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DIFFRACTION OF PLANAR ELECTROMAGNETIC WAVES BY A SLOT

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ABSTRACT

Edge-wave methods are used to estimate the amplitude of electromagnetic-wave coupling through an aperture slot in a plane screen. The high-frequency approximation is used to calculate far-field values. Computed results show good agreement with experimental data.

Key Words: Edge-wave methods, EM coupling, aperture

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Introduction

An understanding of the basic mechanisms by which an electromagnetic signal penetrates an aperture in a shielded enclosure is prerequisite to an investigation of EMR or EMP interaction with devices and subsystems. Of all the aperture configurations that could be considered, the two-dimensional slot in a conducting screen is probably the most elementary. However, the mathematical analysis associated with this elementary geometry is not at all trivial. Although considerable progress has recently been made in obtaining closed-form solutions for slot diffraction[1], the usual approach is to consider separately the two complementary cases in which the slot width is either much less or much greater than a wavelength. The method of moments[2,3] is useful for the former case, and asymptotic techniques can be used to treat the latter.* This paper is concerned with the latter case: that is, with the so-called high-frequency approximation.

The diffraction of electromagnetic waves by a slot in a conducting plane and the dual problem of diffraction by a conducting strip are classic problems in electromagnetic theory and have been often investigated. In fact, this particular aperture is one of the few shapes for which an exact solution can be specified. Unfortunately, the solution is given in terms of Mathieu functions, one of the little known and more complicated functions of mathematical physics. Calculations are made even more difficult by the fact that solutions appear as very slowly convergent series, with the result that the method fails if the characteristic aperture dimensions exceed one or two wavelengths of the incident radiation.

However, by use of asymptotic techniques[4,5], expressions for far fields can be obtained for aperture widths as narrow as one or two wavelengths. These expressions satisfy both the principle of reciprocity and the boundary conditions in the plane of the slot. Although the work discussed here follows the approach described by Kashkind and Vainshteyn[5], the orientation of the present study is both computational and experimental as constrasted with the referenced papers[4,5] which are theoretical. The primary objective of this paper is to establish, on the basis of experimental data, a confidence level for applicability of the asymptotic theory. Because the ranges of applicability of the asymptotic and exact theory do overlap, both methods should be in agreement in this range. Therefore the simple asymptotic theory could be used to check the more complicated exact solutions.

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*Use of moment methods at high frequencies is not feasible with generally available computers because of the very large core storage required to invert the matrix.

As the size of the aperture increases to many wavelengths, as it would for example in the case of an open bomb bay of an aircraft, it becomes almost impossible to avoid using some approximation for numerical calculations. For this reason it is appropriate to describe briefly the particular asymptotic approximation known as the edge method. This technique is closely related to the approach used by Schwarzschild[6], who first considered a slot as consisting essentially of two diffracting half-planes. The response of each half-plane to a variety of excitations can be calculated by the Sommerfeld Green's function. Then the total field at any point is obtained as the superposition of the fields diffracted independently by each half-plane. As the observer moves many wavelengths away from the plane, the diffracted field from a half-plane takes on the character of waves emanating from the edge (hence the name, edge waves). By using more than two half-planes and introducing Babinet's duality principle, one can calculate the fields diffracted by apertures of various shapes. However, the discussion here is restricted to the slot.

The edge-wave method is treated more fully in the next section. Because the functions associated with the edge method can be approximated by elementary exponential functions and Fresnel integrals, a very simple code can be written to calculate the total diffracted field from the slot. Results obtained using this code are in reasonable agreement with experimental electric-field intensity measurements which will be described. These measurements were made only at 10 GHz.

For simplicity this discussion is restricted to the case of electric polarization (incident electric field parallel to the edges of the slot). For convenience the time variation of the form exp (jwt) is suppressed.

Theory

A plane wave with unit amplitude

$$\Phi^{o} = \exp\left[jk(x \cos \theta_o + y \sin \theta_o)\right], \ (0 < \theta_o < \pi) \quad , \tag{1}$$

where $k = \omega/c$ is the free-space wave number, is incident on a slot of width 2d. The slot is formed by the two half-planes $y = 0$, $x > d$ and $y = 0$, $x < -d$. The edges of the slot are parallel to the Z axis of a cylindrical coordinate system r, θ, z. The r, θ plane is the plane of incidence, with the direction of incidence making the angle θ_o with the positive X axis. Our interest is restricted to the diffracted field in the half-space $y \geq o$ (Figure 1) subject to the Fraunhofer conditions

$$kr >> 1, \ kr >> \rho^2 \quad , \tag{2}$$

where r is the distance from the center of the slot to the observer located at (r, θ) with

$$x = r \cos \theta, \quad y = r \sin \theta \quad , \tag{3}$$

and ρ is a characteristic parameter defined by

$$\rho = kd = 2\pi d/\lambda \quad . \tag{4}$$

It is also convenient to introduce the direction cosines α_o and α given by

$$\alpha_o = \cos\theta_o \quad , \quad \alpha = \cos\theta \quad . \tag{5}$$

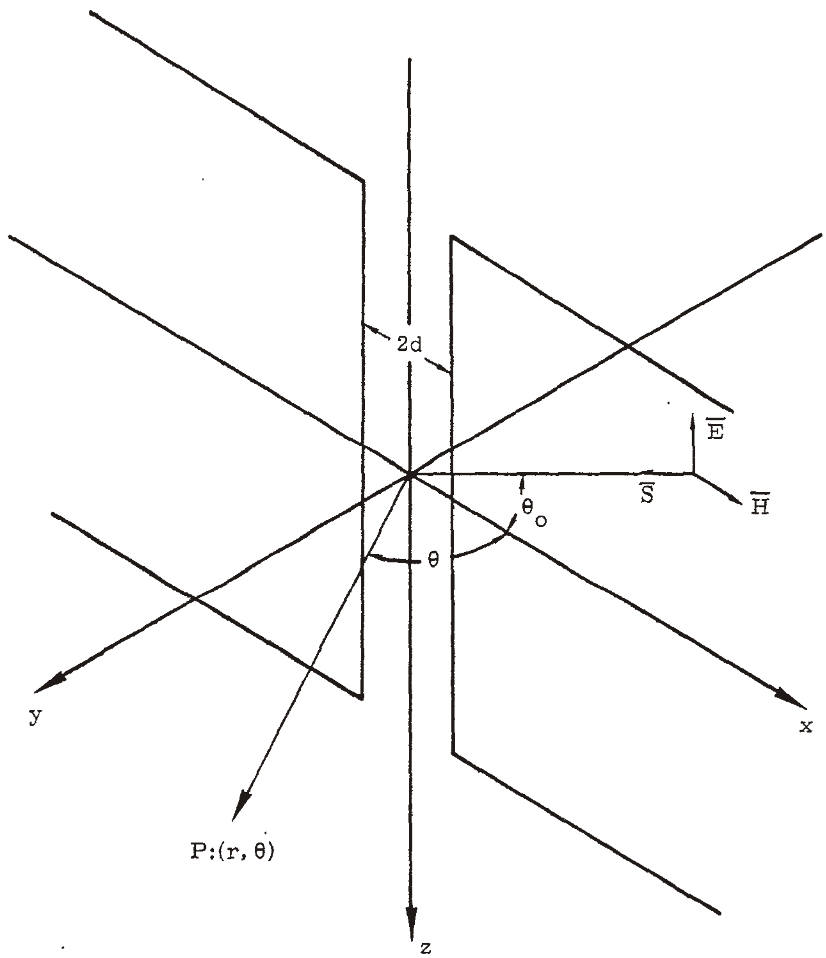

Figure 1. Diffraction of a plane wave by a slot

The diffracted field $\Phi = \Phi(x, y)$ is that solution of the wave equation

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + k^2\Phi = 0 \tag{6}$$

which satisfies the boundary condition

$$\Phi = 0 \text{ for } y = 0, \quad |x| > d \quad . \tag{7}$$

On the slot (i. e., for $|x| < d$) the following conditions of continuity must be satisfied

$$\Phi(x, +0) = \Phi(x, -0), \frac{\partial}{\partial y} \Phi(x, 0) = \frac{\partial}{\partial y} \Phi(x, -0) \quad . \tag{8}$$

Following Khaskind and Vainshteyn[5], Φ is obtained as

$$\Phi = \Phi^{o} + \frac{1}{2\pi j} \int_{-\infty}^{\infty} \psi(\alpha_o, \alpha) \exp\left\{ jk\left[x\alpha + |y| \sqrt{1 - \alpha^2}\right]\right\} \frac{d\alpha}{\sqrt{1 - \alpha^2}} \tag{9}$$

where ψ, given by

$$\psi(\alpha_o, \alpha) = \Psi(\alpha_o, \alpha) e^{-j\rho\alpha} + \Psi(-\alpha_o, -\alpha) e^{j\rho\alpha} \quad , \tag{10}$$

must be so chosen as to satisfy (7) and (8). Appropriate forms for Ψ will be considered shortly.

In the upper half-space $y \geq 0$ at sufficiently large distances from the slot, the field reduces to a cylindrical wave[7]

$$\Phi = \psi(\alpha_o, \alpha) \exp\left[j(kr - 3\pi/4)\right] / \sqrt{2\pi kr} \quad . \tag{11}$$

According to the principle of reciprocity, the field does not change if the positions of the source and point of observation are interchanged; therefore, ψ must satisfy the condition

$$\psi(\alpha_o, \alpha) = \psi(-\alpha, -\alpha_o) \quad . \tag{12}$$

The cylindrical wave (11) should also satisfy the boundary condition (7) which leads to the relationships

$$\psi(\alpha_o, \pm 1) = 0 \quad \text{and} \tag{13}$$

$$\psi(\pm 1, \alpha) = 0 \tag{14}$$

The problem as formulated corresponds to a plane electromagnetic wave which is polarized along the slot in a perfectly conducting plane.

If $\rho >> 1$, a zero-order solution can be obtained by the method of edge waves. This method reduces to the superposition of two waves emanating from the edges of the half planes. In this case (10) can be written in the form

$$\psi^{(o)}(\alpha_o, \alpha) = \Psi_o(\alpha_o, \alpha)\, e^{-j\rho\alpha} + \Psi_o(-\alpha_o, -\alpha) e^{j\rho\alpha} \; , \tag{15}$$

where[5]

$$\Psi_o(\beta, \delta) = -\frac{\sqrt{1+\beta}\sqrt{1-\delta}}{\delta - \beta}\, e^{j\rho\beta} \tag{16}$$

If (15) is substituted into (11) the diffracted field has the very simple form

$$\Phi = -\frac{\sqrt{1+\alpha_o}\sqrt{1-\alpha}}{(\alpha - \alpha_o)\sqrt{2\pi kr}} \exp\left[j(\rho\alpha_o - \rho\alpha + kr - 3\pi/4)\right]$$

$$+ \frac{\sqrt{1-\alpha_o}\sqrt{1+\alpha}}{(\alpha - \alpha_o)\sqrt{2\pi kr}} \exp\left[j(-\rho\alpha_o + \rho\alpha + kr - 3\pi/4)\right] \; . \tag{17}$$

The first term on the right side of (17) represents the cylindrical wave radiated by the edge y = 0, x = d, and the second term represents the wave radiated by the edge y = 0, x = -d (Figure 2.) The disadvantage of (17) is that it fails to satisfy the boundary conditions (13), (14).

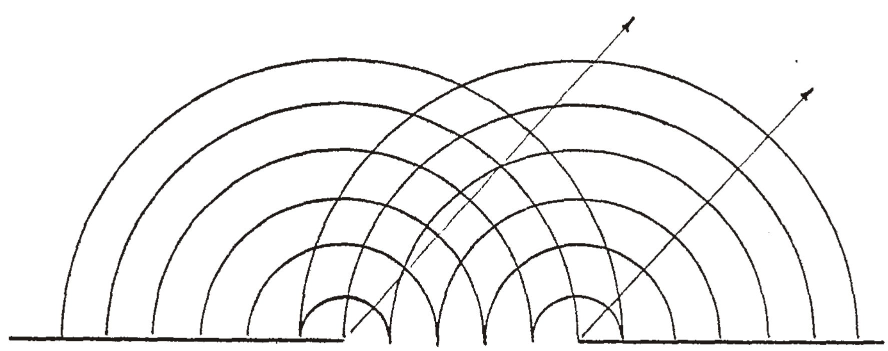

Figure 2. Formation of diffracted field by superposition of edge waves

In order to satisfy the boundary conditions, it is necessary to take into account the fact that the wave radiated from the edge of the left half-plane is diffracted by the right half-plane, and conversely. This modification can be included in the present notation by writing (10) in the form[7]

$$\psi^{(1)}(\alpha_o, \alpha) = \Psi_o(\alpha_o, \alpha)\, G(\rho,\ \alpha_o)\, G(\rho,\ -\alpha)\, G(\rho,\ -\alpha)\, e^{-j\rho\alpha}$$

$$+ \Psi_o(-\alpha_o,\ -\alpha)\, G(\rho,\ -\alpha_o)\, G(\rho,\ \alpha)\, e^{j\rho\alpha} \quad , \tag{18}$$

where G is the Sommerfeld Green's function given by

$$G(\rho, \eta) = \sqrt{2}\, e^{-j\pi/4} \left\{ C\left[2\sqrt{\pi\rho(1-\eta)}\right] + jS\left[2\sqrt{\pi\rho(1-\eta)}\right] \right\} \quad . \tag{19}$$

Here C and S, defined by the expressions

$$C(z) = \int_o^z \cos\frac{\pi}{2}t^2\, dt \text{ and } S(z) = \int_o^z \sin\frac{\pi}{2}t^2\, dt \quad , \tag{20}$$

and the real and imaginary parts, respectively, of the complex Fresnel integral. If (18) is substituted into (11) the diffracted field is obtained as

$$\Phi = -\frac{\sqrt{1+\alpha_o}\sqrt{1+\alpha}}{(\alpha-\alpha_o)\sqrt{2\pi kr}}\, G(\rho,\ \alpha_o)\, G(\rho,\ -\alpha)\, \exp\left[j(\rho\alpha_o - \rho\alpha + kr - 3\pi/4)\right]$$

$$+ \frac{\sqrt{1-\alpha_o}\sqrt{1+\alpha}}{(\alpha-\alpha_o)\sqrt{2\pi kr}}\, G(\rho,\ -\alpha_o)\, G(\rho,\ \alpha)\, \exp\left[j(-\rho\alpha_o + \rho\alpha + kr - 3\pi/4)\right] \quad . \tag{21}$$

According to (19), $G(\rho,\ 1) = 0$, thereby satisfying the boundary conditions (13), (14). A comprehensive discussion of the boundary condition has been given by Weinstein[7], who describes a rigorous mathematical foundation for the method of edge waves.

Fortran codes were written for the evaluation of (17) and (21). Because only the envelope of the RF carrier illuminating the slot can be observed conveniently, the code actually computes the relative intensity

$$I_R = 20 \log_{10} \left| \Phi/\Phi^o \right| \text{ dB} \quad ; \tag{22}$$

that is, the diffracted-field amplitude is normalized with respect to the amplitude of the field incident on the slot. The programs were run on a CDC 6600 computer, and plots were generated by means of a Stromberg Datagraphix 4460 plotter.

Experimental Arrangement

A cursory experimental survey was carried out at Sandia Laboratories to check the validity of the approximations described in this report. The experimental geometry is essentially that illustrated in Figure 1. A slot was cut in the center of a conducting plane consisting of a rectangular sheet of 0.48-cm-thick ST-6 aluminum with dimensions of 2.39 by 2.24 m. The plane formed one wall of a shielded enclosure whose depth was 2.10 m. The inner walls of the shielded enclosure, excepting the wall containing the slot, were covered with a pyramidal, carbon-loaded, polyurethane-foam absorbing material to form an anechoic chamber. The absorbing material reduces unwanted reflections from the enclosure walls and preserves approximate free-space conditions on the shadow side of the conducting plane.

Electric-field intensity measurements using a special miniature E-field sensor[8] were made inside the shielded enclosure on the shadow side of the conducting screen. The sensor is basically a short dipole roughly 1.5 cm in length and 0.02 cm in diameter. A subminiature microwave diode is located at the center of the dipole to convert the RF energy to a DC signal at the measurement point. It is generally accepted[9] that a short dipole probe does not appreciably perturb the field to be measured. Inaccuracies due to the presence of the probe should be less than 0.5 dB at points in the far-field region and away from the edges of the slot.

The sensor was supported by a column of plastic foam (2 lb/ft^3 density) whose intrinsic impedance approaches that of free space. The foam support was attached to a wooden positioning fixture covered with absorbing material. The fixture and supporting column allowed rapid three-dimensional positioning of the probe with a minimum perturbation of the field being measured. The output of the sensor was fed through a 60-cm length of carbon-impregnated plastic line to the base of the probe support, where it connected to a shielded, twisted pair cable leading to a microvolt-ammeter. The carbon-impregnated line passes the DC signal but is essentially transparent to high-frequency energy. The plastic line therefore does not disturb the field being measured, nor does it introduce unwanted RF energy into the measuring system. Pickup of RF energy by the shielded, twisted pair cable was minimized by running the cable underneath the absorbing material. The minimum probe output that could be measured with the instrumentation system used was on the order of 20 μV, corresponding to a field intensity of approximately 0.4 V/m.

Far-field data were obtained for polarizations parallel to the axis of the slot and for both normal and oblique incidence. Denoting the free-space wavelength of the incident radiation by λ, the normalized width of the slot was 2λ, and the frequency was 10 GHz. The far-field variation of the electric intensity was measured on the shadow side of the conducting plane as a function of the polar angle of the radius vector from the center of the slot to the point of observation (see Figure 1). The radial distance was kept constant (nominally 50λ) as the probe was rotated about the Z axis.

The transmitting antenna used to produce the incident wave was a pyramidal horn. The distance from the transmitting antenna to the conducting plane was made as large as possible so that far-field, plane-wave radiation conditions were approximated. For all measurements reported in this paper the minimum distance (along the wave normal) between the transmitting antenna and the center of the slot was 40 λ. On the basis of free-space measurements reported elsewhere[3, 10], it is believed that the maximum variation in field magnitude over the width of the slot at the origin of the coordinate system (Figure 1) in the absence of the conducting material would be better than ± 0.3 dB from the median value.

The magnitude of the electric field incident on the slot, E_{inc}, was taken to be the field that would exist at the aperture location in the absence of the plane and was calculated assuming free-space conditions:

$$E_{inc} = (30\, P_T G_T / R^2)^{1/2} , \tag{23}$$

where P_T is the power delivered to the transmitting antenna, G_T is the gain of the antenna and R is the distance from the antenna to the plane. The measured value of the field at a distance of 10 λ from the transmitting antenna was found to be within 0.2 dB of the value calculated from the above equation.

The overall accuracy of the measurements is estimated to be better than ± 1.5 dB. Inaccuracies in positioning the probe contributed most to the experimental error. Some results of the experiments are described in the next section.

Experimental and Computational Results

Figure 3 shows a typical plot of the amplitude of the diffracted field (zero-order approximation) normalized with respect to the field incident on the slot as a function of the polar angle θ of the radius vector from the center of the slot to the point of observation (Figure 1). The exciting field is normally incident to the plane of the slot. In conformity with the experimental arrangement described above, the slot half-width is one wavelength, and the length of the radius vector to the point of observation is 50 wavelengths.

The same parametric values were used in Figure 4, a first-order approximation, as in Figure 3. However, the value of Φ given by (21) was used in the computation, rather than the value given by (17). Inclusion of the Sommerfeld Green's functions in the code apparently does not grossly affect the calculated field intensities. Instead, there are seen superimposed on the smooth curve of Figure 3 the usual characteristic alternations in extrema which one associates with the Fresnel integral. The similarity between corresponding figures is not unexpected because as ρ increases without limit, G approaches unity and (21) approaches (17). It should be noted that the effects of the inclusion of the Sommerfeld Green's functions in the expression for Φ is more pronounced for values of θ near 0 and π.

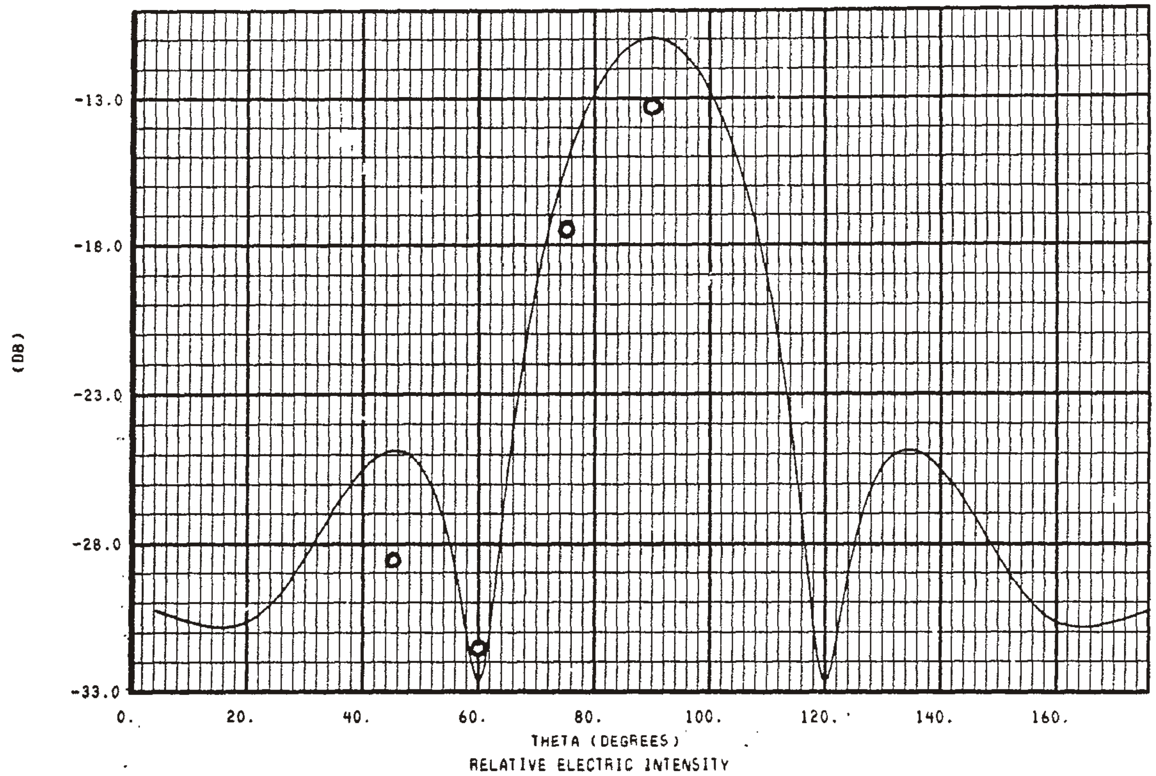

Figure 3. Zero order, high frequency, far field ($r = 50\lambda$) approximation of wave diffracted by a 2λ slot (exciting field normally incident)

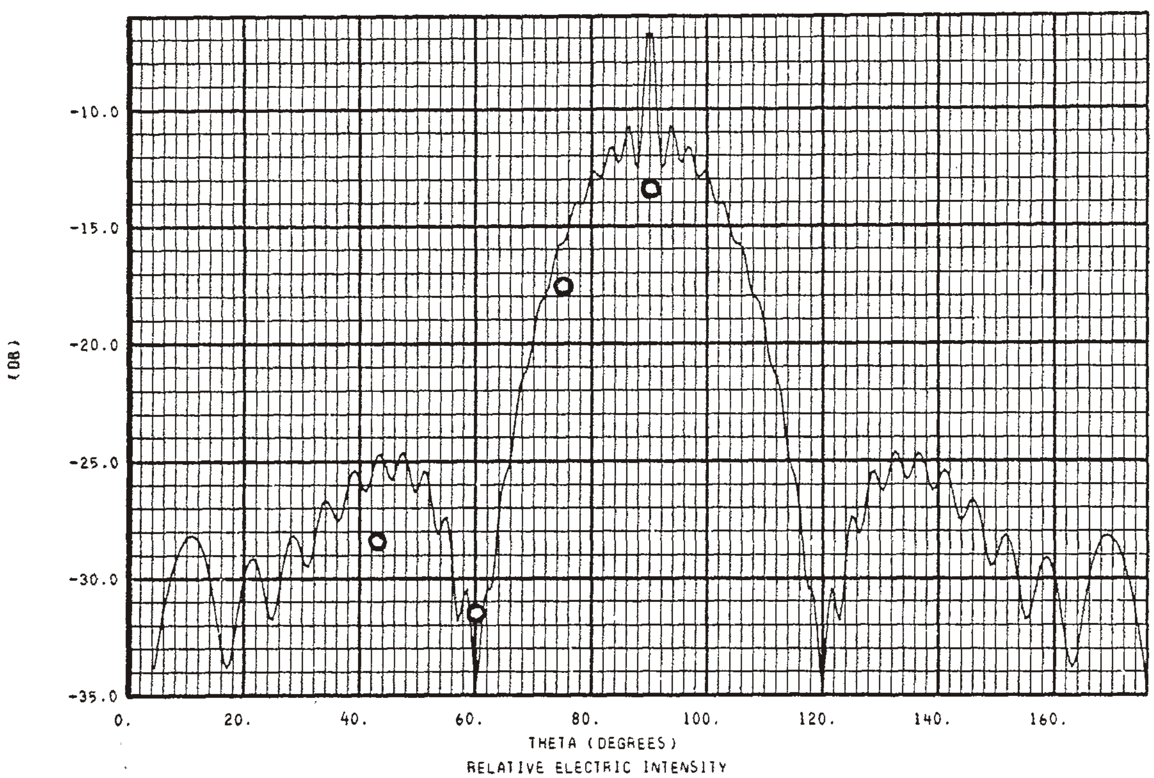

Figure 4. First order, high frequency, far field ($r = 50\lambda$) approximation of wave diffracted by a 2λ slot (exciting field normally incident)

Unless one is interested in values of electric intensity for points near the diffracting screen, (for example $\theta < 20^{\circ}$, or $\theta > 160^{\circ}$) the simpler code should provide adequate accuracy for most applications. The implication here is quite startling. Apparently reasonable estimates of diffracted-field intensities can, under some circumstances, be obtained by use of a theory in which the boundary conditions are ignored.

Data points indicated on Figures 3 and 4 are the experimentally measured values. Because of prior commitments, use of the experimental facility was severely restricted. Consequently only a few data points were taken. The closest approach to the conducting plane was 35 λ. This occurred at $\theta = 45^{\circ}$.

Unfortunately, since the experiment was performed before the code was successfully executed, no attempt was made to map the fields in those regions that the calculations would indicate as being of particular interest. A priori choice of the observations at 45° and 60°, where the function shows a secondary lobe, represents a fortuitous selection.

In general, the agreement between the measured values and values computed by use of the zero-order approximation can be characterized as being within 3 dB. Comparable agreement is seen in Figure 4 for the higher-order approximation. It would, of course, be interesting to repeat the experiment to see whether the tertiary lobe at 10° in Figure 4 could be observed.

Figures 5 and 6 show plots of the normalized amplitude of the diffracted field for an exciting field incidence of 60° (see Figure 1). Slot half-width and radial distance to the probe was 1 λ and 50 λ, respectively. Agreement between computed and measured values shown in Figure 5 seem to be slightly better than the agreement indicated in Figure 3. Again, the agreement can be considered to be within 3 dB.

A comparison of Figures 3 and 5 shows a shift of the main lobe because of the obliquity of incidence. Also, comparing Figures 4 and 6, the characteristic Fresnel perturbations associated with the first-order theory are more prominent if the exciting wave is obliquely incident. Figure 7, a zero order approximation, has been included to show the shift in main and subsidiary lobes as the obliquity increases to 30 degrees.

In summary, the experimental measurements reported here, although sparse, show reasonably good agreement with theory. The measurements tend to support the conclusion that the elementary edge-wave theory can provide 3-dB estimates of the diffracted field within the range of its applicability (far field, wide slot).

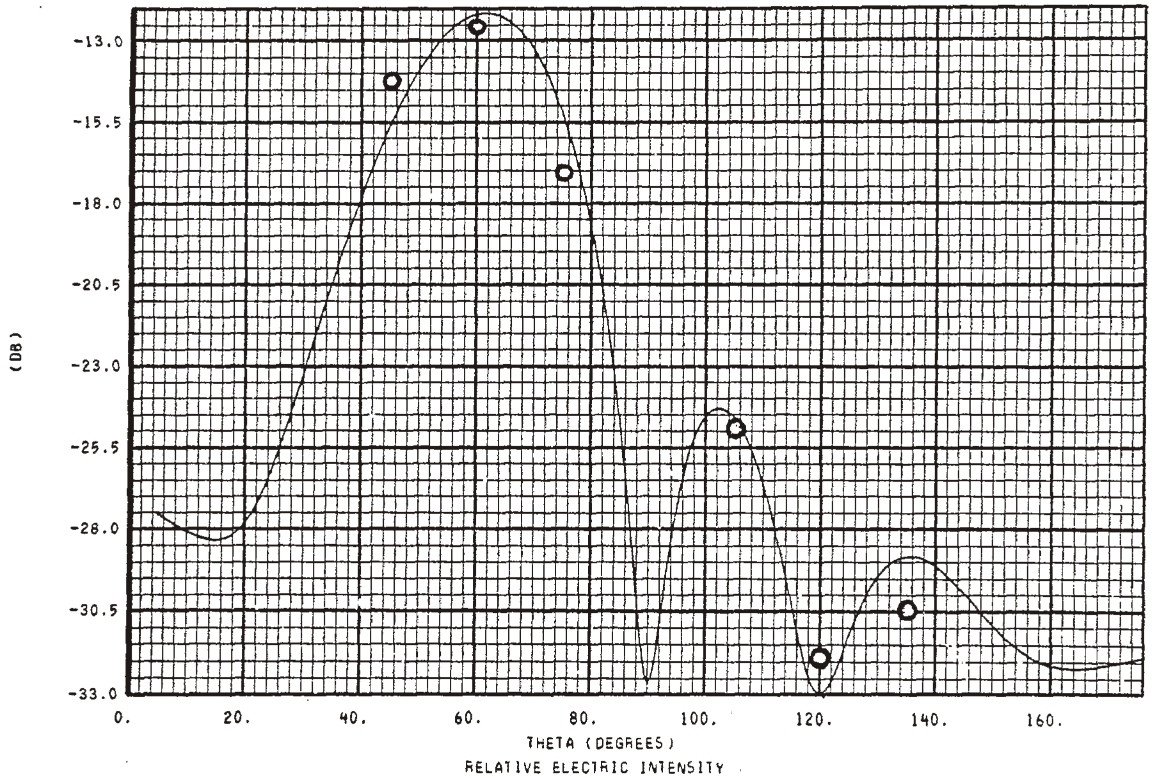

Figure 5. Zero order, far field ($r = 50\lambda$) approximation of wave diffracted by a 2λ slot (exciting field incidence of 60°)

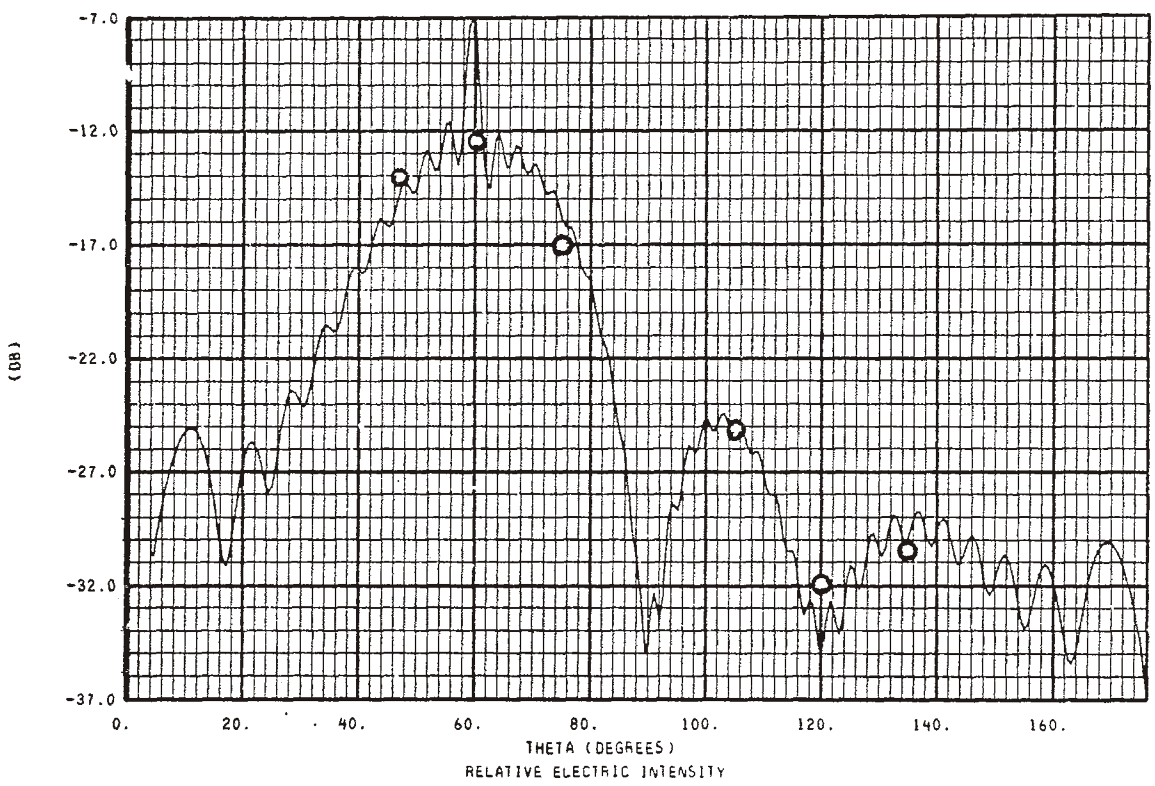

Figure 6. First order, far field ($r = 50\lambda$) approximation of wave diffracted by a 2λ slot (exciting field incidence of 60°)

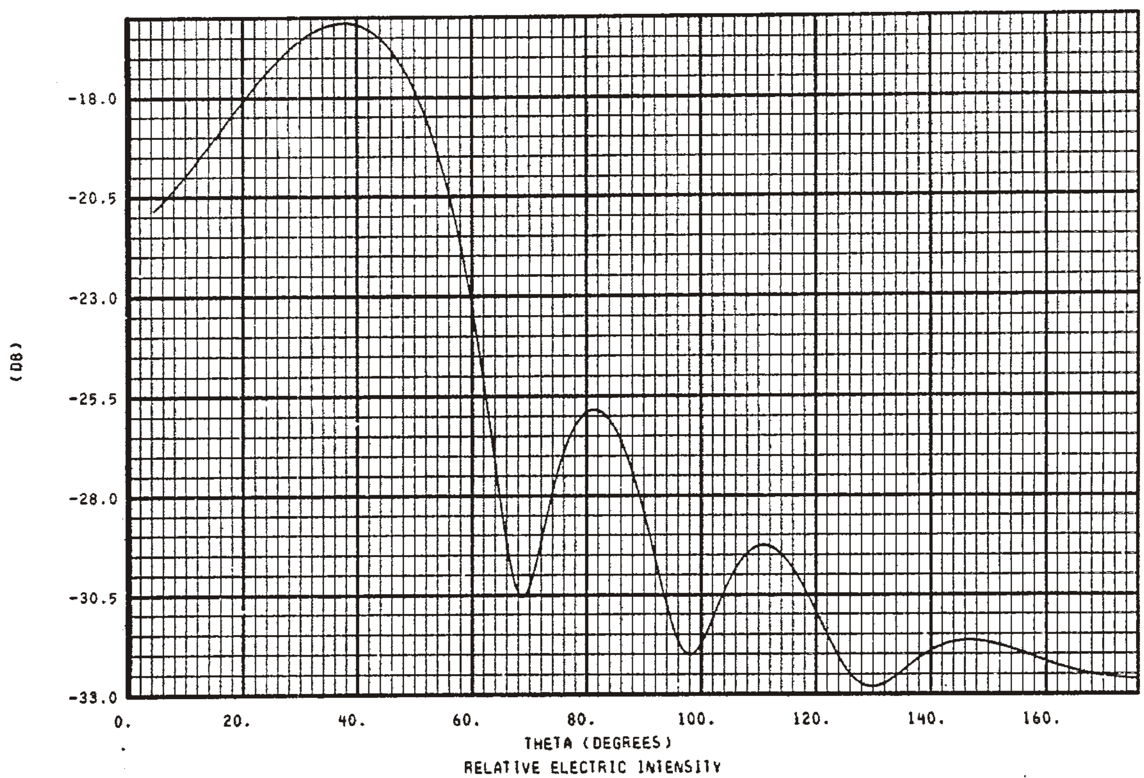

Figure 7. Zero order, far field ($r = 50\lambda$) approximation of wave diffracted by a 2λ slot (exciting field incidence of 30°)

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