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MULTICONDUCTOR ANTENNA TRANSMISSION LINES WITH ARBITRARILY POSITIONED LOAD IMPEDANCES IN AN INCIDENT FIELD

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SUMMARY

The problem considered is that of a multiple conductor transmission line with arbitrarily positioned impedances in series with each wire. The objective is to determine the currents in each of the loads in terms of the amplitude of the incident electric field. The groundwork is laid in this paper for a very general theory for N wire structures when the impedances are situated at the centers of the conductors. If the load elements are in echelon rather than centrally positioned, it appears necessary to abandon the simultaneous integral equation approach employed and resort to the use of two network theorems: superposition and compensation. It is demonstrated that the results depend on a synthesis of antenna and transmission line behavior. This work is an extension to the theory of end-loaded conductors published previously in the G-EMC Transactions.

MULTICONDUCTOR ANTENNA TRANSMISSION LINES WITH ARBITRARILY POSITIONED LOAD IMPEDANCES IN AN INCIDENT FIELD

Introduction

This paper is a sequel to an earlier investigation undertaken to determine the currents in the load impedances of a multiconductor transmission line excited by an incident plane wave electromagnetic field.[1] In the preceding paper the load elements—consisting of combinations of resistors, inductors, and capacitors—are assumed to be in series with the wires at their ends (see Figure 1, Reference 1). For this location of the lumped circuit elements in the structure, only transmission line currents flow in the impedances, because the antenna current vanishes at the ends of the wires. If now the loads are located elsewhere in the conductors, the impedances carry antenna as well as transmission line currents. This fact complicates the problem of calculating the total current in each load. It has been the experience of the writers that it is not too difficult an undertaking to obtain the currents flowing in N load impedances centrally oriented in the conductors of a multiple-wire antenna transmission line for parallel incidence of the electric field. However, for staggered loads the problem becomes more tenuous.

Some of the principles needed for the analysis of multiconductor antenna transmission lines in an incident field for arbitrarily positioned load impedances have been elucidated elsewhere.[1-3] Familiarity of the reader with the contents of the referenced papers is assumed. In the present paper attention is directed, in the interest of brevity, toward finding the currents in the loads of two-conductor configurations excited by an incident plane wave electromagnetic field.

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[1] C. W. Harrison, Jr., "Generalized Theory of Impedance Loaded Multiconductor Transmission Lines in an Incident Field," IEEE Transactions on Electromagnetic Compatibility, Vol EMC-14, No. 2, pp 56-63, May 1972.

[2] C. W. Harrison, Jr., "Reducing the Response of Single-Phase Transmission Lines to Electrical Noise," IEEE Transactions on Electromagnetic Compatibility, Vol EMC-14, No. 2, pp 79-81, May 1972 (Correspondence Item).

[3] C. W. Harrison, Jr., "Bounds on the Load Currents of Exposed One- and Two-Conductor Transmission Lines Electromagnetically Coupled to a Rocket," IEEE Transactions on Electromagnetic Compatibility, Vol EMC-14, No. 1, pp 4-9, February 1972.

The Short-Circuit Current in a Two-Conductor Antenna Transmission Line

Consider a two-wire transmission line with short-circuited terminations, as illustrated by Figure 1. The line lies in the y0z plane, with conductors parallel to the z axis. The wires are of length 2h and of radius a, and the spacing is b. The incident electric field E_z^{inc} is polarized parallel to the axis of the wires and arrives at the azimuth angle ϕ, measured from the positive x axis.

Following Reference 2, the equations for determining the currents in this circuit are as follows:

Conductor 1

$$J_d(z) + I_1^E(z)\psi_a + I_2^E(z)\psi_b = -j\frac{4\pi}{\zeta_0}\left[C_1 \cos\beta z + \frac{E_z^{inc}}{\beta}\exp\left(j\frac{\beta b}{2}\sin\phi\right)\right]. \qquad (1)$$

Conductor 2

$$J_d(z) + I_1^E(z)\psi_b + I_2^E(z)\psi_a = -j\frac{4\pi}{\zeta_0}\left[C_2 \cos\beta z + \frac{E_z^{inc}}{\beta}\exp\left(-j\frac{\beta b}{2}\sin\phi\right)\right].$$

Here $J_d(z)$ has the same significance as in Reference 2;

$$\psi_a = 2\ln\left(\frac{d}{a}\right) \;;\; \psi_b = 2\ln\left(\frac{d}{b}\right) \;;\quad \beta = 2\pi/\lambda \qquad (2)$$

is the radian wave number; λ is the wavelength of the field E_z^{inc}; C_1 and C_2 are constants, and ζ_0 is the characteristic impedance of space. In the rationalized mks system of units used in this paper, $\zeta_0 \simeq 120\pi$ ohms.

The right-hand sides of (1) and (2) are proportional to the vector potentials on the surfaces of the conductors established by the currents flowing in them. These currents are, of course, excited by the incident field E_z^{inc}. The boundary condition at each end of the line is

$$\left[\frac{\partial A_1(z)}{\partial z} - \frac{\partial A_2(z)}{\partial z}\right] = 0\,, \quad z = \pm h \qquad (3)$$

where $A_1(z)$ and $A_2(z)$ are the vector potentials on wires 1 and 2, respectively.

By using (1) - (3), it is a simple matter to show that the short-circuit currents in conductors 1 and 2 of the transmission line pictured in Figure 1 are

$$I_1^E(z) = \frac{I_T^E(z)}{2} + \frac{2\, E_z^{inc} \sin\left(\frac{\beta b}{2} \sin \phi\right)}{\beta Z_c} \tag{4}$$

$$I_2^E(z) = \frac{I_T^E(z)}{2} - \frac{2\, E_z^{inc} \sin\left(\frac{\beta b}{2} \sin \phi\right)}{\beta Z_c} \ . \tag{5}$$

Here Z_c is the characteristic impedance of the line;

$$Z_c = \frac{\zeta_0}{\pi} \ln\left(\frac{b}{a}\right) \ . \tag{6}$$

Also, $I_T^E(z) = I_1^E(z) + I_2^E(z)$ is the current flowing at the point z in an unloaded receiving dipole of effective radius $d = \sqrt{ab}$ for parallel incidence of the electric field.

Determination of the Currents in the Conductors of an Antenna Transmission Line Driven by Two Generators

Figure 2 portrays a two-conductor antenna transmission line center driven by two impedanceless generators. The dimensions of this circuit are the same as those of the circuit pictured in Figure 1. In the previous section of the paper the currents $I_1^E(z)$ and $I_2^E(z)$ were found with V_1 and V_2 suppressed. In this section it is proposed to determine $I_1^V(z)$ and $I_2^V(z)$ with the incident field E_z^{inc} suppressed. The simultaneous integral equations applicable to the circuit of Figure 2 are as follows:

<u>Conductor 1</u>

$$J_d(z) + I_1^V(z)\psi_a + I_2^V(z)\psi_b = -j\,\frac{4\pi}{\zeta_0}\left(C_1 \cos \beta z + \frac{V_1}{2} \sin \beta |z|\right) . \tag{7}$$

<u>Conductor 2</u>

$$J_d(z) + I_1^V(z)\psi_b + I_2^V(z)\psi_a = -j\,\frac{4\pi}{\zeta_0}\left(C_2 \cos \beta z + \frac{V_2}{2} \sin \beta |z|\right) . \tag{8}$$

$J_d(z)$ is the same as in Reference 2 except that now the current $I_T^V(z) = I_1^V(z) + I_2^V(z)$ occurs under the integral sign instead of $I_T^E(z) = I_1^E(z) + I_2^E(z)$.

By adding (7) and (8) and setting $\psi_a = -\psi_b$ so that $d = \sqrt{ab}$, one obtains

$$J_d(z) = -j\,\frac{4\pi}{\zeta_0}\left[\left(\frac{C_1 + C_2}{2}\right)\cos\beta z + \left(\frac{V_1 + V_2}{4}\right)\sin\beta|z|\right]. \tag{9}$$

Equation (9) is in standard form; i.e.,

$$J_d(z) = -j\,\frac{4\pi}{\zeta_0}\left(C\cos\beta z + \frac{V_k}{2}\sin\beta|z|\right) \tag{10}$$

inasmuch as $I_T^v(\pm h) = 0$.

The solution of (10) is

$$I_T^v(0) = \frac{V_k}{Z_d}\ . \tag{11}$$

It follows from (9) - (11) that

$$I_T^v(0) = \frac{V_1 + V_2}{2Z_d}\ . \tag{12}$$

In (11) and (12), Z_d is the driving point impedance of a symmetrical center-driven dipole of half-length h and effective radius d. Tables for Z_d are available in numerous places in the literature.[4,5] By subtracting (8) from (7) and applying (3), it is a simple matter to show that at z = 0,

$$I_1^v(0) = \frac{I_T^v(0)}{2} - \frac{j}{Z_c}\left(\frac{V_1 - V_2}{2}\right)\cot\beta h \tag{13}$$

and

$$I_2^v(0) = \frac{I_T^v(0)}{2} + \frac{j}{Z_c}\left(\frac{V_1 - V_2}{2}\right)\cot\beta h\ , \tag{14}$$

where $I_T^v(0)$ is given by (12).

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[4] R. W. P. King, "Tables of Antenna Characteristics" IFI Plenum, New York, 1971.

[5] R. W. P. King, Theory of Linear Antennas, Chapter 2, pp 169-179, Harvard University Press, 1956.

The Currents in the Load Impedances of an Antenna Transmission Line When the Loads are at the Centers of the Conductors

The objective is to find the currents in the center-loading impedances caused by the incident field E_z^{inc}. Refer to Figure 3. By use of the compensation theorem, one may write

$$V_1 = -I_1(0)Z_{L1} \tag{15}$$

and

$$V_2 = -I_2(0)Z_{L2} , \tag{16}$$

where $I_1(0)$ is the current in the lumped impedance Z_{L1}, and $I_2(0)$ is the current in the lumped impedance Z_{L2}. Also, by use of the superposition theorem,

$$I_1(0) = I_1^V(0) + I_1^E(0) \tag{17}$$

$$I_2(0) = I_2^V(0) + I_2^E(0) . \tag{18}$$

Equations (12) - (18) are now solved for $I_1(0)$ and $I_2(0)$. The results are

$$I_1(0) = \frac{G + H}{D} \tag{19}$$

$$I_2(0) = \frac{J + L}{D} , \tag{20}$$

where

$$\left.\begin{aligned}
G &= 2 Z_d \left[I_1^E(0) + I_2^E(0)\right]\left[Z_c \tan \beta h - j Z_{L2}\right] \\
H &= Z_c \tan \beta h \left[I_1^E(0) - I_2^E(0)\right]\left[2 Z_d + Z_{L2}\right] \\
J &= 2 Z_d \left[I_1^E(0) + I_2^E(0)\right]\left[Z_c \tan \beta h - j Z_{L1}\right] \\
L &= -Z_c \tan \beta h \left[I_1^E(0) - I_2^E(0)\right]\left[2 Z_d + Z_{L1}\right] . \\
D &= \left[2 Z_d + Z_{L2}\right]\left[Z_c \tan \beta h - j Z_{L1}\right] \\
&\quad + \left[2 Z_d + Z_{L1}\right]\left[Z_c \tan \beta h - j Z_{L2}\right] .
\end{aligned}\right\} \tag{21}$$

Also, by using (4) and (5),

$$I_1^E(0) + I_2^E(0) = I_T^E(0) \tag{22}$$

$$I_1^E(0) - I_2^E(0) = \frac{4 E_z^{inc} \sin\left(\frac{\beta b}{2} \sin \phi\right)}{\beta Z_c} . \tag{23}$$

It is of interest to observe that when $Z_{L2} = 0$,

$$I_1(0) = \frac{2 Z_c I_1^E(0)}{2 Z_c + \left(\frac{Z_{L1} Z_c}{2 Z_d}\right) - j Z_{L1} \cot \beta h} . \tag{24}$$

This result is easily verified. The short-circuit current multiplied by the impedance looking back into a network gives the open-circuit voltage V_{0c}. As illustrated by Figure 4, the open-circuit voltage drives a circuit consisting of the source impedance in series with the load impedance. Thus

$$V_{0c} = I_1^E(0) Z_{in1} \tag{25}$$

and

$$I_1(0) = \frac{V_{0c}}{Z_{in1} + Z_{L1}} = \frac{I_1^E(0) Z_{in1}}{Z_{in1} + Z_{L1}} . \tag{26}$$

The well-known formula for the input admittance of a two-wire folded dipole is[6]

$$Y_{in1} = \frac{1}{4 Z_d} - j \frac{\cot \beta h}{2 Z_c} . \tag{27}$$

As before Z_d is the driving point impedance of a symmetrical center-driven dipole of half-length h and effective radius $d = \sqrt{ab}$. The source impedance is then

$$Z_{in1} = 1/Y_{in1} = \frac{4 Z_c Z_d}{Z_c - j2 Z_d \cot \beta h} . \tag{28}$$

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[6] C. W. Harrison, Jr., "Folded Antennas," PhD Dissertation, Harvard University, Cambridge, Massachusetts, 1954.

Substituting (28) into (26) yields (24) as anticipated.

The short-circuit currents $I_n^E(0)$ can be found for any N conductor antenna transmission line by solving the applicable simultaneous integral equations. Also, the driving point currents can be found for the same structure by using similar techniques. It follows that the load currents, for center-loaded configurations, can be found for N conductor circuits in terms of the incident field. It is assumed that transmission line theory is not violated; i.e., $\beta q << 1$, where q is the maximum transverse dimension of the antenna transmission line. Also, it is required that the inequality $q << h$ be satisfied.

Staggered Load Impedances in a Two-Conductor Antenna Transmission Line Excited by an Incident Field

The circuit to be discussed (in a semiquantitative way because of its complexity) is illustrated by Figure 5f. The currents in the loads Z_{L1} and Z_{L2} excited by the incident field E_z^{inc} are determined by the use of the superposition and compensation theorems. This requires introduction of circuits illustrated by 5a to 5f in Figure 5. Evidently, when the load impedances are in the center of the wires, as in Figure 3, the problem can be treated in the manner set forth here. Earlier in the paper the authors presented a technique of solution that may be applied to N conductors, provided that the antenna transmission line is center loaded.

To obtain the currents I_1 and I_2 flowing in Z_{L1} and Z_{L2}, respectively, requires complete analysis of the individual circuits a to e in Figure 5. To facilitate solution of these problems, it is convenient to introduce three coordinate systems. The load Z_{L1} is located at $z = 0$, the load Z_{L2} is located at $z' = 0$, and the center of the structure is located at $z'' = 0$. The distance between Z_{L1} and Z_{L2} is ℓ. The relation between the coordinates is

$$
\begin{aligned}
z &= z' - \ell \\
z &= z'' + \left(\frac{h_2 - h_1}{2}\right) \\
z'' &= z' - \ell - \left(\frac{h_2 - h_1}{2}\right) .
\end{aligned}
\tag{29}
$$

The ends of the structure are located at $z = -h_1$, $z = h_2$; $z' = -(h_1 - \ell)$, $z' = h_2 + \ell$; and $z'' = -(h_1 + h_2)/2$; $z'' = (h_1 + h_2)/2$. These results follow from (29).

The assumed current directions are indicated in each of the drawings constituting Figure 5. It follows that the currents I_1 and I_2 are given by

$$I_1 = I_{1a}^E + I_b^V + I_d^V + I_f^V - I_h^V \tag{30}$$

and

$$I_2 = I_{2a}^E + I_c^v - I_e^v + I_g^v + I_i^v \; . \tag{31}$$

I_{1a}^E and I_{2a}^E are functions of E_z^{inc} and are computed from such expressions as (4) and (5) for an antenna of effective radius $d = \sqrt{ab}$. All other currents appearing in (30) and (31) are functions of V_1 or V_2. These voltages are eliminated by use of the compensation theorem; i.e.,

$$V_1 = -I_1 Z_{L1} \tag{32}$$

$$V_2 = -I_2 Z_{L2} \; . \tag{33}$$

Thus (30) and (31) become simultaneous equations involving the desired currents I_1 and I_2.

The authors now consider the circuits appearing in Figure 5 individually.

As indicated above, I_{1a}^E is obtained from (4), and I_{2a}^E from (5). The current I_T^E must be known at $z'' = -(h_2 - h_1)/2$ and at $z'' = -\ell - (h_2 - h_1)/2$ for an unloaded receiving and scattering antenna[7] of radius $d = \sqrt{ab}$.

Figure 5b represents an asymmetrical dipole. Note that the placement of shorting bars across the top of the generators and across the bottom does not alter the circuit. The effective voltage is $V_1/2$, and each generator carries half the current. Accordingly,

$$I_b^v = \frac{V_1}{4Z_{d1}} \; . \tag{34}$$

Here Z_{d1} is the impedance of an asymmetrical dipole[8] of leg lengths h_1 and h_2 and effective radius $d = \sqrt{ab}$.

The current I_c^v at $z = -\ell$ in Figure 5b is obtained directly from the formula for the current along an asymmetrical dipole.[8] Note that I_c^v is one-half the total current in the structure at $z = -\ell$.

[7]Liang-Chi Shen, "A Simple Theory of Receiving and Scattering Antennas," IEEE Transactions on Antennas and Propagation, Vol AP-18, No. 1, pp 112-114, January 1970.

[8]Liang-Chi Shen, Tai Tsun Wu, and Ronold W. P. King, "A Simple Formula of Current in Dipole Antennas," IEEE Transactions on Antennas and Propagation, Vol AP-16, No. 5, pp 542-547, September 1968.

The circuit pictured in Figure 5c is a transmission line. From simple transmission line theory one obtains

$$I_d^V = - \frac{jV_1}{Z_c(\tan \beta h_1 + \tan \beta h_2)} , \tag{35}$$

where Z_c is given by (6).

The current I_e^V in the circuit of Figure 5c is also obtained from simple transmission line considerations. The internal impedance of the generator of voltage V_1 is

$$Z_g = j Z_c \tan \beta h_2 . \tag{36}$$

The input impedance of the line looking down from the generator terminals is

$$Z_{in} = j Z_c \tan \beta h_1 . \tag{37}$$

The equivalent circuit is shown in Figure 6.

Clearly,

$$I_s = \frac{-jV_1}{Z_c (\tan \beta h_1 + \tan \beta h_2)} \tag{38}$$

$$V_s = I_s Z_{in} = \frac{V_1 \tan \beta h_1}{\tan \beta h_1 + \tan \beta h_2} . \tag{39}$$

The current I_e^V at distance ℓ from the generator is

$$I_e^V = I_s \cos \beta\ell - j \frac{V_s}{Z_c} \sin \beta\ell . \tag{40}$$

Substituting (38) and (39) into (40) yields

$$I_e^V = -j \frac{V_1}{Z_c} \left[\frac{\cos \beta\ell + \sin \beta\ell \tan \beta h_1}{\tan \beta h_1 + \tan \beta h_2} \right] , \tag{41}$$

provided that the transmission line is dissipationless.

The current I_g^v occurring in the circuit pictured in Figure 5d is given by

$$I_g^v = \frac{V_2}{4Z_{d2}} , \tag{42}$$

where Z_{d2} is the driving point impedance of an asymmetrical dipole of effective radius $d = \sqrt{ab}$ and leg lengths $h_2 + \ell$ and $h_1 - \ell$.[8] The current I_f^v at $z' = \ell$ is obtained from the formula for the current in an asymmetrical dipole. Again note that I_f^v is one-half the total current in the structure at the cross section under consideration.

Again circuit e in Figure 5 is a transmission line. By analogy with circuit c in Figure 5, one obtains

$$I_i^v = -j \frac{V_2}{Z_c \left[\tan \beta (h_2 + \ell) + \tan \beta (h_1 - \ell)\right]} \tag{43}$$

and

$$I_h^v = -j \frac{V_2}{Z_c} \left[\frac{\cos \beta\ell + \sin \beta\ell \tan \beta(h_2 + \ell)}{\tan \beta(h_2 + \ell) + \tan \beta(h_1 - \ell)}\right] . \tag{44}$$

This completes determination of the individual currents constituting I_1 and I_2, Figure 5f. It is interesting to observe that among other factors the load currents depend on the input impedances of two asymmetrically driven dipoles.

The evaluation of the ten component currents in the circuit would normally be done by use of a computer.

Conclusions

A general theory has been developed for the response of an N conductor impedance-loaded transmission line to an incident electric field when the loads are centrally positioned. Simultaneous integral equations for the currents appropriate to antennas driven by lumped generators, as well as antennas driven by incident fields, are employed. When the loads are in echelon, a general theory does not appear to be feasible. The solution of the problem is effected for a two-conductor configuration by using the superposition and compensation theorems. The modes of asymmetrically driven dipoles and transmission lines are involved. Extension of the theory to more than two conductors becomes a Herculean task.

APPENDIX

If $\beta h \leq 3\pi/2$ and $\Omega = 2 \ln\left(\frac{2h}{d}\right) \geq 8$, the current $I_T^E(z)$ appearing in (4) and (5) may be computed from the formula

$$I_T^E(z) \simeq j \frac{4\pi}{\zeta_0} \frac{E_z^{inc}}{\beta} \left[\frac{\cos \beta z - \cos \beta h}{\psi_{du} \cos \beta h - \psi_u(h)}\right], \tag{45}$$

where

$$\psi_{du} = (1 - \cos \beta h)^{-1} \int_{-h}^{h} (\cos \beta z' - \cos \beta h)\left[K(0, z') - K(h, z')\right] dz' \tag{46}$$

$$\psi_u(h) = \int_{-h}^{h} (\cos \beta z' - \cos \beta h) K(h, z')\, dz' \tag{47}$$

with

$$K(z, z') = (\exp j\beta R)/R \tag{48}$$

$$R = \sqrt{(z - z')^2 + d^2}\ . \tag{49}$$

The half-length of the antenna is h in the notation employed here. One should anticipate need for a computer to determine $I_T^E(z)$. The impedance and current distribution along electrically short, moderately thin asymmetrical antennas driven by a generator (in contrast to an incident field) may be determined by reference to the literature.[9, 10]

At high frequencies, when the antenna transmission line becomes electrically long, the brilliant work of Shen[7-8] should be utilized to find the currents along dipole receiving and scattering antennas,[7] as well as the impedance and current distribution along asymmetrical dipoles.[8]

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[9] R. W. P. King and T. T. Wu, "The Cylindrical Antenna with Arbitrary Driving Point," IEEE Transactions on Antennas and Propagation, Vol AP-13, No. 5, pp 710-718, September 1965.

[10] C. W. Harrison, Jr., C. D. Taylor, E. A. Aronson, and E. E. O'Donnell, "On the Driving Point Impedance of an Asymmetrical Dipole," IEEE Transactions on Antennas and Propagation, Vol AP-14, No. 6, pp 794-795, November 1966.

The following remarks apply exclusively to Reference 7. The assumed, but suppressed, time dependence employed in Reference 7 is exp $(-i\omega t)$. The writers prefer the time dependence exp $(j\omega t)$. Accordingly, replace i by -j throughout the theory. For Equation (3) to be dimensionally correct, multiply it by $E_z^{inc}\lambda$. Set $\theta_i = \pi/2$ and note that log means ln. Also, replace z by $|z|$ in Equation (3), since $I_{s\infty}(z) = I_{s\infty}(-z)$. Using the notation employed in the present paper, Equation (8) of Reference 7 should be written

$$I_T^E(z'') = I_{s\infty}(z'') + C_{s1} I_\infty(z'' + h) + C_{s2} I_\infty(z'' - h) \ . \tag{50}$$

From (29) or Figure 5a of the present paper, it is noted that $h \rightarrow (h_1 + h_2)/2$ and $-h \rightarrow -(h_1 + h_2)/2$. These values are used for h in Equation (9), Reference 7. Interest centers in obtaining I_T^E at the first position, $z'' = -(h_2 - h_1)/2$, and I_T^E at the second position, $z'' = -\ell - (h_2 - h_1)/2$ (Figure 5a of the present paper). It follows that $z'' + h = h_1$ and $z'' - h = -h_2$ for the first position. Also, for the second position, $z'' + h = -\ell + h_1$ and $z'' - h = -\ell - h_2$. Evidently $z \rightarrow z''$ in Equation (3) of Reference 7.

The following remarks apply exclusively to Reference 8. Replace i by -j and multiply Equation (6) by V_1 or V_2 as appropriate. Also in Equation (6) replace z by $|z|$ and note that $-\pi < \mathrm{Im}[\ln f(z)] \leq \pi$. Again observe that log means ln. The asymmetrical dipole pictured in Figure 5b of the present paper requires no modification in notation, and may be applied directly to obtain Z_{d1} and I_c^V.

To obtain Z_{d2} and I_f^V, note that $-h_1 \rightarrow -(h_1 - \ell)$ and $h_2 \rightarrow h_2 + \ell$. Refer to Figure 5d of the present paper. Hence Equation (11) in Reference 8 becomes

$$I_f^V(z') = I_\infty(z') + C_d I_\infty(h_1 - \ell + z') + C_u I_\infty(h_2 + \ell - z') \ . \tag{51}$$

C_d and C_u are obtained from Equations (14) and (15) of Reference 8 by replacing h_1 by $(h_1 - \ell)$ and h_2 by $(h_2 + \ell)$. Obviously, $z \rightarrow z'$ in Equation (6), and the driving point is at $z' = 0$. The current I_f^V is calculated at $z' = \ell$. This theory yields excellent results when a generator is not closer than 0.15λ to either end of the dipole.

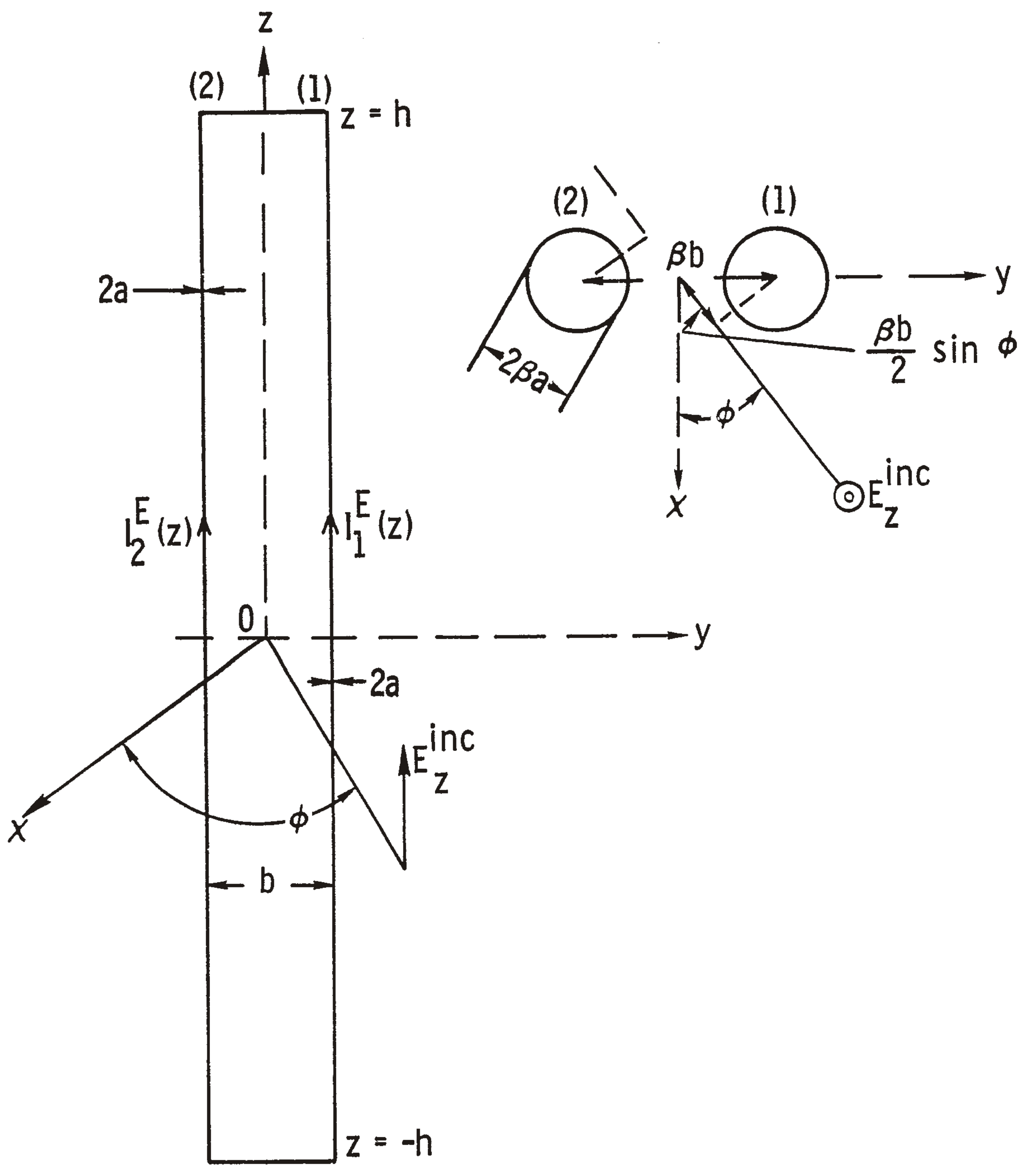


Figure 1. Diagram Used in Determining the Short-Circuit Current in a Two-Wire Antenna Transmission Line

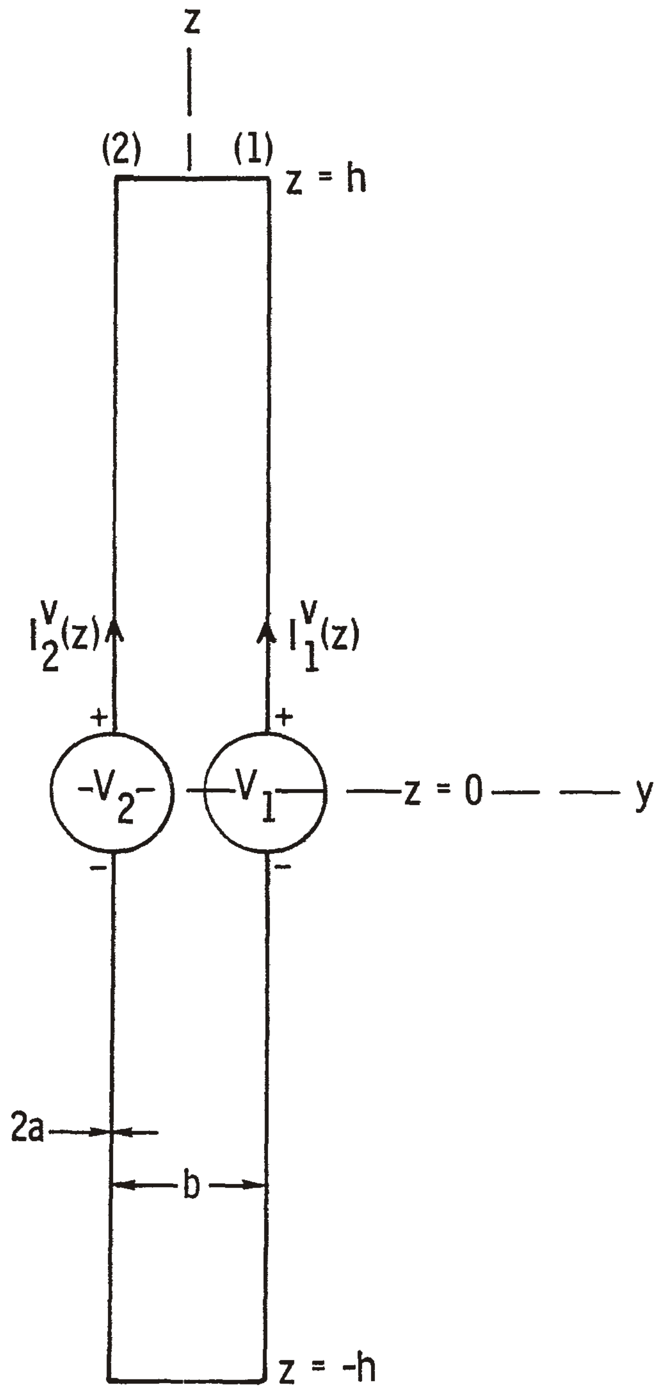


Figure 2. Antenna Transmission Line Driven at the Center of Each Conductor by Impedanceless Generators

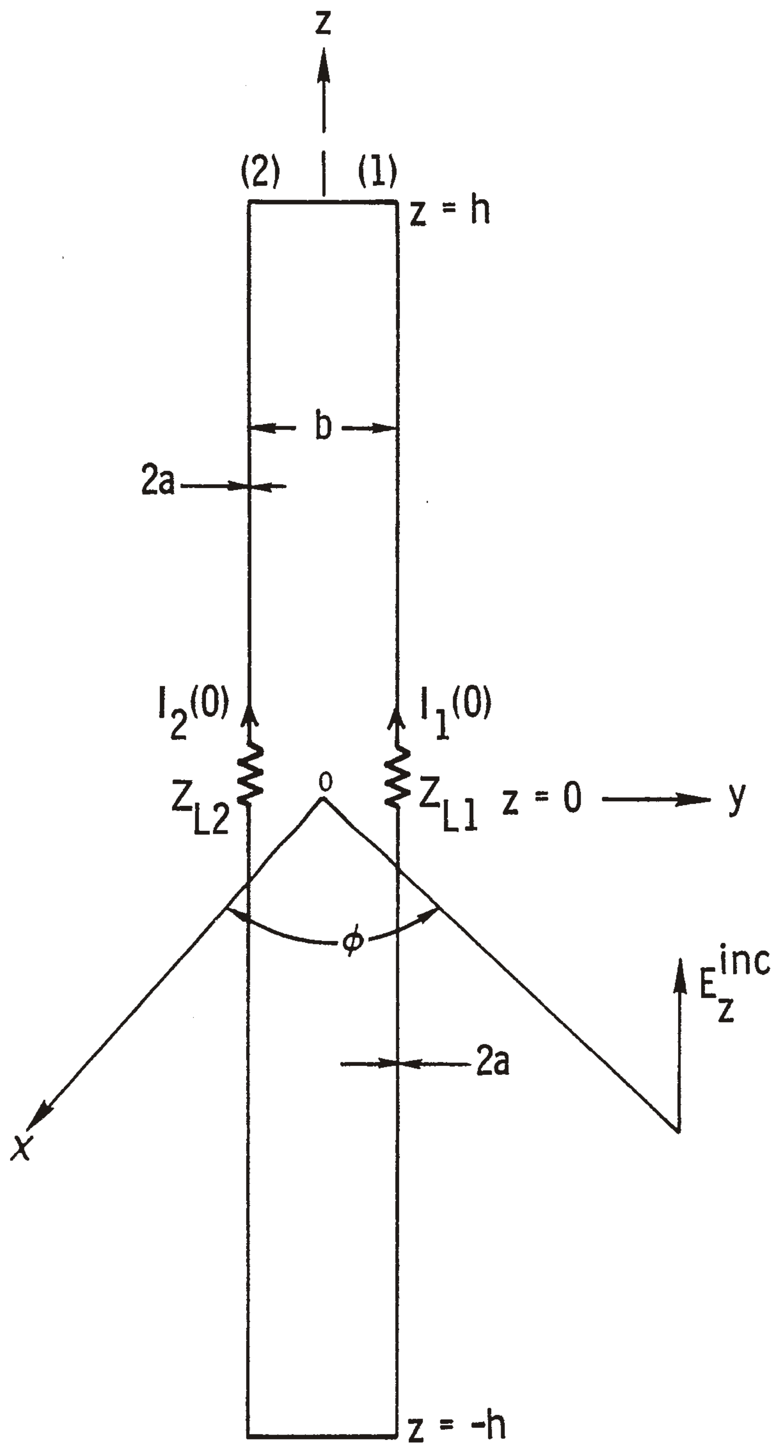
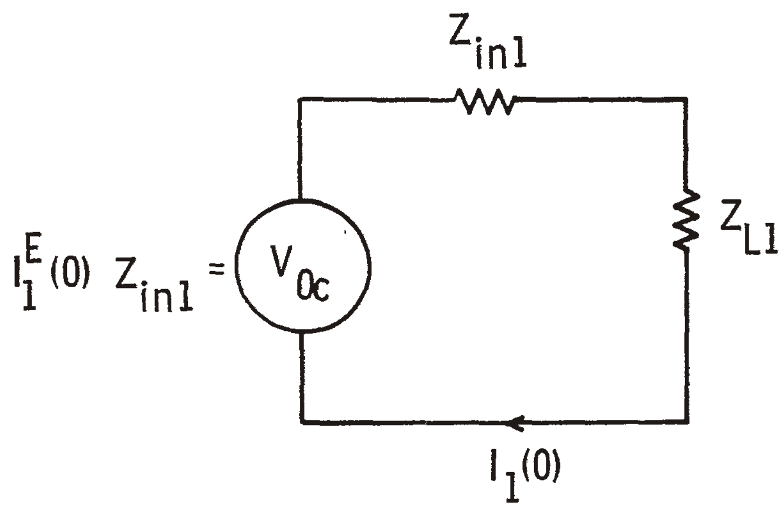
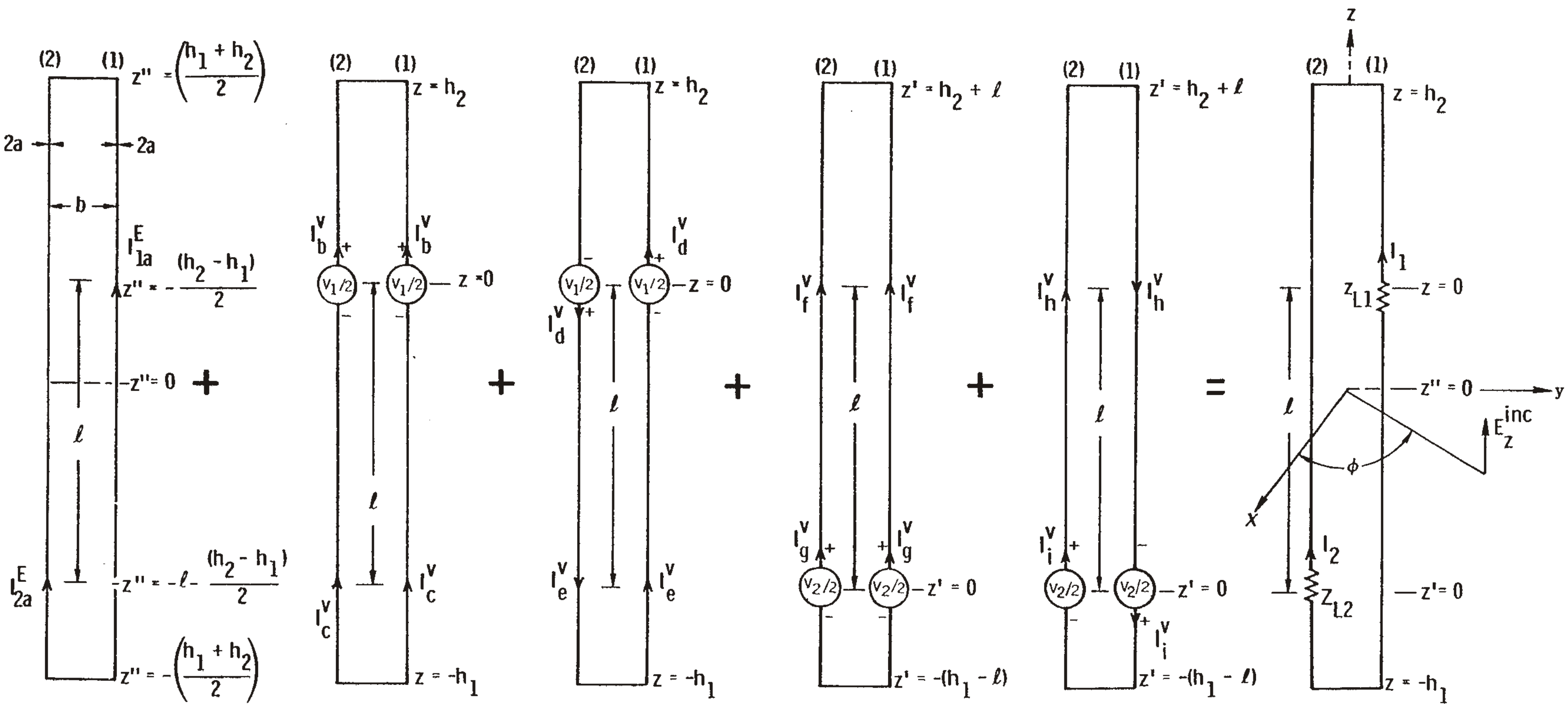


Figure 3. Center-Loaded Two-Conductor Antenna Transmission Line



$$I_1(0) = \frac{I_1^E(0)\, Z_{in1}}{Z_{in1} + Z_{L1}}$$

Figure 4. The Equivalent Circuit of an Impedance-Loaded Folded Dipole



a. Short-Circuit Current for Incident Field
b. Asymmetrical Dipole
c. Transmission Line
d. Asymmetrical Dipole
e. Transmission Line
f. Impedance-Loaded Antenna Transmission Line

Figure 5. The Use of Superposition to Determine the Load Currents When the Impedances Are in Echelon

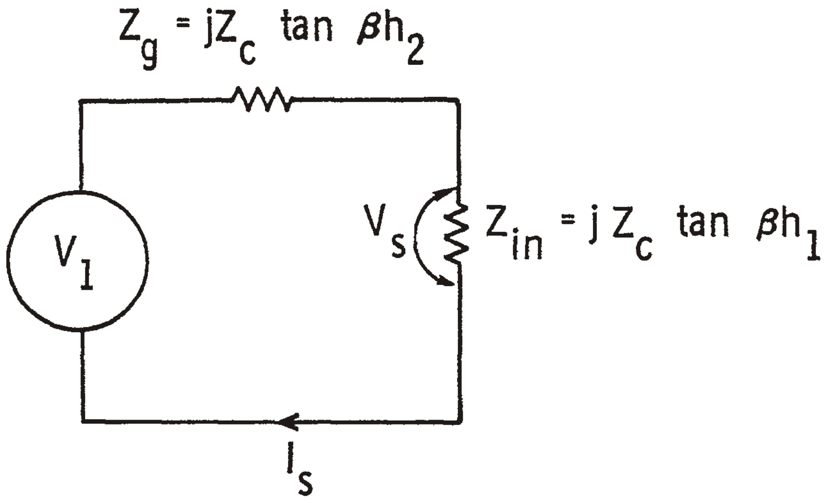


Figure 6. Circuit for Determining the Current I_e^v in Figure 5c