\cos\omega$ and simplifying the above result, we finally arrive at

$$\underset{\sim}{X}_{PO}(\phi^i,\omega) = \underset{\sim}{X}^i_{PO}(\phi^i,\omega) - \underset{\sim}{X}^r_{PO}(\phi^i,\omega) = -\tan\frac{\omega-\phi^i}{2} + \tan\frac{\omega+\phi^i}{2} \quad . \tag{22}$$

It is worthwhile to mention that X_U, as defined in the following equation, is bounded at the shadow boundaries

$$\underset{\sim}{X}_U(\phi^i,\omega) = \underset{\sim}{X}(\phi^i,\omega) - \underset{\sim}{X}_{PO}(\phi^i,\omega) \quad . \tag{23}$$

$\underset{\sim}{X}_U(\phi^i,\phi)$ could be called the Ufimstev diffraction coefficient [13] and will be used in Section IV.

After substituting (19) into (15), deforming the path to the steepest descent path P_s and taking into account the pole contributions, we arrive at the following result:

$$\underset{\sim}{u}^s(\rho,\phi) = \frac{i}{4\pi}\int_{P_s} \underset{\sim}{X}(\phi^i,\omega)\, e^{ik\rho\cos(\omega-|\phi|}\, d\omega + \begin{cases} -e^{-ik\rho\cos(\phi^i+|\phi|)} & \pi-\phi^i < |\phi| < \pi \\ 0 & 0<|\phi|<\pi-\phi \end{cases} \tag{24}$$

where the upper term is valid for the region $\pi - \phi^i < |\phi| < \pi$, and the lower term is valid for the region $0 < |\phi| < \pi - \phi^i$.

Having determined the scattered field u^s, we now can find the total field u^t from (2) which may be written in the following alternative form:

$$u^t = u^g + u^d \tag{25}$$

where u^g is the geometrical optic term defined as:

$$u^g = \theta(\varepsilon^i)\, u^i + \theta(\varepsilon^r)\, u^r \quad . \tag{26}$$

In the above equation, θ is the unit step function, ε^i and ε^r are illumination indicators for the incident and reflected fields

$$\varepsilon^i = \begin{cases} +, & \text{for } -(\pi-\phi^i) < \phi < \pi \\ -, & \text{otherwise,} \end{cases} \tag{27}$$

and

$$\varepsilon^r = \begin{cases} +, & \text{for } \pi-\phi^i < \phi < \pi \\ - & \text{otherwise,} \end{cases} \tag{28}$$

and

$$\underset{\sim}{u}^r = u^r(\rho\cos\phi, \rho\sin\phi) = -e^{-ik\rho\cos(\phi+\phi^i)} \tag{29}$$

i.e., the reflected field. In defining ε^i and ε^r, it is assumed that $0 < \phi^i < \pi$, although generalization for all angles of ϕ^i is quite straightforward. Moreover, u^d in (25) is the edge diffracted field expressed as:

$$u^d(\rho\cos\phi, \rho\sin\phi) = \underset{\sim}{u}^d(\rho,\phi) = \frac{i}{4\pi}\int_{P_s} \underset{\sim}{X}(\phi^i,\omega)\, e^{ik\rho\cos(\omega-|\phi|)}\, d\omega \quad . \tag{30}$$

The integration in (30) may be evaluated asymptotically using the standard saddle-point integration technique [14]. For large $k\rho$ and observation angles away from the shadow boundaries, u^d may be computed to give

$$\underset{\sim}{u}^d(\rho,\phi) = X(\phi^i,\phi)\, g(k\rho) + O[(k\rho)^{-3/2}] \quad k\rho \to \infty \quad ,$$

where

$$g(k\rho) = \frac{e^{i(k\rho+\pi/4)}}{2\sqrt{2\pi k\rho}} \quad , \tag{32}$$

i.e., the first term of the asymptotic expansion of $\frac{i}{4} H_0^{(1)}(k\rho)$.

Substituting u^g and u^d from (26) and (31), respectively, into (25), the asymptotic form of the total field may finally be written as:

$$u^t = \theta(\varepsilon^i)\, u^i + \theta(\varepsilon^r)\, u^r + \underset{\sim}{X}(\phi^i,\phi)\, g(k\rho) + O\,[(k\rho)^{-3/2}] \quad . \tag{33}$$

The above result is precisely Keller's GTD solution for the half-plane diffraction by an incident plane wave. It may be noted that according to (17) both the observation and incident angles appear in the argument of $\underset{\sim}{X}(\phi^i,\phi)$ in a symmetric manner.

Clearly, (33) is not valid for the observation angles correspond to the shadow boundaries, where (30) must be evaluated more carefully. For instance, exactly at the shadow boundaries the saddle point of (30) and the poles of $\underset{\sim}{X}(\phi^i,\omega)$ coincide and special care must be exercised to correctly evaluate the integral asymptotically. For the problem at hand, i.e., plane wave incident, integration (30) can be performed exactly in terms of the Fresnel integral by employing the following result [4]:

$$\frac{i}{4\pi}\int_{P_s} \sec\frac{1}{2}(\omega-\phi^i)\, e^{ik\rho\cos(\omega-\phi)}\, d\omega = \pm\, e^{-ik\rho\cos(\phi-\phi^i)} F\left(\pm\sqrt{2k\rho}\,\cos\frac{\phi-\phi^i}{2}\right), \tag{34}$$

with the upper sign for $\cos\frac{\phi-\phi^i}{2} > 0$ and the lower sign for $\cos\frac{\phi-\phi^i}{2} < 0$. In (34), F is the Fresnel integral with the following definition:

$$F(\tau) = \frac{e^{-i\pi/4}}{\sqrt{\pi}} \int_{\tau}^{\infty} e^{it^2}\, dt \quad . \tag{35}$$

From (35) it can easily be shown that

$$F(\tau) + F(-\tau) = 1 \quad . \tag{36}$$

Using (34), one can then evaluate (30) in terms of the Fresnel integral, and finally construct u^t with the help of (36) resulting in the following equation:

$$u^t = F(\xi^i)\, u^i + F(\xi^r)\, u^r \tag{37}$$

where u^i and u^r are evaluated at the observation point and

$$\xi^i = -\sqrt{2k\rho}\, \cos \frac{\phi - \phi^i}{2} \tag{38a}$$

$$\xi^r = \sqrt{2k\rho}\, \cos \frac{\phi + \phi^i}{2} \quad . \tag{38b}$$

Equation (37) is valid for $-\pi < \phi^i < \pi$ and $-\pi < \phi < \pi$. This equation demonstrates the classical solution of the half-plane diffraction by an incident plane wave originally obtained by Sommerfeld. The asymptotic expansion of u^t as derived in (33) may be constructed directly from (37) by simply using the first two terms of the asymptotic series expansion of Fresnel integral shown here as:

$$F(\tau) \sim \theta(-\tau) + e^{i\tau^2} \frac{e^{i\pi/4}}{2\pi\tau} \sum_{n=0}^{\infty} \Gamma(n + \tfrac{1}{2})\, (i\tau^2)^{-n} \qquad |\tau| >> 0,$$

where Γ is the Gamma function and θ is the unit step function.

In reviewing the material presented in this section, we note that its principal contribution has been the introduction of the spectral diffraction coefficient, which - in turn - is shown to be associated with the integral representation of the scattered and total fields. In contrast to Keller's coefficients, the infinities in the spectral coefficients at shadow and reflection boundaries do not lead to infinite fields. The equivalence between the GTD results and those derived from the spectral representation for observation angles not close to the shadow boundaries has been established. In the next few sections, we will illustrate the broad nature of the spectral concept and its versatility of application by considering more general incident waves and complex structures than the half-plane illuminated by a plane wave.

III. DIFFRACTION OF AN ARBITRARY FIELD BY A HALF-PLANE

The problem of diffraction of an arbitrary incident field by a half-plane is of great interest in applied electromagnetism. As an example, one is often interested in solving the problem of radiation or scattering from an antenna mounted on a conductive body with sharp edges; or for an antenna with a given pattern function, which is mounted close to the earth and is radiating in the presence of a protrusion or a hill that may be modeled by a knife edge. It should be mentioned that the principal modification for the case of an arbitrary incident field shows up essentially in the neighborhood of the incident and reflected shadow boundaries, where Keller's representation is not valid because of infinities in Keller's coefficient. In various uniform theories these infinities are eliminated either by additive or multiplicative factors that cancel these infinities at the appropriate angles and yield finite results. However, since each uniform theory is based on its own Ansatz, the final results are not necessarily the same. Thus, the solution to this problem is of considerable theoretical importance, since it also provides, in some special cases, a reliable means for comparing and testing the validity of various Ansatz that form the basis of available uniform asymptotic techniques.

A search through the literature reveals that there has not been a detailed analysis of the half-plane diffraction, due to an arbitrary incident field, until quite recently. The half-plane diffraction problem of the radiated field by an isotropic line source has been discussed by Clemmow [15] and Born and Wolf [4], and others. A more general case has been analyzed by Khestanov [16], but this work does not provide any specific results of the behavior of the diffracted field, but only an integral representation, which is not evaluated explicitly. Recently, a few terms of the asymptotic representation of the diffracted field of an anisotropic line source have been given by Lee and Deschamps [17], using the uniform asymptotic theory of Ahluwalia, Lewis, and Boersma [1].

In this section, we employ the results of Section II and develop an expression for the diffracted field by a half-plane due to an arbitrary incident field (with no caustics). Furthermore, the construction of the asymptotic expression of the diffracted field at the shadow boundaries will be presented in detail for an anisotropic line source. Results will be compared with other available data.

A. Construction of the Solution

In this section, we again assume that there is no z-dependence; hence, a two-dimensional diffraction problem will be considered. Any field radiated from a given source distribution and in a domain outside the source region satisfies the two-dimensional Helmholtz equation. It is well known that this radiated field can always be expressed in terms of the spectrum of plane waves in the following fashion:

$$u^i(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U^i(\beta)\ e^{-i\beta x - \sqrt{\beta^2 - k^2}\, y}\, d\beta, \qquad y \leq 0 \tag{40}$$

where y is measured along the normal to the surface on which $u^i(x,0)$ is measured. For instance, that surface may be chosen as the dashed surface shown in Fig. 2. It is also noticed that $U^i(\beta) = \mathcal{F}[u^i(x,0)]$. Introducing the change of variables $x = \rho \cos \phi$, $y = \rho \sin \phi$ and $\beta = -k \cos \eta$ into (40), $u^i(x,y)$ may then be

$$\underset{\sim}{u}^i(\rho,\phi) = u^i(\rho \cos \phi, \rho \sin \phi) = \frac{1}{2\pi} \int_{i\infty}^{\pi - i\infty} k \sin \eta\ \underset{\sim}{U}^i(\eta)\ e^{ik\rho \cos(\eta-\phi)}\, d\eta, \tag{41}$$

where $\underset{\sim}{U}^i(\eta) = U^i(-k \cos \eta)$. The integrand of (41) represents a plane wave propagating in η direction with an amplitude of $\frac{1}{2\pi} k \sin \eta\ \underset{\sim}{U}^i(\eta)$.

Our task is to find the total diffracted field by a half-plane due to the incident field $u^i(x,y)$. To do this, we first determine the scattered field of a (spectral) plane wave from (16), weight it with the spectral amplitude coefficient of the same wave, i.e., $\frac{1}{2\pi} k \sin \eta\ \underset{\sim}{U}^i(\eta)$, and finally integrate it over the entire spectrum. This procedure leads to the following representation of the scattered field:

$$u^t(\rho,\phi) = \int_{i\infty}^{\pi-i\infty} \left\{ e^{-ik\rho\,\cos(\phi-\eta)}\,F(\xi^i) - e^{-ik\rho\,\cos(\phi-\eta)}\,F(\xi^r) \right\} \frac{k}{2\pi}\,\sin\eta\,U^i(\eta)\,d\eta, \tag{44}$$

where

$$\xi^i = -\sqrt{2k\rho}\,\cos\frac{\phi-\eta}{2} \tag{45a}$$

$$\xi^r = \sqrt{2k\rho}\,\cos\frac{\phi+\eta}{2} \tag{45b}$$

Equation (44) is the complete representation of the solution of the total field diffracted by a half-plane illuminated by an arbitrary incident field. It is further noticed that (44) represents a superposition integral. This integral may be evaluated asymptotically by first sorting out the dominant exponential parts of the integrand and then expanding the rest of the integrand in terms of the Taylor series. This procedure will be worked out in detail in the next section where more specific examples are treated.

Since the Taylor expansion of the Fresnel integral will be used for asymptotic evaluation of (44), it is given by the following formula:

$$F(\tau) = \frac{e^{i\tau^2}}{2} \sum_{n=0}^{\infty} \frac{e^{-i\frac{n\pi}{4}}}{\Gamma(\frac{n}{2}+1)} (-\tau)^n \qquad \tau \to 0 \quad . \tag{46}$$

B. Anisotropic <u>Line Source</u>

In this section, we will examine the usefulness of the formulas developed in the previous section. Consider a line source with a non-uniform pattern. This line source may be thought of as the radiated field from a source distribution located far away from the half-plane. The geometry of the half-plane with a nonisotropic line source is shown in Fig. 2. We notice that s_1 and s_2 are the coordinates of the line source (a phase center for a given source distribution far away from the edge) and s is the separation between the source and the edge of the half-plane. Index "0" is used to denote the source coordinate system erected at the source line (s_1, s_2), and angle ϕ^i is shown in the figure.

The radiated field from a given source distribution $p(x_0', y_0')$ may be expressed as:

$$u^i(x_0,y_0) = \int p(x_0',y_0')\ g_0(x_0',y_0' \mid x_0,y_0)\ dx_0'\ dy_0' \ , \tag{47}$$

where $g_0(x_0',y_0' \mid x_0,y_0) = \frac{i}{4} H_0^{(1)}(k \mid \vec{\rho}_0 - \vec{\rho}_0\,')$ is the Green's function of the two-dimensional Helmholtz operator. For large values of ρ_0, compared with the region occupied by the source, (47) may be written as:

$$\underset{\sim}{u}^i(\rho_0,\phi_0) = u^i(\rho_0 \cos\phi_0, \rho_0 \sin\phi_0) = \frac{e^{i(k\rho_0+\frac{\pi}{4})}}{2\sqrt{2\pi k\rho_0}} \int p(x_0',y_0')\ e^{-ik\rho_0 \hat{\rho}_0 \cdot \vec{\rho}_0\,'}\ dx_0'\ dy_0' \ . \tag{48}$$

If we define P as the double Fourier transform of $p(x',y')$, (48) will then take the following form of which $\underset{\sim}{P}(\phi_0)$ is commonly called the Pattern Function:

$$\underset{\sim}{u}^i(\rho_0,\phi_0) = \frac{e^{i(k\rho_0+\frac{\pi}{4})}}{2\sqrt{2\pi k\rho_0}}\ P(-k\cos\phi_0, -k\sin\phi_0) = \frac{e^{i(k\rho_0+\frac{\pi}{4})}}{2\sqrt{2\pi k\rho_0}}\ \underset{\sim}{P}(\phi_0). \tag{49}$$

Since we are interested in obtaining the transform of the incident field, i.e., $U^i(\beta,y_0) = F[u^i(x_0,y_0)]$, we may use the convolution theorem to express the Fourier transform of (47) as:

$$U^i(\beta,y_0) = \int F[p]\ \frac{e^{-\gamma \mid y_0 - y_0' \mid}}{2\gamma}\ dy_0' \ , \tag{50}$$

where $\gamma = \sqrt{\beta^2 - k^2}$, with Re $\gamma > 0$ and $I_m \gamma < 0$. For large negative values of y_0, such that $y_0 - y_0' < 0$ for all y_0', (50) may be rewritten in the following fashion:

$$U^i(\beta,y_0) = \frac{e^{\gamma y_0}}{2\gamma} \int F[p]\ e^{-i\sqrt{k^2-\beta^2}\ y_0'}\ dy_0' = \frac{e^{\gamma y_0}}{2\gamma}\ P(\beta, -\sqrt{k^2 - \beta^2}). \tag{51}$$

It is noted that in (51) the integral has a Fourier transform character, hence, $P(\beta,-\sqrt{k^2-\beta^2})$ is the double Fourier transform of $p(x_0',y_0')$, namely, the same function as introduced in (50). Evaluating the transform of the incident field at the half-plane, i.e., $y_0 = -s_2$, and shifting the origin to the edge of the half-plane, i.e., $x_0 = -s_1$, we obtain the following result for the transform of the incident field:

$$U^i(\beta) = U^i(\beta, -s_2) = \frac{e^{-\gamma s_2 - i\beta s_1}}{2\gamma} P(\beta,-\sqrt{k^2-\beta^2}) \quad . \tag{52}$$

After introducing the change of variables $\beta = -k \cos \eta$, $s_1 = s \cos \phi_s$ and $s_2 = s \sin \phi_s$, one can finally express (52) as follows:

$$\underset{\sim}{U}^i(\eta) = U^i(-k \cos \eta) = \frac{e^{iks \cos(\eta-\phi^i)}}{-2ik \sin \eta} \underset{\sim}{P}(-\pi+\eta) \quad . \tag{53}$$

It is apparent that $\underset{\sim}{P}(-\pi+\eta)$ is the same functional introduced in (49) as $\underset{\sim}{F}(\phi_0)$. One may now conclude that the transform of the incident field (49) can be constructed directly from the pattern function $\underset{\sim}{P}(\phi_0)$ by replacing ϕ_0 with $-\pi+\eta$ as in (53).

Next, we substitute (53) into (44), and obtain the following expression for the total diffracted field by the half-plane illuminated by an incident field given in (47):

$$\underset{\sim}{u}^t(\rho,\phi) = \frac{1}{2\pi}\int_{i\infty}^{\pi-i\infty} \left\{e^{-ik\rho \cos(\phi-\eta)}\, F(\xi^i) - e^{-ik\rho \cos(\phi+\eta)}\, F(\xi^r)\right\} \tag{54}$$

$$\cdot \frac{i}{2} \underset{\sim}{P}(-\pi+\eta)\, e^{iks \cos(\eta-\phi^i)} d\eta$$

The above expression for the total is uniformly valid for all observation angles ϕ as well as distance ρ.

Next, we proceed to evaluate (4) asymptotically for large values of ξ^i and ξ^r which correspond to the observation directions different from the shadow boundary directions. This is done by employing (9), which may be rewritten in the first two terms as:

$$F(\tau) = \theta(-\tau) + \hat{F}(\tau) \qquad |\tau| >> 0, \tag{55}$$

where

$$\hat{F}(\tau) = e^{i\frac{\pi}{4}} \frac{e^{i\tau^2}}{2\sqrt{\pi}\tau} + O(\tau^{-3}) \quad . \tag{56}$$

Substituting (55) into (54) and deforming the path of integration to the steepest descent path, we finally derive the following expression for the total diffracted field:

$$\underset{\sim}{u}^t(\rho,\phi) = \theta(\varepsilon^i)\, \underset{\sim}{u}^i(\rho_0,\varphi_0) + \theta(\varepsilon^r)\, \underset{\sim}{u}^r(\rho_1,\phi_1) + \underset{\sim}{X}(\phi^i,\phi)\, g(k\rho)\, \underset{\sim}{u}^i(s,-\pi+\phi^i) + O[(k\rho)^{-2}] \quad , \tag{57}$$

where the definitions of $\theta(\varepsilon^i)$, $\theta(\varepsilon^r)$, $g(k\rho)$ and $\underset{\sim}{X}(\phi^i,\phi)$ are the same as in (33), $\underset{\sim}{u}^i$ is defined in (49), and $\underset{\sim}{u}^r$ is the field radiated from the fictitious image source, as shown in Fig. 2. Equation (57) has the same form as the result obtained using the GTD. Obviously, (57) does not hold at the shadow boundaries where either $\xi^i = 0$ or $\xi^r = 0$, since (55) can not be used for small values of τ; hence, special care must be exercised to evaluate (54) for small values of ξ^i and ξ^r.

As an example, we consider the case where the observation direction coincides with the incident shadow boundary direction, i.e., $\phi = -(\pi - \phi^i)$. For this case, $\xi^i = 0$ and $\xi^r \neq 0$ at the saddle point. To evaluate (54) asymptotically, one can still replace $F(\xi^r)$ by its asymptotic expression from (55) and calculate the integral via the steepest descent technique to give:

$$\int_{i\infty}^{\pi-i\infty} -\frac{i}{4\pi} e^{ik\rho\cos(\phi^i+\eta)}\, F(\xi^r)\, \underset{\sim}{P}(-\pi+\eta)\, e^{iks\cos(\eta-\phi^i)}\, d\eta = -\underset{\sim}{X}^r(\phi^i,-\pi+\phi^i)\, g(k\rho)\, \underset{\sim}{u}^i(s,-\pi+\phi^i) \quad , \tag{58}$$

where $\underset{\sim}{X}^r$ has been defined in (18b). The evaluation of (54) for the part containing $F(\xi^i)$ is more involved than that which was used to obtain (58).

It should be noted that $\xi^{i} = -\sqrt{2k\rho}\cos\frac{\phi - \eta}{2}$ goes to zero for the observation angle $\phi = -(\pi - \phi^{i})$ and the saddle point $\eta = \phi^{i}$. Therefore, (55) can no longer be used to evaluate (54) asymptotically.

The procedure for calculating the integral

$$I = \int_{i\infty}^{\pi - i\infty} \frac{i}{4\pi} F\left[\sqrt{2k\rho}\sin\frac{\eta - \phi^{i}}{2}\right] \underset{\sim}{P}(-\pi + \eta)\, e^{ik(s+\rho)\cos(\eta - \phi^{i})}\, d\eta \qquad (59)$$

will be described in what follows. The above integral has been constructed by placing $\varphi = -(\pi - \varphi^{i})$ into (54) for the term containing $F(\xi^{i})$. At this point, it may be tempting to conclude that for large $k(s + \rho)$ only the leading term of the saddle-point type of expansion of the integral in (59) will suffice to yield the k^{-1} term in the final result. That this is incorrect, will soon be evident from the discussion below. In order to perform a complete asymptotic evaluation of (59), we first expand the integrand $F\underset{\sim}{P}$ in terms of the Taylor series around $\eta = \varphi^{i}$, and then evaluate each term of the infinite series using the steepest descent technique. This procedure is called the Complete Asymptotic Expansion in the reference [14]. Here, we only express the final result, namely,

$$I(\Omega) = \int_{SDP} f(\eta)\, e^{ik\Omega\cos(\eta - \phi^{i})}\, d\eta = e^{ik\Omega - i\frac{\pi}{4}} \sum_{n=0}^{\infty} e^{-i\frac{n\pi}{2}} \frac{f^{(2n)}}{(2n)!} \left(\frac{2}{k\Omega}\right)^{n+\frac{1}{2}} \Gamma\left(n + \frac{1}{2}\right)$$

$$\text{as } \Omega \to \infty \qquad (60)$$

where

$$f^{(2n)} = \left.\frac{\partial^{2n} f}{\partial \eta^{2n}}\right|_{\eta=\phi^{i}} \quad \text{and} \quad f(\eta) = \sum_{n=0}^{\infty} \frac{f^{(n)}}{n!} (\eta - \phi^{i})^{n} \quad .$$

The Taylor expansion of the integrand $F\underset{\sim}{P}$ may be constructed by first using the formula (46), resulting in:

$$F\left[\sqrt{2k\rho}\ \sin\frac{\eta-\phi^i}{2}\right] = \frac{e^{ik\rho}\ e^{-ik\rho\cos(\eta-\phi^i)}}{2} \sum_{n=0}^{\infty} \frac{e^{-i\frac{n\pi}{4}}}{\Gamma\left(\frac{n}{2}+1\right)} \left[-\sqrt{2k\rho}\ \sin\frac{\eta-\phi^i}{2}\right]^n$$

$$= \frac{e^{ik\rho}\ e^{-ik\rho\cos(\eta-\phi^i)}}{2} \sum_{n=0}^{\infty} \frac{e^{-in\frac{\pi}{4}}}{\Gamma\left(\frac{n}{2}+1\right)}$$

$$\cdot \left[-\sqrt{2k\rho} \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)!} \left(\frac{\eta-\phi^i}{2}\right)^{(2m+1)}\right]^n \tag{61}$$

and then expressing $\underset{\sim}{P}$ as:

$$\underset{\sim}{P}(-\pi+\eta) = \sum_{n=0}^{\infty} \frac{\underset{\sim}{P}^{(n)}(-\pi+\phi^i)}{n!} (\eta-\phi^i)^n \quad . \tag{62}$$

According to (60), only the even terms of the expansion of $F\underset{\sim}{P}$ are needed. The sum of even terms of the Taylor expansion of $F\underset{\sim}{P}$, say $(F\underset{\sim}{P})_e$, is given by

$$(F\underset{\sim}{P})_e = \frac{1}{2}\, e^{ik\rho}\ e^{-ik\rho\cos(\eta-\phi^i)}$$

$$\cdot \left\{\underset{\sim}{P}^{(0)} \sum_{n=0}^{\infty} \frac{e^{-in\frac{\pi}{2}}}{\Gamma(n+1)} (2k\rho)^n \left(\frac{\eta-\phi^i}{2}\right)^{2n} + \underset{\sim}{P}^{(1)} \sum_{n=1}^{\infty} 2\, \frac{e^{-i(2n-1)\frac{\pi}{4}}}{\Gamma\left(n+\frac{1}{2}\right)} (-1)^{2n-1}\right.$$

$$\left. \cdot\ (2k\rho)^{n-\frac{1}{2}} \left(\frac{\eta-\phi^i}{2}\right)^{2n} + \ldots \right\} \quad . \tag{63}$$

Substituting (63) into (59) and using (60), one derives the result:

$$I = \underset{\sim}{P}^{(0)} \frac{e^{ik(\rho+d)+i\frac{\pi}{4}}}{4\sqrt{2\pi ks}} \cdot \frac{1}{\sqrt{\pi}} \sum_{n=0}^{\infty} e^{-in\pi} \frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(n+1)} \left(\frac{\rho}{s}\right)^n$$

$$+ \underset{\sim}{P}^{(1)} \frac{e^{ik\rho+i\frac{\pi}{4}}}{\sqrt{2\pi k\rho}} \frac{e^{ikd+i\frac{\pi}{4}}}{2\sqrt{2\pi ks}} \sum_{n=0}^{\infty} e^{-in\pi} \left(\frac{\rho}{s}\right)^n + O[k^{-3/2}] \tag{64}$$

through the application of the following identities

$$(1 + x)^{-\frac{1}{2}} = \frac{1}{\sqrt{\pi}} \sum_{n=0}^{\infty} e^{-in\pi} \frac{\Gamma\left(n + \frac{1}{2}\right)}{\Gamma(n + 1)} x^n \tag{65}$$

and

$$(1 + x)^{-1} = \sum_{n=0}^{\infty} e^{-in\pi} x^n \quad , \tag{66}$$

one may further simplify (64) to obtain:

$$I = \frac{1}{2} \frac{e^{ik(\rho+s)+i\frac{\pi}{4}}}{2\sqrt{2\pi k(\rho + s)}} \underset{\sim}{P}(-\pi + \phi^i) + \frac{e^{ik\rho+i\frac{\pi}{4}}}{2\sqrt{2\pi k\rho}} \cdot \frac{e^{iks+i\frac{\pi}{4}}}{2\sqrt{2\pi ks}} \frac{2\rho}{\rho + s} \underset{\sim}{P}'(-\pi + \phi^i) + O(k^{-3/2}) \tag{67}$$

where $\underset{\sim}{P}'(-\pi + \phi^i) = \underset{\sim}{P}^{(1)}(-\pi + \phi^i) = \left.\frac{\partial}{\partial \phi_0} \underset{\sim}{P}(\phi_0)\right|_{\phi_0=-\pi+\phi^i}$, that is, the angular derivative of the pattern function.

Using (58) and (67), the total field $\underset{\sim}{u}^t$ may finally take the following value at the incident shadow boundary (SB^i):

$$\underset{\sim}{u}^t(\rho,-\pi + \varphi^i) = \frac{1}{2} \underset{\sim}{u}^i(\rho + s,-\pi + \phi^i) + g(k\rho) \frac{e^{iks+i\frac{\pi}{4}}}{2\sqrt{2\pi ks}} \frac{2\rho}{\rho + s} \underset{\sim}{P}'(-\pi + \phi^i)$$

$$- \underset{\sim}{X}^r(\phi^i,-\pi + \varphi^i)\, g(k\rho)\, \underset{\sim}{u}^i(s,-\pi + \phi^i) + O(k^{-3/2}) \quad . \tag{68}$$

Similarly, one can determine the total field $\underset{\sim}{u}^t$ at the reflected shadow boundary (SB^r). The important feature of (68) is the appearance of the second term in the r.h.s. of (68), which depends on the angular derivative of the pattern function. This term, obviously, vanishes for an isotropic line source where the pattern function is constant. The result shown in (68) agrees completely with that of the uniform asymptotic theory of Ahluwalia, Lewis and Boersma [1] exactly at the shadow boundaries, though it does not mean that the observed agreement also holds in the vicinity of the shadow boundaries. More importantly, (68) shows the discrepancy of the uniform theory of Kouyoumjian and Pathak [2], when applied in determining the diffracted field by an anisotropic line source before their slope diffraction modification [18] was introduced.

IV. DIFFRACTION BY AN APERTURE IN AN INFINITE SCREEN

In this section, we discuss the problem of three-dimensional scalar diffraction by a convex aperture of arbitrary shape in an infinite screen. This study has a long history in the literature pertaining to electromagnetic and acoustic diffraction phenomena. In 1891, Kirchhoff was possibly the first to present a reasonably good mathematical description of the problem, which was later improved by Braunbeck and others. A critical review of many different diffraction formulations has been presented by Bouwkamp [19], and a comparative study of Kirchhoff's formulation, Braunbeck's work and the Geometrical Theory of Diffraction (GTD) has been done by Keller and his associates [20]. GTD is the modified version of the Geometrical Optics (GO) developed by Keller [9] in 1962; at the same time, the modified version of the Physical Optics (PO), called the Physical Theory of Diffraction (PTD), was developed by Ufimtsev [18]. The GTD and PTD formulations have been the essential core of many new developments in the high frequency diffraction analyses. Many different uniform theories have recently been suggested to overcome the difficulties occurring in GTD at the shadow boundaries and caustics. As mentioned in the introduction, the Uniform Theory of Kouyoumjian and Pathak (UKP) [2] introduces a multiplicative transition function to circumvent these difficulties at the shadow boundaries, and the Uniform Asymptotic Theory (UAT) [1], which has been suggested by Ahluwalia, Lewis and Boersma, gives a finite value to GTD at the shadow boundaries. Interestingly, both uniform theories use the function $\hat{F}(\tau)$, introduced in (56), as their transition function in a completely different manner. The caustic difficulties have been partially alleviated with the application of the Equivalent Current Method (ECM) introduced by

Millar [21], Ryan and Peters [22] and its modification by Knott and Senior [23]. The determination of the field at caustics has also been studied by Ludwig [24] and others. Some difficulties in applying PTD for complex scatterers have been discussed and resolved by Mitzner [25].

Our goals in this section are to apply the techniques developed earlier and to construct a solution to the aperture problem, which is valid everywhere, including the shadow boundaries and the caustic directions. An added feature of the approach is that it is able to bring out the subtle relationships among GTD, PTD and ECM in a unified fashion. The analysis is again carried out, primarily in the spectral domain; thus, this may be regarded as yet another application of the Spectral Theory of Diffraction (STD).

The geometry of a convex aperture in an infinite screen is shown in Fig. 3. The screen S and the aperture A lie in the x-y plane, and Γ, which possesses continuous tangents, represents the rim of the aperture. We let the incident field u^i be a plane wave with the wave vector $k\hat{z}$, which takes the following form:

$$u^i = e^{ikz} \tag{69}$$

The case in which the incident field impinges obliquely on the structure has been investigated by the authors in a separate paper [26].

The total diffracted field u^t may be split as follows:

$$u^t = u^s + u^i, \tag{70}$$

where u^s is the scattered field. Here, as in previous cases, we assume that the total field u^t is zero on the screen, and therefore, the problem at hand may be called the diffraction by an aperture in a soft screen. Since the scattered field u^s is generated by the induced discontinuity of the total field in the screen, u^s satisfies the following equation:

$$(\partial_x^2 + \partial_y^2 + \partial_z^2 + k^2)\, u^s = \partial_z u^t \Big|_{0-}^{0+} \delta(z). \tag{71}$$

It is noted that $\partial_z u^t \Big|_{0-}^{0+} = \partial_z u^s \Big|_{0-}^{0+}$. Introducing $g_0 = e^{i|\vec{R} - \vec{R}'|}/4\pi|\vec{R} - \vec{R}'|$ as the Green's function of the three-dimensional Helmholtz operator, one finds that

$$(\partial_x^2 + \partial_y^2 + \partial_z^2 + k^2)\, g_0(x,y,z|x',y',z') = -\,\delta(x - x')\,\delta(y - y')\,\delta(z - z'). \tag{72}$$

Using the convolution theorem, one can easily verify that u^s is determined from the following equation:

$$u^s = -\int_S \partial_z u^t \Big|_{0-}^{0+} g_0(x,y,z|x',y',0)\,dx'dy' \tag{73}$$

u^s may further be decomposed as

$$u^s = u^d + u^0, \tag{74}$$

where u^d is the diffracted field and u^0 is the scattered field when the aperture is closed. One may readily observe that

$$u^0 = \begin{Bmatrix} -u^i \\ u^r = -u^i(-z) \end{Bmatrix}_{\gtrless}, \tag{75}$$

where u^r is simply the reflected field and symbols > and < refer to the regions $z > 0$ and $z < 0$, respectively. Using (73), (74) and (75), the total diffracted field u^t may also be expressed as:

$$u^t = u^f + u^{PO}, \tag{76}$$

where u^{PO} is the physical optics field and takes the following value:

$$u^{PO} = -\int_S \partial_z u^0 \Big|_{0-}^{0+} g_0 \, dx'dy' + u^i = u^i + u^0 + \int_A \partial_z u^0 \Big|_{0-}^{0+} g_0 \, dx'dy' \tag{77}$$

and u^f designates the fringe diffracted field. It is noticed that

$\partial_z u^0 \Big|_{0-}^{0+}$ over S is the same as the induced discontinuity of the physical optics field and has the value $\partial_z u^0 \Big|_{0-}^{0+} = -2\partial_z u^i \Big|_{z=0}$ over S.

Our task is now to determine u^f, which is written as

$$u^f = -\int_S \left[\partial_z u^t \Big|_{0-}^{0+} - \partial_z u^0 \Big|_{0-}^{0+} \right] g_0 \, dx'dy'. \tag{78}$$

The quantity in the bracket in (78) which is the fringe field discontinuity, is mainly confined around the rim of the aperture. To obtain u^f from (78), we first transform all of the quantities into the spectral domain by the following definition:

$$G_0 = F[g_0] = \int_{-\infty}^{\infty} g_0 \, e^{i(\alpha x' + \beta y')} \, dx'dy' = \frac{e^{-\gamma|z|}}{2\gamma}, \tag{79}$$

where $\gamma = \sqrt{\alpha^2 + \beta^2 - k^2}$. The above result can easily be verified by Fourier transforming (72) and imposing the radiation condition. We next define the Fourier transform of the induced discontinuities as:

$$D_U(\alpha,\beta) = F\{[-\partial_z u^t \Big|_{0-}^{0+} + \partial_z u^0 \Big|_{0-}^{0+}] \,\theta(S)\}$$

$$= \int_S \left\{ -\partial_z u^t \Big|_{0-}^{0+} + \partial_z u^0 \Big|_{0-}^{0+} \right\} e^{i(\alpha x' + \beta y')} \, dx'dy' \tag{80a}$$

$$D(\alpha,\beta) = F[-\partial_z u^t \Big|_{0-}^{0+}], \tag{80b}$$

$$D_{PO}(\alpha,\beta) = \mathcal{F}\left\{[- \partial_z u^0 \Big|_{0-}^{0+}]\ \theta(S)\right\}. \tag{80c}$$

where $\theta(S)$ has value one on S and zero otherwise. It is now apparent that the Fourier transform of u^f is

$$U^f = D_U G_0 \tag{81}$$

To determine D_U, which eventually allows one to obtain u^f, an orthogonal edge-coordinate system is introduced. As shown in Fig. 3, every point of the screen S can be uniquely determined by its coordinate values (τ,σ), where σ is an arc length measured from a reference point on the curve Γ and τ is a distance measured on the outward normal of the curve Γ from the curve to the point of interest. If the curve Γ is expressed by its parametric representation

$$\Gamma: \begin{cases} x = f(t) \\ y = g(t) \end{cases} \quad t\varepsilon[t_i,t_f], \tag{82}$$

then an arbitrary point of the screen can be coordinated as

$$\begin{cases} x = f(t) + \dfrac{g'(t)}{\ell(t)}\ \tau \\ \\ y = g(t) - \dfrac{f'(t)}{\ell(t)}\ \tau, \end{cases} \tag{83}$$

where $\ell(t) = \sqrt{[f'(t)]^2 + [g'(t)]^2}$ and prime denotes the derivative with respect to the parameter t. One may notice that $\left(\dfrac{g'(t)}{\ell(t)}, -\dfrac{f'(t)}{\ell(t)}\right)$ defines the components of the outward unit normal. Using transformation (83), the differential area is readily found to be:

$$dxdy = (1 + \frac{\tau}{\rho})\ d\tau d\sigma = (1 + \frac{\tau}{\rho})\ell(t)\ d\tau dt, \tag{84}$$

where ρ is the radius of curvature of Γ, i.e.,

$$\rho = \frac{[\ell(t)]^3}{g''f' - f''g'}. \tag{85}$$

By employing transformation (83) in the evaluation of (80), one arrives at:

$$D_U(\alpha,\beta) = \int_{t_i}^{t_f} e^{i[\alpha f(t) + \beta g(t)]} X_0(\alpha,\beta)\ell(t)dt, \tag{86a}$$

where

$$X_0(\alpha,\beta) = \int_0^\infty \left[- \partial_z u^t \Big|_{0-}^{0+} + \partial_z u^0 \Big|_{0-}^{0+} \right] (1 + \frac{\tau}{\rho}) \; e^{i\tau[\alpha g'(t) - \beta f'(t)]/\ell(t)} d\tau. \tag{86b}$$

By substituting (86a) into (81) and performing the inverse Fourier transformation, one finds that

$$u^f = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} D_U(\alpha,\beta) \frac{e^{- i(\alpha x + \beta y) - \gamma|z|}}{2\gamma} d\alpha d\beta. \tag{87}$$

Introducing the spherical coordinate system (R,θ,ϕ) and replacing $x = R \sin\theta \cos\phi$ and $y = R \sin\theta \sin\phi$ into (87), we may evaluate u^f for large values of R (far field) to obtain (see Felsen and Marcuvitz [14], p. 439),

$$u^f(\vec{R}) = D_U\ (-k \sin\theta \cos\phi,\ -k \sin\theta \sin\phi) \frac{e^{ikR}}{4\pi R} = \underset{\sim}{D}_U(\theta,\phi)\ g_0(kR),\ R \gg 0 \tag{88}$$

The above evaluation is valid as long as the diffraction coefficient in the kernel of (83) is well behaved at the observation angle (θ,ϕ). For the diffraction coefficient D_U given in (82), this is always the case. However, if we were to use either D or D_{PO} in our representation in place of D_U, a direct substitution of $\alpha \to -k \sin\theta \cos\phi$, $\beta \to -k \sin\theta \cos\phi$ would not be valid and the integral would have to be handled in a similar manner as outlined in Section II.

It is worthwhile to mention, that up to this stage, everything is exact and formally valid for an arbitrary incident field u^i. However, since an exact expression for $-\partial_z u^t \Big|_{0-}^{0+}$ and, hence, of $D_U(\alpha,\beta)$ in (86), is not known, it is necessary to introduce an approximation, that allows us to evaluate (86b) in a reasonable fashion. To illustrate the procedure, let us assume for simplicity that the incident field is a normally incident plane wave, as described in (69). We further assume that we are dealing with the high frequency region, i.e., the dimensions of the aperture are large compared to the incident wavelength. Under the latter assumption, one may replace the induced discontinuity in u^t on the screen, i.e.,

$\left[-\partial_z u^t \Big|_{0-}^{0+} + \partial_z u^0 \Big|_{0-}^{0+}\right] (1 + \frac{\tau}{\rho})$, with the discontinuities that would have been induced if a local half-plane had been erected tangent to the curve Γ and was placed in the plane of the screen. This approximation, which has been used very extensively in the high frequency diffraction analysis, is indeed the core of GTD and PTD. Introducing this approximation and comparing (86b) with (12) and (6a), one arrives at:

$$X_0(\alpha,\beta) \sim X[(\alpha g' - \beta f')/\ell] - X_{PO}[(\alpha g' - \beta f')/\ell] = X_U[(\alpha g' - \beta f')/\ell], \tag{89}$$

where X and X_{PO} are those terms previously defined in (13) and (21), respectively. It should be pointed out that in deriving (89)a different orientation for the coordinate system has been used for the local half plane than that employed in Section II. Taking this difference between the two systems of coordinates into account and remembering that our interest lies in the normally incident case, we may simplify (13) and (14) to obtain

$$X[\alpha] = -\frac{2k^{1/2}\sqrt{\alpha + k}}{\alpha} \tag{90}$$

and

$$X_{PO}[\alpha] = -\frac{2k}{\alpha}. \tag{91}$$

Substituting (89) into (86a), one finally arrives at:

$$D_U(\alpha,\beta) = \int_{t_i}^{t_f} e^{i[\alpha f(t) + \beta g(t)]} X_U[(\alpha g'(t) - \beta f'(t))/\ell(t)]\ell(t)dt. \tag{92}$$

The above formula resembles the formulation used in the equivalent edge method (ECM). In this method, a fictitious line edge-current was introduced using GTD diffraction coefficients, in order to circumvent the caustic difficulty. In our procedure, the derivation in (88) proceeds very naturally, such that there is no need to introduce an artificial current line source. In his monograph, Ufimtsev derived an expression for a circular disk, which is similar to (86b), however, with the order of integration interchanged. His construction was carreid out in the space domain and was directed toward the calculation of the far field. The expression he derived is identical to (86a) with α and β replaced by $-k \sin\theta \cos\phi$ and $-k \sin\theta \sin\phi$, respectively. Ufimtsev evaluated (86a) asymptotically using the stationary phase method and then determined (86b) at stationary points using the expression of the far field, diffracted by a half plane. Next, he used an interpolation scheme to determine the diffracted field near and at the caustic directions. In conclusion, Ufimtsev's construction (PTD) is largely based on intuitive consideration, as he used a switched order of integration in contrast to (86a).

We evaluate (92) asymptotically for large values of k using the stationary phase method after substituting $\alpha = -k \sin\theta \cos\phi$ and $\beta = -k \sin\theta \sin\phi$ into (92) and assuming that $\theta \neq 0$. The stationary points (bright points) are the roots of the following equation:

$$\alpha f'(t) + \beta g'(t) = 0. \tag{93}$$

It is easily understood from (93) that the directions of the lines from the stationary points to the observation point (at infinity) and the tangent to the curve Γ, at the stationary points, are perpendicular to each other. In other words, the diffracted and incident rays lie on the diffraction cone, which is degenerated to a plane for our case. This is the result which was first stated by Keller in his GTD technique [9]. For a convex aperture, there are two values of t which satisfy (93). This is true if α and β do not vanish simultaneously, i.e., $\theta \neq 0$. We denote these two values by t_1 and t_2 and apply the standard stationary phase technique to the integral (88) arriving at:

$$D_U(\alpha,\beta) = \sum_{j=1}^{2} \sqrt{2\pi}\, \left|\alpha f''(t_j) + \beta g''(t_j)\right|^{-1/2} \chi_U[(\alpha g'(t_j) - \beta f'(t_j))/\ell(t_j)]$$

$$\cdot\, \ell(t_j)\, e^{i[\alpha f(t_j) + \beta g(t_j)] \pm i\frac{\pi}{4}} + O(k^{-1}), \tag{94}$$

where $\pm$ are selected according to the sign of $\alpha f''(t_j) + \beta g''(t_j)$. In calculating (92), it has already been noticed that the contributions of the end points cancel each other, due to the fact that $f(t_i) = f(t_f)$ and $g(t_i) = g(t_f)$. The same relations also hold for their derivatives. Substituting (94) into (88), one can finally determine the asymptotic value of u^f.

Having determined u^f, we next evaluate the asymptotic form of u^t, defined in (76), by first constructing the asymptotic evaluation of u^{PO} given in (73). Following the same procedure which led to (87), one readily obtains

$$u^{PO} = u^i + \frac{1}{4\pi^2} \int_{-\infty}^{\infty} D_{PO}(\alpha,\beta)\, \frac{e^{-i(\alpha x + \beta y) - \gamma|z|}}{2\gamma}\, d\alpha d\beta, \tag{95}$$

where $D_{PO}(\alpha,\beta)$ has already been defined in (80). Unlike $D_u(\alpha,\beta)$, $D_{PO}(\alpha,\beta)$ is a singular function and, hence, care must be exercised in evaluating the integral appearing in (95). Taking into account the pole contribution, we then can express (95) as follows for the far-field region and observation angles $\theta \neq 0$:

$$u^{PO} = u^i + \begin{Bmatrix} -u^i \\ u^r \end{Bmatrix}_{\gtrless} + D_{PO}(-k \sin\theta \cos\phi, -k \sin\theta \sin\phi) \frac{e^{ikR}}{4\pi R}, \tag{96}$$

where $u^r = -u^i(-z)$ is the reflected field and $\gtrless$ corresponds to the regions $z \gtrless 0$. To obtain the asymptotic value of D_{PO}, we follow the same procedure which led us from (86a) to (92) and finally to (94). In doing so, we arrive at

$$D_{PO}(\alpha,\beta) = \text{right hand of (94) after replacing the index U by PO.} \tag{97}$$

Using (96) and (88) and recalling that $X = X_U + X_{PO}$, the total diffracted field u^t may be written finally as:

$$u^t = u^i + \begin{Bmatrix} -u^i \\ u^r \end{Bmatrix}_{\gtrless} + \frac{e^{ikR}}{4\pi R} \sum_{j=1}^{2} \sqrt{2\pi} \left|\alpha f''(t_j) + \beta g''(t_j)\right|^{-1/2}$$

$$\cdot X[(\alpha g'(t_j) - \beta f'(t_j))/\ell(t_j)]\ \ell(t_j)\ e^{i[\alpha f(t_j) + \beta g(t_j)] \pm i\frac{\pi}{4}} + O(k^{-1}), \tag{98}$$

where $\alpha = -k \sin\theta \cos\phi, \beta = -k \sin\theta \sin\phi$ and $X[\cdot]$ is defined in (90). The above result agrees completely with the result obtained using the GTD ray optical construction. As was previously mentioned, (95) is not valid for small values of θ and, in fact, diverges as $\theta \to 0$.

When $\theta \rightarrow 0$, u^t can be determined by employing other techniques instead of the stationary phase method to evaluate (92). To do this, we expand the integrand of (92) in terms of the Taylor series around $\theta = 0$, and then perform the integration term by term. Substituting $\alpha = -k \sin\theta \cos\phi$ and $\beta = -k \sin\theta \sin\phi$ into (92) and using the Taylor series expansion, one finally obtains from (86) the following expression for u^f:

$$u^f = \frac{e^{ikR}}{4\pi R} \int_{t_i}^{t_f} [-1 + A + B + O(k^3 \sin^3 \theta)]\, \ell(t)\, dt \tag{99}$$

where

$$A = \left\{ -\frac{\cos\theta\, g'(t) - \sin\theta\, f'(t)}{4\ell(t)} + ik[\cos\phi\, f(t) + \sin\phi\, g(t)] \right\} \sin\theta, \tag{100a}$$

and

$$B = \left\{ ik \frac{\cos\phi\, g'(t) - \sin\phi\, f'(t)}{4\ell(t)} [\cos\phi\, f(t) + \sin\phi\, g(t)] \right.$$

$$\left. - \frac{1}{8}\left[\frac{\cos\phi\, g'(t) - \sin\phi\, f'(t)}{\ell(t)}\right]^2 + \frac{1}{2}k^2 [\cos\phi\, f(t) + \sin\phi\, g(t)]^2 \right\} \sin^2\theta. \tag{100b}$$

The physical optics field contribution, i.e., (77) can also be simplified for small values of θ to give

$$u^{PO} = \frac{e^{ikR}}{4\pi R} \int_A \left\{ -2ik - 2k^2 (x' \cos\phi + y' \sin\phi) \sin\theta \right.$$

$$\left. + ik^3 (x' \cos\phi + y' \sin\phi)^2 \sin^2\theta + O(k^4 \sin^3\theta) \right\} dx'dy', \tag{101}$$

where, as defined in (69), $u^i = e^{ikz}$ and $u^r = -e^{-ikz}$. It may be noticed that exactly at the caustic direction where $\theta = 0$, the total field u^t takes the following form

$$u^t = u^{PO} + u^f = -[2ikA + L]\, \frac{e^{ikR}}{4\pi R}, \tag{102}$$

where A and L are the area and circumference of the aperture, respectively.

B. Diffraction by an Elliptic Aperture

In order to demonstrate the generality of the procedure described in the last section, we now go on to consider the diffracted field by an elliptic aperture due to a normally incident plane wave. The aperture is characterized by its axes such that 2a and 2b are its major and minor axes, respectively. One may then parametrize the rim of this aperture as

$$\begin{cases} x = f(t) = a \cos t \\ y = g(t) = b \sin t \end{cases} \qquad t \varepsilon [0, 2\pi] \quad . \tag{103}$$

Substituting (103) into (98) and simplifying the result, the total diffracted field in the region $z > 0$ and for values of $\theta \neq 0$ can readily be shown to be

$$u^t = \sqrt{\frac{1}{\pi k}}\, abA^{-3/2} \sin^{-3/2}\theta \left\{ -e^{ikA\sin\theta - i\frac{\pi}{4}} \cos\frac{1}{2}\left(\frac{\pi}{2} - \theta\right) + e^{-ikA\sin\theta + i\frac{\pi}{4}} \cos\frac{1}{2}\left(\frac{\pi}{2} + \theta\right) \right\} \frac{e^{ikR}}{4\pi R} + 0[k^{-1}] \tag{104}$$

where $A = (a^2 \cos^2 \phi + b^2 \sin \phi)^{1/2}$. This result agrees with the one obtained by Jones [12] using a rather elaborate construction of MacDonald, and also with the GTD formula.

For small values of θ, i.e., for observation angles along the caustic direction, it is necessary to use (99) and (101) to determine the diffracted field. For the case $a = b$, i.e., a circular aperture, the resultant field expressions are rather simple and are given by

$$u^f = [-2\pi a + (-\frac{\pi a}{8} + \frac{i\pi}{8} ka^2 + \frac{\pi}{2} k^2 a^3) \sin^2 \theta] \frac{e^{ikR}}{4\pi R} \tag{105}$$

and

$$u^{PO} = [-2i\pi ka^2 + \frac{i\pi}{4} k^3 a^4 \sin^2 \theta] \frac{e^{ikR}}{4\pi R} \quad . \tag{106}$$

For this special case of a circular aperture (77) can be integrated exactly to give

$$u^{PO} = -4\pi ika^2 \frac{J_1(ka \sin \theta)}{ka \sin \theta} \frac{e^{ikR}}{4\pi R}, \tag{107}$$

where J_1 is the first-order Bessel function. As one expects, (106) is indeed the small argument expansion of (107).

V. DIFFRACTION BY A SEMI-INFINITE CYLINDER

The geometry of a semi-infinite hollow cylinder with the convex cross section Γ is shown in Fig. 4. The cross section Γ is parametrized as (81), and we assume that an incident plane wave impinges on the cylinder, axially, i.e.,

$$u^i = e^{-ikz} \quad . \tag{108}$$

Once again this case is of interest, since the axial direction coincides with the line caustic, where conventional GTD results give rise to fictitious infinities. As in (70), we split the total diffracted field u^t into the incident field u^i and the scattered field u^s, where the latter satisfies the following equation:

$$\left(\partial_x^2 + \partial_y^2 + \partial_z^2 + k^2\right) u^s = \partial_n u^t \Big|_{0-}^{0+} \delta(S) \quad . \tag{109}$$

In (109), ∂_n denotes the differentiation along the outward normal of the cylinder and $\delta(S)$ is the delta distribution defined on the surface of the cylinder S. Upon employing the Green's function defined in (68), one may express the scattered field u^s as follows:

$$u^s = -\int_S \partial_n u^t \Big|_{0-}^{0+} g_0 \, da' \quad , \tag{110}$$

where da' is the differential area of the cylinder.

To determine u^s from (110), we first introduce a three-dimensional Fourier transformation by the following definition

$$G_0 = F[g_0] = \int_{-\infty}^{\infty} g_0 \, e^{i(\alpha x'+\beta y'+\gamma z')} \, dx' \, dy' \, dz' = \frac{1}{\alpha^2 + \beta^2 + \gamma^2 - k^2} \quad . \tag{111}$$

The above result can readily be verified by Fourier transformation of (72). We next define the Fourier transform of the induced discontinuity (or surface current) as:

$$D(\alpha,\beta,\gamma) = \int_{-\infty}^{\infty} -\partial_n u^t \Big|_{0-}^{0+} \delta(S)\ e^{i(\alpha x'+\beta y'+\gamma z')}\ dx'\ dy'\ dz'$$

$$= \int_{t_i}^{t_f} e^{i[\alpha f(t)+\beta g(t)]}\ X_0(\gamma)\ \ell(t)\ dt \quad , \tag{112}$$

where $\ell(t) = \sqrt{[f'(t)]^2 + [g'(t)]^2}$ and

$$X_0(\gamma) = \int_{-\infty}^{0} -\partial_n u^t \Big|_{0-}^{0+} e^{i\gamma z'}\ dz' \quad . \tag{113}$$

Using the fact that $u^s = DG_0$ and performing the inverse Fourier transformation, it is then found that

$$u^s = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} D(\alpha,\beta,\gamma)\ \frac{e^{-i(\alpha x+\beta y+\gamma z)}}{\alpha^2 + \beta^2 + \gamma^2 - k^2}\ d\alpha\ d\beta\ d\gamma \quad . \tag{114}$$

The above formulation may further be simplified after substituting $x = R \sin\theta \cos\phi$, $y = R \sin\theta \sin\phi$ and $z = R\cos\theta$ into (114), and performing the γ integration. The result for large values of R (far field) and in the region $z > 0$ is given by

$$u^s = D(-k \sin\theta \cos\phi, -k \sin\theta \sin\phi)\ \frac{e^{ikR}}{4\pi R} \tag{115}$$

where $D(\alpha,\beta) = D\left(\alpha,\beta,-\sqrt{k^2 - \alpha^2 - \beta^2}\right)$. Unlike in (77b), D is bounded for $\theta = 0$ in (115) because the caustic direction and shadow boundaries are separated for this case.

To evaluate (112) which eventually allows us to find (115), we employ the same approximation that was used in the previous section in connection with (86b). Namely, we assume that in the high frequency region the induced discontinuity in the cylinder, i.e., $-\partial_n u^t\Big|_{0-}^{0+}$, can be replaced with the discontinuity that would have been induced if a local half plane had been erected tangent to the cylinder. Introducing this approximation and comparing (113) with (12), one arrives at:

$$X_0(\alpha) \sim X[\alpha] = \frac{2i\sqrt{2k}}{\sqrt{\alpha - k}} \tag{116}$$

where $X[\cdot]$ is previously defined in (13). Substituting (116) into (112) and using the fact that (116) is independent of the integration variable t, one obtains

$$D(\alpha,\beta) = X[-\sqrt{k^2 - \alpha^2 - \beta^2}] \int_{t_i}^{t_f} e^{i[\alpha f(t)+\beta g(t)]} \, \ell(t) \, dt \quad . \tag{117}$$

The above simplification is not possible when the incident field does not propagate in the axial direction. In this case X will be a function of t and remains in the integral as X_U in (92).

Substituting (117) into (115) and following the same procedures which led to (98), we obtain the total field u^t in the region $z > 0$ and for $\theta \neq 0$ as:

$$u^t = u^i + \frac{e^{ikR}}{4\pi R} X[-\sqrt{k^2 - \alpha^2 - \beta^2}] \sum_{j=1}^{2} \sqrt{2\pi} \, |\alpha f''(t_j) + \beta g''(t_j)|^{1/2} \, \ell(t_j)$$
$$\cdot \, e^{ik[\alpha f(t_j)+\beta g(t_j)| \pm i\frac{\pi}{4}} + O(k^{-1}) \quad , \tag{118}$$

where $\alpha = -k \sin\theta \cos\phi$, $\beta = -k \sin\theta \sin\phi$, and t_1 and t_2 are the roots of (93). The plus and minus signs in (118) are determined according to the sign of $\alpha f''(t_j) + \beta g''(t_j)$.

For small angles of θ, we use the Taylor series expansion to evaluate (117). The final result for the total diffracted field u^t in the region $z > 0$ is

$$u^t = u^i + \frac{e^{ikR}}{4\pi R} \frac{2}{\cos\frac{\theta}{2}} \int_{t_i}^{t_f} \Big\{ 1 - ik[\cos\phi\, f(t) + \sin\phi\, g(t)]$$

$$- \frac{k^2}{2} [\cos\phi\, f(t) + \sin\phi\, g(t)]^2 \sin^2\theta + O[k^3 \sin^3\theta] \Big\} \ell(t)\, dt \quad . \tag{119}$$

Interestingly, exactly at the caustic direction where $\theta = 0$, the above formula can be further simplified to give

$$u^t = u^i + 2L \frac{e^{ikR}}{4\pi R} \quad , \tag{120}$$

where L denotes the circumference of the cross section of the cylinder.

A. Diffraction by a Semi-infinite Circular Cylinder

We use (118) and (119) to determine the diffracted field by a circular cylinder illuminated by a plane wave propagating along the axial direction. If the radius is denoted by a, the cross section may then be parameterized as:

$$\begin{cases} x = f(t) = a\cos t \\ y = g(t) = a\sin t \end{cases} \qquad t\varepsilon[0,2\pi] \quad . \tag{121}$$

Substituting (121) into (118) and (119) and simplifying the result, one finally arrives at:

(i) for large values of θ

$$u^t = u^i + \frac{2}{\cos\frac{\theta}{2}} \sqrt{\frac{2\pi a}{k\sin\theta}} \left\{ e^{ika\sin\theta - i\frac{\pi}{4}} + e^{-ika\sin\theta + i\frac{\pi}{4}} \right\} \frac{e^{ikR}}{4\pi R} \tag{122}$$

(ii) for small values of θ

$$u^t = u^i + \frac{1}{\cos\frac{\theta}{2}} \left\{4\pi a - \pi k^2 a^3 \sin^2\theta + O[k^4 \sin^4\theta]\right\} \frac{e^{ikR}}{4\pi R} \quad . \tag{123}$$

We may obtain a uniform expression for the diffracted field by simply substituting (121) into (117) and performing the integration. This substitution is done to derive

$$u^t = u^i + \frac{4\pi a}{\cos\frac{\theta}{2}} J_0(ka \sin\theta) \frac{e^{ikR}}{4\pi R} , \quad 0 \leq \theta \leq \frac{\pi}{2} \quad , \tag{124}$$

where J_0 is the zero-order Bessel function. *We may notice that (122) and (123) are the asymptotic expansion and small argument representation of (124), respectively.*

VI. CONCLUSIONS

The motivation behind this paper has been to introduce the Spectral Theory of Diffraction, or STD, which is an approach for constructing a solution for high frequency diffraction problems. It is shown that the scattered field, when expressed in the spectral domain, has the correct behavior at the shadow boundaries and caustic directions (even when there is a confluence of these two) and reduces to Keller's results away from these angles. The case of non-planar illumination has been treated and the results have been compared with other uniform theories. The problems of diffraction by apertures and semi-infinite cylinders has been discussed using the concepts of STD, and the formulations obtained in this manner have been compared with that of Ufimtsev's theory. It is hoped that because of its versatility and uniform nature STD will find future applications to a broad class of problems related to high frequency diffraction phenomena.

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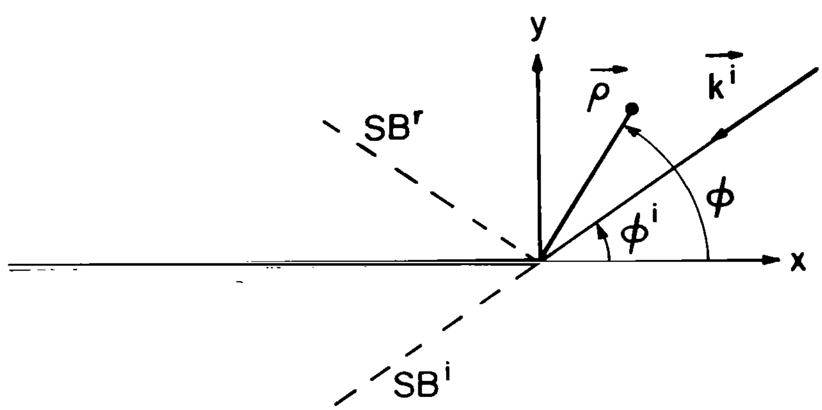

Fig. 1 Diffraction of a plane wave by a half-plane

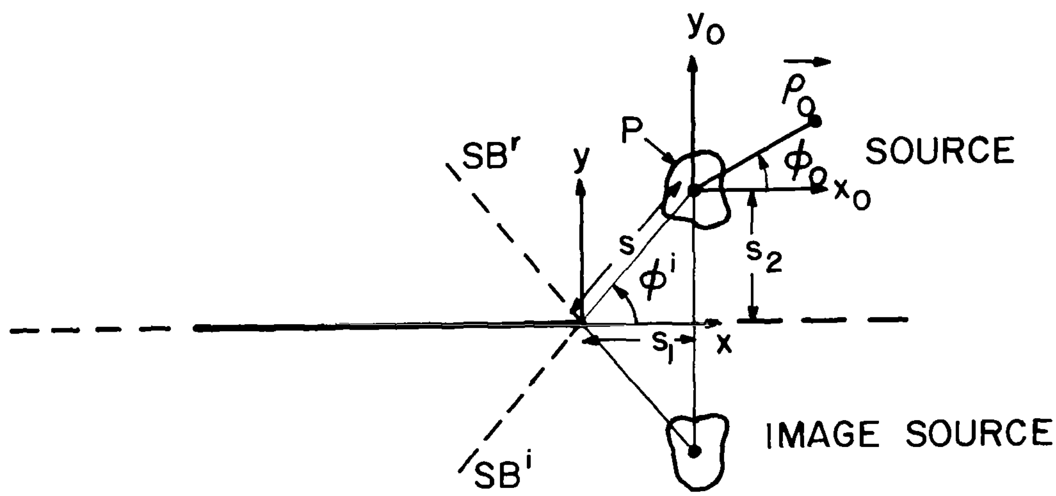

Fig. 2 Diffraction of an orbitrary Field by a half-plane

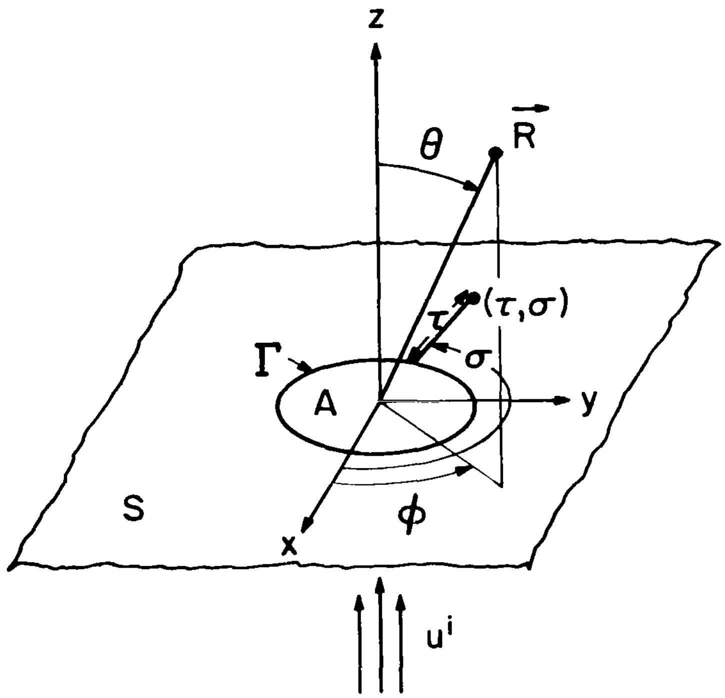

Fig. 3 Diffraction by an Aperture in an infinite screen

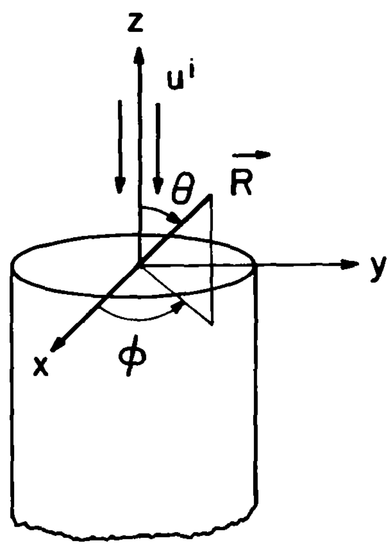

Fig. 4 Diffraction by a semi-infinite cylinder