INTERACTION NOTE

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A SIMPLE PROCEDURE FOR ESTIMATING THE CURRENT INDUCED ON CYLINDER-LIKE CONDUCTORS ILLUMINATED BY A UNIT STEP ELECTROMAGNETIC PULSE

by

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ABSTRACT

A simple procedure is developed for estimating the current induced on cylinder-like conductors that are illuminated by a unit step electromagnetic pulse. The procedure is illustrated by determining the current induced on an aircraft with and without a long trailing wire antenna.

PRECIS

It is shown that for a class of cylinder-like conducting objects a set of simple analytical formulas may be used to obtain the induced currents when the objects are illuminated by a step electromagnetic pulse. First the conducting object is represented by a right circular cylinder. Thus the object must at least be similar to a cylinder. If the cross section of the object is not circular then an ellipse is fitted to the cross section as closely as possible so that an effective circular cylinder radius may be defined

$$a = \frac{1}{2}(a_e + b_e)$$

where a_e is the length of the semimajor axis of the cross section ellipse and b_e the corresponding length of the semiminor axis. If the cross section of the cylinder-like object varies over the axial length (as for a prolate spheroid for example) then an average radius is used. The foregoing procedure is valid only for thin cylinders, i. e. when $L^2 >> a^2$, where L is the total axial length.

Second the axial current is obtained by using

$$\frac{\ell n(L/a)}{L\,H_o} I(z,t)\Bigg|_{\substack{t=\frac{L}{c}\tau \\ z=Lu}} = C_1 \quad , \text{ a constant}$$

where $I(z,t)$ is the current through the cross section at z and time t, and H_o is the amplitude of the incident magnetic field. The constant C_1

depends on τ and u and may be determined from Figures 1 and 2 for u = 0.5 and u = 0.25, respectively. For example, from Figure 1 (u = .5) taking τ = 0.5 one obtains C_1 = 3.5. Note that

$$E_o = \zeta H_o$$

where E_o is the amplitude of the incident electric field and $\zeta = \sqrt{\mu/\varepsilon}$ is the intrinsic wave impedance of the ambient medium.

The late time behavior of the induced axial current is simply an exponentially damped harmonic oscillation. An analytic expression for the damping constant is

$$\alpha = \frac{2c}{L} \frac{0.46}{\ell n(L/a) - 1.723}$$

and for the frequency of the oscillation is

$$f = \frac{c}{2L} \left[1 - \frac{0.25}{\ell n(L/a) - 1.723} \right]$$

For a rough estimate of the variation of the axial current with z, one may assume that the current decreases linearly from the middle of the structure to a zero value at the ends of the structure.

Third, the surface current density on the conducting object at a given cross section may be obtained by dividing the axial current by the circumference of the cross section.

To illustrate the use of the foregoing formulas the current induced on an aircraft will subsequently be obtained--the effect of the aircraft wings on the current is ignored. The current is obtained for the aircraft with and without a long trailing wire antenna.

ANALYSIS

Consider a cylindrical antenna, with length L and radius a, to be illuminated by an electromagnetic field. The axial current induced on the antenna may be expressed

$$\tilde{I}(z,\omega) = \frac{\tilde{E}_o L}{\zeta} f(kL,\ z/L,\ a/L) \tag{1}$$

where $k = \omega/c = 2\pi/\lambda$ is the radian wave number, z is the axial coordinate, and E_o is the complex amplitude of the incident wave with the assumed but suppressed harmonic time dependence $e^{j\omega t}$.

For pulse illumination (1) may be interpreted as the Fourier transform of the induced current while $\tilde{E}_o(\omega)$ is the Fourier transform of the incident pulse. Hence

$$I(z,t) = \frac{L}{\zeta\sqrt{2\pi}} \int_{-\infty}^{\infty} E_o(\omega)\ f(kL,z/L,a/L)e^{j\omega t}\ d\omega \tag{2}$$

If the pulse is a unit Heaviside step, for example, then

$$\tilde{E}_o(\omega) = \frac{1}{j\omega} = \frac{L}{c}\frac{1}{jkL} \tag{3}$$

Using (3) in (2) yields

$$I(z,t) = \frac{L}{\zeta\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{jkL}\ f(kL,z/L,a/L)e^{jkL(ct/L)}\ d(\omega L/c) \tag{4}$$

Letting $\xi = kL$ in (4) yields

$$I(z,t) = \frac{L}{\zeta\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{j\xi}\ f(\xi,z/L,a/L)e^{j\xi(ct/L)}\ d\xi \tag{5}$$

or

$$\frac{1}{L} I(Lu, L\tau/c) = \frac{1}{\zeta\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{j\xi} f(\xi, u, a/L) e^{j\xi\tau} d\xi \tag{6}$$

Thus for a constant a/L

$$\frac{1}{L} I(z,t)\Big|_{t=\frac{L}{c}\tau,\ z = Lu}$$

is independent of length and the following observations may be made

Peak current α L

Rise time α L

For thin antennas Hallen[1] has shown that the radial dependence of the induced current is simply the factor $[\ell n(L/a)]^{-1}$. Hence

$$\frac{\ell n(L/a)}{L} I(z,t)\Big|_{t=\frac{L}{c}\tau, z=Lu} = \text{constant} \tag{7}$$

is independent of the antenna length and radius, provided the antenna is thin, i. e. $L^2 >> a^2$. Using the numerical data obtained by Sassman[2] to check (7) in predicting the variation of the peak current with a change in antenna radius yields

$$\frac{\ell n(L/a)}{L} I(z,t)\Big|_{\substack{t = \frac{L}{2c} \\ z = L/2}} = \begin{cases} 3.59\ H_o & L/a = 20 \\ 3.50\ H_o & L/a = 200 \\ 3.46\ H_o & L/a = 2000 \end{cases} \tag{8}$$

where H_o is the amplitude of the magnetic field (in A/m) of the incident step pulse. Hence (7) is sufficiently accurate to use for engineering purposes.

The late time response of the current induced by a step incident pulse is obtained by Sassman to be a damped sinusoidal oscillation with the exponential damping $\exp(-\alpha t)$. Analytical solutions for the frequency of this oscillation and the damping factor α have been obtained by Lee and Leung.[3] They are

$$f_{res} = \frac{c}{2L}\left[1 - \frac{0.25}{\ln(L/a) - 1.723}\right] \tag{9}$$

$$\alpha = \frac{2c}{L}\,\frac{0.46}{\ln(L/a) - 1.723} \tag{10}$$

Also of interest is the variation of the induced current with position along the linear antenna. Again relying on Sassman's results, it is found that for thin antennas the induced current is maximum at the center and then decreases almost linearly from the center, approaching zero at the ends of the antenna. For thicker antennas this almost linear variation still occurs; however, the current does not approach zero at the antenna ends since Sassman considered the antenna to have flat end caps supporting radial currents.

Actually the equation derived for the current, (8), indicates that the damping constant is inversely proportional to L. But from (10) it is seen that the damping constant also depends somewhat upon the antenna radius a. Hence (8) is accurate only when $\alpha t < 1$. For figures 1 and 2 the $L/a \doteq 200$.

NUMERICAL EXAMPLE

As a practical illustration of the foregoing development consider an aircraft with and without a long trailing wire antenna. It is of interest to determine the current induced on the aircraft when it is illuminated by a step pulse. In order to get at least a rough estimate of the effect of adding the trailing wire to the aircraft, the presence of the aircraft wings is ignored.

The aircraft without the trailing wire antenna is represented by a circular cylinder with length $L_1 = 70$ m and radius $a_1 = 3$ m. And the aircraft with the trailing wire antenna attached is represented by a cylinder with length $L_2 = 8600$ m and radius $a_2 = 0.0024$ m. The radius of the structure is chosen to be the radius of the trailing wire alone, since the trailing wire antenna being used for VLF communication is considerably longer than the aircraft.[4]

A comparison of the currents induced on the aircraft with and without the presence of the trailing wire antenna is shown in Table 1.

Table 1: Properties of the Current Induced on an Aircraft by an Incident Step Pulse. (E_o is the amplitude of the incident electric field.)

	Aircraft w/o Trailing Wire	Aircraft w Trailing Wire
Peak Current at $z = L_1/4$	0.1 E_o Amp.	0.01 E_o Amp.
Peak Current at $z = L_1/2$	0.2 E_o Amp.	0.02 E_o Amp.
Peak Current at $z = 3L_1/4$	0.1 E_o Amp.	0.03 E_o Amp.
Peak Current at $z = L_1$	0 Amp.	0.04 E_o Amp.
Rise Time	0.12 μs	14 μs
Ringing freq.	1.8 MHz	17 KHz
Damping Cons.	2.8×10^6/s	2.4×10^3/s

REFERENCES

1. E. Hallén, "Theoretical Investigations into the Transmitting and Receiving Qualities of Antennas," Nova Acta Regiae Societatis Scientiarium Upsaliensis, Series IV, Vol. II, no. 4, pp 1-44, 1938.

2. R. W. Sassman, "The Current Induced on a Finite Perfectly Conducting, Solid Cylinder in Free Space by an Electromagnetic Pulse," AFWL Interaction Note 11, July 1967.

3. S. W. Lee and B. Leung, "The Natural Resonance Frequency of a Thin Cylinder and Its Application to EMP Studies," AFWL Interaction Note 96, February 1972.

4. C. W. Harrison, Jr., "Radiation from an Antenna Trailing an Aircraft--VLF Case," Sandia Laboratories Report SC-TM-71 0221B, Albuquerque, N. M., April 1971.

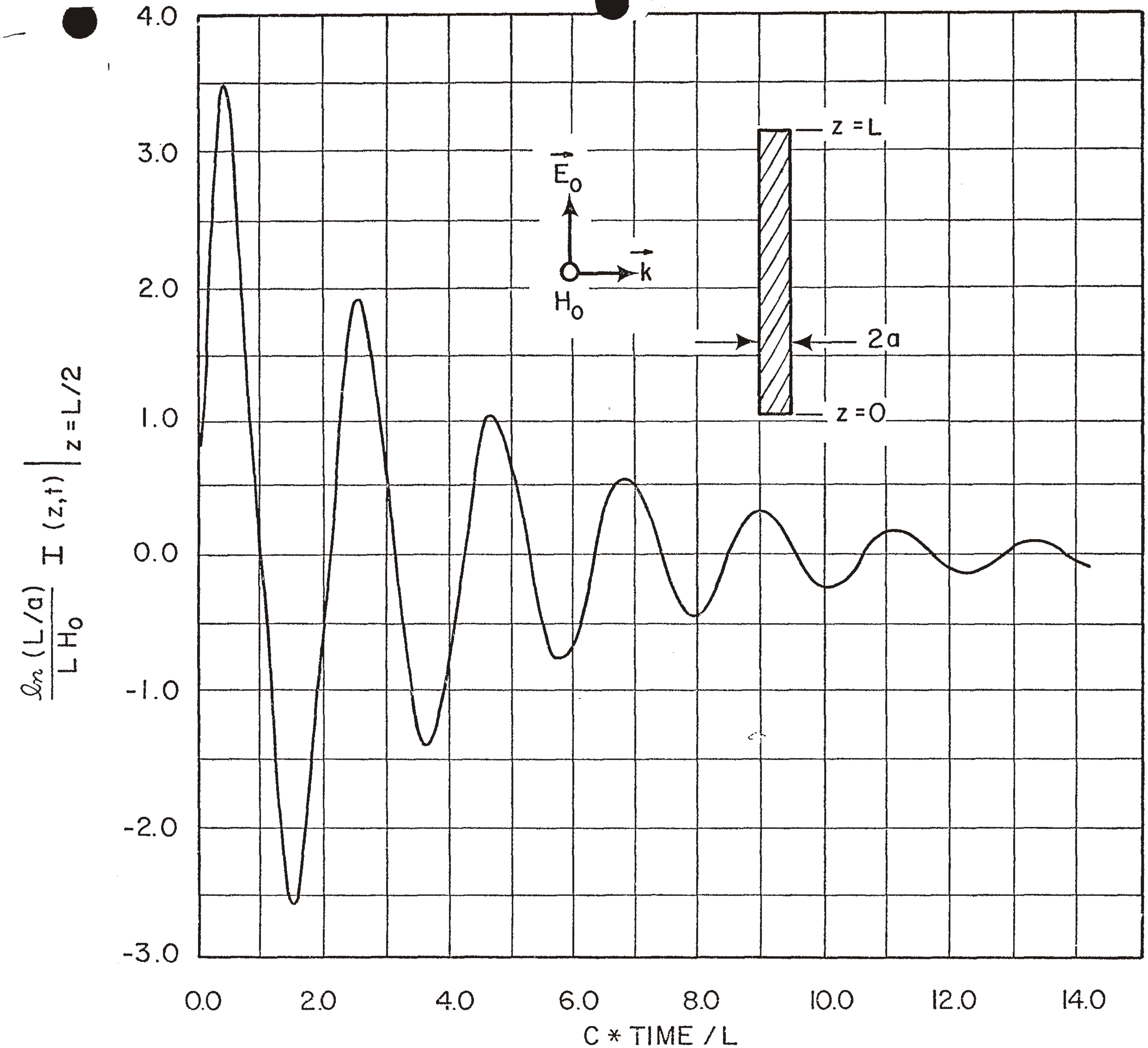
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
ℓn (L/a) / L Ho I (z,t) |z=L/2
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
C * TIME / L
E0
k
H0
z = L
2a
z = 0

Figure 1

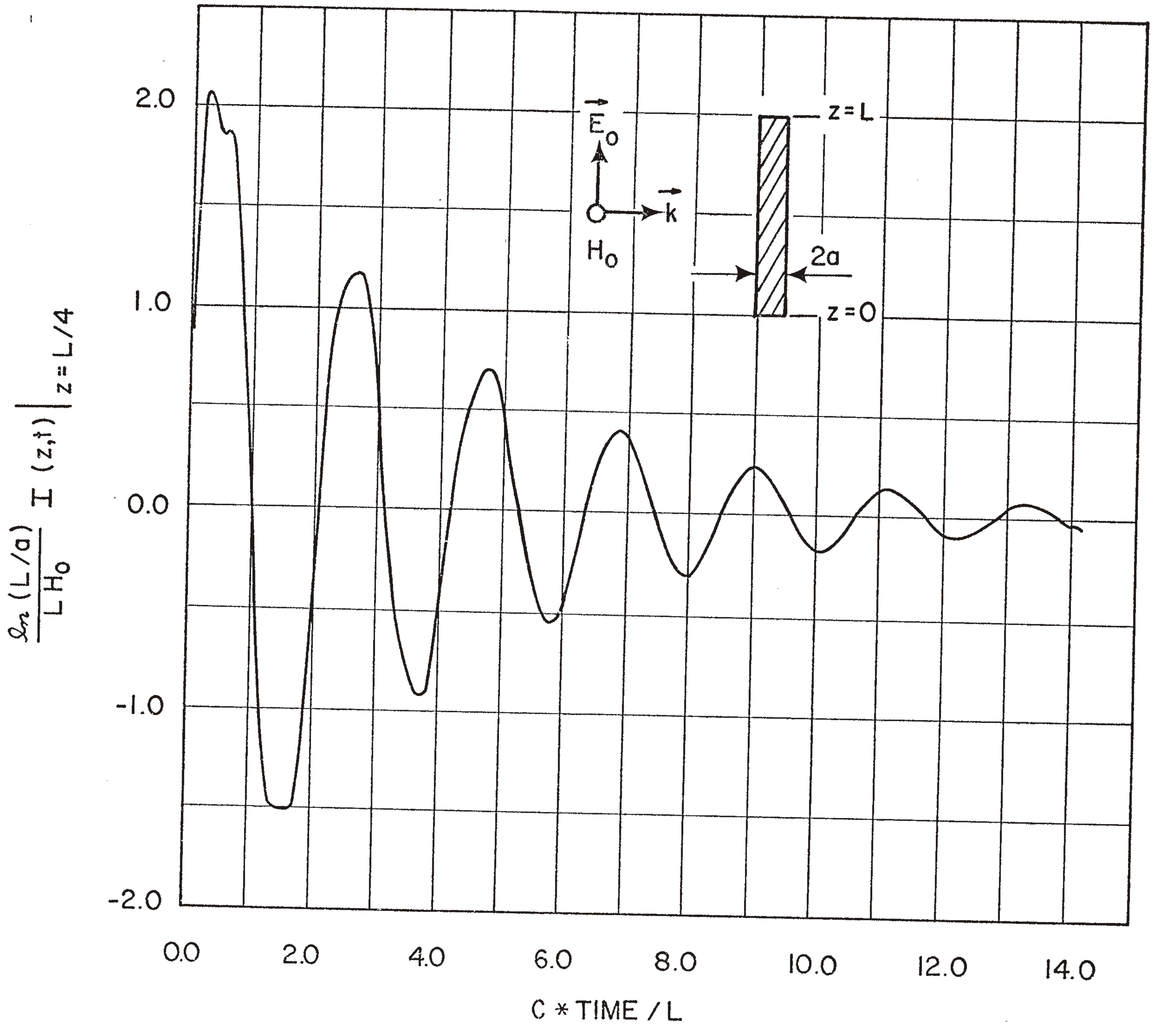
2.0
1.0
0.0
-1.0
-2.0
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
C * TIME / L
ln (L/a) / L H₀ I (z,t) |z=L/4
E₀
k
H₀
z=L
z=0
2a

Figure 2