Interaction Notes

Note 239

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Currents Induced on Metal/Dielectric Structures for TM Plane Wave Incidence

Yeongming Hwang
W. D. Burnside

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Abstract

The induced surface current density on a two-dimensional air foil type wing by a Transverse Magnetic wave is studied in this report. The reaction concept is employed to formulate an integral equation for the structure. The surface current density is expanded with subsectional bases. The dielectric body is modeled with polarization current which is then expanded in terms of subarea bases. By enforcing reaction tests with an array of electric test sources which are the same as the expansion bases (Galerkin's method), the moment method is employed to reduce the integral equation to a system of simultaneous linear equations. Inversion of the matrix equation yields the current distribution.

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CONTENTS

SECTION I
INTRODUCTION

The objective of this report is to investigate the effect of the dielectric loaded edge of a two-dimensional air foil type wing on the surface current and charge density induced by a transverse magnetic (TM) wave. Rumsey's reaction concept[1] is employed to formulate an integral-equation solution for the dielectric metal structure. The dielectric section can be attached to the conducting body. In the reaction formulation, the surface current density on the conducting surface and the polarization current in the dielectric body are expanded with suitable bases. By enforcing reaction tests with an array of electric test sources which are the same as those of expansion bases (Galerkin's method), the moment method is employed to reduce the integral equation to a system of linear equations. Numerical solution of this system yields a stationary result for the samples of the current distribution.

There have been several published papers regarding the moment method in electromagnetic problems. A good review paper was given by Richmond[2]. He has also developed a wire grid array model for cylinders with transverse-magnetic wave[3] incident and a piecewise-sinusoidal reaction formulation for electromagnetic radiation and scattering problems involving cylinders with non-circular cross section for the transverse-electric incident wave. Wang et al.[4] extended the reaction formulation from two-dimensional electromagnetic problems to three-dimensional problems in which the surface current density on the conducting surface is expanded with suitable bases, an electric surface monopole and surface dipole. In the case of a thin dielectric-coated conducting body, the polarization current is related to the surface current density on the surface of the conducting body.

This report extends the reaction formulation to include the scattered dielectric body. Section II presents the theory outline of the reaction concept. In Section III, the relevant bases for a two-dimensional electromagnetic problem with time harmonic TM case are given and discussed in detail. The magnetic field has no Z-dependence. H_z vanishes everywhere and the time dependence $e^{j\omega t}$ is understood and suppressed (the Z-axis is parallel with the axis of the cylinder). Evaluation of the self and mutual reactances and excitation voltages are considered in Section IV. Application to some special cases are given in Section V and the results are in excellent agreement with the exact classical solutions which involve an infinite series of cylindrical mode functions. Finally, the induced surface current density on a dielectric loaded edge of two-dimensional air foil type wing is given in this chapter.

The results for the surface current and charge densities induced on the fuselage of a 747 and B-1 aircraft are presented in the frequency and time domain in Reference [8] for an electromagnetic pulse incident.

SECTION II
THE REACTION INTEGRAL FORMULATION

The exterior scattering problem is considered in this section. Let us assume that the impressed electric and magnetic current $(\bar{J}^i, \bar{M}^i)$ generate the electric and magnetic field intensity $(\bar{E}, \bar{H})$ in the presence of a conducting body and a dielectric body such as shown in Fig. 1.

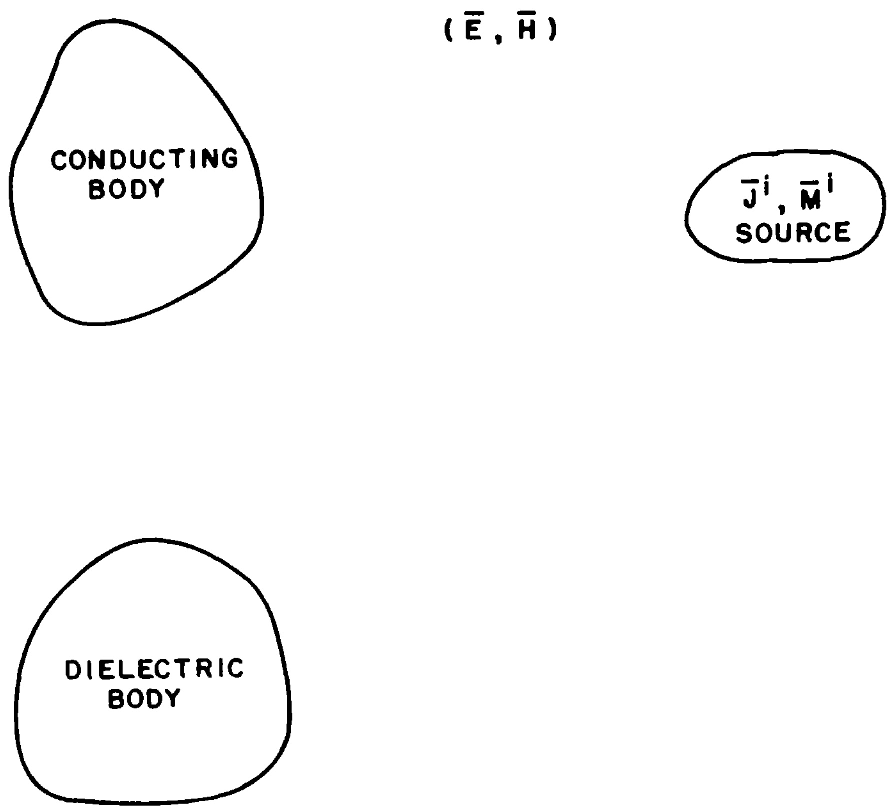

Fig. 1. The source $(\bar{J}^i, \bar{M}^i)$ generates the field $(\bar{E}, \bar{H})$ with scatterers.

From the surface-equivalence theorem of Schelkunoff[5] and the volume equivalence theorem of Rhodes[6], the total field at any point exterior to the scatterers is the sum of the incident field $(\bar{E}^i \bar{H}^i)$ generated by $(\bar{J}^i, \bar{M}^i)$ and the scattered field $(\bar{E}^s, \bar{H}^s)$ due to the surface current densities

(1) $\bar{J}^s = \hat{n} \times \bar{H}$

(2) $\bar{M}^s = \bar{E} \times \hat{n}$

on the close surface of the conducting body and the equivalent electric current density

(3) $\bar{J}^{eq} = j\omega(\varepsilon - \varepsilon_0)\,\bar{E}$

in the dielectric body. This result is shown in Fig. 2.

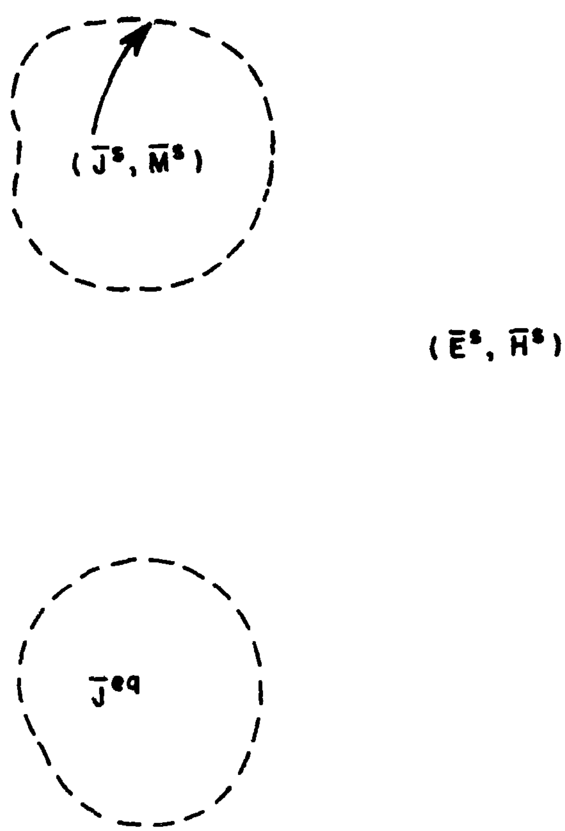

Fig. 2. The scattered field may be generated by $(\bar{J}^s, \bar{M}^s)$ and $\bar{J}^{eq}$ in free space.

For a finite conducting body, the magnetic surface current density can be related to the surface impedance Z_s as

(4) $$\overline{M}^s = Z_s \, \overline{J}^s \times \hat{n} \quad .$$

Note that $\overline{M}^s$ vanishes if the scatterer is a perfect conductor.

Placing an electric test source $\overline{J}^t$ in the interior region of the conducting body and applying the reaction theorem[1], it can be shown that

(5) $$-\oint\!\!\oint_S (\overline{J}^s \cdot \overline{E}^t - \overline{M}^s \cdot \overline{H}^t)ds - \iiint_V (\overline{J}^{eq} \cdot \overline{E}^t)dv$$

$$= \iiint_V (\overline{J}^i \cdot \overline{E}^t - \overline{M}^i \cdot \overline{H}^t)dv,$$

where $(\overline{E}^t, \overline{H}^t)$ is the free-space field of the test source. Equation (5) states that the interior test source in the conducting body has zero reaction with the other sources. However, if the test source is placed in the dielectric body, the reaction between the interior test source in the dielectric body and the other sources is no longer zero. The reaction theorem in this case, gives

(6) $$-\oint\!\!\oint_S (\overline{J}^s \cdot \overline{E}^t - \overline{H}^t \cdot \overline{M}^s)ds - \iiint_V (\overline{J}^{eq} \cdot \overline{E}^t)\, dv$$

$$= \iiint_V (\overline{J}^i \cdot \overline{E}^t - \overline{M}^i \cdot \overline{H}^t)dv + \iiint_V (\overline{J}^t \cdot \overline{E} - \overline{M}^t \cdot \overline{H})dv.$$

We represent the surface current density distribution as

(7) $$\overline{J}^s = \sum_{\ell=1}^{L} I_\ell \, \overline{J}^s_\ell$$

and the equivalent electric current density as

(8) $$\bar{J}^{eq} = \sum_{m=1}^{M} I_m \bar{J}_m^{eq}$$

where the complex constants I_ℓ (or I_m) are samples of the function $\bar{J}^s$ (or $\bar{J}^{eq}$). The vector functions $\bar{J}_\ell$ (or $\bar{J}_{\ell m}$) are basis functions, subsectional bases, or expansion functions. If L's $\bar{J}^{t(s)}$ and M's $\bar{J}^{t(eq)}$ are placed in the interior region of the conducting body and dielectric body, respectively, we can obtain the simultaneous linear equations

(9) $$\sum_{\ell=1}^{L} I_\ell C_{\ell j} + \sum_{m=1}^{M} I_m C_{mj} = A_j \qquad \text{with } j=1,\cdots,L,LN,\cdots,L+M$$

where

(10a) $$C_{\ell j} = -\iint_\ell \bar{J}_\ell^s \cdot [\bar{E}_j^t - (\hat{n} \times \bar{H}_j^t) Z_s] \, ds,$$

(10b) $$C_{mj} = -\iiint_m \bar{J}_m^{eq} \cdot \bar{E}_j^t \, dv,$$

and

(11) $$A_j = \iint \bar{J}^i \cdot \bar{E}_j^t \, ds = \iint_j \bar{J}_j^t \cdot \bar{E}^i \, ds \qquad \text{for } 1 \le j \le L$$

where $\bar{E}_j^t$ is the free-space field generated by $\bar{J}_j^{t(s)}$ and

(12a) $$C_{\ell j} = -\iint_\ell \bar{J}^s \cdot [\bar{E}_j^t - (\hat{n} \times \bar{H}_j^t) Z_s] \, ds$$

(12b) $$C_{mj} = -\iiint_m \bar{J}_m^{eq} \cdot \bar{E}_j^t \, dv - \frac{\delta_{mj}}{j\omega(\varepsilon - \varepsilon_0)} \iiint_V \bar{J}_j^{t(eq)} \cdot \bar{J}_m^{eq} \, dv dv'$$

in which

$$\delta_{mj} = \begin{cases} 1, & m=j \\ 0, & m \neq j \end{cases}$$

and

(13) $$A_j = \iiint_V \bar{J}_j^{eq} \cdot \bar{E}^i \, dv \qquad \text{for } L<j\leq L+M$$

where $\bar{E}_j^t$ is the free-space field generated by $\bar{J}_j^{t(eq)}$. For computational speed and storage of computer, it will be advantageous to choose the test function of the same size, shape and functional form as the expansion function. The impedance matrix $C_{\ell j}$ is symmetric and we got a well-conditioned set of simultaneous linear equations.

SECTION III
ELECTRIC TEST SOURCE AND EXPANSION MODES

In the preceding section, the reaction integral equation reduces to a system of simultaneous linear equations via the moment method in two steps. First the unknown current distribution is expanded with basis functions. Test sources with the same functional form are then employed to perform the reaction test. In the remainder of this report two types of test sources are employed:

A. Longitudinal Strip Sources

For a two-dimensional conducting body illuminated by a TM incident wave, the electric current density induced on the conducting surface is in the longitudinal direction. Thus a suitable choice of the bases is the longitudinal strip source[9].

Consider the strip source illustrated in Fig. 3. This source has an electric current distribution $\overline{J} = \hat{z}\, J(x')$ located on the x'z' plane. This strip source has width h and infinite length in z-direction and radiates in free space. It can be shown that the fields are given as follows:

(14) $$\overline{E} = -\frac{k\eta}{4}\int_0^h \overline{J}\, H_0^{(2)}\,(k|\overline{\rho}-\overline{\rho}'|)\, dx'$$

(15) $$\overline{H} = -\frac{jk}{4}\int_0^h \overline{J} \times \hat{\rho}\, H_1^{(2)}\,(k\rho)\, dx'$$

where

(16a) $$h = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \quad ,$$

(16b) $$k = \omega\sqrt{\mu\varepsilon} \quad ,$$

(16c) $\eta = \frac{\mu}{\varepsilon}$

and $H_n^{(2)}$ represents the cylindrical Hankel function of the second kind.

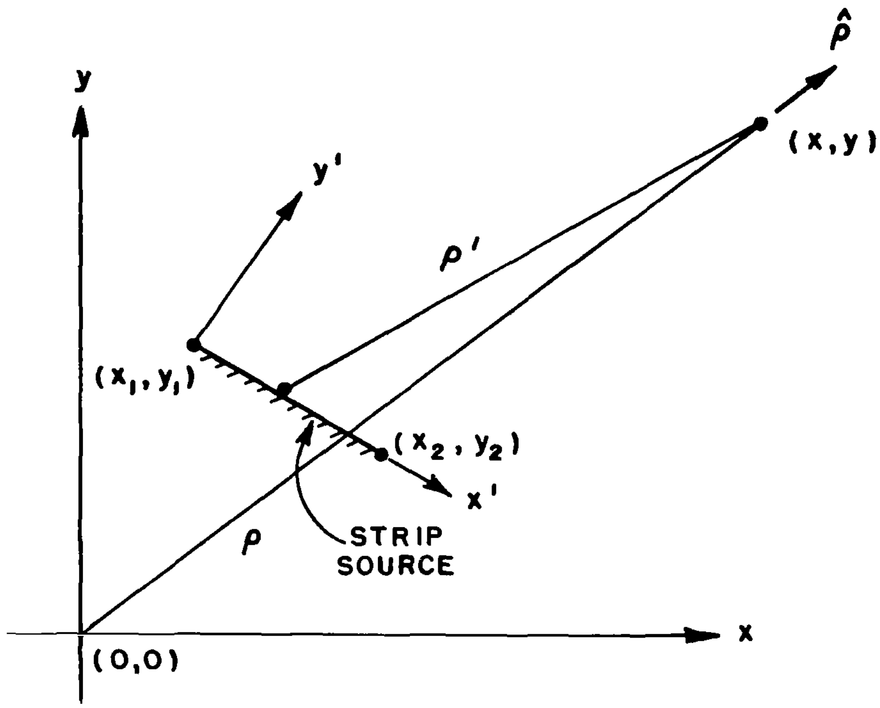

Fig. 3. An electric strip source radiates in free space.

If the electric strip source has a uniform distribution

(17) $\overline{J}(x') = \hat{z}$,

the Eqs. (14) and (15) can be written as

(18) $E_z = - \frac{k\eta}{4} \int_0^h H_0^{(2)} (k|\overline{\rho}-\overline{\rho}'|) \, dx'$

(19) $\overline{H} = - \frac{jk}{4} \int_0^h \hat{z} \times \hat{\rho} \, H_1^{(2)} (k|\overline{\rho}-\overline{\rho}'|) \, dx'$.

B. Infinitely Long (Longitudinal) Square Cell Sources

In the case of a dielectric body illuminated by a TM incident wave, the electric current density induced in the dielectric body is in the longitudinal direction. Thus, the longitudinal square cell is chosen as a suitable basis in the dielectric body, such as depicted in Fig. 4.

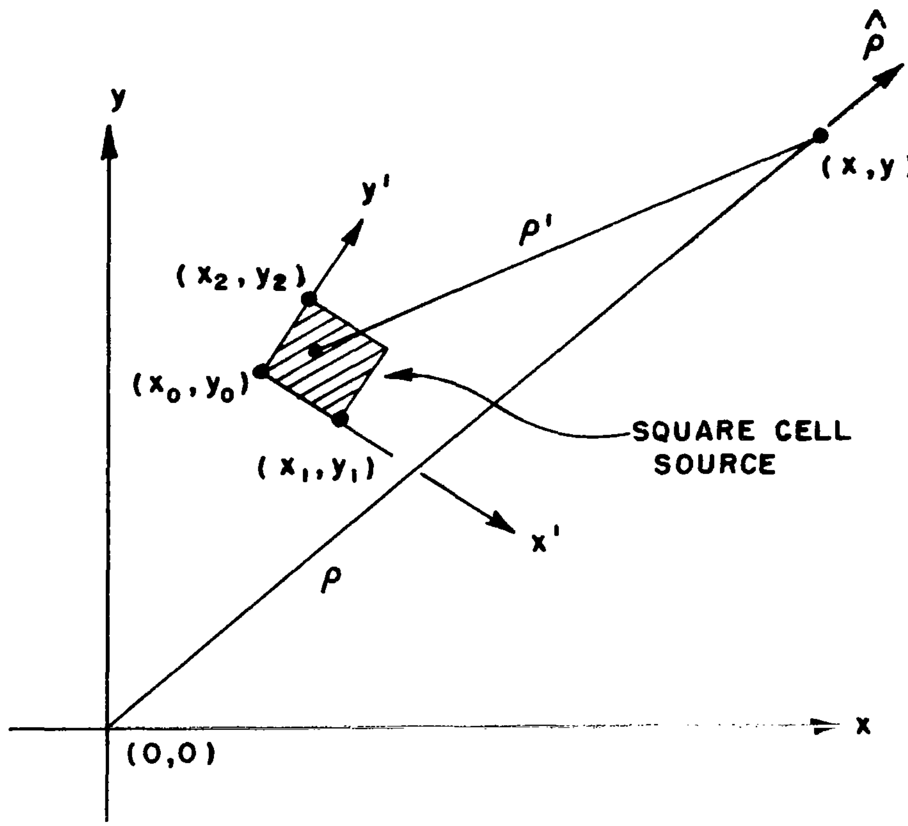

Fig. 4. An electric square cell source radiates in free space.

If the electric square cell source has a uniform distribution over the cell, i.e.,

$$(20) \qquad \bar{J}(x',y') = \hat{z} \quad , \quad \begin{cases} 0 \le x' \le h \\ 0 \le y' \le h \end{cases} .$$

The free space fields are

$$(21) \qquad E_z = -\frac{k\eta}{4} \int_0^h dx' \int_0^h dy' \; H_0^{(2)} \left(k|\bar{\rho}-\bar{\rho}'|\right)$$

and

$$(22) \qquad \bar{H} = -\frac{jk}{4} \int_0^h dx' \int_0^h dy' \; \hat{z} \times \hat{\rho} \; H_1^{(2)} \left(k|\bar{\rho}-\bar{\rho}'|\right).$$

SECTION IV
THE IMPEDANCE MATRIX AND THE EXCITATION COLUMN

A. The Impedance Matrix

From the viewpoint of the reaction theorem, the complex number $C_{\ell j}$ (or C_{mj}) in Eq. (10) (or Eq. (12)) represents the reaction between the sources ℓ (or m) and j. This reaction is given by

$$(23) \qquad C_{\ell j} = - \int_{\ell} \bar{E}_j^t \cdot \bar{J}_\ell^s \, d\ell'$$

for electric strip sources and

$$(24) \qquad C_{mj} = - \iint_m \bar{E}_j^t \cdot \bar{J}_m^{eq} \, ds'$$

for electric square cell sources. $C_{\ell j}$ involves a double integral; whereas, C_{mj} involves quadruple integral. When ℓ, m≠ j, the reaction is obtained via numerical integration techniques. The self reaction C_{jj} is evaluated in the following way. For an electric strip source, it is shown that

$$(25) \qquad C_{jj} = \frac{k\eta}{4} \int_{-\Delta C_j/2}^{\Delta C_j/2} dx' \int_{-\Delta C_j/2}^{\Delta C_j/2} dx \, H_o^{(2)} (k|x-x'|)$$

$$\sim \frac{k\eta}{4} (\Delta C_j)^2 \left[1 - j \frac{2}{\pi} \log_e \frac{\gamma k \Delta C_j}{2e}\right]$$

where the small argument approximation for the Hankel function

$$(26) \qquad H_o^{(2)} (Z) \sim 1 - j \frac{2}{\pi} \log_e \frac{\gamma Z}{2}$$

is used. $\gamma = 1.781$, C_j is the width of the electric strip source and e is 2.718. For an electric square cell source, the square subarea of the integral is approximated by a circular one of the same area. Using the addition theorem for Hankel functions one finds that

$$(27) \qquad C_{jj} = \frac{k\eta}{4}\int_0^a \rho'\, d\rho' \int_0^{2\pi} d\phi' \int_0^a \rho d\rho \int_0^{2\pi} d\phi'\; H_0\,(k|\bar{\rho}-\bar{\rho}'|)$$

$$= \pi^2 k \int_0^a \rho' d\rho' \int_0^a \rho d\rho \; x \begin{cases} H_0^{(2)}\,(k\rho')\, J_0(k\rho) & , \quad \rho<\rho' \\ J_0\,(k\rho')\, H_0^{(2)}(k\rho) & , \quad \rho>\rho' \end{cases} .$$

Thus, the quadruple integral with a singular integrand reduces to a double integral with a non-singular integrand which can then be integrated numerically.

B. The Excitation Column

The complex voltages A_j in Eqs. (11) and (13) give the "excitation column" or "excitation vector" in the system of linear equations $[C_{j\ell}]\ [I_\ell] = [A_j]$. These voltages are independent of the surface impedance Z_s of the conducting body and the permittivity of the dielectric body. If the line source is located at a great distance from the scatterer, the incident field $(\bar{E}^i, \bar{H}^i)$ may be regarded as a plane wave with

$$(28) \qquad \bar{E}^i = \hat{z}\, E_o\, e^{jk(x \cos\phi_i + y \sin\phi_i)}$$

where ϕ_i is the incident angle of the source and E_o is the incident electric field intensity at the origin of an arbitrarily chosen coordinate system. When this incident plane wave illuminates a strip source at point (x_{j1}, y_{j1}) and point $x_{j2}, y_{j2})$ such as shown in Fig. 5, the integral of Eq. (11) gives

$$(29) \qquad A_j = C_j\, e^{jk\psi_j} \frac{\sin kt}{t} ,$$

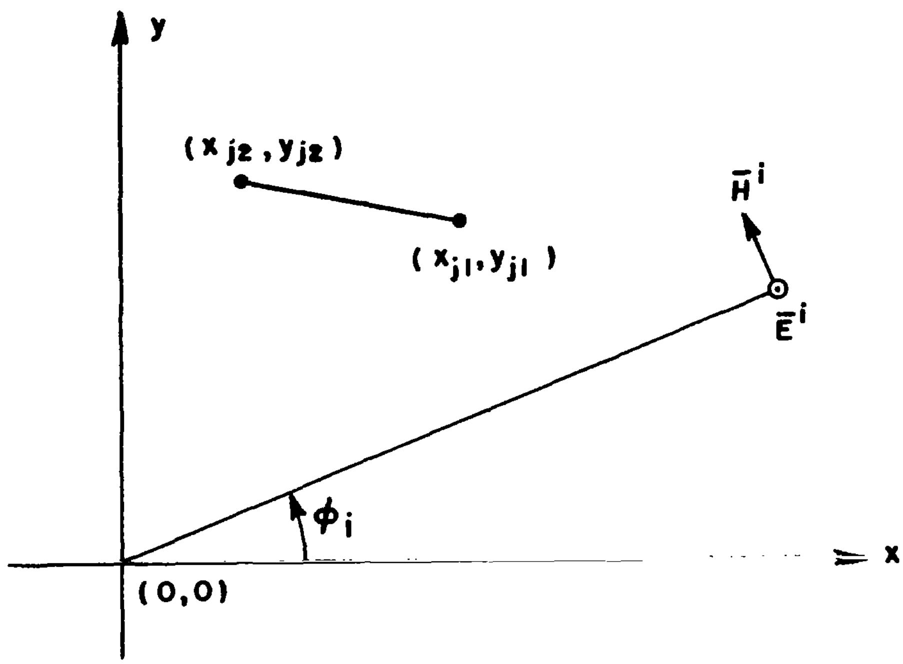

Fig. 5. A plane wave illuminates a strip source.

where

(30a) $$C_j = \sqrt{(x_{j2}-x_{j1})^2 + (y_{j2}-y_{j1})^2}$$

(30b) $$\psi_j = \frac{1}{2}\,[(x_{j1}+x_{j2})\cos\phi_i + (y_{j1}+y_{j2})\sin\phi_i],$$

and

(30c) $$t = \frac{1}{2}\,[(x_{j2}-x_{j1})\cos\phi_i + (y_{j2}-y_{j1})\sin\phi_i].$$

When the plane wave illuminates a rectangular cell (see Fig. 6),

(31) $$A_j = C_{jx}\, C_{jy}\, e^{j \frac{k}{2} [(x_{j1}+x_{j2})\cos\phi_i + (y_{j1}+y_{j2})\sin\phi_i]} \cdot \frac{\sin\frac{k_x}{2}}{\frac{k_x}{2}} \cdot \frac{\sin\frac{k_y}{2}}{\frac{k_y}{2}} ,$$

where

(32a) $$C_{jx} = \sqrt{(x_{j1}-x_{jo})^2 + (y_{j1}-y_{jo})^2} ,$$

(32b) $$C_{jy} = \sqrt{(x_{j2}-x_{jo})^2 + (y_{j2}-y_{jo})^2} ,$$

(32c) $$k_x = k[(x_{j1}-x_{jo})\cos\phi_i + (y_{j1}-Y_{jo})\sin\phi_i]$$

and

(32d) $$k_y = k[(x_{j2}-x_{jo})\cos\phi_i + (y_{j2}-y_{jo})\sin\phi_i].$$

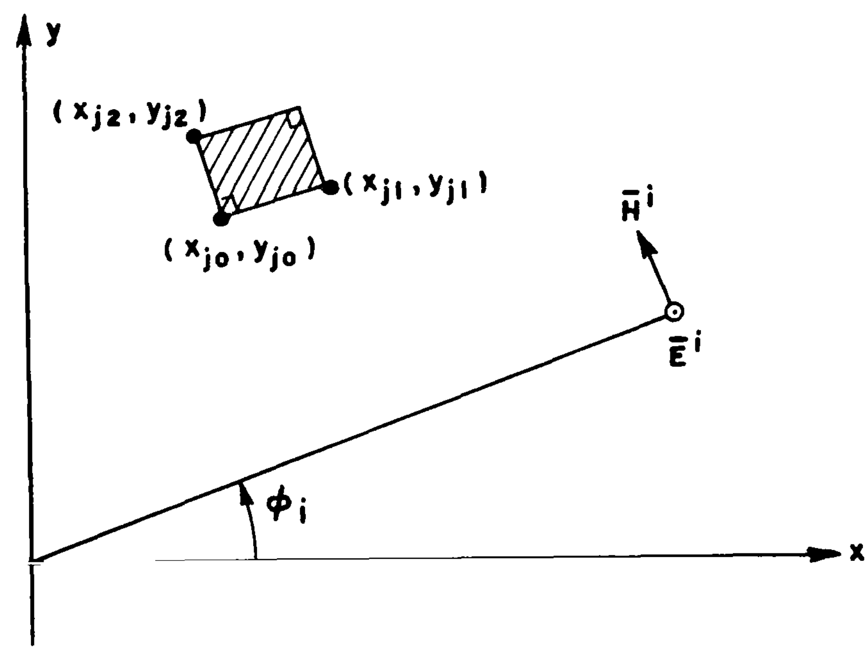

Fig. 6. A plane wave illuminates a rectangular cell.

SECTION V
APPLICATIONS AND NUMERICAL RESULTS

In the preceding sections, the reaction theorem is used to formulate the integral equation for the induced current density on the scattering body. Suitable basis and test sources have been chosen for a TM electromagnetic wave incident on the metal dielectric body. The basis functions are chosen the same as the test sources (Galerkin method). The self impedance is considered separately while the mutual impedance between them is computed numerically via the modified Gaussian quadrupture integration scheme. In the following, the reaction integral will be applied to some special cases and then compared with the exact solution in terms of infinite series of cylindrical mode functions. Finally the induced surface current density on a two-dimensional wing model, in which the leading edge includes a limited amount of dielectric material, will be investigated.

A. A Thin-Dielectric Coated, Perfectly-Conducting Circular Cylinder

Consider a dielectric-coated, perfectly-conducting cylinder illuminated by an electric line source. Let $(\vec{J}^s, \vec{M}^s)$ denote the surface current density induced on the conducting surface and $\vec{J}^{eq}$ denotes the polarization current density in the dielectric layer. The circular cylinder is approximated by a polygonal cylinder with N segments such that the perimeter of the polygonal cylinder is equal to that of the original circular cylinder.

The approximate polygonal cylinder is shown in Fig. 7. For a TM source illuminating a thin-dielectric coated, perfectly-conducting cylinder, the polarization current density can be related to the surface current density as follows[9]:

(33) $\bar{J}^{eq} = [(\frac{\varepsilon_o - \varepsilon}{\varepsilon}) k_1 \sin k_1 \xi] \bar{J}^s$

where ε is the permittivity of the dielectric layer, ε_o is the permittivity of free space, k_1 is the propagation constant in the dielectric layer and ξ is the distance measured normally outward from the conducting surface. Figure 8 shows the backscattering echo width of the dielectric-coated circular cylinder versus the diameter of the coated perfectly-conducting cylinder. The backscattering echo width is defined as

$$\lim_{\rho \to \infty} 2\pi\rho \frac{|\bar{E}^s|^2}{|\bar{E}^i|^2} .$$

The result shown in Fig. 8 is in good agreement with those computed by the eigenfunction solution.

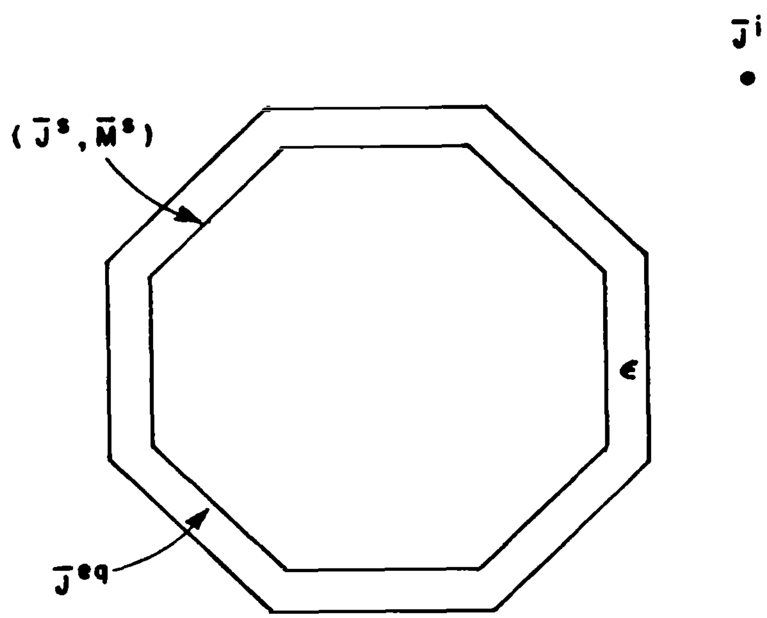

Fig. 7. Dielectric-coated, conducting polygonal cylinder illuminated by a parallel electric line source.

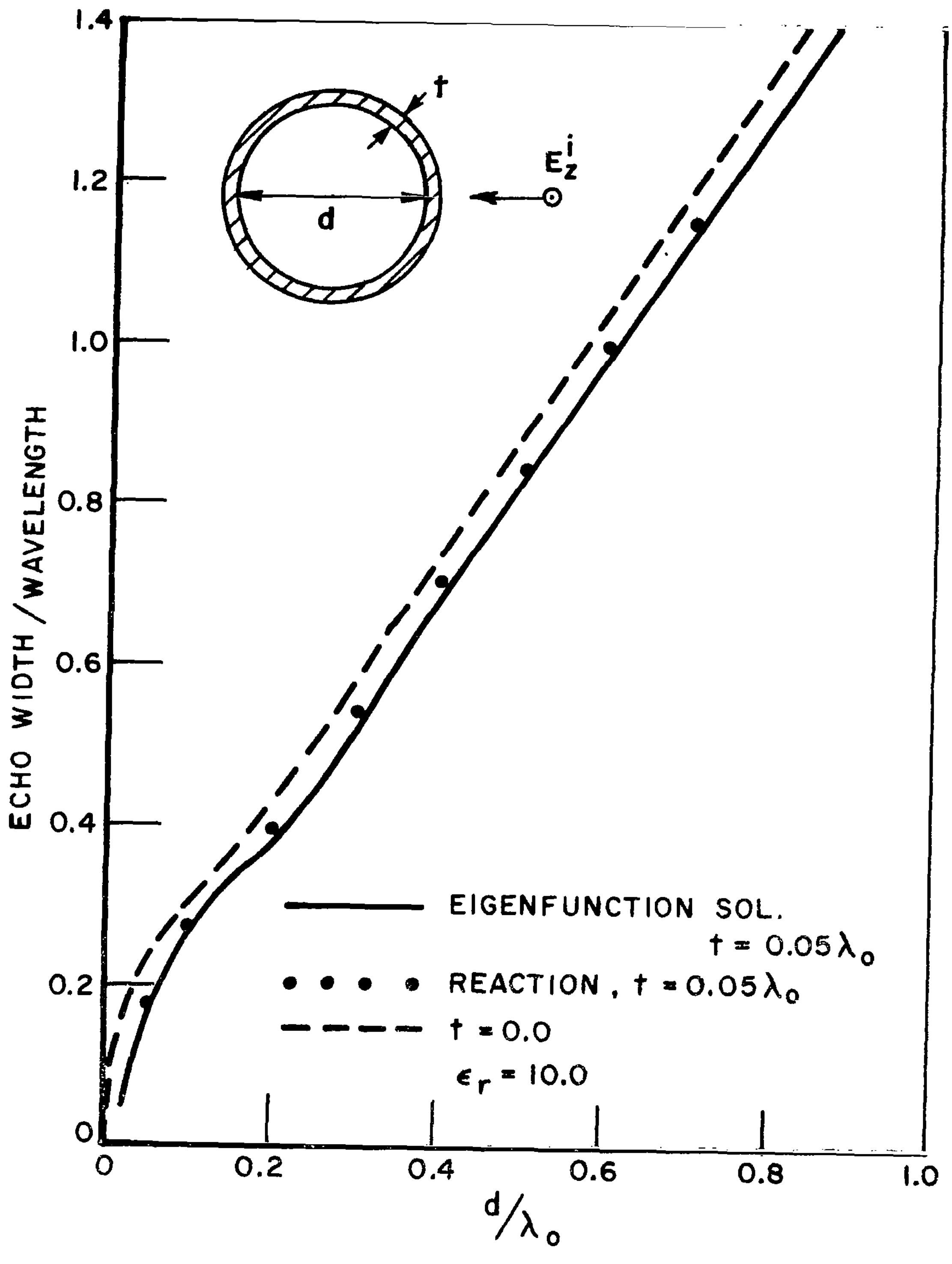

Fig. 8. Backscattering echo width of dielectric-coated circular cylinder.

If the polarization current density in the dielectric layer is not related to the surface current density, then a square cell is used as the base and test source in the dielectric body. Figure 9 shows the normalized induced current density on the perfectly-conducting surface of a dielectric coated cylinder with a diameter $d = 0.2\lambda_o$, where λ_o is the free space wavelength. 16 strip bases on the perfectly-conducting surface and 16 square cells representing the entire dielectric layer are used in the reaction integral. Thus a system of 32 simultaneous linear equations is generated. The computed surface current density is in excellent agreement with those obtained via the thin-dielectric coating model discussed before.

B. Scattering by a Dielectric Cylinder

A circular dielectric cylindrical shell is shown in Fig. 10. A square cell is used as the expansion mode and test source. Figure 11 shows the distance scattering pattern for a circular dielectric cylindrical shell illuminated by a TM plane wave. 34 square cells are used to approximate the cylindrical shell. Figure 11 shows that the results are in excellent agreement with the exact classical solution which involves an infinite series of cylindrical mode functions.

C. A Two-Dimensional Foil Type Wing

In this last section the reaction integral is used to analyze the induced surface current density on a two-dimensional air foil type wing. The leading edge of the wing includes a limited amount of dielectric material such as shown in Fig. 12. The effect of the dielectric-loaded leading edge is taken into account by the mutual reactance between the square cell source in the dielectric body and the strip source on the perfectly-conducting surface. The TM plane wave has grazing incidence on the leading edge of the wing. The observation point is chosen at the base of the dielectric region (see

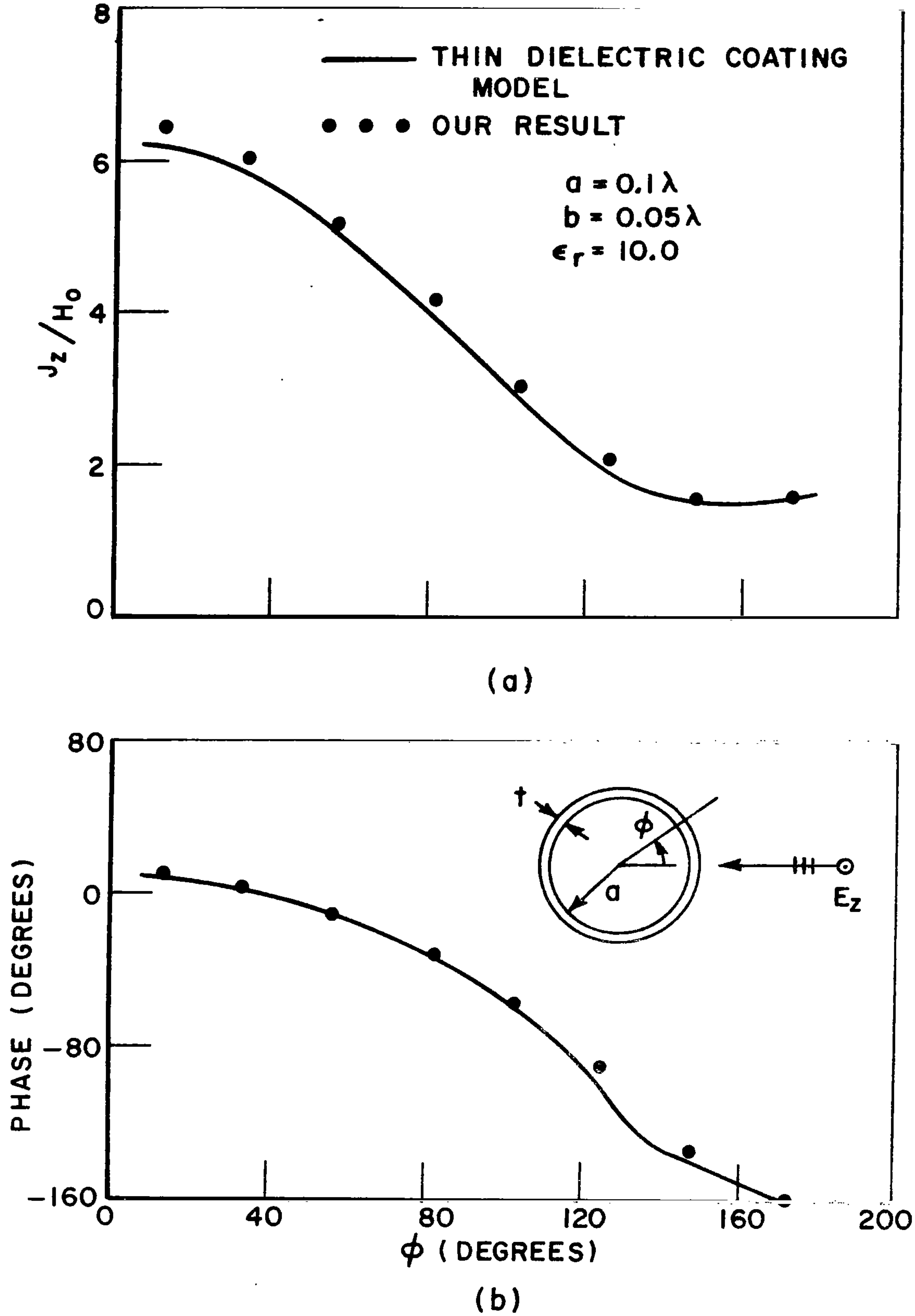

Fig. 9. The surface current density J_z/H_o.

Fig. 12). The normalized induced current density J_z/H_o versus frequency on the wing is shown in Fig. 13, where H_o is the magnetic field intensity of the incident field.

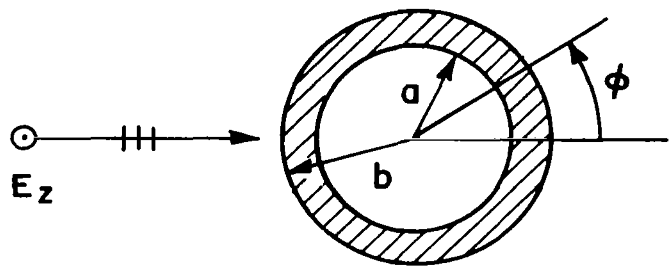

Fig. 10. A circular dielectric cylindrical shell with plane wave incident.

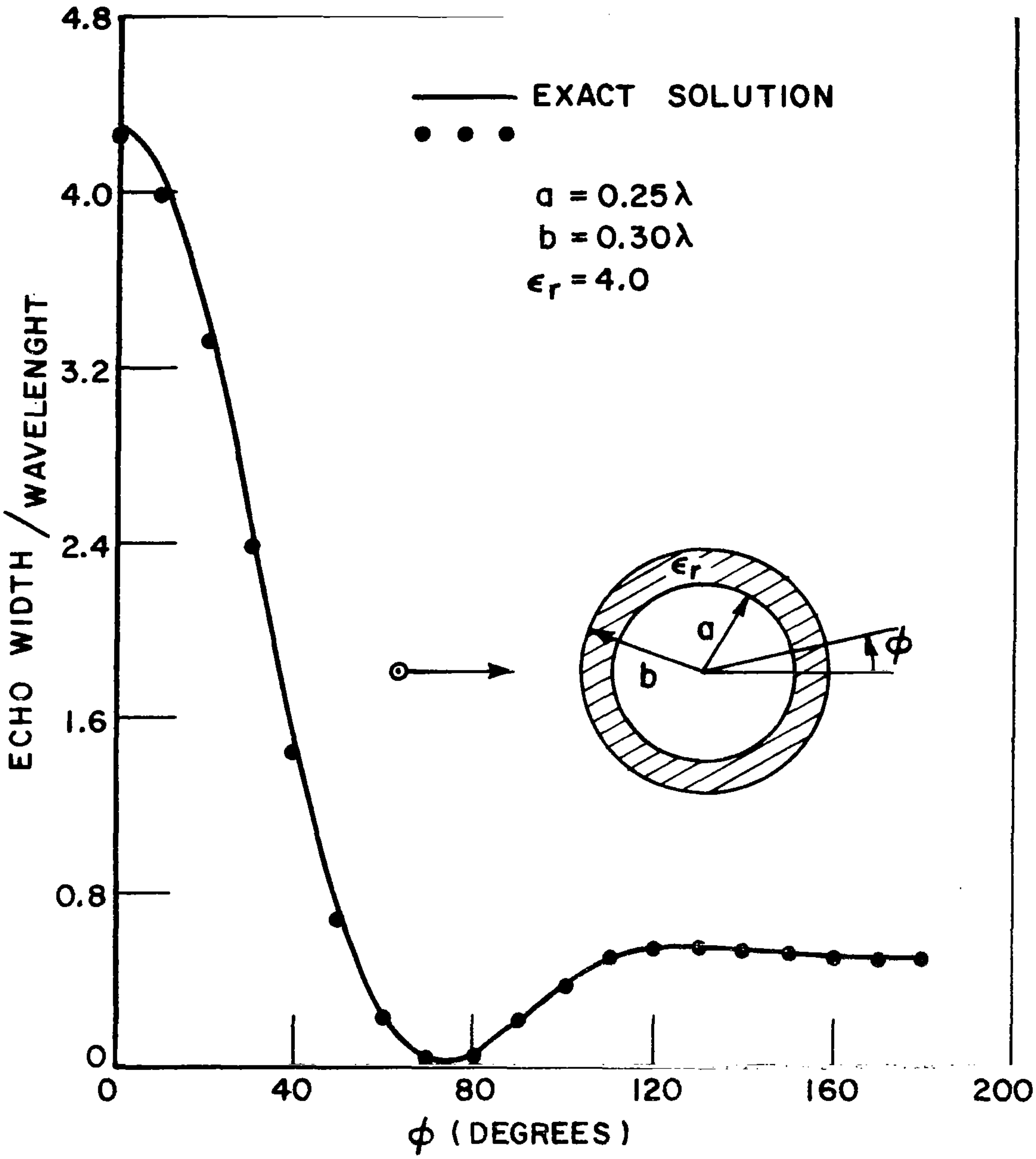

Fig. 11. Distant scattering pattern of circular dielectric cylindrical shell with plane wave incident.

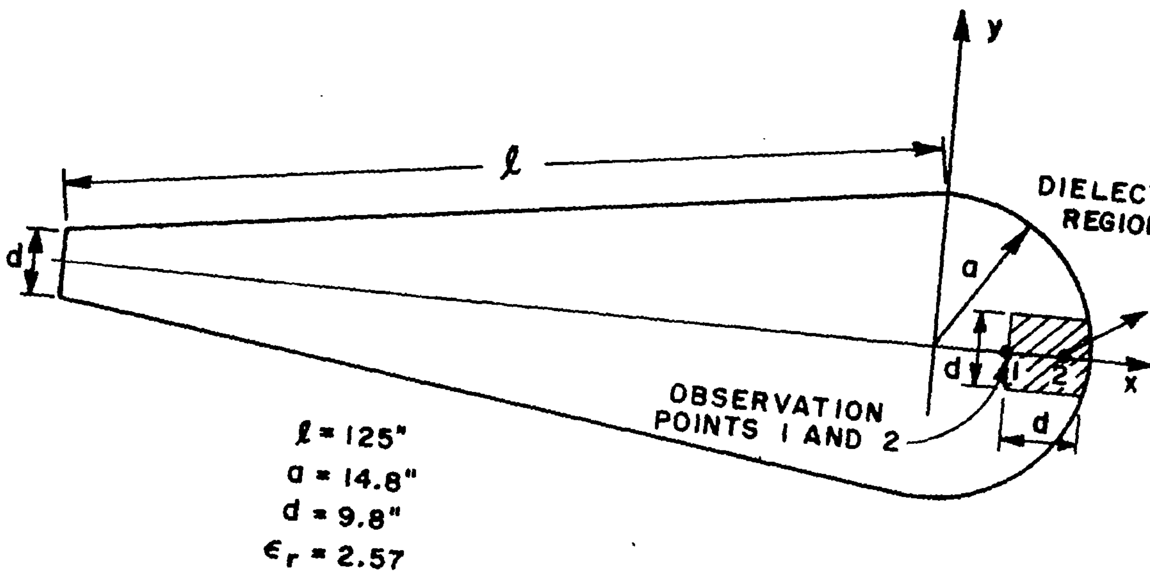

Fig. 12. A two-dimensional foil type wing model.

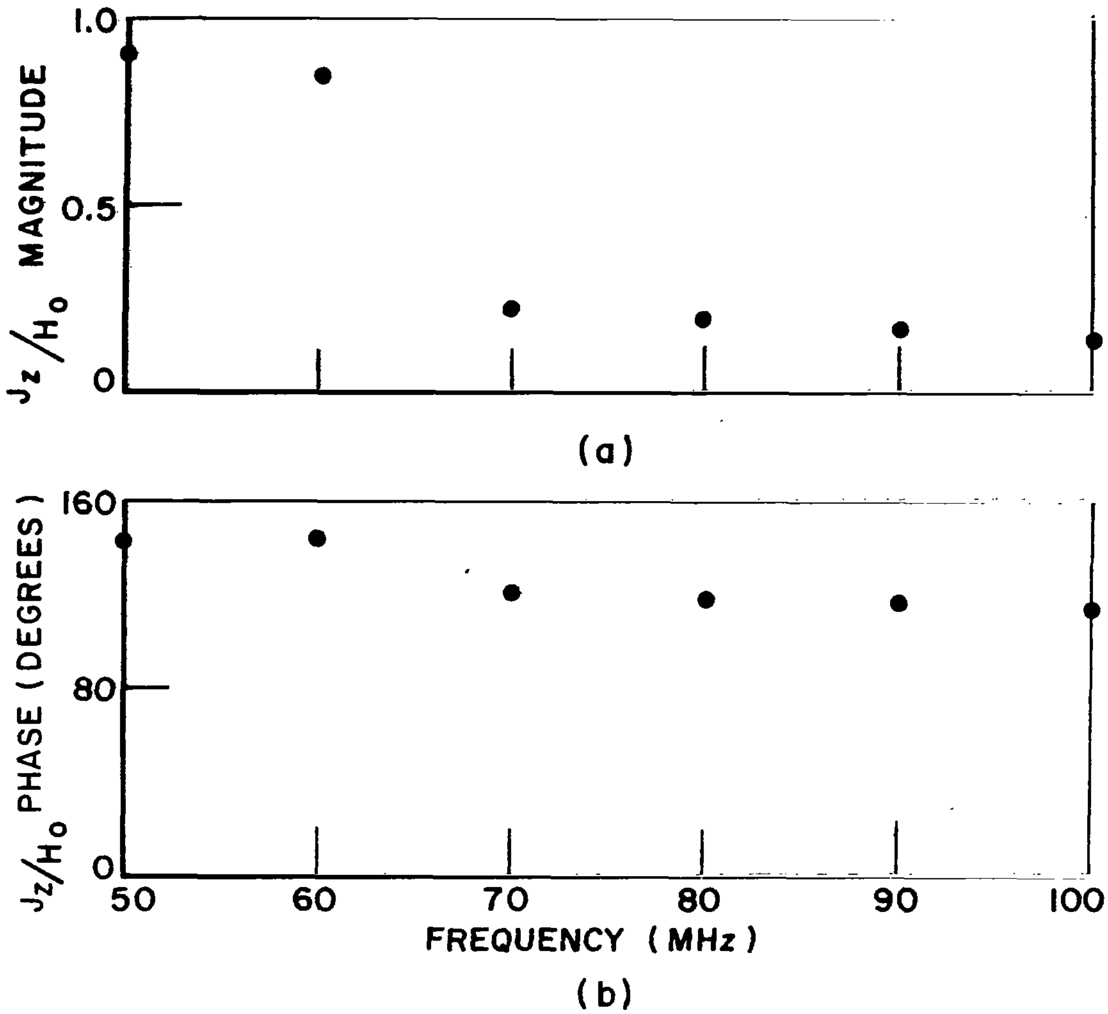

Fig. 13. The induced current density J_z/H_o at point 1.

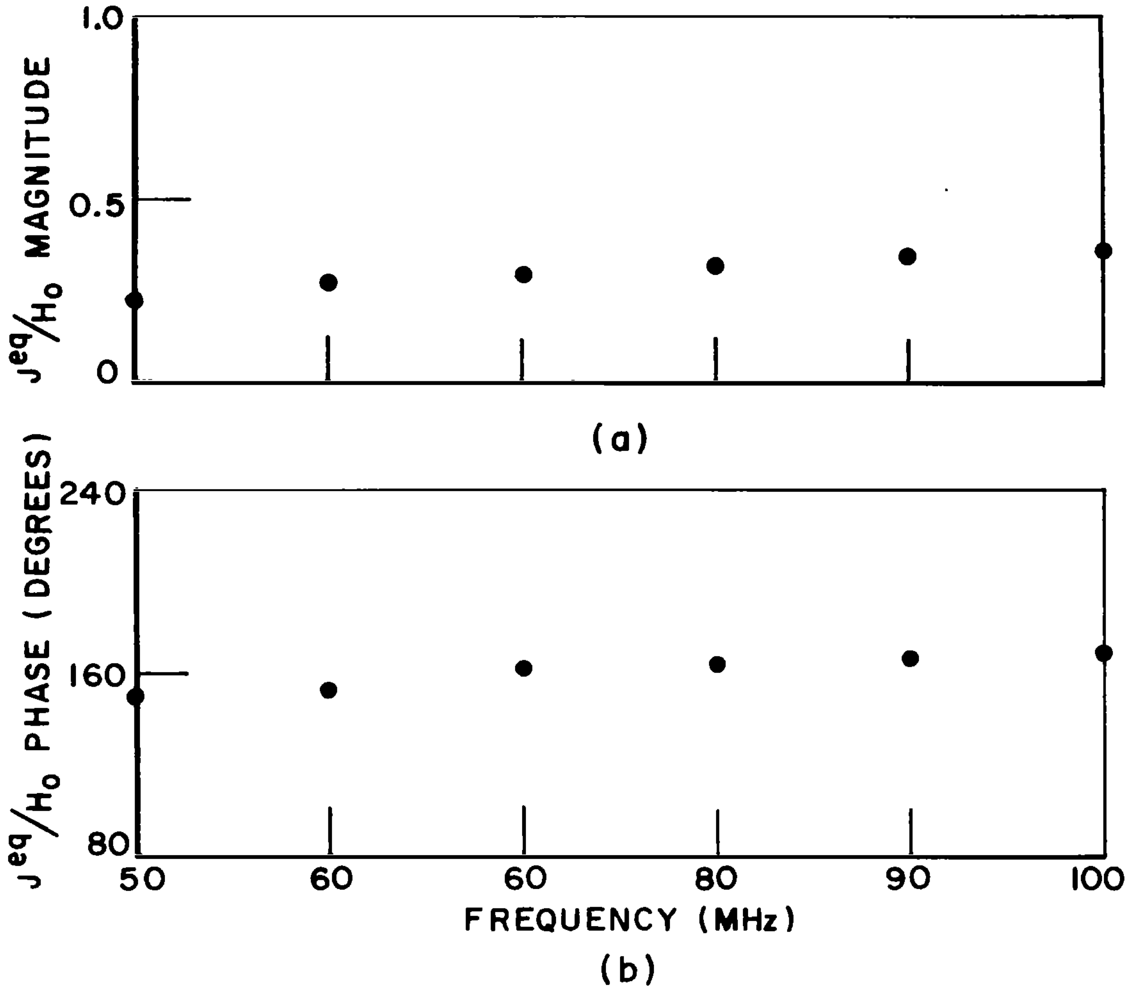

Fig. 14. The polarization current J^{eq}/H_o at point 2.

SECTION VI
SUMMARY AND DISCUSSIONS

The reaction concept and Galerkin's method are employed to develop an integral equation for the surface current density induced on a metal/dielectric body. For the two-dimensional TM case, the surface current density is expanded with strip bases while the polarization current is expanded with square cell bases. Reaction tests are enforced with electric test sources. This reaction-Galerkin technique yields accurate results as well as a symmetric impedance matrix for electromagnetic scattering problems. The theory is then applied to a two-dimensional foil type wing in which the leading edge includes a small amount of dielectric material. The effect of the dielectric-loaded foil type wing on the induced surface current density is then investigated.

In general, the above theory is limited by the capacity and accuracy of a computer to solve a system of simultaneous linear equations. A hybrid method[7], which combines the geometrical theory of diffraction and the moment method, can be used to reduce the unknowns in the system of linear equation. One can, then, solve scattering problems for higher frequencies or large objects in terms of wavelength. The theories discussed so far can be applied to treat the TE illumination in two-dimensional or three-dimensional electromagnetic problems.

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