Interaction Notes

Note 209

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Peak Current Estimates: Cylinders in Free Space with Extensions to Other Structures

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ABSTRACT

A method has been developed to estimate both the transient response of straight wire scatterers and the peak current induced on a straight wire. Extensions of the method include the effects of loading and the treatment of more complex structures. A sample work sheet and the necessary response curves are provided for peak current estimates. Accuracies on the order of 10% can be obtained, as demonstrated by several examples.

ACKNOWLEDGEMENT

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INTRODUCTION

The transient behavior of conducting cylinders in free space can be determined by referring to handbook data,[1,2] by using an approach such as the singularity expansion method (SEM),[4] by using lumped parameter network (LPN) models of an antenna,[3] or by solving an integral equation using the method of moments.[5,6,7] While these techniques are effective, they do have shortcomings for the engineer who needs a quickly formed, easily obtainable, and reasonably accurate estimate of peak current. Unambiguous application of handbook data is restricted to the range of parameters spanned by the data. Often, it is difficult to estimate the errors arising from extrapolation beyond the given range of parameters. The SEM and LPN approaches require a working familiarity with these methods, require knowledge of the resonance characteristics of the object, but can be used to provide estimates of the details of structure response in addition to peak current at some additional effort over the present method. The use of moment methods is more convenient than the SEM for the analysis of complex geometries, but it relies on access to a modern, high speed computer (necessitating some expertise in running computer codes). Finally, all of these solution procedures fail to provide the non-EM expert with a feeling for the important problem parameters or the effects of varying these parameters. If detail of transient characteristics are desired, the inconveniences of these methods must be tolerated. However, for the engineer who needs an estimate of the structure response (so that he can then decide if a detailed analysis is warranted), an alternative procedure is available: a simple method of calculating peak currents on various structures (which can be modeled by straight wires) for arbitrary input waveforms and determining the effects of loading. This method is not meant to replace the techniques described above, but it is intended to provide easily obtainable estimates of peak currents. This note emphasizes using the method by providing the needed curves, sample problems, a sample work sheet

(Appendix A), and directions on how to use the work sheet. Theoretical development, including references to similar work, will be the topic of another paper. Examples of extending this procedure to structures more complicated than straight wires is provided in Appendix B. A recent note by Taylor[12] develops another technique for simple current estimates based on the first term of the singularity expansion (lowest order pole pair of the conducting cylinder). The results of the present note compare favorably with Taylor's. Taylor's method is based on late time properties while ours is based on early time properties.

FORMULATION

For the free-field excitations employed in EMP interaction and coupling studies, peak currents usually occur during the early time response of a structure. Buildup of current may be limited by reflections or because of the driving field dying to zero before the first reflection occurs. For wire structures, the wire diameter influences the induced current as well as the frequency content and magnitude of the incident field. Generally, "fatter" wires develop higher currents. The procedures discussed below were developed for straight wires and properly account for these characteristics. Extension of the method to more complex structures hinges upon approximating the given structure by wire, cylinder, or set of wires or cylinders. The examples given in Appendix B show how this may be done.

The peak current estimates can be formed by performing the integral

$$I_{peak} = A \int_0^T e(t)\, dt, \tag{1}$$

where e(t) is the free field tangent to the wire in volts/meter. The current estimation procedure is then reduced to finding the values of T and A. For broadside incidence, the value of T is typically chosen as the time a reflection from the structure is seen at the center of the structure. The value of A can be determined by the two approaches discussed below.

IMPULSE RESPONSE METHOD

Background

Using the convolution theorem, the current excited on an infinite wire at time T is

$$i(T) = \int_{-\infty}^{\infty} h(T - t)\, e(t)\, dt, \tag{2}$$

where e(t) is the temporal variation of the incident field tangent to the wire, and h(t) is the impulse response of the infinite wire.[8,9] This h(t) depends on the wire radius, a. The dependence can be accounted for by plotting h as a function of time normalized by a/c as shown in Fig. 1[9,10], where $c = 3 \times 10^8$ m/s^{-1}. Because electromagnetic waves travel at the speed of light, this is also the impulse response of the current on a finite wire up until the time that the first reflection is seen. Consequently, the h(t) shown in Fig. 1 is sufficient for calculating the peak current. The value A is obtained from h(t) by approximating the convolution (Eq. 2), i.e., by using the value of h(t) at an average time. This average time, τ, is chosen midway between the time e(t) reaches full strength (rise time, t_1) and the time that the integral is terminated (t_2):

$$I_{peak} \approx h(\tau) \int_0^{t_2} e(t)\, dt, \tag{3}$$

where

$$\tau = \frac{t_2 - t_1}{2} . \tag{4}$$

The selection of t_1 and t_2 is illustrated by the examples. Consequently, calculating I_{peak} involves:

- Approximating the actual structure by a straight wire.
- Finding $c\tau/a$ where $c = 3 \times 10^8$ m/s and a = wire radius.

Then

$$I_{peak} \approx h(c\tau/a) \int_0^{t_2} e(t)\, dt.$$

Example

Let us calculate the peak current on 1.0-m-long wire with a diameter of 0.01347 m. The wire is illuminated by a unit step plane wave incident broadside on the wire with an electric field parallel to the wire. The numbered steps below parallel the numbers on the sample worksheet in Appendix A, and the completed work sheet for this example is given as Example 1 in Appendix B.

1. The actual geometry is described above — sketch this on the work sheet in Fig. A.
2. Approximate the geometry of Fig A by a straight wire. In this example, no approximation is required, but if an approximation is made, sketch it in Fig. B.
3. Sketch the driving e(t) in Fig. C.
4. Enter a = 0.01347/2 = 0.00674 m.

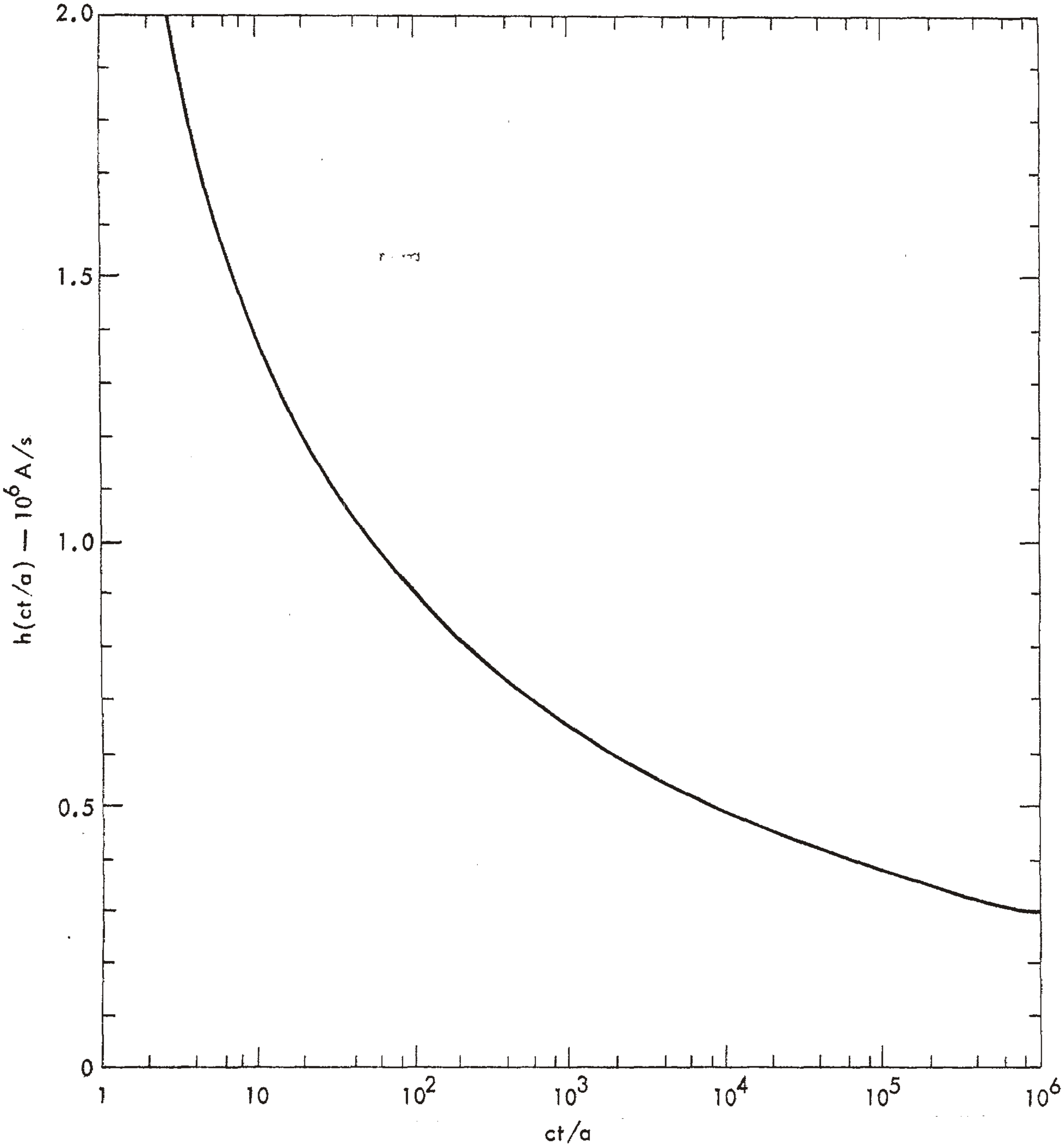


Fig. 1. Impulse response of the current on an infinite wire when illuminated from **broadside** by a plane wave. Tick marks on the log scale are at the values, 1, 2, 4, 6, 8, 10, 20, 40, etc.

5. Enter L = 1.0 m.
6. Calculate $t_r = \frac{L}{2c} = 1.67 \times 10^{-9}$ s.
7. Plot this value on Fig. C to aid in deciding how to estimate $\int_0^T e(t)\,dt$ and also to decide the value of t_2 for step 10b.
8. Sketch the approximate e(t). In our example, we will use the unit step without approximation.
9. Calculate $\int_0^T e(t)\,dt = 1.67 \times 10^{-9}$ V · s/m.
10. Use the left column for the impulse response method. The right hand column is discussed later.
    a. Enter $t_1 = 0$ s (i.e., a step).
    b. Enter $t_2 = 1.67 \times 10^{-9}$ s (see Example 11, there $t_2 \neq t_r$.
    c. Calculate $\tau = 0.833 \times 10^{-9}$ s.
    d. Calculate $c\tau/a = 37.1$.
11. Read $h(37.1) = 1.06 \times 10^6$ A/s from Fig. 1.
12. Calculate $I_{sc} = h(37.1) \int_0^T e(t)\,dt = 1.77$ mA.
13. In our case, we have no load, so we need not correct for loading.
14. $I_L = I_{sc} = 1.77$ mA (for comparison, the Sandia handbook yields 1.78 mA). Note that steps 13 and 14 permit a rough estimation for the effects of loading. Application of these two steps is illustrated by Example 5 in Appendix B.

SURGE ADMITTANCE METHOD

Background

This procedure employs the real part of the input admittance to an infinite wire driven as an antenna[9,11] (Fig. 2). The development of this method is not as straightforward as the impulse response procedure, but it can be shown that

$$I_{peak} \approx cG(aF) \int_0^T e(t)\,dt, \quad (5)$$

where $c = 3 \times 10^8$ m/s, F is an "effective" frequency and a is the wire radius, as before. The value of T is chosen in the same way as in the impulse response method. To date, an expression for F that is valid for a wide set of cases has not been found. Consequently, this method tends to provide less accuracy than the impulse response method. In spite of this, the surge admittance method is considered here to provide a check on the other method. Several guides can be given to the choice of F:

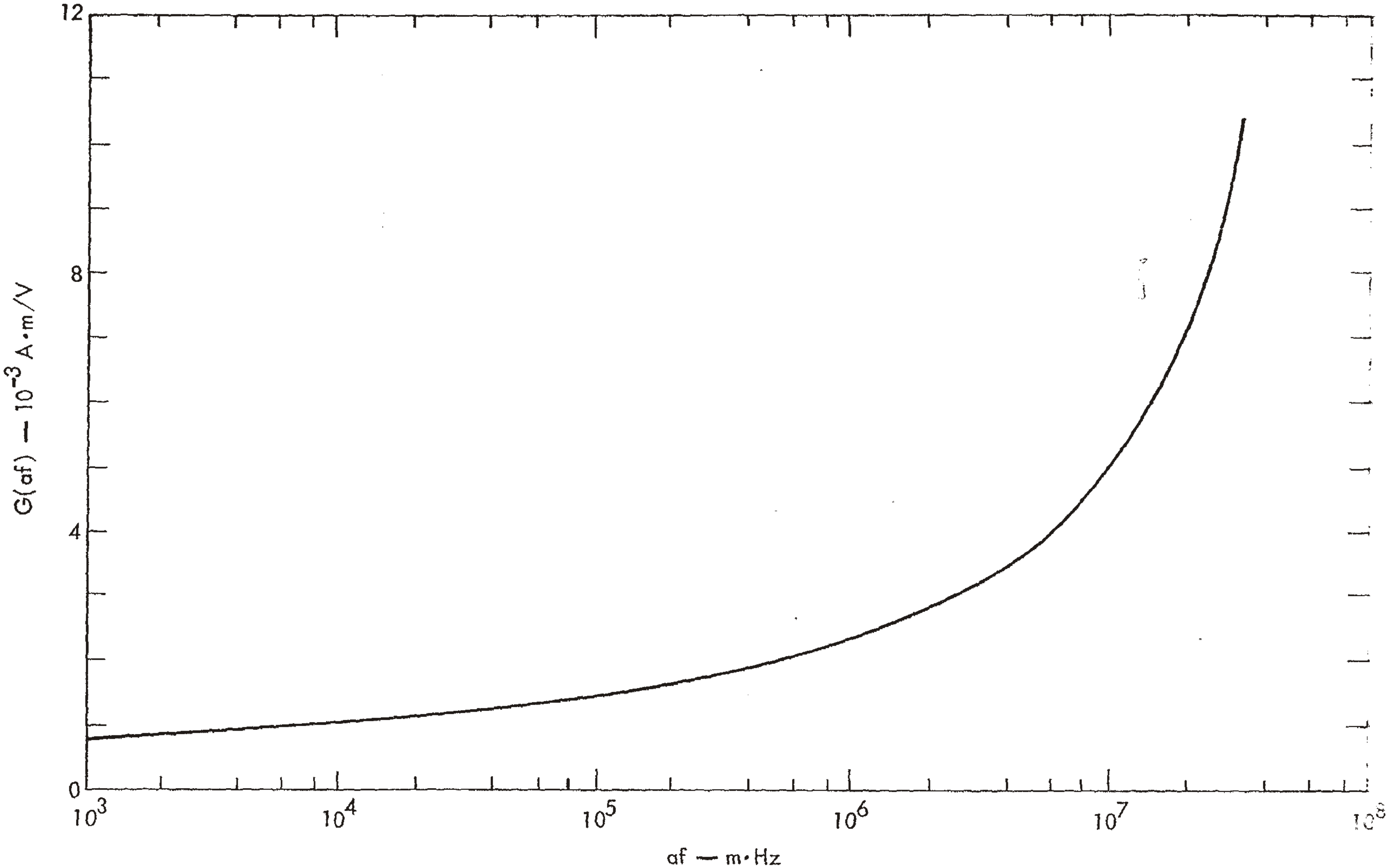


Fig. 2. Real part of the input admittance to an infinite wire antenna. Tick marks on the log scale are at the values $1 \cdot 10^3$, $2 \cdot 10^3$, $4 \cdot 10^3$, $6 \cdot 10^3$, $8 \cdot 10^3$, $1 \cdot 10^4$, $2 \cdot 10^4$, $4 \cdot 10^4$, etc.

- If the incident field is a very short pulse, use the upper frequency where the spectrum of the input is done ~10 db from its peak value (Example 3, Appendix B).
- If the incident field rises, then returns to zero (or undershoots) at a time on the order of t_r (Step 6 on the work sheet), approximate F by $1/2t_0$ where t_0 is the time of the first zero crossing of e(t) (Example 11, Appendix B). F can also be approximated by $1/2t_t$ if the first zero crossing has not yet been reached.
- For other cases, use step 10 on the work sheet, as illustrated by Example 2, Appendix B. Here, an "average" frequency is chosen. Note that a scheme has not yet been devised for every rapid rise time function (i.e., step function).

Another advantage of the surge admittance method is demonstrated by rewriting Eq. 5:

$$I_{peak} = \frac{1}{Z} \int_0^T e(t)\, c dt. \tag{6}$$

Choosing T = L/2c and replacing cdt by dl,

$$I_{peak} = \frac{1}{Z} \int_0^{L/2} e\left(\frac{l}{c}\right) dl, \tag{7}$$

which can be approximated if e(t) ≈ E (a constant):

$$I_{peak} \sim \frac{E \cdot \frac{L}{2}}{Z} . \tag{8}$$

Consequently, a quick (but crude) estimate of I_{peak} can be formed by multiplying the field strength of the incident wave by half the structure length and dividing by an impedance. This impedance ranges from 200 Ω for fat cylinders to ~1000 Ω for thin wires. After making an educated guess of Z, we can make rough estimates of peak current using Eq. 8 (which is easily memorized).

Example

Consider a typical four-term exponential EMP wave impinging broadside on a straight wire 180 m long with a radius of 0.1 m. The numbers below parallel the numbers in Example 2 (Appendix B).

1. Sketch the geometry in Fig. A.
2. We will use the geometry of Fig. A.
3. A rough sketch of the EMP wave is shown in Fig. C.
4. Enter a = 0.1 m.
5. Enter L = 180 m.

6. Calculate $t_r = 3.0 \times 10^{-7}$ s.
7. Plot this value on Fig. C.
8. Approximate e(t) and sketch on Fig. D. Refer to Fig. D, Example 2.
9. Calculate $\int_0^T e(t)\,dt = 1.28 \times 10^{-2}$ V · s/m, using $T = t_r$.
10. Use the right hand column for the surge admittance method. The left hand side of Example 2 has also been completed for comparison.
   a. Enter $t_1 = 10^{-8}$ and calculate $f_1 = 2.5 \times 10^7$ Hz.
   b. Enter $t_2 = 3.0 \times 10^{-7}$ and calculate $f_2 = 8.33 \times 10^5$ Hz
   c. Calculate $F = \frac{1}{2}(f_1 + f_2) = 1.29 \times 10^7$ Hz.
   d. Calculate $aF = 1.29 \times 10^6$ m-Hz.
11. Find G $(1.29 \times 10^6) = 2.5 \times 10^{-3}$ A-m/V from Fig. 2.
12. Calculate $I_{sc} = cG(aF) \int_0^T e(t)\,dt$

$$= (3 \times 10^8)(1.29 \times 10^6)(1.28 \times 10^{-2}) \text{ A.}$$
$$= 9600 \text{ A.}$$

13. No loading.
14. $I_L = I_{sc} = 9600$ A. For comparison, solution by an integral equation approach yielded 8800 A.

EXTENSIONS

In this section, several simple extensions are considered which allow the method to be used on a wider class of structures. Examples of these extensions are provided in Appendix B. Simple extensions follow.

- When $\vec{E}$ is not parallel to the wire axis, use e(t) cosθ, where θ is the angle between $\vec{E}$ and the wire axis.
- For structures containing wires that meet at a junction, one of two procedures can be used:
  - Approximate the structure by a single, fat cylinder.
  - Simply add the single wire currents together. This gives an overestimate because reflections from the junction are not considered. Care must be taken, however, because from an electromagnetic standpoint, the structure may appear to look like a solid surface; using the first method may give better results.
- To account for loading, an approximate formula can be used (this formula holds in the frequency domain, but it is only approximate here).

$$I_{peak} = \frac{G_L}{G_L + G_{eff}} I_{sc}$$

In this formula, I_{sc} is the total current found if $G_E = \infty$ (i.e., short circuit conditions) and $G_{eff} = I_{sc}/c \int_0^T e(t)\,dt$. For a single wire, $G_{eff} = G(aF)$ (Fig. 2, step 11), but G_{eff} will be larger if the second procedure has been used for structures containing several wires.

EXAMPLES

In addition to the two previous examples, the examples in Appendix B illustrate the type of structures that can be analyzed and the approaches that give good results.

Table 1. Summary of examples.

Example	Calculated (A)	h(t) est. (A)	Error (%)	G(f) est. (A)	Error (%)
1	1.78×10^{-3}	1.77×10^{-3}	-0.6	—	—
2	8800	9400	6.8	9600	9.1
3	20.6×10^{-2}	18.7×10^{-3}	-9.2	18.3×10^{-3}	-11.2
4	2500	1470	-2.0	1050	-30.0
5	2200	2440	13.0	2840	29.1
6	2200	2130	-3.0	1560	-29.1
7 I_{sc}	3670	3800	3.5	4630	26.2
I_L	1150	1020	-11.0	1000	-13.0
8 I_{sc}	3670	6500	77.0	4210	14.7
I_L	1150	1070	-7.0	980	-14.8
9	1500	1480	-1.0	1020	-32.0
10	5700	5670	-0.5	6320	10.9
11	27 000	34 200	27.0	24 700	-8.5
		Average error	+7.2%		-4.0%

The error in estimating a peak current induced on a real-life structure (using the procedure outlined here) consists of two parts:

- Errors in approximating the structure by a wire.
- Errors arising from estimating the response of this wire.

The errors in the first category, which tend to be the largest, are not included in Table 1. Using the h(ct/a) method, if a "reasonable" problem is under consideration, the second category error appears to be on the order of 8%. Larger errors may be encountered when considering complex geometries. In any event, the First-cut calculations should be accurate enough to determine if a more complete coupling analysis of the structure is required.

REFERENCES

1. D. E. Merewether, J. A. Cooper, and R. L. Parker, Eds., Electromagnetic Pulse Handbook for Missiles and Aircraft in Flight, Sandia Laboratories Rept. SC-M-71-0346 (1972), pp. 15-37.
2. W. C. Emberson, EMP Preferred Test Procedures, (Illinois Institute of Technology Research Institute, Urbana, Illinois, 1972).
4. F. M. Tesche, "On the Singularity Expansion Method as Applied to Electromagnetic Scattering from Thin-Wires," Interaction Note 102, April 1972.
5. E. P. Sayre and R. F. Harrington, "Transient Response of Straight Wire Scatterers and Antennas," in Proc. 1968 IEEE Intern. Antenna Propagation Symp., (Boston, Mass., 1968).
6. E. K. Miller and M. L. Van Blaricam, "The Short-Pulse Response of the Straight Wire," IEEE Trans. Antennas and Prop. AP-21, 396 (1973).
7. J. A. Landt, E. K. Miller, and F. J. Deadrick, Time-Domain Models of Thin-Wire Antennas and Scatterers, Lawrence Livermore Laboratory, Interaction Note 206, November 1973.
8. R. F. Harrington, Time-Harmonic Electromagnetic Fields, (McGraw-Hill, New York, 1961), pp. 232-238.
9. E. K. Miller and J. A. Landt, "Transient Response of the Infinite Cylindrical Antenna and Scatterer," (in preparation).
10. P. R. Barnes, "The Axial Current on an Infinitely Long, Perfectly Conducting Cylinder in Free Space by a Transient Electromagnetic Plane Wave," Air Force Weapons Laboratory Interaction Note 64, Kirtland AFB, New Mexico, 1971.
11. E. K. Miller, "Admittance Dependence of the Infinite Cylindrical Antenna Upon Exciting Gap Thickness," Radio Science, 2 (12); 1431-1435 (1967).
12. C. D. Taylor, "A Simple Procedure for Estimating the Current Induced on Cylinder-Like Conductors Illuminated by a Unit Electromagnetic Pulse," Interaction Note 176, July 1973.

APPENDIX A. SAMPLE WORKSHEET

Date	Problem	Engineer

Fig. A. Actual Geometry

Fig. B. Wire Approximation

1. Sketch actual geometry in Fig. A. ________
2. Sketch the wire approximation in Fig. B. ________
3. Sketch the driving e(t) in Fig. C. ________
4. Wire radius a = ________
5. Wire length L = ________
6. Time of reflection $t_r = \frac{L}{2c}$ = $(c = 3 \times 10^8 \text{ m/s})$ ________
7. Plot t_r on Fig. C. ________
8. Sketch the approximate e(t) in Fig. D. ________
9. Estimate $\int_0^T e(t)\, dt$ = ________

Fig. C. Incident Field e(t)

Fig. D. Approximate e(t)

$h\left(\frac{c\tau}{a}\right)$ approach

10. Find $\frac{c\tau}{a}$
    a. Rise time t_1 = ________
    b. Decay time t_2 = ________
    c. $\tau = \frac{t_2 - t_1}{2}$ = ________
    d. $\frac{c\tau}{a}$ = ________
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right)$ = ________
12. $I_{sc} = h\left(\frac{c\tau}{a}\right)\int_0^T e(t)\, dt$ = ________
13. Loading $G_L = \frac{1}{Z_L}$ = ________
    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\, dt}$ = ________
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = ________

G(aF) approach

10. Find aF
    a. $f_1 = \frac{1}{4t_1}$ = ________
    b. $f_2 = \frac{1}{4t_2}$ = ________
    c. $F = \frac{f_1 + f_2}{2}$ = ________
    d. aF = ________
11. From graph G(aF) = ________
12. $I_{sc} = cG(aF)\int_0^T e(t)\, dt$ = ________
13. Loading $G_L = \frac{1}{Z_L}$ = ________
    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\, dt}$ = ________
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = ________

Date | Problem: Example 1 | Engineer

For comparison, I_{sc} = 1.78 mA from Sandia H.B.

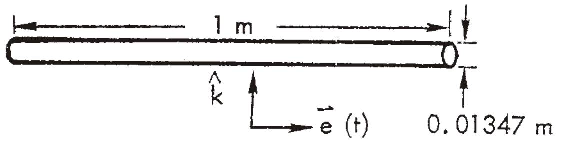


Fig. A. Actual Geometry

1. Sketch actual geometry in Fig. A. ______
2. Sketch the wire approximation in Fig. B. ______
3. Sketch the driving e(t) in Fig. C. ______
4. Wire radius a = 0.00674
5. Wire length L = 1.0
6. Time of reflection $t_r = \frac{L}{2c}$ = ($c = 3 \times 10^8$ m/s) 1.67×10^{-9}
7. Plot t_r on Fig. C. ______
8. Sketch the approximate e(t) in Fig. D. ______
9. Estimate $\int_0^T e(t)\,dt$ = 1.67×10^{-9}

Fig. B. Wire Approximation

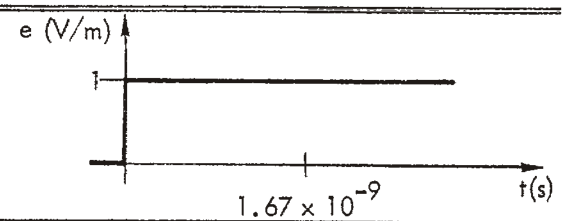


Fig. C. Incident Field e(t)

Fig. D. Approximate e(t)

$h\left(\frac{c\tau}{a}\right)$ approach

10. Find $\frac{c\tau}{a}$
    a. Rise time t_1 = 0.
    b. Decay time t_2 = 1.67×10^{-9}
    c. $\tau = \frac{t_2 - t_1}{2}$ = 0.833×10^{-9}
    d. $\frac{c\tau}{a}$ = 37.1
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right)$ = 1.06×10^{6}
12. $I_{sc} = h\left(\frac{c\tau}{a}\right)\int_0^T e(t)\,dt$ 1.77 mA
13. Loading $G_L = \frac{1}{Z_L}$ = ______

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = ______
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = 1.77 mA

G(aF) approach

10. Find aF
    a. $f_1 = \frac{1}{4t_1}$ = ______
    b. $f_2 = \frac{1}{4t_2}$ = ______
    c. $F = \frac{f_1 + f_2}{2}$ = ______
    d. aF = ______
11. From graph G(aF) = ______
12. $I_{sc} = cG(aF)\int_0^T e(t)\,dt$ = ______
13. Loading $G_L = \frac{1}{Z_L}$ = ______

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = ______
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = ______

Date	Problem Example 2	Engineer

For comparison, I_{sc} = 8800 A calculated.

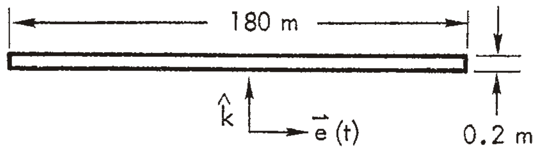


Fig. A. Actual Geometry

Fig. B. Wire Approximation

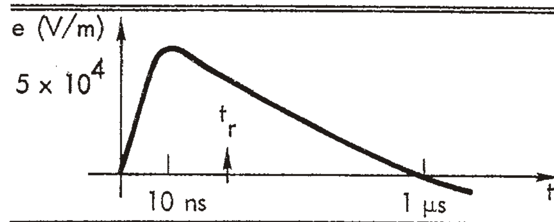


Fig. C. Incident Field e(t)

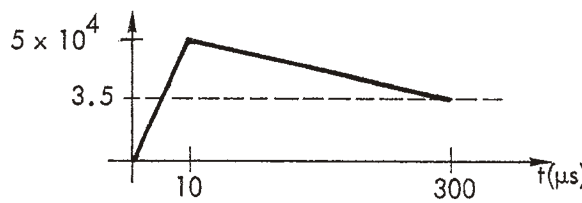


Fig. D. Approximate e(t)

1. Sketch actual geometry in Fig. A. ______
2. Sketch the wire approximation in Fig. B. ______
3. Sketch the driving e(t) in Fig. C. ______
4. Wire radius a = 0.1
5. Wire length L = 180.
6. Time of reflection $t_r = \frac{L}{2c}$ = ($c = 3 \times 10^8$ m/s) 3.0×10^{-7}
7. Plot t_r on Fig. C. ______
8. Sketch the approximate e(t) in Fig. D. ______
9. Estimate $\int_0^T e(t)\,dt$ = 1.28×10^{-2}

$h\left(\frac{c\tau}{a}\right)$ approach		$G(aF)$ approach	
10. Find $\frac{c\tau}{a}$		10. Find aF	
a. Rise time t_1 =	10^{-8}	a. $f_1 = \frac{1}{4t_1}$ =	2.5×10^{7}
b. Decay time t_2 =	3.0×10^{-7}	b. $f_2 = \frac{1}{4t_2}$ =	8.33×10^{5}
c. $\tau = \frac{t_2 - t_1}{2}$ =	1.45×10^{-7}	c. $F = \frac{f_1 + f_2}{2}$ =	1.29×10^{7}
d. $\frac{c\tau}{a}$ =	435	d. aF =	1.29×10^{6}
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right)$ =	0.73×10^{6}	11. From graph G(aF) =	2.5×10^{-3}
12. $I_{sc} = h\left(\frac{c\tau}{a}\right)\int_0^T e(t)\,dt$ =	9400 A	12. $I_{sc} = cG(aF)\int_0^T e(t)\,dt$ =	9600 A
13. Loading $G_L = \frac{1}{Z_L}$ =		13. Loading $G_L = \frac{1}{Z_L}$ =	
$G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ =		$G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ =	
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ =	9400 A	14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ =	9600 A

Date | Problem Example 3 | Engineer

For comparison, an experiment yielded ~ 1500 A

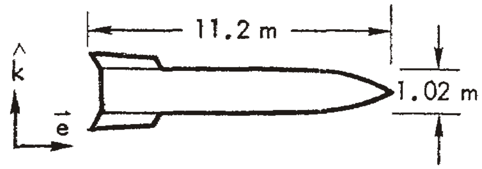


Fig. A. Actual Geometry

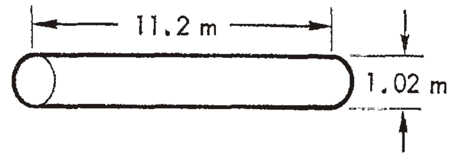


Fig. B. Wire Approximation

1. Sketch actual geometry in Fig. A. ____
2. Sketch the wire approximation in Fig. B. ____
3. Sketch the driving e(t) in Fig. C. ____
4. Wire radius a = *0.51*
5. Wire length L = *11.2*
6. Time of reflection $t_r = \frac{L}{2c} =$ ($c = 3 \times 10^8$ m/s) *1.87×10^{-8}*
7. Plot t_r on Fig. C. ____
8. Sketch the approximate e(t) in Fig. D. ____
9. Estimate $\int_0^T e(t)\,dt =$ *7.0×10^{-4}*

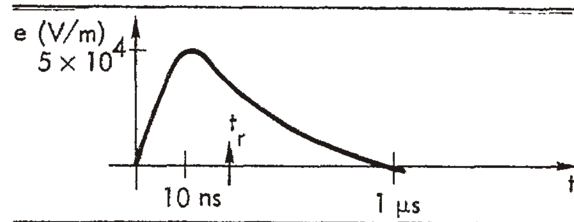


Fig. C. Incident Field e(t)

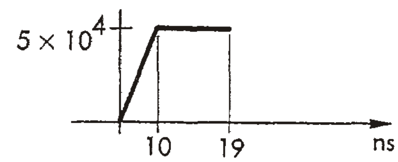


Fig. D. Approximate e(t)

$h\left(\frac{c\tau}{a}\right)$ approach

10. Find $\frac{c\tau}{a}$
    a. Rise time $t_1 =$ *10^{-8}*
    b. Decay time $t_2 =$ *1.9×10^{-8}*
    c. $\tau = \frac{t_2 - t_1}{2} =$ *0.45×10^{-8}*
    d. $\frac{c\tau}{a} =$ *2.65*
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right) =$ *2.1×10^6*
12. $I_{sc} = h\left(\frac{c\tau}{a}\right)\int_0^T e(t)\,dt =$ *1470 A*
13. Loading $G_L = \frac{1}{Z_L} =$ ____

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt} =$ ____
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc} =$ *1470 A*

G(aF) approach

10. Find aF
    a. $f_1 = \frac{1}{4t_1} =$ *2.5×10^7*
    b. $f_2 = \frac{1}{4t_2} =$ *1.32×10^7*
    c. $F = \frac{f_1 + f_2}{2} =$ *1.91×10^7*
    d. $aF =$ *9.7×10^6*
11. From graph $G(aF) =$ *5×10^{-3}*
12. $I_{sc} = cG(aF)\int_0^T e(t)\,dt =$ *1050 A*
13. Loading $G_L = \frac{1}{Z_L} =$ ____

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt} =$ ____
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc} =$ *1050 A*

Date | Problem Example 4 | Engineer

For comparison, 20.6 mA calculated.

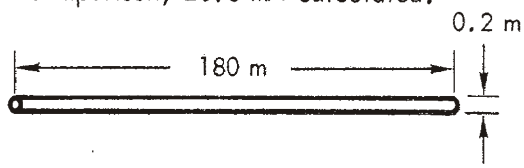
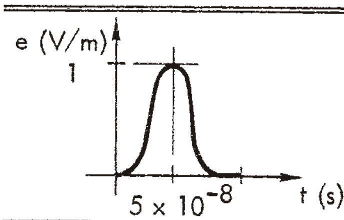


Fig. A. Actual Geometry

Fig. B. Wire Approximation

1. Sketch actual geometry in Fig. A. ______
2. Sketch the wire approximation in Fig. B. ______
3. Sketch the driving e(t) in Fig. C. ______
4. Wire radius a = *0.1*
5. Wire length L = *180*
6. Time of reflection $t_r = \frac{L}{2c}$ = (c = 3×10^8 m/s) *3×10^{-7}*
7. Plot t_r on Fig. C. ______
8. Sketch the approximate e(t) in Fig. D. ______
9. Estimate $\int_0^T e(t)\,dt = \frac{\sqrt{\pi}}{a} =$ *1.97×10^{-8}*



$e(t) = \exp(-a^2 t^2)$
$a = 9 \times 10^7$
10 db cutoff at
3.4×10^7 Hz

Fig. C. Incident Field e(t)

Fig. D. Approximate e(t)

$h\left(\frac{c\tau}{a}\right)$ approach

10. Find $\frac{c\tau}{a}$
    a. Rise time t_1 = *5×10^{-8}*
    b. Decay time t_2 = *10^{-7}*
    c. $\tau = \frac{t_2 - t_1}{2}$ = *2.5×10^{-8}*
    d. $\frac{c\tau}{a}$ = *75*
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right)$ = *0.95×10^6*
12. $I_{sc} = h\left(\frac{c\tau}{a}\right)\int_0^T e(t)\,dt$ = *18.7 mA*
13. Loading $G_L = \frac{1}{Z_L}$ = ______

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = ______
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = *18.7 mA*

G(aF) approach

10. Find aF
    a. $f_1 = \frac{1}{4t_1}$ = ______
    b. $f_2 = \frac{1}{4t_2}$ = ______
    c. $F = \frac{f_1 + f_2}{2}$ = *$f_{-10\,db}$* = *3.4×10^7*
    d. aF = *3.4×10^6*
11. From graph G(aF) = *3.1×10^{-3}*
12. $I_{sc} = cG(aF)\int_0^T e(t)\,dt$ = *18.3 mA*
13. Loading $G_L = \frac{1}{Z_L}$ = ______

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = ______
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = *18.3 mA*

Date	Problem Example 5	Engineer

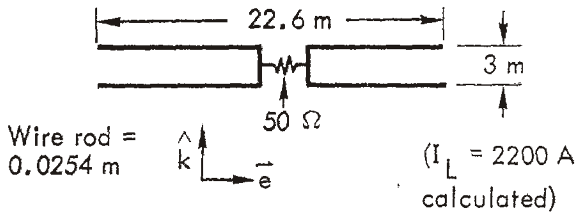


Fig. A. Actual Geometry

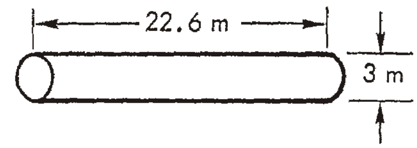


Fig. B. Wire Approximation

1. Sketch actual geometry in Fig. A.
2. Sketch the wire approximation in Fig. B.
3. Sketch the driving e(t) in Fig. C.
4. Wire radius a = *1.5*
5. Wire length L = *22.6*
6. Time of reflection $t_r = \frac{L}{2c}$ = *3.76×10^{-8}*
   ($c = 3 \times 10^8$ m/s)
7. Plot t_r on Fig. C.
8. Sketch the approximate e(t) in Fig. D.
9. Estimate $\int_0^T e(t)\,dt$ = *1.63×10^{-3}*

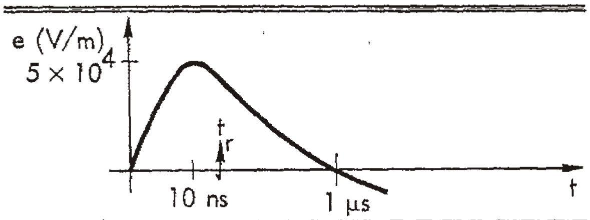


Fig. C. Incident Field e(t)

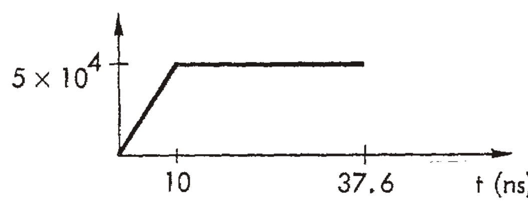


Fig. D. Approximate e(t)

$h\left(\frac{c\tau}{a}\right)$ approach

10. Find $\frac{c\tau}{a}$
    a. Rise time t_1 = *10^{-8}*
    b. Decay time t_2 = *3.76×10^{-8}*
    c. $\tau = \frac{t_2 - t_1}{2}$ = *1.38×10^{-8}*
    d. $\frac{c\tau}{a}$ = *2.76*
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right)$ = *2.0×10^{6}*
12. $I_{sc} = h\left(\frac{c\tau}{a}\right)\int_0^T e(t)\,dt$ = *3260 A*
13. Loading $G_L = \frac{1}{Z_L}$ = *20×10^{-3}*
    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = *6.7×10^{-3}*
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = *2440 A*

G(aF) approach

10. Find aF
    a. $f_1 = \frac{1}{4t_1}$ = *2.5×10^{7}*
    b. $f_2 = \frac{1}{4t_2}$ = *6.65×10^{6}*
    c. $F = \frac{f_1 + f_2}{2}$ = *1.58×10^{7}*
    d. aF = *2.4×10^{7}*
11. From graph G(aF) = *8.2×10^{-3}*
12. $I_{sc} = cG(aF)\int_0^T e(t)\,dt$ = *4010 A*
13. Loading $G_L = \frac{1}{Z_L}$ = *20×10^{-3}*
    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = *8.2×10^{-3}*
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = *2843 A*

Date | Problem Example 6 | Engineer

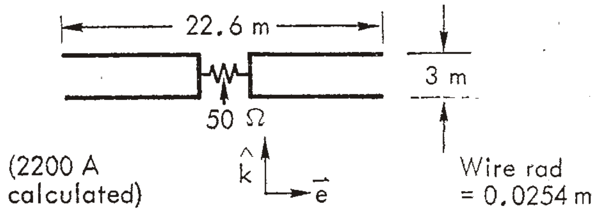


Fig. A. Actual Geometry

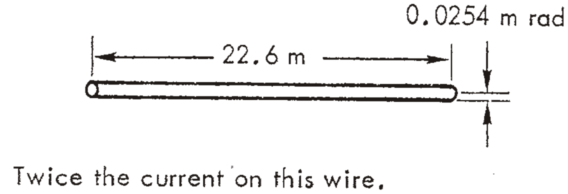


Fig. B. Wire Approximation

1. Sketch actual geometry in Fig. A. ______
2. Sketch the wire approximation in Fig. B. ______
3. Sketch the driving e(t) in Fig. C. ______
4. Wire radius a = 0.0254
5. Wire length L = 22.6
6. Time of reflection $t_r = \frac{L}{2c}$ = 3.76×10^{-8}
   ($c = 3 \times 10^8$ m/s)
7. Plot t_r on Fig. C. ______
8. Sketch the approximate e(t) in Fig. D. ______
9. Estimate $\int_0^T e(t)\,dt$ = 1.63×10^{-3}

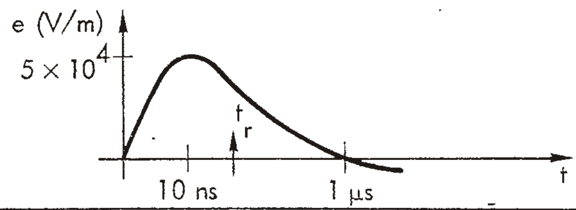


Fig. C. Incident Field e(t)

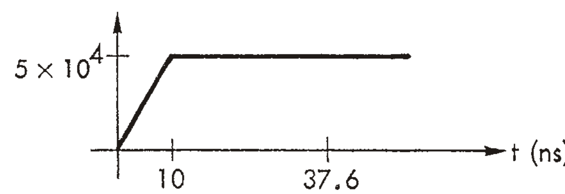


Fig. D. Approximate e(t)

$h\left(\frac{c\tau}{a}\right)$ approach

10. Find $\frac{c\tau}{a}$
    a. Rise time t_1 = 10^{-8}
    b. Decay time t_2 = 3.76×10^{-8}
    c. $\tau = \frac{t_2 - t_1}{2}$ = 1.38×10^{-8}
    d. $\frac{c\tau}{a}$ = 163
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right)$ = $2 \times 0.85 \times 10^{6}$
12. $I_{sc} = h\left(\frac{c\tau}{a}\right) \int_0^T e(t)\,dt$ = 2770 A
13. Loading $G_L = \frac{1}{Z_L}$ = 20×10^{-3}
    $G_{eff} = \frac{I_{sc}}{c \int_0^T e(t)\,dt}$ = 5.67×10^{-3}
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = 2130 A

G(aF) approach

10. Find aF
    a. $f_1 = \frac{1}{4t_1}$ = 2.5×10^{7}
    b. $f_2 = \frac{1}{4t_2}$ = 6.65×10^{6}
    c. $F = \frac{f_1 + f_2}{2}$ = 1.58×10^{7}
    d. aF = 4×10^{5}
11. From graph G(aF) = $2 \times 1.9 \times 10^{-3}$
12. $I_{sc} = cG(aF) \int_0^T e(t)\,dt$ = 1858 A
13. Loading $G_L = \frac{1}{Z_L}$ = 20×10^{-3}
    $G_{eff} = \frac{I_{sc}}{c \int_0^T e(t)\,dt}$ = 3.8×10^{-3}
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = 1561 A

Date | Problem Example 7 | Engineer

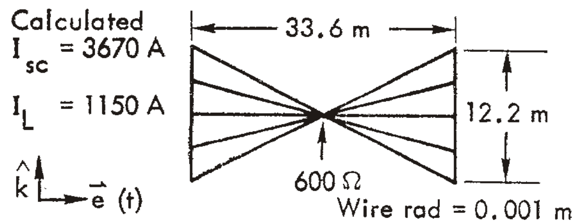


Fig. A. Actual Geometry

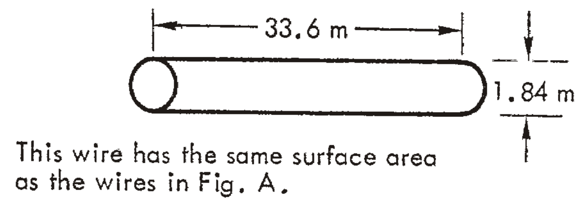


Fig. B. Wire Approximation

1. Sketch actual geometry in Fig. A.
2. Sketch the wire approximation in Fig. B.
3. Sketch the driving e(t) in Fig. C.
4. Wire radius a = 0.97
5. Wire length L = 33.6
6. Time of reflection $t_r = \frac{L}{2c}$ = ($c = 3 \times 10^8$ m/s) 5.6×10^{-8}
7. Plot t_r on Fig. C.
8. Sketch the approximate e(t) in Fig. D.
9. Estimate $\int_0^T e(t)\,dt$ = 2.55×10^{-3}

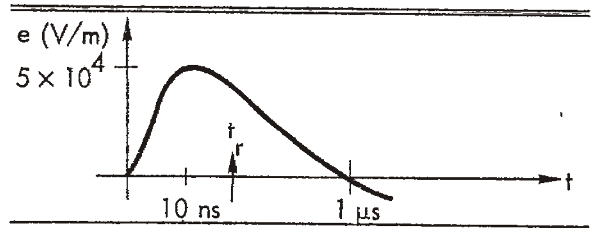


Fig. C. Incident Field e(t)

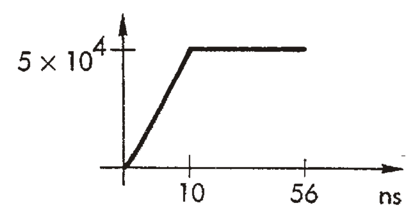


Fig. D. Approximate e(t)

$h\left(\frac{c\tau}{a}\right)$ approach

10. Find $\frac{c\tau}{a}$
    - a. Rise time t_1 = 10^{-8}
    - b. Decay time t_2 = 5.6×10^{-8}
    - c. $\tau = \frac{t_2 - t_1}{2}$ = 2.3×10^{-8}
    - d. $\frac{c\tau}{a}$ = 7.1
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right)$ = 1.49×10^6
12. $I_{sc} = h\left(\frac{c\tau}{a}\right)\int_0^T e(t)\,dt$ = 3800 A
13. Loading $G_L = \frac{1}{Z_L}$ = 1.67×10^{-3}

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = 4.53×10^{-3}
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = 1023 A

G(aF) approach

10. Find aF
    - a. $f_1 = \frac{1}{4t_1}$ = 2.5×10^7
    - b. $f_2 = \frac{1}{4t_2}$ = 4.5×10^6
    - c. $F = \frac{f_1 + f_2}{2}$ = 1.48×10^7
    - d. aF = 1.44×10^7
11. From graph G(aF) = 6.05×10^{-3}
12. $I_{sc} = cG(aF)\int_0^T e(t)\,dt$ = 4630 A
13. Loading $G_L = \frac{1}{Z_L}$ = 1.67×10^{-3}

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = 6.05×10^{-3}
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = 1000 A

Date	Problem Example 8	Engineer

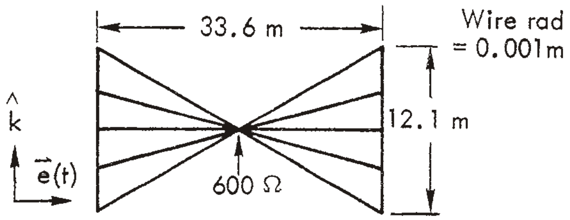


Fig. A. Actual Geometry

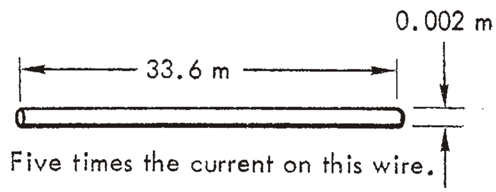


Fig. B. Wire Approximation

1. Sketch actual geometry in Fig. A. ______
2. Sketch the wire approximation in Fig. B. ______
3. Sketch the driving e(t) in Fig. C. ______
4. Wire radius a = 0.001
5. Wire length L = 33.6
6. Time of reflection $t_r = \frac{L}{2c}$ = ($c = 3 \times 10^8$ m/s) 5.6×10^{-8}
7. Plot t_r on Fig. C. ______
8. Sketch the approximate e(t) in Fig. D. ______
9. Estimate $\int_0^T e(t)\,dt$ = 2.55×10^{-3}

See page 18

Fig. C. Incident Field e(t)

See page 18

Fig. D. Approximate e(t)

$h\left(\frac{c\tau}{a}\right)$ approach

10. Find $\frac{c\tau}{a}$
    - a. Rise time t_1 = 10^{-8}
    - b. Decay time t_2 = 5.6×10^{-8}
    - c. $\tau = \frac{t_2 - t_1}{2}$ = 2.3×10^{-8}
    - d. $\frac{c\tau}{a}$ = 6900
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right)$ = $5 \times 0.5/ \times 10^{6}$
12. $I_{sc} = h\left(\frac{c\tau}{a}\right)\int_0^T e(t)\,dt$ = 6500 A
13. Loading $G_L = \frac{1}{Z_L}$ = 1.67×10^{-3}

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = 8.5×10^{-3}
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = 1067 A

G(aF) approach

10. Find aF
    - a. $f_1 = \frac{1}{4t_1}$ = 2.5×10^{7}
    - b. $f_2 = \frac{1}{4t_2}$ = 4.5×10^{6}
    - c. $F = \frac{f_1 + f_2}{2}$ = 1.48×10^{7}
    - d. aF = 1.48×10^{4}
11. From graph G(aF) = $5 \times 1.1 \times 10^{-3}$
12. $I_{sc} = cG(aF)\int_0^T e(t)\,dt$ = 4207 A
13. Loading $G_L = \frac{1}{Z_L}$ = 1.67×10^{-3}

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = 5.5×10^{-3}
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = 980 A

Date | Problem Example 9 | Engineer

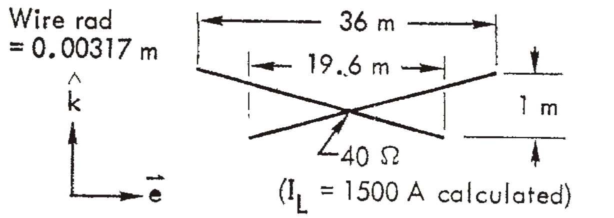


Fig. A. Actual Geometry

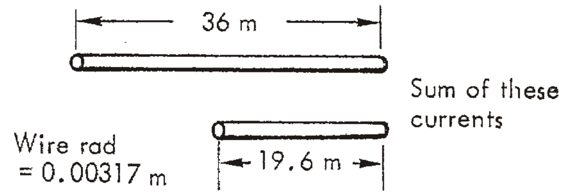


Fig. B. Wire Approximation

1. Sketch actual geometry in Fig. A. ______
2. Sketch the wire approximation in Fig. B. ______
3. Sketch the driving e(t) in Fig. C. ______
4. Wire radius a = *0.00317*
5. Wire length L = *36, 19.8*
6. Time of reflection $t_r = \frac{L}{2c}$ = ($c = 3 \times 10^8$ m/s) *6×10^{-8}, 3.3×10^{-8}*
7. Plot t_r on Fig. C. ______
8. Sketch the approximate e(t) in Fig. D. ______
9. Estimate $\int_0^T e(t)\,dt$ = *2.75×10^{-3}, 1.4×10^{-3}*

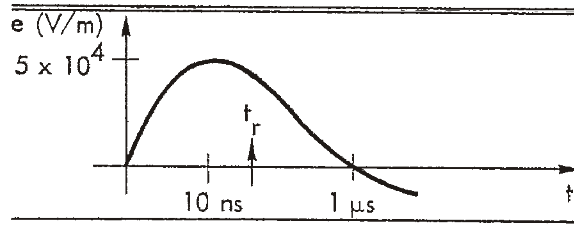


Fig. C. Incident Field e(t)

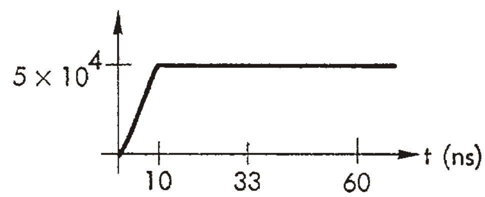


Fig. D. Approximate e(t)

$h\left(\frac{c\tau}{a}\right)$ approach

10. Find $\frac{c\tau}{a}$
   a. Rise time t_1 = *10^{-8}*
   b. Decay time t_2 = *6×10^{-8}, 3.3×10^{-8}*
   c. $\tau = \frac{t_2 - t_1}{2}$ = *2.5×10^{-8}, 1.65×10^{-8}*
   d. $\frac{c\tau}{a}$ = *2366, 1562*
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right)$ = *0.58×10^{6}, 0.63×10^{6}*
12. $I_{sc} = h\left(\frac{c\tau}{a}\right)\int_0^T e(t)\,dt$ = *1595 + 882*
13. Loading $G_L = \frac{1}{Z_L}$ = *25×10^{-3}*

   $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = *1.98×10^{-3}*
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = *2295 A*

(or 1480 A using long arm)

G(aF) approach

10. Find aF *(for long arm only)*
   a. $f_1 = \frac{1}{4t_1}$ = *2.5×10^{7}*
   b. $f_2 = \frac{1}{4t_2}$ = *4.17×10^{6}*
   c. $F = \frac{f_1 + f_2}{2}$ = *1.46×10^{7}*
   d. aF = *4.6×10^{4}*
11. From graph G(aF) = *1.3×10^{-3}*
12. $I_{sc} = cG(aF)\int_0^T e(t)\,dt$ = *1072 A*
13. Loading $G_L = \frac{1}{Z_L}$ = *25×10^{-3}*

   $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = *1.3×10^{-3}*
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = *1020 A*

Date | Problem Example 10 | Engineer

Stick model of fuselage and wings:

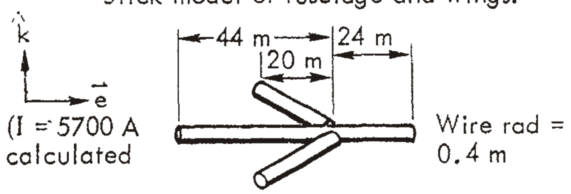


Fig. A. Actual Geometry

From experience of Example 9, use

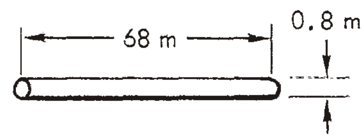


Fig. B. Wire Approximation

1. Sketch actual geometry in Fig. A. ______
2. Sketch the wire approximation in Fig. B. ______
3. Sketch the driving e(t) in Fig. C. ______
4. Wire radius a = *0.4*
5. Wire length L = *68*
6. Time of reflection $t_r = \frac{L}{2c} =$ ($c = 3 \times 10^8$ m/s) *1.13×10^{-7}*
7. Plot t_r on Fig. C. ______
8. Sketch the approximate e(t) in Fig. D. ______
9. Estimate $\int_0^T e(t)\,dt =$ *5.4×10^{-3}*

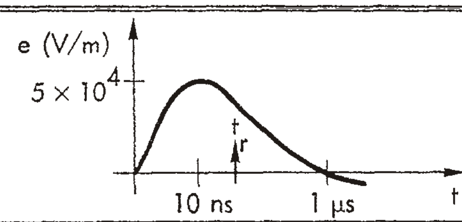


Fig. C. Incident Field e(t)

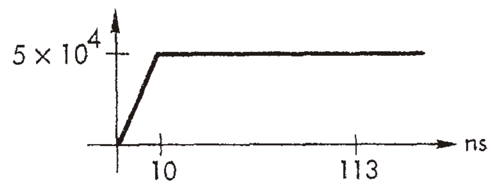


Fig. D. Approximate e(t)

$h\left(\frac{c\tau}{a}\right)$ approach

10. Find $\frac{c\tau}{a}$
   - a. Rise time $t_1 =$ *10^{-8}*
   - b. Decay time $t_2 =$ *1.13×10^{-7}*
   - c. $\tau = \frac{t_2 - t_1}{2} =$ *5.15×10^{-8}*
   - d. $\frac{c\tau}{a} =$ *39*
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right) =$ *1.05×10^6*
12. $I_{sc} = h\left(\frac{c\tau}{a}\right)\int_0^T e(t)\,dt =$ *5670 A*
13. Loading $G_L = \frac{1}{Z_L} =$ ______

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt} =$ ______
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc} =$ *5670 A*

G(aF) approach

10. Find aF
   - a. $f_1 = \frac{1}{4t_1} =$ *2.5×10^7*
   - b. $f_2 = \frac{1}{4t_2} =$ *2.2×10^6*
   - c. $F = \frac{f_1 + f_2}{2} =$ *1.36×10^7*
   - d. aF = *5.4×10^6*
11. From graph G(aF) = *3.9×10^{-3}*
12. $I_{sc} = cG(aF)\int_0^T e(t)\,dt =$ *6318 A*
13. Loading $G_L = \frac{1}{Z_L} =$ ______

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt} =$ ______
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc} =$ *6318 A*

10/21/785

APPENDIX B. EXAMPLES

Date | Problem Example 11 | Engineer

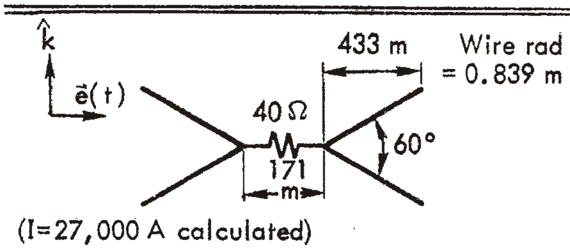


Fig. A. Actual Geometry

Twice the current on this wire.

Fig. B. Wire Approximation

1. Sketch actual geometry in Fig. A. ______
2. Sketch the wire approximation in Fig. B. ______
3. Sketch the driving e(t) in Fig. C. ______
4. Wire radius a = *0.839*
5. Wire length L = *1037*
6. Time of reflection $t_r = \frac{L}{2c}$ = (c = 3×10^8 m/s) *1.72×10^{-6}*
7. Plot t_r on Fig. C. ______
8. Sketch the approximate e(t) in Fig. D. ______
9. Estimate $\int_0^T e(t)\,dt$ = *2.5×10^{-2}*

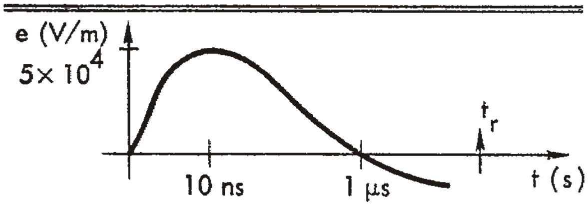


Fig. C. Incident Field e(t)

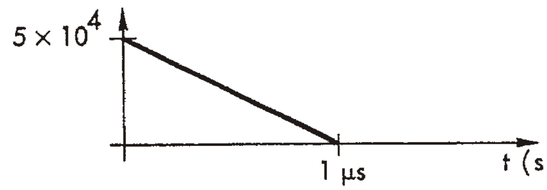


Fig. D. Approximate e(t)

$h\left(\frac{c\tau}{a}\right)$ approach

10. Find $\frac{c\tau}{a}$
    a. Rise time t_1 = *10^{-8}*
    b. Decay time t_2 = *10^{-6}*
    c. $\tau = \frac{t_2 - t_1}{2}$ = *5×10^{-7}*
    d. $\frac{c\tau}{a}$ = *178*
11. From Fig. 1 $h\left(\frac{c\tau}{a}\right)$ = *$2 \times 0.83 \times 10^{6}$*
12. $I_{sc} = h\left(\frac{c\tau}{a}\right)\int_0^T e(t)\,dt$ = *41,500 A*
13. Loading $G_L = \frac{1}{Z_L}$ = *25×10^{-3}*

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = *5.3×10^{-3}*
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = *34,240 A*

G(aF) approach

10. Find aF *(est. period as $2 \times t_2$)*
    a. $f_1 = \frac{1}{4t_1}$ = ______
    b. $f_2 = \frac{1}{4t_2}$ = ______
    c. $F = \frac{f_1 + f_2}{2}$ = *est. 5×10^5*
    d. aF = *4.2×10^5*
11. From graph G(aF) = *$2 \times 1.9 \times 10^{-3}$*
12. $I_{sc} = cG(aF)\int_0^T e(t)\,dt$ = *28,500 A*
13. Loading $G_L = \frac{1}{Z_L}$ = *25×10^{-3}*

    $G_{eff} = \frac{I_{sc}}{c\int_0^T e(t)\,dt}$ = *3.8×10^{-3}*
14. $I_L = \frac{G_L}{G_L + G_{eff}} I_{sc}$ = *24,740 A*