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On Transient Electromagnetic Excitation of a Rectangular Cavity Through an Aperture

David K. Cheng
C. A. Chen
Syracuse University
Syracuse, New York 13210

Abstract

Induction theorem is used to formulate the problem of transient electromagnetic excitation of a rectangular cavity through an aperture. New variables are introduced to convert the governing second-order differential equation into a set of first-order equations which correspond to normalized state equations. This conversion will result in faster convergence in the numerical solution. Moment method is employed to solve the equations subject to specified boundary conditions. Cavity fields are expressed in terms of subsectional expansion functions with time-dependent coefficients, and external fields are represented as superpositions of plane waves. These fields are properly matched at the aperture. The procedure for evaluating a typical expansion-coefficient vector by singularity-expansion method is outlined. The formulation takes into account the effect of reflections from cavity walls on the aperture field and does not require that the aperture be small.

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I. INTRODUCTION

The study of the transient field behavior inside a conducting cavity due to excitation through an aperture by an incident electromagnetic pulse (EMP) is important because it is relevant to the understanding of the shielding effectiveness of actual situations. For example, the problem of electromagnetic penetration through door slits into aircraft and other installations due to strong radiation sources is of considerable practical interest.

Past investigations on EMP excitation of cavity-backed apertures have largely dealt with small openings and have neglected the effect of cavity reflections on the aperture field distribution. For small openings the quasi-static method is used to determine the fictitious magnetic current and charge distributions in the aperture. Equivalent electric and magnetic dipoles are defined, and their radiated fields determined with the aid of scalar and vector potentials.[1-8] The fields in the cavity are customarily expanded in terms of unperturbed normal modes. Some are steady-state time-harmonic solutions,[1-6] and tedious Fourier transformation of frequency-domain results would be required in order to find transient responses. The quasi-static approximation can not be applied when the aperture is not small and when early-time responses are important.

In neglecting the effect of the reflections from cavity walls on the aperture field distribution, one essentially treats the external and internal portions of the problem separately. Since cavity dimensions obviously play an important part in the total problem, this approach may result in significant errors.

In this report we propose to avoid the quasi-static approximation and to solve the internal and external portions of the problem simultaneously. The small-aperture assumption is not implied, and use is made of the induction theorem[9-10] in determining the scattered fields. The problem will be formulated in the transformed s-domain. For external field problems, such as those involving dipole radiation[11] and conducting-body scattering, frequency-domain and time-domain responses can be related by replacing the wavenumber k by s/c where s is the Laplace-transform variable and c the speed of light. However, this is not permissible in an internal field problem involving waveguides or cavities because the phase velocities of the various waveguide and cavity modes are different and are dependent on geometrical dimensions.

New variables will be introduced to convert the governing second-order differential equation into a set of first-order equations which correspond to normalized state equations. The field within the cavity will be expanded in terms of suitably chosen subsectional expansion functions with variable coefficients and the field outside the cavity expressed as a superposition of plane-wave fields. The cavity and the external fields are matched at the aperture where a fictitious magnetic current exists. A combined field expression containing the unknown expansion coefficients is obtained. To determine these coefficients the moment method[12] is used to convert the first-order equations into matrix equations. It will be shown that the typical coefficient matrix can be expressed in a form for which the singularity-expansion method[13] can be used to advantage.

The report contains the details of the theoretical formulation and the procedure of solution for the problem concerning transient electromagnetic excitation of a rectangular cavity through an aperture. Much computational work is involved in the application of the moment method and the singularity-expansion method. It is planned that appropriate numerical results will be presented in a subsequent report after work in this regard is completed.

II. FORMULATION OF THE PROBLEM

We consider the problem of a rectangular aperture in an infinite conducting plane backed by a rectangular conducting box, as shown in Fig. 1. An incident transient electromagnetic wave ($\vec{E}^i$, $\vec{H}^i$) impinges normally on the plane and the aperture. The problem is to determine the scattered field in the y > 0 region and the field penetrated through the aperture into the conducting cavity.

II-1. Equivalent Aperture Magentic Current

We shall invoke the induction theorem for the solution of this problem. Figure 2(a) represents a simplified 2-dimensional view of the original problem. ($\vec{E}_c$, $\vec{H}_c$) and ($\vec{E}_s$, $\vec{H}_s$) are, respectively, the cavity field and the external scattered field. In order to determine these unknown fields, we consider the case when the aperture is covered by a conductor. The entire region to the left of the infinite plane will have a null field and, according to the induction theorem, a magnetic current $\vec{M}_o$ on the right surface of the conducting plane will support a different scattered field ($\vec{E}_s^o$, $\vec{H}_s^o$), as shown in Fig. 2(b), where

$$\begin{aligned} \vec{M}_o &= \vec{E}_s^o \times \hat{n} \\ &= \hat{n} \times \vec{E}^i \\ &= \hat{y} \times \vec{E}^i \ . \end{aligned} \tag{1}$$

For a normally incident plane wave ($\vec{E}^i$, $\vec{H}^i$), the scattered field ($\vec{E}_s^o$, $\vec{H}_s^o$) from an infinite conducting plane without an aperture is easily determined. The null field to the left of the plane will be maintained if the plane is removed and a magnetic current $2\vec{M}_o$ exists in its place which will result in a field $(\vec{E}^i + \vec{E}_s^o, \vec{H}^i + \vec{H}_s^o)$ in the $y > 0$ region, as shown in Fig. 2(c).

Subtracting the fields in Fig. 2(c) from those in Fig. 2(a), we obtain the problem in Fig. 2(d). The magnetic current $\vec{M}$ in the aperture is

$$\vec{M} = -2\vec{M}_o = -\hat{y} \times \vec{E}^i \tag{2}$$

which supports the field $(\vec{E}_c, \vec{H}_c)$ inside the cavity and a field $(\vec{E}_s - \vec{E}_s^o, \vec{H}_s - \vec{H}_s^o)$ to the right of the infinite plane. We note that the region in which the difference field $(\vec{E}_s - \vec{E}_s^o, \vec{H}_s - \vec{H}_s^o)$ exists is source-free and that the tangential component of the electric field is required to vanish on conducting walls.

II-2. Transformed Governing Equations

For the problem in Fig. 2(d), we start from the two Maxwell's curl equations

$$\vec{\nabla} \times \vec{E} = -\mu_o \frac{\partial \vec{H}}{\partial t} - \vec{M} \tag{3}$$

$$\vec{\nabla} \times \vec{H} = \varepsilon_o \frac{\partial \vec{E}}{\partial t} . \tag{4}$$

Taking the Laplace transform of Eqs. (3) and (4), we obtain

$$\vec{\nabla} \times \tilde{\vec{E}} = -\mu_o s \tilde{\vec{H}} - \tilde{\vec{M}} \tag{5}$$

$$\vec{\nabla} \times \tilde{\vec{H}} = \varepsilon_o s \tilde{\vec{E}} \tag{6}$$

where a tilde (~) over a quantity denotes the Laplace transform of that quantity.

Let $\tilde{\vec{F}}$ be the Laplace transform of an electric vector potential $\vec{F}$ such that

$$\tilde{\vec{E}} = - \vec{\nabla} \times \tilde{\vec{F}} . \tag{7}$$

Combining Eqs. (5) to (7) and using the Lorentz gauge, we have an inhomogeneous Helmholtz equation:

$$\vec{\nabla}^2 \tilde{\vec{F}} - \mu_o \varepsilon_o s^2 \tilde{\vec{F}} = - \tilde{\vec{M}} , \tag{8}$$

Solution of Eq. (8) for $\tilde{\vec{F}}$ will give $\tilde{\vec{E}}$ from Eq. (7) and $\tilde{\vec{H}}$ from

$$\tilde{\vec{H}} = \frac{1}{\mu_o s} \vec{\nabla} \times \vec{\nabla} \times \tilde{\vec{F}} \tag{9}$$

in regions where $\vec{M}$ is zero.

Assuming an incident plane wave with the electric field polarized in the z-direction;* i.e.,

$$\tilde{\vec{E}}^i = \hat{z} E_z^i , \tag{10}$$

the Laplace transform of Eq. (2) becomes

$$\tilde{\vec{M}} = - 2\hat{x} \tilde{E}_z^i = \hat{x} \tilde{M}_x \tag{11}$$

which has only an x-component. The x-component of Eq. (8) is then

$$\vec{\nabla}^2 \tilde{F}_x - \mu_o \varepsilon_o s^2 \tilde{F}_x = - \tilde{M}_x \delta(y) \tag{12}$$

where $\delta(y)$ is a Dirac delta function. From Eqs. (7) and (9), we have

$$\tilde{E}_x = 0 \tag{13}$$

$$\tilde{E}_y = - \frac{\partial}{\partial z} \tilde{F}_x \tag{14}$$

$$\tilde{E}_z = \frac{\partial}{\partial y} \tilde{F}_x \tag{15}$$

$$\tilde{H}_x = - \frac{1}{\mu_o s} \left[\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] \tilde{F}_x \tag{16}$$

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*There is little loss in generality by specifying the polarization of the incident wave since the shape and orientation of the aperture are still arbitrary.

$$\tilde{H}_y = \frac{1}{\mu_o s} \frac{\partial^2}{\partial x \partial y} \tilde{F}_x \tag{17}$$

$$\tilde{H}_z = \frac{1}{\mu_o s} \frac{\partial^2}{\partial x \partial z} \tilde{F}_x \; . \tag{18}$$

The coupling between the fields inside and outside the cavity will be accounted for by the consideration of the boundary conditions at the aperture.

The second-order differential equation (12) can be represented as a set of first-order equations by defining new quantities $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ such that

$$- \frac{\partial}{\partial x} \tilde{F}_x(r,s) = s \; \tilde{u}(r,s) \tag{19}$$

$$- \frac{\partial}{\partial y} \tilde{F}_x(r,s) = s \; \tilde{v}(r,s) \tag{20}$$

and

$$- \frac{\partial}{\partial z} \tilde{F}_x(r,s) = s \; \tilde{w}(r,s) \tag{21}$$

where r is the space variable. We have, from Eq. (12),

$$\frac{\partial}{\partial x} \tilde{u}(r,s) + \frac{\partial}{\partial y} \tilde{v}(r,s) + \frac{\partial}{\partial z} \tilde{w}(r,s) = - \mu_o \varepsilon_o s \; \tilde{F}_x(r,s) + \frac{1}{s} \tilde{M}_x \; . \tag{22}$$

Comparing Eqs. (21) and (20) with Eqs. (14) and (13) respectively, we see that

$$\tilde{E}_y = s \; \tilde{w} \tag{23}$$

and

$$\tilde{E}_z = - s \; \tilde{v} \; . \tag{24}$$

The introduction of $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ and the use of the first-order equations will result in significantly faster convergence in the numerical solution.

III-3. Operator Equations

In order to write the first-order equations (19) to (22) in a succinct form, we define

$$L = \begin{bmatrix} 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{\partial}{\partial x} & 0 & 0 & 0 \\ -\frac{\partial}{\partial y} & 0 & 0 & 0 \\ -\frac{\partial}{\partial z} & 0 & 0 & 0 \end{bmatrix} \tag{25}$$

$$P = \begin{bmatrix} -\mu_o \varepsilon_o & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{26}$$

$$\tilde{f}(r,s) = \begin{bmatrix} \tilde{F}_x \\ \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} \tag{27}$$

and

$$\tilde{e}_g(s) = \begin{bmatrix} \frac{1}{s} \tilde{M}_x \\ 0 \\ 0 \\ 0 \end{bmatrix} . \tag{28}$$

We can then write Eqs. (19) to (22) as

$$L\ \tilde{f}(r,s) = s\ P\ \tilde{f}(r,s) + \tilde{e}_g(r,s) . \tag{29}$$

Note that the inverse Laplace transform of Eq. (29) is a set of normalized state equations in the four state variables F_x, u, v, and w.

Let an inner product be defined as

$$<\tilde{f}_1,\tilde{f}_2> = \int \tilde{f}_1^T \cdot \tilde{f}_2 \, d\tau$$

$$= \int (\tilde{F}_{x1}\tilde{F}_{x2} + \tilde{u}_1\tilde{u}_2 + \tilde{v}_1\tilde{v}_2 + \tilde{w}_1\tilde{w}_2)d\tau \qquad (30)$$

where the superscript T denotes transposition and the volume integration is carried over the rectangular cavity and the half-space to the right of the infinite plane.

$$<\tilde{f}_1, L\tilde{f}_2> = \int \{\tilde{F}_{x1} (\frac{\partial}{\partial x} \tilde{u}_2 + \frac{\partial}{\partial y} \tilde{v}_2 + \frac{\partial}{\partial z} \tilde{w}_2)$$

$$- \tilde{u}_1 \frac{\partial}{\partial x} \tilde{F}_{x2} - \tilde{v}_1 \frac{\partial}{\partial y} \tilde{F}_{x2} - \tilde{w}_1 \frac{\partial}{\partial z} \tilde{F}_{x2}\}d\tau \ . \qquad (31)$$

By making use of the relation

$$\frac{\partial}{\partial x} (\tilde{F}_{x1}\tilde{u}_2 - \tilde{u}_1\tilde{F}_{x2}) = \tilde{F}_{x1} \frac{\partial}{\partial x} \tilde{u}_2 + \tilde{u}_2 \frac{\partial}{\partial x} \tilde{F}_{x1}$$

$$- \tilde{F}_{x2} \frac{\partial}{\partial x} \tilde{u}_1 - \tilde{u}_1 \frac{\partial}{\partial x} \tilde{F}_{x2} \qquad (32)$$

and similar ones for $\frac{\partial}{\partial y} (\tilde{F}_{x1}\tilde{v}_2 - \tilde{v}_1\tilde{F}_{x2})$ and $\frac{\partial}{\partial z} (\tilde{F}_{x1}\tilde{w}_2 - \tilde{w}_1\tilde{F}_{x2})$, Eq. (31) can be rewritten as

$$<\tilde{f}_1, L\tilde{f}_2> = \int \{\tilde{F}_{x2}(\frac{\partial}{\partial x} \tilde{u}_1 + \frac{\partial}{\partial y} \tilde{v}_1 + \frac{\partial}{\partial z} \tilde{w}_1) - \tilde{u}_2 \frac{\partial}{\partial x} \tilde{F}_{x1}$$

$$- \tilde{v}_2 \frac{\partial}{\partial y} \tilde{F}_{x1} - \tilde{w}_2 \frac{\partial}{\partial z} \tilde{F}_{x1}\}d\tau$$

$$+ \int \{\frac{\partial}{\partial x} (\tilde{F}_{x1} \tilde{u}_2 - \tilde{u}_1\tilde{F}_{x2}) + \frac{\partial}{\partial y} (\tilde{F}_{x1}\tilde{v}_2 - \tilde{v}_1\tilde{F}_{x2})$$

$$+ \frac{\partial}{\partial z} (\tilde{F}_{x1}\tilde{w}_2 - \tilde{w}_1\tilde{F}_{x2})\}d\tau \ . \qquad (33)$$

We note that the first volume integral on the right-hand side of Eq. (33) is exactly the inner product $<L\tilde{f}_1, \tilde{f}_2>$. Each term in the integrand of the second volume integral can be written as a divergence operation; hence the second volume integral can be changed to a surface integral by virtue of the divergence theorem. For example:

$$\int \frac{\partial}{\partial x} (\tilde{F}_{x1}\tilde{u}_2 - \tilde{u}_1\tilde{F}_{x2})d\tau = \int \vec{\nabla} \cdot \hat{x} (\tilde{F}_{x1}\tilde{u}_2 - \tilde{u}_1\tilde{F}_{x2})d\tau$$

$$= \oint (\tilde{F}_{x1}\tilde{u}_2 - \tilde{u}_1\tilde{F}_{x2}) \hat{x} \cdot \hat{n}da \qquad (34)$$

where $\hat{n}$ is an outward unit vector normal to the surface da. Thus, the second volume integral on the right-hand side of Eq. (33) can be converted to a surface integral as follows:

$$\int \{\frac{\partial}{\partial x} (\tilde{F}_{x1}\tilde{u}_2 - \tilde{u}_1\tilde{F}_{x2}) + \frac{\partial}{\partial y} (\tilde{F}_{x1}\tilde{v}_2 - \tilde{v}_1\tilde{F}_{x2}) + \frac{\partial}{\partial z} (\tilde{F}_{x1}\tilde{w}_2 - \tilde{w}_1\tilde{F}_{x2})\}d\tau$$

$$= \oint \{(\tilde{F}_{x1}\tilde{u}_2 - \tilde{u}_1\tilde{F}_{x2}) \hat{x} \cdot \hat{n} + (\tilde{F}_{x1}\tilde{v}_2 - \tilde{v}_1\tilde{F}_{x2})\hat{y} \cdot \hat{n} + (\tilde{F}_{x1}\tilde{w}_2 - \tilde{w}_1\tilde{F}_{x2})\hat{z} \cdot \hat{n}\}da. \qquad (35)$$

Now the boundary conditions in Fig. 1 require that $\tilde{E}_y$ vanish at $x = 0$, a and at $z = 0$, c and that $\tilde{E}_z$ vanish at $x = 0$, a and at $y = -c$, 0 except at the aperture. In view of Eqs. (23) and (24), $\tilde{w}$ and $\tilde{v}$ satisfy the same boundary conditions as $\tilde{E}_y$ and $\tilde{E}_z$ respectively. The surface integral over the right half-space is zero as $r \to \infty$; hence, the surface integral on the right-hand side of Eq. (34) will vanish if $\tilde{F}_x$ is zero at $x = 0$ and a. Consequently, the second volume integral on the right-hand side of Eq. (33) is zero and

$$<\tilde{f}_1, L\tilde{f}_2> = <L\tilde{f}_1, \tilde{f}_2> . \qquad (36)$$

Equation (36) is a statement of the self-adjointness of the operator L. In numerical computation the expansion functions for $\tilde{F}_x$ will be chosen in such a way that $\tilde{F}_x$ is zero at $x = 0$ and $x = a$.

III. SOLUTION BY MOMENT METHOD

The method of moments[12] will be used to solve the operator equation (29) for the problem at hand. Four steps are involved here. First, the space inside the cavity is divided into subsections and suitable expansion functions are chosen over the subsections. The elements of the unknown vector $\tilde{f}(r,s)$ in Eq. (27) are then expressed in terms of the expansion functions within the cavity. Second, the field in the y > 0 region is expressed as a superposition of plane waves. Third, the cavity and the half-space fields are matched at the aperture. Fourth, inner products are taken so that the matrix equations for the unknown expansion coefficients are obtained. These steps are developed below.

III-1. Expansions Functions for Cavity Field

Assume that the space within the cavity is suitably subdivided in the x, y, and z directions and expansion functions $F_{x(i,j,k)}(r)$, $u_{(i,j,k)}(r)$, $v_{(i,j,k)}(r)$, and $w_{(i,j,k)}(r)$ are chosen over the subsections. The expansion functions must satisfy the required boundary conditions. For convenience, we define the following column vectors:

$$f^F_{(i,j,k)}(r) = \begin{bmatrix} F_{x(i,j,k)}(r) \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad f^u_{(i,j,k)}(r) = \begin{bmatrix} 0 \\ u_{(i,j,k)}(r) \\ 0 \\ 0 \end{bmatrix}$$

$$f^v_{(i,j,k)}(r) = \begin{bmatrix} 0 \\ 0 \\ v_{(i,j,k)}(r) \\ 0 \end{bmatrix}, \quad f^w_{(i,j,k)}(r) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_{(i,j,k)}(r) \end{bmatrix} . \tag{37}$$

In view of Eq. (27), we can then write the expanded form of $\tilde{f}(r,s)$ inside the cavity as

$$\tilde{f}(r,s) = \sum_{i,j,k} \{\tilde{\alpha}_{(i,j,k)}(s)\, f^{F}_{(i,j,k)}(r) + \tilde{\beta}_{(i,j,k)}(s)\, f^{u}_{(i,j,k)}(r) + \tilde{\gamma}_{(i,j,k)}(s)\, f^{v}_{(i,j,k)}(r) + \tilde{\delta}_{(i,j,k)}(s)\, f^{w}_{(i,j,k)}(r)\}. \quad (38)$$

Note that the expansion functions F_x, u, v, and w are functions of position only and that the inverse transformation of $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\delta}$ will yield the time-varying expansion coefficients.

III-2. Plane-Wave Representation for Field in Half-Space

In the half-space $y \geq 0$, $\tilde{E}_z$ can be expressed as a superposition of plane waves

$$\tilde{E}_z = \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{g}^E(k_x,k_z) e^{-j(k_x x + k_y y + k_z z)} dk_x dk_z \quad (39)$$

where

$$jk_y = \sqrt{(s/c)^2 + k_x^2 + k_z^2} \quad (40)$$

and the new quantity $\tilde{g}^E(k_x,k_z)$ can be determined from the boundary condition at the aperture. At y = 0, we have

$$\tilde{E}_z\Big|_{y=0} = \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{g}^E(k_x,k_z) e^{-j(k_x x + k_z z)} dk_x dk_z$$

$$= \begin{cases} -s \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\tilde{m}_{x(i,n_y,k)}(s)] v_{(i,n_y,k)}(x,0,z), & \text{at aperture} \\ 0, & \text{elsewhere} \end{cases} \quad (41)$$

where n_y is the subsection number for the cavity in the y direction at y = 0, j = 1 being assigned to the first subsection at y = - b. It has been assumed in Eq. (41) that the magnetic current $\tilde{M}_x$ in Eq. (11), which represents the discontinuity in $\tilde{E}_z$ at the aperture, has been expanded in terms of the expansion functions $v_{(i,n_y,k)}(x,0,z)$:

$$\tilde{M}_x = \sum_{i,k} \tilde{m}_{x(i,n_y,z)}(s)\; v_{(i,n_y,k)}(x,0,z) \; . \tag{42}$$

From Eq. (41), $\tilde{g}^E(k_x,k_z)$ can be determined by an inverse Fourier transformation.

$$\tilde{g}^E(k_x,k_z) = - s \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\, \tilde{m}_{x(i,n_y,k)}(s)] \cdot \iint\limits_{\text{aperture}} v_{(i,n_y,k)}(x,0,z)\; e^{j(k_x x + k_z z)}\, dxdz$$

$$= - s \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\, \tilde{m}_{x(i,n_y,k)}(s)] G^V(k_x,k_z) \tag{43}$$

where

$$G^V(k_x,k_z) = \iint\limits_{\text{aperture}} v_{(i,n_y,k)}(x,0,z)\; e^{j(k_x x + k_z z)}\, dxdz \; . \tag{44}$$

Substituting Eq. (43) in Eq. (39), we can write $\tilde{E}_z$ in the $y \geq 0$ region as

$$\tilde{E}_z(r,s) = - \frac{s}{4\pi} \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\, \tilde{m}_{x(i,n_y,k)}(s)] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G^V(k_x,k_z)\; e^{-j(k_x x + k_y y + k_z z)}\, dk_x dk_z \; . \tag{45}$$

Using Eqs. (45), (19) to (21), and (24), we obtain the following expressions for $\tilde{F}_x$, $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ in the $y \geq 0$ region.

$$\tilde{F}_x(r,s) = \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\, \tilde{m}_{x(i,n_y,k)}(s)]\; \tilde{F}'_{x(i,n_y,k)}(x,y,z,s) \tag{46}$$

with

$$\tilde{F}'_{x(i,n_y,k)}(x,y,z,s) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{s}{k_y} G^V(k_x,k_z)e^{-j(k_x x+k_y y+k_z z)} dk_x dk_z \quad (47)$$

$$\tilde{u}(r,s) = \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\tilde{m}_{x(i,n_y,k)}(s)]\tilde{u}'_{(i,n_y,k)}(x,y,z,s) \quad (48)$$

with

$$\tilde{u}'_{(i,n_y,k)}(x,y,z,s) = -\frac{j}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\frac{k_x}{k_y})G^V(k_x,k_z)e^{-j(k_x x+k_y y+k_z z)} dk_x dk_z \quad (49)$$

$$\tilde{v}(r,s) = \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\tilde{m}_{x(i,n_y,k)}(s)]\tilde{v}'_{(i,n_y,k)}(x,y,z,s) \quad (50)$$

with

$$\tilde{v}'_{(i,n_y,k)}(x,y,z,s) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G^V(k_x,k_z)e^{-j(k_x x+k_y y+k_z z)} dk_x dk_z \quad (51)$$

and

$$\tilde{w}(r,s) = \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\tilde{m}_{x(i,n_y,k)}(s)]\tilde{w}'_{(i,n_y,k)}(x,y,z,s) \quad (52)$$

with

$$\tilde{w}'_{(i,n_y,k)}(x,y,z,s) = -\frac{j}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\frac{k_z}{k_y})G^V(k_x,k_z)e^{-j(k_x x+k_y y+k_z z)} dk_x dk_y . \quad (53)$$

III-3. Field-Matching at Aperture

At the boundary y = 0 between the cavity and the half-space, there is a discontinuity in $\tilde{E}_z$ due to the existence of the equivalent magnetic current, as given in Eq. (41). Matching $\tilde{F}_x$, ũ, and ṽ at the aperture by setting j = n_y in Eq. (38) and y = 0 in Eqs. (46), (48), and (52), we obtain the following equations:

$$\sum_{i,k} \tilde{\alpha}_{(i,n_y,k)}(s)F_{x(i,n_y,k)}(x,0,z)$$
$$= \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\tilde{m}_{x(i,n_y,k)}(s)]\tilde{F}'_{x\,(i,n_y,k)}(x,0,z,s) \quad (54)$$

$$\sum_{i,k} \tilde{\beta}_{(i,n_y,k)}(s) u_{(i,n_y,k)}(x,0,z)$$

$$= \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s} \tilde{m}_{x(i,n_y,k)}(s)] \tilde{u}'_{(i,n_y,k)}(x,0,z,s) \qquad (55)$$

and

$$\sum_{i,k} \tilde{\delta}_{(i,n_y,k)}(s) w_{(i,n_y,k)}(x,0,z)$$

$$= \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s} \tilde{m}_{x(i,n_y,k)}(s)] \tilde{w}'_{(i,n_y,k)}(x,0,z,s). \qquad (56)$$

An important task of this problem is the determination of the expansion coefficients $\tilde{\alpha}_{(i,n_y,k)}(s)$, $\tilde{\beta}_{(i,n_y,k)}(s)$, and $\tilde{\delta}_{(i,n_y,k)}(s)$ in terms of $\tilde{\gamma}_{(i,n_y,k)}(s)$ and $\tilde{m}_{x(i,n_y,k)}(s)$. To this end we take the inner products of Eqs. (54) to (56) with $F_{x(i',n_y,k')}(x,0,z)$, $u_{(i',n_y,k')}(x,0,z)$, and $w_{(i',n_y,k')}(x,0,z)$, respectively, over the aperture and obtain

$$\sum_{i,k} \tilde{\alpha}_{(i,n_y,k)}(s) \langle F_{x(i',n_y,k')}, F_{x(i,n_y,k)} \rangle$$

$$= \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s} \tilde{m}_{x(i,n_y,k)}(s)] \langle F_{x(i',n_y,k')}, \tilde{F}'_{x(i,n_y k)} \rangle \qquad (57)$$

$$\sum_{i,k} \tilde{\beta}_{(i,n_y,k)}(s) \langle u_{(i',n_y,k')}, u_{(i,n_y,k)} \rangle$$

$$= \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s} \tilde{m}_{x(i,n_y,k)}(s)] \langle u_{(i',n_y,k')} \tilde{u}'_{(i,n_y,k)} \rangle \qquad (58)$$

and

$$\sum_{i,k} \tilde{\delta}_{(i,n_y,k)}(s) \langle w_{(i',n_y,k')}, w_{(i,n_y,k)} \rangle$$

$$= \sum_{i,k} [\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s} \tilde{m}_{x(i,n_y,k)}(s)] \langle w_{(i',n_y,k')}, \tilde{w}'_{(i,n_y,k)} \rangle . \qquad (59)$$

It is convenient to write Eqs. (57) to (59) in a matrix form by introducing column matrices $\{\tilde{\alpha}_{(i,n_y,k)}(s)\}$, $\{\tilde{\beta}_{(i,n_y,k)}(s)\}$, $\{\tilde{\gamma}_{(i,n_y,k)}(s)\}$, $\{\tilde{\delta}_{(i,n_y,k)}(s)\}$, and $\{\frac{1}{s}\tilde{m}_{x(i,n_y,k)}(s)\}$. We have

$$\{\tilde{\alpha}_{(i,n_y,k)}(s)\} = [A]\{\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\tilde{m}_{x(i,n_y,k)}(s)\} \tag{60}$$

$$\{\tilde{\beta}_{(i,n_y,k)}(s)\} = [B]\{\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\tilde{m}_{x(i,n_y,k)}(s)\} \tag{61}$$

and

$$\{\tilde{\delta}_{(i,n_y,k)}(s)\} = [D]\{\tilde{\gamma}_{(i,n_y,k)}(s) + \frac{1}{s}\tilde{m}_{x(i,n_y,k)}(s)\} \tag{62}$$

where the square matrices [A], [B], and [D] are defined in terms of inner products evaluated over the aperture.

$$[A] = [<F_{x(i',n_y,k')}, F_{x(i,n_y,k)}>]^{-1}[<F_{x(i',n_y,k')}, \tilde{F}'_{x(i,n_y,k)}>] \tag{63}$$

$$[B] = [<u_{(i',n_y,k')}, u_{(i,n_y,k)}>]^{-1}[<u_{(i',n_y,k')}, \tilde{u}'_{(i,n_y,k)}>] \tag{64}$$

$$[D] = [<w_{(i',n_y,k')}, w_{(i,n_y,k)}>]^{-1}[<w_{(i,'n_y,k')}, \tilde{w}'_{(i,n_y,k)}>] \; . \tag{65}$$

III-4. Combined Field Expression

We can now combine Eqs. (38), (46), (48), (50), (52), and (54) to (56) and write a combined field expression for the transformed vector $\tilde{f}(r,s)$ defined in Eq. (27) that holds inside the cavity, in the $y > 0$ half-space, as well as in the aperture:

$$\tilde{f}(r,s) = \sum_{\substack{i,k \\ j\neq n_y}} \{\tilde{\alpha}_{(i,j,k)}(s)\ f^F_{(i,j,k)}(r) + \tilde{\beta}_{(i,j,k)}(s)\ f^u_{(i,j,k)}(r)$$

$$+ \tilde{\gamma}_{(i,j,k)}(s)\ f^v_{(i,j,k)}(r) + \tilde{\delta}_{(i,j,k)}(s)\ f^w_{(i,j,k)}(r)\}$$

$$+ \sum_{\substack{i,k \\ j=n_y}} \tilde{\gamma}_{(i,n_y,k)}(s)\ \tilde{f}_{n_y}(r,s) + \sum_{\substack{i,k \\ j=n_y}} \frac{1}{s}\, \tilde{m}_{x(i,n_y,k)}(s)\ \tilde{f}_{m_x}(r,s) \qquad (66)$$

where

$$\tilde{f}_{n_y}(r,s) = \begin{Bmatrix} \tilde{F}'_{x(i,n_y,k)}(r,s) + \sum_{i',k'} A_{(i',k';i,k)}\ F_{x(i',n_y,k')}(x,y,z) \\ \tilde{u}'_{(i,n_y,k)}(r,s) + \sum_{i',k'} B_{(i',k';i,k)}\ u_{(i',n_y,k')}(x,y,z) \\ \tilde{v}'_{(i,n_y,k)}(r,s) + v_{(i,n_y,k)}(x,y,z) \\ \tilde{w}'_{(i,n_y,k)}(r,s) + \sum_{i',k'} D_{(i',k';i,k)}\ w_{(i',n_y,k')}(x,y,z) \end{Bmatrix} \qquad (67)$$

and

$$\tilde{f}_{m_x}(r,s) = \begin{Bmatrix} \tilde{F}'_{x(i,n_y,k)}(r,s) + \sum_{i',k'} A_{(i',k';i,k)}\ F_{x(i',n_y,k')}(x,y,z) \\ \tilde{u}'_{(i,n_y,k)}(r,s) + \sum_{i',k'} B_{(i',k';i,k)}\ u_{(i',n_y,k')}(x,y,z) \\ \tilde{v}'_{(i,n_y,k)}(r,s) \\ \tilde{w}'_{(i,n_y,k)}(r,s) + \sum_{i',k'} D_{(i',k';i,k)}\ w_{(i',n_y,k')}(x,y,z) \end{Bmatrix} \qquad (68)$$

where $A_{(i',k';i,k)}$, $B_{(i',k';i,k)}$, and $D_{(i',k';i,k)}$ are, respectively, typical elements in the square matrices [A], [B], and [D] defined in Eqs. (63) to (65).

III-5. The Matrix Equations

Preparatory to solving for the expansion coefficients $\tilde{\alpha}_{(i,j,k)}(s)$, $\tilde{\beta}_{(i,j,k)}(s)$, $\tilde{\gamma}_{(i,j,k)}(s)$, and $\tilde{\delta}_{(i,j,k)}(s)$ in Eq. (66), it is expedient to arrange their values for the different indices in a column-matrix form and represent them simply as $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\delta}$ respectively. Substituting Eq. (66) in Eq. (29) and taking the inner product of the resulting equation with respect to $f^F_{(i,j,k)}$, $f^u_{(i,j,k)}$, $f^v_{(i,j,k)}$ and $f^w_{(i,j,k)}$ defined in Eq. (37) (with respect to $\tilde{f}_{n_y}$ defined in Eq. (67) when $j = n_y$), we obtain the following matrix equation:

$$\left[\begin{array}{c|c|c|c} 0 & \ell^{Fu}_{mn} & \ell^{Fv}_{mn} & \ell^{Fw}_{mn} \\ \hline \ell^{uF}_{mn} & 0 & \ell^{uv}_{mn} & 0 \\ \hline \ell^{vF}_{mn} & \ell^{vu}_{mn} & \ell^{vv}_{mn} & \ell^{vw}_{mn} \\ \hline \ell^{wf}_{mn} & 0 & \ell^{wv}_{mn} & 0 \end{array}\right] \left[\begin{array}{c} \tilde{\alpha} \\ \hline \tilde{\beta} \\ \hline \tilde{\gamma} \\ \hline \tilde{\delta} \end{array}\right] = s \left[\begin{array}{c|c|c|c} p^{FF}_{mn} & 0 & p^{Fv}_{mn} & 0 \\ \hline 0 & p^{uu}_{mn} & p^{uv}_{mn} & 0 \\ \hline p^{vF}_{mn} & p^{vu}_{mn} & p^{vv}_{mn} & p^{vw}_{mn} \\ \hline 0 & 0 & p^{wv}_{mn} & p^{ww}_{mn} \end{array}\right] \left[\begin{array}{c} \tilde{\alpha} \\ \hline \tilde{\beta} \\ \hline \tilde{\gamma} \\ \hline \tilde{\delta} \end{array}\right]$$

$$+ \left[\begin{array}{c} q^F \\ \hline q^u \\ \hline q^v \\ \hline q^w \end{array}\right] \left\{\frac{1}{s}\, \tilde{m}_{x(i',n_y,k')}(s)\right\} + \left[\begin{array}{c} 0 \\ \hline 0 \\ \hline \tilde{C}(s) \\ \hline 0 \end{array}\right] \qquad (69)$$

where C(s) is a column matrix

$$\{\tilde{C}(s)\} = \begin{cases} 0 \quad , \quad j \neq n_y \\ \{<\tilde{F}'_x, \frac{1}{s}\tilde{M}_x(s)>\} \ , \end{cases} \qquad (70)$$

and ℓ_{mn}'s, p_{mn}'s, and q's are themselves matrices arising from inner products. The expressions for ℓ_{mn}'s, p_{mn}'s, and q's are given in the Appendix. m and n are indices locating the position of a particular subsection over which an inner product is taken.

$$m = i + (j-1)n_x + (k-1)n_x n_y \tag{71}$$

$$n = i' + (j' - 1)\, n_x + (k' - 1)n_x n_y \tag{72}$$

where n_x and n_y are the numbers of subsections in the x and y directions respectively. We note again that Eq. (69) is in fact a set of transformed state equations in $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\delta}$.

IV. FIELD EVALUATION BY SINGULARITY-EXPANSION METHOD

IV-1. A Typical Equation for Expansion Coefficient

Although the component matrices in Eq. (69), as detailed in the Appendix, appear highly complex, we note that those (ℓ_{mn}^{uv}, ℓ_{mn}^{vu}, ℓ_{mn}^{vv}, ℓ_{mn}^{vw}, p_{mn}^{Fv}, p_{mn}^{uv}, p_{mn}^{vF}, p_{mn}^{vu}, p_{mn}^{vw}, and p_{mn}^{wv}) representing the coupling between cavity and external fields because of the existence of the aperture are sparse, having only a few nonzero elements. The unknown coefficient matrices $\{\tilde{\alpha}(s)\}$, $\{\tilde{\beta}(s)\}$, $\{\tilde{\gamma}(s)\}$, and $\{\tilde{\delta}(s)\}$ can be solved from Eq. (69). Typically an equation of the following form is obtained.

$$[\tilde{Z}(s)]\ \{\tilde{\alpha}(s)\} = [\tilde{H}(s)]\ \{\frac{1}{s}\ \tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\} \tag{73}$$

where $[\tilde{Z}(s)]$ and $[\tilde{H}(s)]$ are very complicated square matrices containing ℓ_{mn}'s, p_{mn}'s, and q_{mn}'s in Eq. (69), and $\{\tilde{K}(s)\}$ is in general not the same as $\{\tilde{C}(s)\}$ in Eq. (70). However, because of the sparseness of many of the component matrices, the numerical evaluation of $[\tilde{Z}(s)]$ and $[\tilde{H}(s)]$ is not very difficult. This is especially so when the aperture is a narrow slot. The equations for the other unknown expansion coefficients $\{\tilde{\beta}(s)\}$, $\{\tilde{\gamma}(s)\}$, and $\{\tilde{\delta}(s)\}$ are of the same form as the equation for $\{\tilde{\alpha}(s)\}$ in Eq. (73).

In the following section the procedure for solving such an equation by the singularity-expansion method is outlined.

IV-2. Solution by Singularity Expansion

The singularity-expansion method for solving transient electromagnetic boundary-value problems was first formalized by Baum.[13] It expresses the solution in terms of natural frequencies, natural modes and coupling coefficients, and the time-domain response is a summation of singularity terms. The natural frequencies and natural modes are independent of the incident-wave parameters which affect only the coupling coefficients.

Consider Eq. (73) from which the typical expansion coefficient matrix $\{\tilde{\alpha}(s)\}$ is to be determined. We write

$$\{\tilde{\alpha}(s)\} = [\tilde{Z}(s)]^{-1} \left([\tilde{H}(s)] \{\frac{1}{s} \tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\}\right) . \tag{74}$$

Let s_α be the zeros of $|\tilde{Z}(s)|$ or the roots of the equation

$$\det[\tilde{Z}(s)] = 0 . \tag{75}$$

In circuit-theory terminology, $[\tilde{Z}(s)]$ corresponds to the system impedance matrix and s_α are the natural frequencies. $[\tilde{Z}(s)]^{-1}$ can be expanded in a partial-fraction form as follows:

$$[\tilde{Z}(s)]^{-1} = \sum_\alpha \frac{[R_\alpha]}{s - s_\alpha} \tag{76}$$

where the constant square matrix $[R_\alpha]$ is the system residue matrix at the pole s_α. $[R_\alpha]$ can be written as the product of a natural mode vector $\{R_\alpha^m\}$ and the transpose of a coupling vector $\{R_\alpha^c\}$:[11,13]

$$[R_\alpha] = \{R_\alpha^m\} \{R_\alpha^c\}^T \tag{77}$$

where $\{R_\alpha^m\}$ is a solution of the equation

$$[Z(s_\alpha)] \{R_\alpha^m\} = 0 \tag{78}$$

and $\{R_\alpha^c\}$ is a solution of

$$[Z(s_\alpha)]^T \{R_\alpha^c\} = 0 .$$

A close examination of the composition of the matrices $[\tilde{Z}(s)]$ and $[\tilde{H}(s)]$ reveals that their poles coincide and therefore cancel. We have, from Eqs. (74), (76) and (77),

$$\{\tilde{\alpha}(s)\} = \sum_{\alpha} \frac{\{R_{\alpha}^{m}\}\{R_{\alpha}^{c}\}^{T}}{s - s_{\alpha}} \left([\tilde{H}(s)]\ \{\frac{1}{s}\, \tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\}\right) . \tag{79}$$

Now define

$$[\tilde{H}(s)]\ \{\frac{1}{s}\, \tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\} = \tilde{N}(s) \quad \{\tilde{V}_{o}(s)\} \tag{80}$$

where $\{\tilde{V}_o(s)\}$ is the excitation vector when the incident wave is a pulse. We can then write Eq. (79) as

$$\begin{aligned} \{\tilde{\alpha}(s)\} &= \sum_{\alpha} \frac{\{R_{\alpha}^{m}\}\{R_{\alpha}^{c}\}^{T}}{s - s_{\alpha}}\, \tilde{N}(s)\ \{\tilde{V}_{o}(s)\} \\ &= \sum_{\alpha} \frac{\{R_{\alpha}^{m}\}}{s-s_{\alpha}}\, \tilde{\eta}_{\alpha}(s)\ \tilde{N}(s) \end{aligned} \tag{81}$$

where

$$\tilde{\eta}_{\alpha}(s) = \{R_{\alpha}^{c}\}^{T}\{\tilde{V}_{o}(s)\} \tag{82}$$

is called the coupling coefficient.[13] We note that $\tilde{N}(s)$ itself may contain poles in the finite plane, but this fact does not result in any serious difficulty.

We are now in a position to write the expressions for the field distributions within the cavity. From Eq. (38),

$$\begin{aligned} \tilde{F}_{x}(x,y,z,s) &= \sum_{i,j,k} \tilde{\alpha}_{(i,j,k)}(s)\ F_{x(i,j,k)}(x,y,z) \\ &= \{\tilde{\alpha}(s)\}^{T}\ \{F_{x(i,j,k)}(x,y,z)\} \end{aligned} \tag{83}$$

which, in view of Eq. (81), becomes

$$\tilde{F}_x(x,y,z,s) = \sum_\alpha \tilde{\eta}_\alpha(s)\ \{R_\alpha^m\}^T\ \{F_{x(i,j,k)}(x,y,z)\}\ (s - s_\alpha)^{-1}\ \tilde{N}(s)$$

$$= \sum_\alpha \tilde{\eta}_\alpha(s)\ \nu_\alpha^F(x,y,z)\ (s - s_\alpha)^{-1}\ \tilde{N}(s)\ . \tag{84}$$

In Eq. (84),

$$\nu_\alpha^F(x,y,z) = \{R_\alpha^m\}^T\ \{F_{x(i,j,k)}(x,y,z)\} \tag{85}$$

is a natural mode for F_x. In a similar manner, we will get

$$\tilde{E}_y(x,y,z,s) = \sum_\alpha \tilde{\eta}_\alpha(s)\ \nu_\alpha^{E_y}(x,y,z)\ (s - s_\alpha)^{-1}\ \tilde{N}(s) \tag{86}$$

and

$$\tilde{E}_z(x,y,z,s) = \sum_\alpha \tilde{\eta}_\alpha(s)\ \nu_\alpha^{E_z}(x,y,z)\ (s - s_\alpha)^{-1}\ \tilde{N}(s) \tag{87}$$

where $\nu_\alpha^{E_y}$ and $\nu_\alpha^{E_z}$ are the natural modes for $\tilde{E}_y$ and $\tilde{E}_z$ respectively. We know from Eq. (13) that $\tilde{E}_x = 0$.

The scattered fields outside the cavity in the $y \geq 0$ region can be found from Eqs. (46) to (53). When the incident waveform is given, the time-domain field distributions can be determined by performing an inverse Laplace transformation.

V. CONCLUDING REMARKS

In this report we have formulated the problem of transient electromagnetic excitation of a rectangular cavity through an aperture by making use of the induction theorem. New variables are introduced to convert the governing second-order differential equation into a set of first-order equations which correspond to normalized state equations. This conversion will result in significantly faster convergence in the numerical solution.

The method of moments is employed to solve the equations subject to the specified boundary conditions. By dividing the cavity region into subsections, the fields within the cavity are expressed in terms of appropriate expansion functions with time-dependent coefficients. The fields in the half-space outside the cavity are represented as superpositions of plane waves. At the aperture, the cavity and external fields are properly matched. Inner products are taken with testing functions and the first-order equations are converted into matrix equations containing the expansion coefficients as unknown column vectors. Evaluation of these expansion-coefficient vectors is a necessary step prior to the determination of field distributions. The procedure for evaluating a typical coefficient vector by the singularity-expansion method is outlined.

In this work the quasi-static approximation is not used, and the aperture is not required to be small. Because the internal and external portions of the problem are solved simultaneously and the boundary conditions at the aperture are satisfied, the effect of the reflections from the cavity walls on the aperture field is taken into account.

To complete the study of the transient behavior of a cavity-backed aperture, some numerical results would be highly desirable.

One must then subdivide the cavity region, choose appropriate expansion functions, form inner products to obtain the matrix equations, solve for the expansion coefficients by the singularity-expansion method, and perform inverse Laplace transformation. Much computation is involved in searching for the complex singularities, in finding the natural-mode and coupling vectors, and in determining the coupling coefficients; but, given time, these objectives are all within reach.

APPENDIX - Expressions for Submatrices in Matrix Equation (69).

We list here the formulas for the matrix elements which appear in Equation (69).

$$[\ell^{Fu}_{mn}] = [<F_{x(i,j,k)}(x,y,z), \frac{\partial}{\partial x} u_{(i',j',k')}(x,y,z)>] \; ; \quad j, j' \neq n_y \qquad \text{(A-1)}$$

$$[\ell^{Fv}_{mn}] = \begin{cases} [<F_{x(i,j,k)}(x,y,z), \frac{\partial}{\partial y} v_{(i',j',k')}(x,y,z)>] \; ; \; j, j' \neq n_y \\ [<F_{x(i,j,k)}(x,y,z), \frac{\partial}{\partial y} v_{(i',j',k')}(x,y,z) \\ \quad + \frac{\partial}{\partial x} \sum_{i'',k''} B_{(i'',k'';i',k')} u_{(i'',n_y,k'')}(x,y,z) \\ \quad + \frac{\partial}{\partial z} \sum_{i'',k''} D_{(i'',k'';i',k')} w_{(i'',n_y,k'')}(x,y,z)>] \; ; \; \begin{matrix} j \neq n_y \\ j' = n_y \end{matrix} \end{cases} \qquad \text{(A-2)}$$

$$[\ell^{Fw}_{mn}] = [<F_{x(i,j,k)}(x,y,z), \frac{\partial}{\partial z} w_{(i',j',k')}(x,y,z)>] \; ; \quad j, j' \neq n_y$$

$$[p^{FF}_{mn}] = [<F_{x(i,j,k)}(x,y,z), - \mu_o \varepsilon_o F_{x(i',j',k')}(x,y,z)>] \; ; \; j, j' \neq n_y \qquad \text{(A-4)}$$

$$[p^{Fv}_{mn}] = \begin{cases} [<F_{x(i,j,k)}(x,y,z), - \mu_o \varepsilon_o \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z)>] ; \; \begin{matrix} j \neq n_y \\ j' = n_y \end{matrix} \\ 0 \; ; \quad j', j' \neq n_y \end{cases} \qquad \text{(A-5)}$$

$$[q^F] = [<F_{x(i,j,k)}(x,y,z), - \mu_o \varepsilon_o \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z)$$

$$- \frac{\partial}{\partial x} \sum_{i'',k''} B_{(i'',k'';i',k')} \, u_{(i'',n_y,k'')}(x,y,z) \qquad \text{(A-6)}$$

$$- \frac{\partial}{\partial z} \sum_{i'',k''} D_{(i'',k'';i',k')} w_{(i'',n_y,k'')}(x,y,z)>] ; \quad \begin{matrix} j \neq n_y \\ j' = n_y \end{matrix}$$

$$[\ell_{mn}^{uF}] = [\langle u_{(i,j,k)}(x,y,z), -\frac{\partial}{\partial x} F_{x(i',j',k')}(x,y,z)\rangle] \; ; \quad j,\, j' \neq n_y \tag{A-7}$$

$$[\ell_{mn}^{uv}] = \begin{cases} 0 \; ; \quad j,\, j' \neq n_y \\ [\langle u_{(i,j,k)}(x,y,z), -\frac{\partial}{\partial x} \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z)\rangle]; & j \neq n_y,\; j' = n_y \end{cases} \tag{A-8}$$

$$[p^{uu}] = [\langle u_{(i,j,k)}(x,y,z),\; u_{(i',j',k')}(x,y,z)\rangle] \; ; \quad j,\, j' \neq n_y \tag{A-9}$$

$$[p_{mn}^{uv}] = \begin{cases} 0 \; ; \quad j,\, j' \neq n_y \\ [\langle u_{(i,j,k)}(x,y,z), \sum_{i'',k''} B_{(i'',k'';i',k')} u_{(i'',n_y,k'')}(x,y,z)\rangle]; & j \neq n_y,\; j'' = n_y \end{cases} \tag{A-10}$$

$$[q^{u}] = [\langle u_{(i,j,k)}(x,y,z),\; s \sum_{i'',k''} B_{(i'',k'';i',k')} u_{(i'',n_y,k'')}(x,y,z)$$

$$+ \frac{\partial}{\partial x} \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z)\rangle]; \quad j \neq n_y,\; j' = n_y \tag{A-11}$$

$$[\ell_{mn}^{vF}] = \begin{cases} [\langle v_{(i,j,k)}(x,y,z), -\frac{\partial}{\partial y} F_{x(i',j',k')}(x,y,z)\rangle]; \quad j,\, j' \neq n_y \\ [\langle \sum_{i'',k''} B_{(i'',k'';i,k)} u_{(i'',n_y,k'')}(x,y,z), -\frac{\partial}{\partial x} F_{x(i',j',k')}(x,y,z)\rangle \\ \quad + \langle v_{(i,n_y,k)}(x,y,z), -\frac{\partial}{\partial y} F_{x(i',j',k')}(x,y,z)\rangle \\ \quad + \langle \sum_{i'',k''} D_{(i'',k'';i,k)} w_{(i'',n_y,k'')}(x,y,z), -\frac{\partial}{\partial z} F_{x(i',j',k')}(x,y,z)\rangle]; & j = n_y,\; j' \neq n_y \end{cases} \tag{A-12}$$

$$[\ell_{mn}^{vu}] = \begin{cases} 0 \; ; \quad j,\, j' \neq n_y \\ [\langle \sum_{i'',k''} A_{(i'',k'';i,k)} F_{x(i'',n_y,k'')}(x,y,z), \frac{\partial}{\partial x} u_{(i',j',k')}(x,y,z)\rangle]; & j = n_y,\; j' \neq n_y \end{cases} \tag{A-13}$$

$$[\ell_{mn}^{vv}] = \begin{cases} [<\sum_{i'',k''} A(i'',k'';i,k)^{F}x(i'',n_y,k'')(x,y,z),\ \frac{\partial}{\partial y} v_{(i',j',k')}(x,y,z)>]; & j=n_y,\ j'\neq n_y \\ 0\ ;\ \ j \neq n_y & \\ [<\sum_{i'',k''} A(i'',k'';i,k)^{F}x(i'',n_y,k'')(x,y,z),\ \frac{\partial}{\partial x} \sum_{i'',k''} B(i'',k'';i',k')^{u}(i',n_y,k')(x,y,z) & \\ \quad + \frac{\partial}{\partial y} v_{(i',j',k')}(x,y,z) + \frac{\partial}{\partial z} \sum_{i'',k''} D(i'',k'';i',k')^{w}(i',n_y,k')(x,y,z)> & \\ \quad + <\sum_{i'',k''} B(i'',k'';i,k)^{u}(i'',n_y,k'')(x,y,z), -\frac{\partial}{\partial x} \sum_{i'',k''} A(i'',k'';i',k')^{F}x(i'',n_y,k'')(x,y,z)> & \\ \quad + <v_{(i,n_y,k)}(x,y,z),\ -\frac{\partial}{\partial y} \sum_{i'',k''} A(i'',k'';i',k')^{F}x(i'',n_y,k'')(x,y,z)> & \\ \quad + <\sum_{i'',k''} D(i'',k'';i,k)^{w}(i'',n_y,k''), -\frac{\partial}{\partial z} \sum_{i'',k''} A(i'',k'';i',k')^{F}x(i'',n_y,k'')(x,y,z)>]; & j,\ j' = n_y \end{cases} \qquad \text{(A-14)}$$

$$[\ell_{mn}^{vw}] = \begin{cases} 0\ ;\ j,\ j' \neq n_y & \\ [<\sum_{i'',k''} A(i'',k'';i,k)^{F}x(i'',n_y,k'')(x,y,z),\ \frac{\partial}{\partial z} w_{(i',j',k')}(x,y,z)>]; & j=n_y,\ j'\neq n_y \end{cases} \qquad \text{(A-15)}$$

$$[p_{mn}^{uF}] = \begin{cases} 0\ ;\ j,\ j' \neq n_y & \\ [<\sum_{i'',k''} A(i'',k'';i,k)^{F}x(i'',n_y,k'')(x,y,z), -\mu_o\varepsilon_o{}^{F}x(i',j',k')(x,y,z)>]; & j=n_y,\ j''\neq n_y \end{cases} \qquad \text{(A-16)}$$

$$[p_{mn}^{vu}] = \begin{cases} 0\ ;\ j,\ j' \neq n_y & \\ [<\sum_{i'',k''} B(i'',k'';i,k)^{u}(i'',n_y,k'')(x,y,z),\ u_{(i',j',k')}(x,y,z)>]; & j=n_y,\ j'\neq n_y \end{cases} \qquad \text{(A-17)}$$

$$[p_{mn}^{vv}] = \begin{cases} [\langle v_{(i,j,k)}(x,y,z),\ v_{(i',j',k')}(x,y,z)\rangle]\ ;\ j,\ j' \neq n_y \\ [\langle v_{(i,n_y,k)}(x,y,z),\ v_{(i',j',k')}(x,y,z)\rangle];\ j = n_y,\ j' \neq n_y \\ [\langle \sum_{i'',k''} A_{(i'',k'';i,k)} F_{x(i'',n_y,k'')}(x,y,z), -\mu_o\varepsilon_o \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z)\rangle \\ \quad + \langle \sum_{i'',k''} B_{(i'',k'';i,k)} u_{(i'',n_y,k'')}(x,y,z),\ \sum_{i'',k''} B_{(i'',k'';i',k')} u_{(i'',n_y,k'')}(x,y,z)\rangle \\ \quad + \langle v_{(i,n_y,k)}(x,y,z),\ v_{(i',n_y,k')}(x,y,z)\rangle \\ \quad + \langle \sum_{i'',k''} D_{(i'',k'';i,k)} w_{(i'',n_y,k'')}(x,y,z),\ \sum_{i'',k''} D_{(i'',k'';i',k')} w_{(i'',n_y,k'')}(x,y,z)\rangle]; \\ \qquad j = j' = n_y \end{cases} \tag{A-18}$$

$$[p_{mn}^{vw}] = \begin{cases} 0\ ;\ j,\ j' \neq n_y \\ [\langle \sum_{i'',k''} D_{(i'',k'';i,k)} w_{(i'',n_y,k'')}(x,y,z),\ w_{(i',j',k')}(x,y,z)\rangle];\ j = n_y,\ j' \neq n_y \end{cases} \tag{A-19}$$

$$[q^v] = \begin{cases} [\langle v_{(i,j,k)}(x,y,z),\ \frac{\partial}{\partial y} \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z)\rangle];\ j \neq n_y,\ j' = n_y \\ [\langle \sum_{i'',k''} A_{(i'',k'';i,k)} F_{x(i'',n_y,k'')}(x,y,z), -\mu_o\varepsilon_o s \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z) \\ \qquad - \frac{\partial}{\partial x} \sum_{i'',k''} B_{(i'',k'';i',k')} u_{(i'',n_y,k'')}(x,y,z) \\ \qquad - \frac{\partial}{\partial z} \sum_{i'',k''} D_{(i'',k'';i',k')} w_{(i'',n_y,k'')}(x,y,z)\rangle \\ \quad + \langle \sum_{i'',k''} B_{(i'',k'';i,k)} u_{(i'',n_y,k'')}(x,y,z),\ s \sum_{i'',k''} B_{(i'',k'';i',k')} u_{(i'',n_y,k'')}(x,y,z) \\ \qquad + \frac{\partial}{\partial x} \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z)\rangle \end{cases} \tag{A-20}$$

$$+ \langle v_{(i,n_y,k)}(x,y,z),\ \frac{\partial}{\partial y} \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z)\rangle$$

$$+ \langle \sum_{i'',k''} D_{(i'',k'';i,k)} W_{(i'',n_y,k'')}(x,y,z),\ s \sum_{i'',k''} D_{(i'',k'';i',k')} W_{(i'',n_y,k'')}(x,y,z)$$

$$+ \frac{\partial}{\partial z} \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z)\rangle]; \quad j=j'=n_y$$

$$\{\tilde{C}(s)\} = \begin{cases} 0; & j \neq n_y \\ [\langle \tilde{F}'_{x(i,n_y,k)}(x,y,z,s),\ \frac{1}{s} \tilde{M}_x(x,y,z,s)\rangle]; & j = n_y \end{cases} \qquad \text{(A-21)}$$

$$[\ell^{wF}_{mn}] = [\langle w_{(i,j,k)}(x,y,z),\ -\frac{\partial}{\partial z} F_{x(i',j',k')}(x,y,z)\rangle]; \quad j,\ j' \neq n_y \qquad \text{(A-22)}$$

$$[\ell^{vv}_{mn}] = \begin{cases} 0; & j, j' \neq n_y \\ [\langle w_{(i,j,k)}(x,y,z),\ -\frac{\partial}{\partial z} \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z)\rangle]; & j \neq n_y,\ j'=n_y \end{cases} \qquad \text{(A-23)}$$

$$[p^{wv}_{mn}] = \begin{cases} 0; & j, j' \neq n_y \\ [\langle w_{(i,j,k)}(x,y,z),\ \sum_{i'',k''} D_{(i'',k'';i',k')} W_{(i'',n_y,k'')}(x,y,z)\rangle]; & j \neq n_y,\ j'=n_y \end{cases} \qquad \text{(A-24)}$$

$$[p^{ww}_{mn}] = [\langle w_{(i,j,k)}(x,y,z),\ w_{(i',j',k')}(x,y,z)\rangle]; \quad j,\ j' \neq n_y \qquad \text{(A-25)}$$

$$[q^{w}] = [\langle w_{(i,j,k)}(x,y,z),\ \frac{\partial}{\partial z} \sum_{i'',k''} A_{(i'',k'';i',k')} F_{x(i'',n_y,k'')}(x,y,z)$$

$$+ s \sum_{i'',k''} D_{(i'',k'';i',k')} W_{(i'',n_y,k'')}(x,y,z)\rangle]; \quad j \neq n_y,\ j'=n_y \qquad \text{(A-26)}$$

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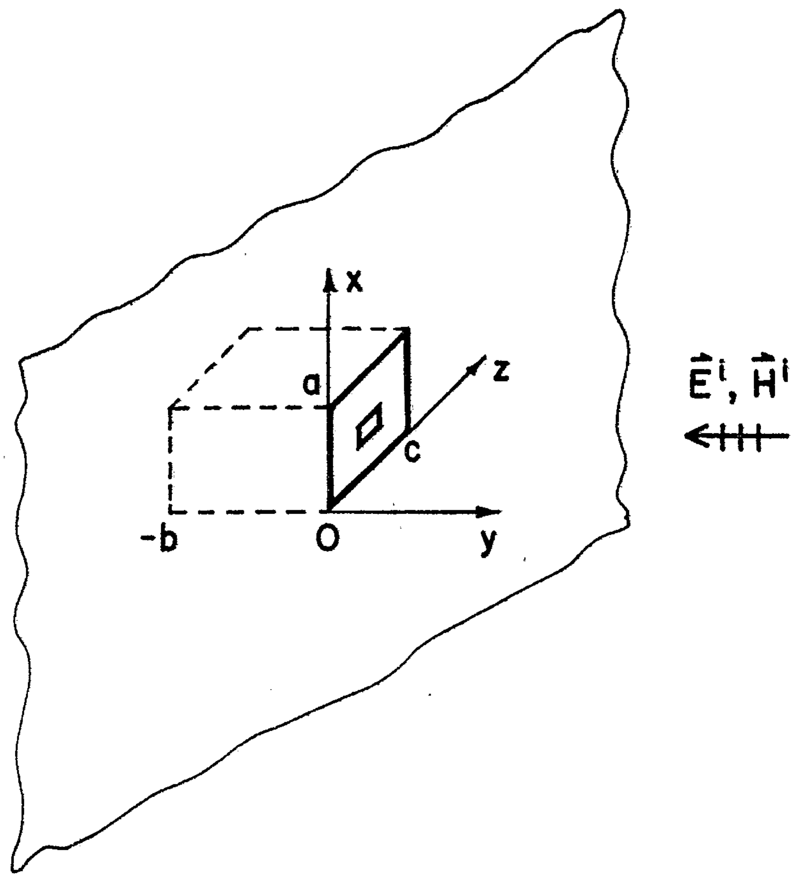

Fig. 1. A cavity-backed aperture problem.

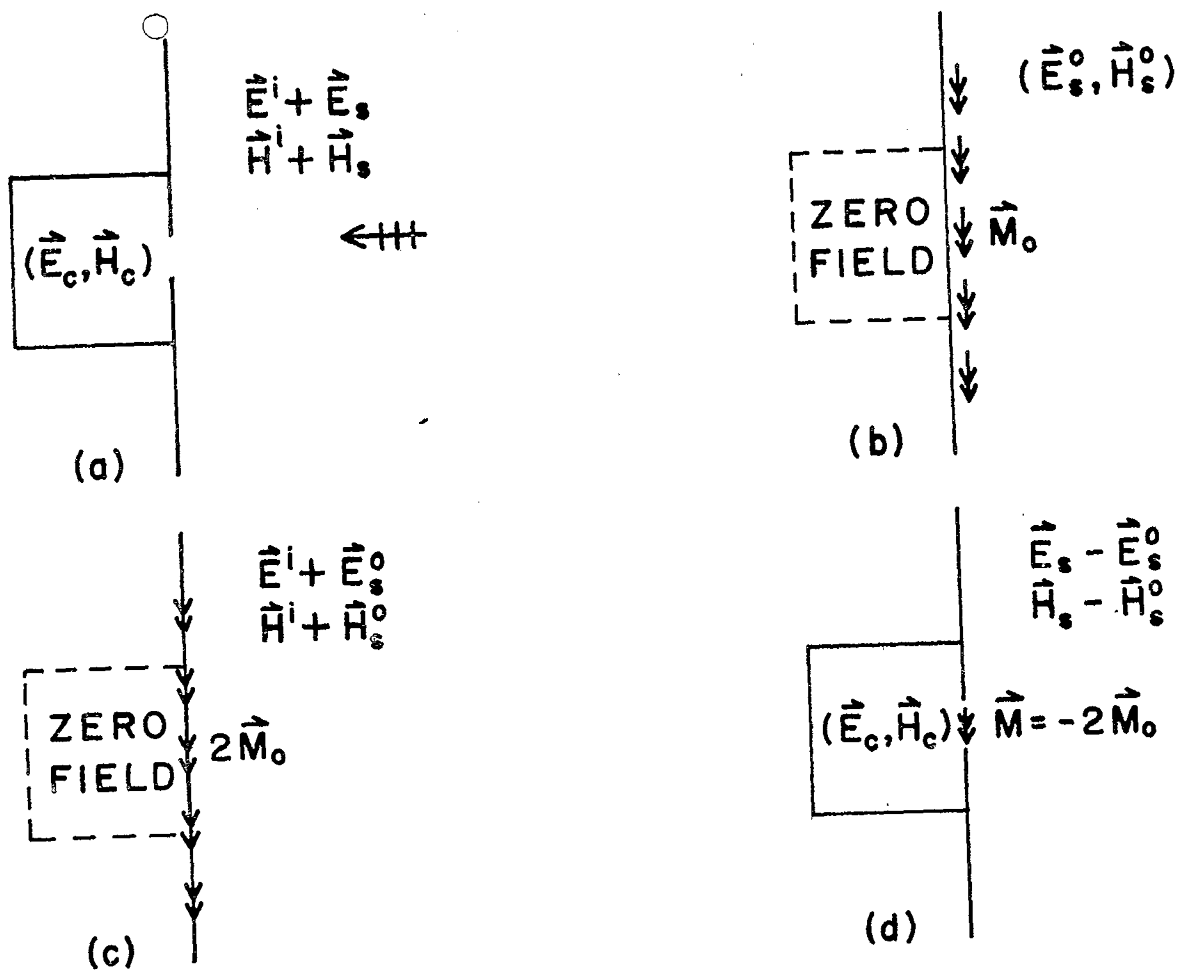

Fig. 2. Application of induction theorem.