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TRANSIENT CURRENTS ON SURFACE CABLES:

2-D FINITE DIFFERENCE CALCULATIONS

by

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TABLE OF CONTENTS

LIST OF FIGURES

Section 1
INTRODUCTION

The electronic components of mobile defense systems (missiles, artillery, communications, etc.) are often interconnected by electrical cables lying on the surface of the earth. To assess the effects of EMP on these systems we must estimate the response of these surface cables to excitation by EMP. This paper is the first of a two-part consideration of this problem. In this paper we examine the adequacy of the transmission line analog of the surface cable. In the second part (published as a companion volume) we make predictions of the currents that can be expected on surface cables excited by high-altitude and ground burst EMP, under various conditions of cable termination, polarization and angle of incidence.

In addressing the first problem we re-examined the current induced on the insulated wire at the earth/air interface. A finite difference solution was developed that will yield the currents induced on the surface cable when uniformly excited by an electric field pulse. Examination of the current predictions made using this computer solution revealed that the longitudinal impedance of the surface cable can be estimated from the solutions for the homogeneous media problem. Further, it was found that the technique used to combine the homogeneous solutions was not as previously suggested by other writers (Ref. 1,2). Predictions made using the prior combination technique are only erroneous by a factor of 3/2; however, for most applications of this of analysis, errors of this magnitude are tolerable.

Section 2

APPROXIMATIONS APPLICABLE TO THE SURFACE CABLE MODEL

By the nomenclature "surface cable model" we shall imply the idealized physical situation shown in Figure 2.1.

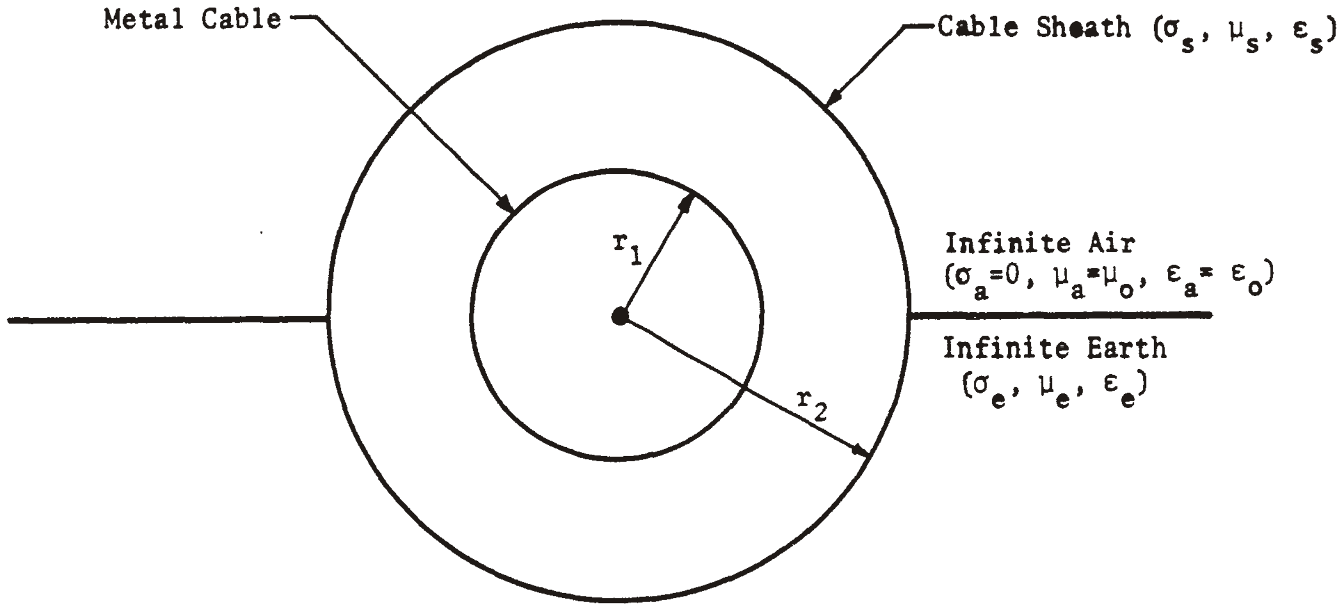

Figure 2.1. Cross-Section of Surface Cable Model

We assume (for simplicty) that the inner cable of radius, r_1, is a perfect conductor. The cable sheath is essentially insulating but could have a conductivity σ_s. The half-space "air" and "earth" media are homogeneous and each of infinite extent.

The prototype model of Figure 2.1 poses, in general, (finite cable length, arbitrary incident field, etc.) a formidable problem for "first principle" analysis. Consequently, as in previous work, there is strong motivation for attempting to utilize a transmission line analog to the surface cable.

In 1965, Whitson and Vance [1] conducted CW experiments (theoretically interpreted via transmission line models) on various cables both fully buried and lying on the surface of the earth. They found good agreement with theory for buried bare wires (at low frequencies), but poor agreement for insulated surface wires. A similar subsequent theoretical treatment by Marston and Graham [2] (again analyzing some experiments of Whitson) led to essentially the same results. The former authors felt that the poor surface cable results were due to intermittent cable contact with the earth. It now appears, however, that the analyses of the experiments may have also suffered from inadequate approximations.

In these previous approximate treatments of the Figure 2.1 model, transmission line impedances for the surface cables have been estimated by plausible modification of the corresponding parameters describing a cable fully buried. Furthermore, the insulated buried cable transmission line parameters were also obtained by plausible modification of the bare wire models. The latter approximations, (especially those of reference [2]) are not appropriate for transient calculations containing important high frequency ($\gtrsim 10^7$ Hertz) components. The choice of a suitable transmission line model for an insulated cable totally embedded in a single medium (earth or air) will be more fully discussed in our subsequent note. Here we primarily utilize a simplified SC model to study the approximate generalization of a single-medium transmission line longitudinal impedance to the two-media surface cable situation of Figure 2.1.

An incremental section of the transmission line analog to the cable of Figure 2.1 is shown in Figure 2.2:

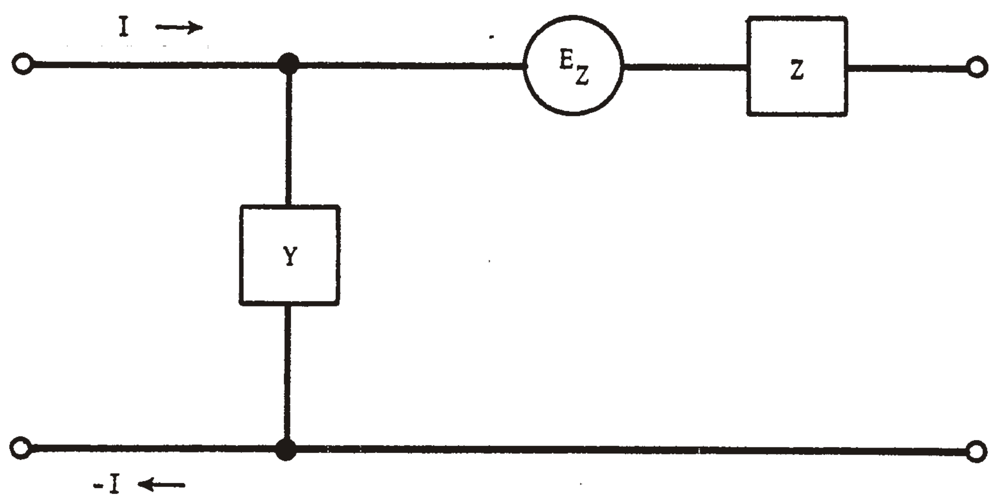

Figure 2.2. Transmission Line Section for the Surface Cable

In Figure 2.2 the transverse admittance, Y, and the longitudinal impedance , Z, are assumed uniformly distributed along the line. A driving incident field along the cable is represented by a distributed generator E_Z. One may then obtain the frequency domain current on the wire by solving (subject to the desired termination conditions) the transmission line equation, as discussed in many texts [3]:

$$\frac{\partial^2 I}{\partial z^2} + YZI = YE_z \quad . \tag{1}$$

The surface cable parameters Y and Z are to be estimated from the corresponding admittance and impedance of the Figure 2.2 model applied to an insulated cable fully buried (infinite depth approximation) in the earth. For the transverse admittance of the surface cable, references [1] and [2] offer the prescription:

$$Y_{\substack{\text{surface}\\\text{cable}}} \approx 1/2 \; Y_{\substack{\text{fully}\\\text{buried}}} \quad . \tag{2}$$

Approximation (2) seems intuitively reasonable, viewing the SC admittance as the inverse of the total (parallel circuit combination) transverse "leakage" impedance through both air and soil. The admittance through the air should be much smaller than that through the earth and hence only the lower semicircle of the surface cable contributes in (2).

Two estimates for the SC longitudinal impedance have been suggested. Whitson and Vance [1] use (later reproduced in the DASA Handbook [4]):

$$Z_{\substack{\text{surface}\\ \text{cable}}} \approx 2Z_{\substack{\text{fully}\\ \text{buried}}} \quad . \tag{3}$$

Marston and Graham [2] adopt the estimate:

$$Z_{\substack{\text{surface}\\ \text{cable}}} \approx Z_{\substack{\text{fully}\\ \text{buried}}} \quad . \tag{4}$$

The approximate validity of explicitly simple estimates such as (3) and (4) may be tested within the framework of a simplified surface cable model as we shall demonstrate in the following section.

Given satisfactory estimates of Y_{SC} and Z_{SC} for the surface cable we could define a propagation constant for the transmission line:

$$h_{SC} = \sqrt{Y_{SC}\ Z_{SC}} \quad . \tag{5}$$

Alternatively, we could utilize the best approximation for Z_{SC} and attempt to estimate h_{SC} directly. These matters, i.e., how best to employ the surface cable approximate parameters in an actual calculation, will be considered in our following note.

Section 3

CALCULATIONS OF TRANSIENT CURRENTS ON A UNIFORMLY EXCITED SURFACE CABLE

Aside from obvious computational merits, a two-dimensional formulation of the SC problem is valuable as a guide to suggestion and evaluation of approximations applicable (perhaps) to a more general situation. We calculate in this section, using two different approaches, the transient response of the Figure 2.1 SC model (infinite length) to an axially uniform electric field incident at the surface of the inner conductor and having the form:

$$E_{inc}(t) = H(t) \text{ volts/meter,} \tag{6}$$

where H(t) is the Heaviside unit step.

3.1 Finite Difference Solution

For our two-dimensional SC model the Maxwell equations are:

$$\frac{\partial H_\phi}{\partial r} + \frac{H_\phi}{r} - \frac{1}{r}\frac{\partial H_r}{\partial \phi} = \sigma E_Z + \varepsilon \frac{\partial E_Z}{\partial t} \quad , \tag{7}$$

$$\frac{1}{r}\frac{\partial E_Z}{\partial \phi} = -\mu \frac{\partial H_r}{\partial t} \quad , \tag{8}$$

and,

$$\frac{\partial E_Z}{\partial r} = \mu \frac{\partial H_\phi}{\partial t} \quad . \tag{9}$$

Utilizing the symmetry of the problem we may define the geometry as shown in Figure 3.1.

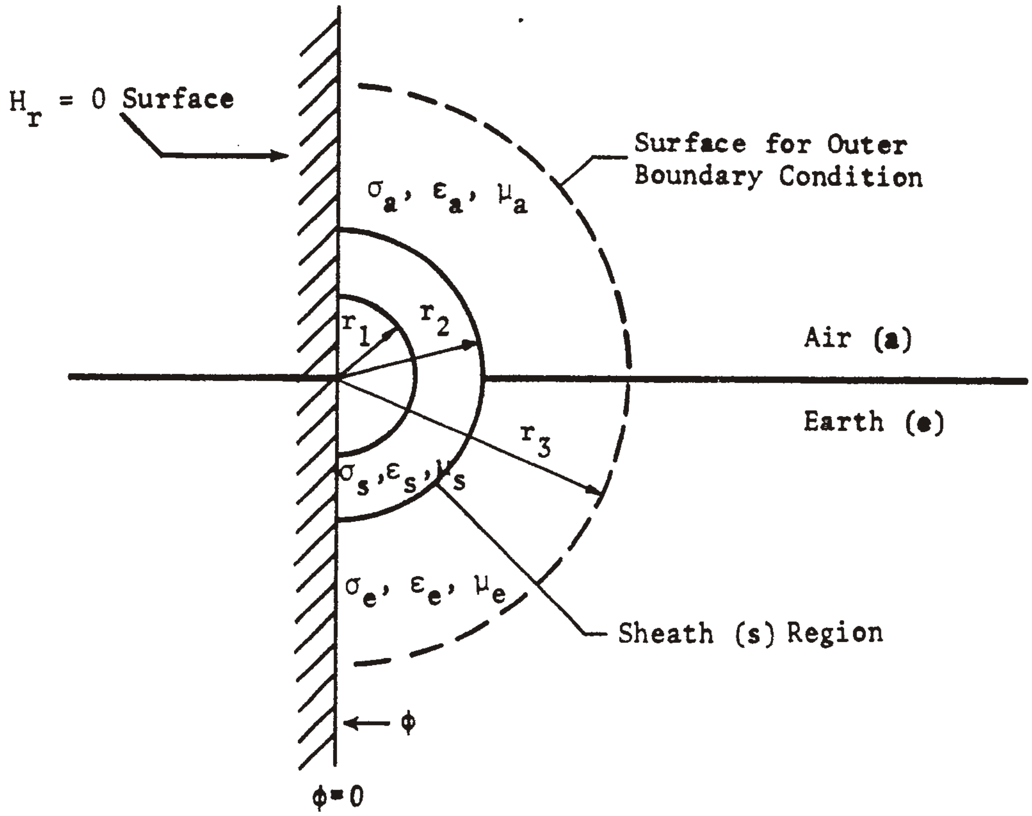

Figure 3.1. Surface Cable Geometry

Within the semicircle of radius r_3 in Figure 3.1 we grid the field coordinates and time as follows:

$$E_Z^n(i, j) = E_Z(t, \phi, r)\Big|_{\substack{t = n\Delta t\quad ;\ n = 1,\ n_{max} \\ \phi = (i-1/2)\Delta\phi;\ i = 1,\ i_{max} \\ r = (j-1)\Delta r;\quad j = 2,\ j_{max}}} \qquad (10)$$

$$H_\phi^n(i, j) = H_\phi(t, \phi, r)\Big|_{\substack{t = (n+1/2)\ \Delta t \\ \phi = (i-1/2)\ \Delta\phi \\ r = (j-1/2)\ \Delta r}} \qquad (11)$$

$$H_r(i,j) = H_r(t,\phi,r)\Big|_{\substack{t=(n+1/2)\Delta t\\ \phi=(i-1)\Delta\phi\\ r=(j-1)\Delta r}} \tag{12}$$

A straightforward differencing [5] of equations (7-9) yields, respectively, the expressions:

$$E_Z^{n+1}(i,j+1)\left[\frac{\sigma}{2}+\frac{\varepsilon}{\Delta t}\right]\Delta r = E_z^{n}(i,j+1)\left[\frac{\varepsilon}{\Delta t}-\frac{\sigma}{2}\right]\Delta r + \frac{1}{\Delta r}\left[H_\phi^n(i,j+1) - H_\phi^n(i,j)\right]$$

$$+\frac{1}{\partial j\Delta r}\left[H_\phi^n(i,j+1) + H_\phi^n(i,j)\right] \tag{13}$$

$$-\frac{1}{j\Delta r\Delta\phi}\left[H_r^n(i+1,j+1) - H_r^n(i,j+1)\right] \quad ;$$

$$H_r^{n+1}(i,j) = H_r^n(i,j) - \frac{\Delta t}{\mu(j-1)\Delta\phi\Delta r}\left[E_Z^{n+1}(i+1,j) - E_Z^{n+1}(i,j)\right] \tag{14}$$

and,

$$H_\phi^{n+1}(i,j) = H_\phi^n(i,j) + \frac{\Delta t}{\mu\Delta r}\left[E_Z^{n+1}(i,j+1) - E_Z^{n+1}(i,j)\right] \quad . \tag{15}$$

By choosing i_{max} odd, the earth-air interface is horizontal on an H_r plane and $\varepsilon\frac{\partial E_Z}{\partial t}$ may be computed inside the media boundaries. The matched vertical boundary in Figure 3.1 represents a zero normal derivative specification.

At the perfectly conducting surface defined by $r_1 = (j_s-1)\Delta r$, the incident field (6) may be assumed nonzero (= 1 volt/m) at the first time step. The boundary condition is therefore:

$$E_Z^{N\geq 1}(i,j_s) = -1 \quad . \tag{16}$$

For times $\lesssim 2(r_3-r_1)/c$ (clear time) the above solution effectively simulates infinite outer boundaries as we desire in the surface cable model.

Consequently, in this note we simply assume a perfectly conducting outer shell with r_3 chosen so as to yield a sufficiently long clear time.

Having solved the problem as described we then need only compute the tangential magnetic field distribution at the wire surface. A first order Taylor series approximation is adequate:

$$H_{tang.}(i)_{r=r_1} \approx \frac{3H_\phi(i,j_s) - H_\phi(i,j_s+1)}{2} \quad . \qquad (17)$$

3.2 Approximate Analytical Solution

Our approximate SC model is based on an exact calculation of the longitudinal impedance characteristic of the cable (driven by an axially uniform applied field) illustrated in Figure 3.2.

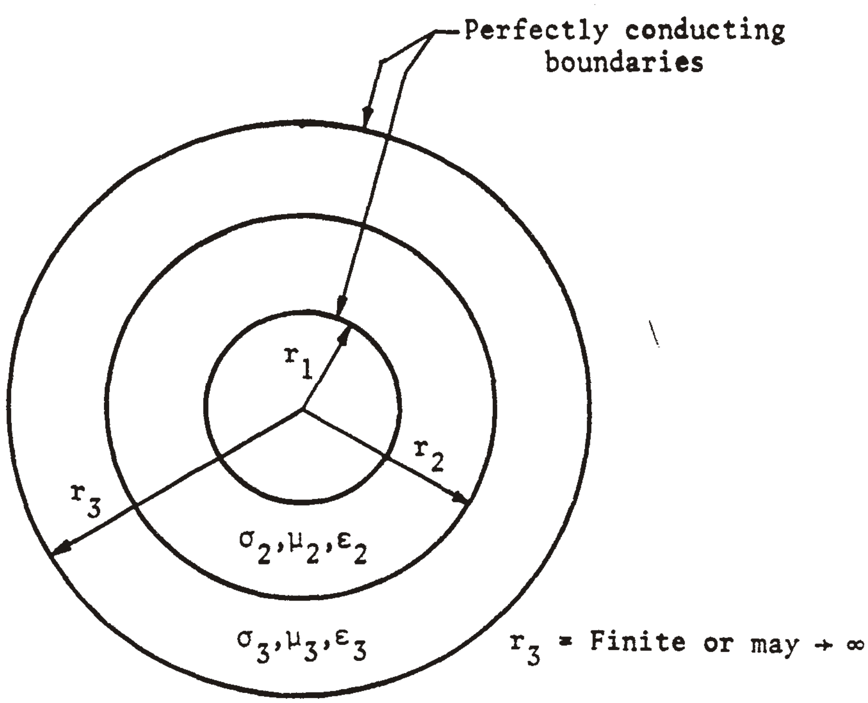

Figure 3.2. Cable Geometry

Our primary interest lies in the case of an infinite outer medium 3, but it is instructive to solve the finite outer boundary problem and then obtain (exactly) the infinite boundary results as a limiting case. It can thus be demonstrated that most of the interesting features of the model occur only in the case of an infinite outer medium. The elementary solution to the boundary value problem appropriate to Figure 3.2 and the various impedance expressions are derived in the Appendix.

The impedance of the Figure 3.2 cable is just

$$Z = \left.\frac{\text{(constant applied field}}{\text{wire current}}\right|_{r=r_1} = \frac{-E_Z^{(2)}(r=r_1)}{2\pi r_1\ H_\phi^{(2)}(r=r_1)} \quad . \tag{18}$$

Explicitly, we find for the infinite outer boundary case (with notations and conventions of the Appendix):

$$Z(\omega) = \frac{i\omega\mu_2}{2\pi r_1 k_2}\left[\frac{Y_o(k_2r_1) + C_\infty J_o(k_2r_1)}{Y_1(k_2r_1) + C_\infty J_1(k_2r_1)}\right] \quad , \tag{19}$$

where,

$$C_\infty = -\frac{Y_o(k_2r_2) - \dfrac{\mu_3k_2}{\mu_2k_3}\dfrac{H_o^{(2)}(k_3r_2)}{H_1^{(2)}(k_3r_2)}\ Y_1(k_2r_2)}{J_o(k_2r_2) - \dfrac{\mu_3k_2}{\mu_2k_3}\dfrac{H_o^{(2)}(k_3r_2)}{H_1^{(2)}(k_3r_2)}\ J_1(k_2r_2)} \quad . \tag{20}$$

If $E_{inc}(\omega)$ is the frequency domain input pulse, then the time response of the cable may be obtained from (18), (19), and the law of Biot and Savart.

$$I(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{E_{inc}(\omega)}{Z(\omega)}\, e^{i\omega t}\, d\omega \quad . \tag{21}$$

Given the step pulse (6) we may evaluate (21) numerically utilizing the shift theorem:

$$I(t) = \frac{e^{\alpha t}}{2\pi} \int_{-\infty}^{\infty} \frac{e^{i\omega t}\, d\omega}{(\alpha+i\omega)\; Z(\omega-i\alpha)} \quad . \tag{22}$$

Over the time range of interest we have obtained excellent numerical transforms (Section 3.3) from (22), with an offset $\alpha = 10^8$.

Now let us estimate the corresponding surface cable (Figure 2.1) response from the above result. Taking into account both the earth and air media, perhaps the simplest estimate of the surface cable current $I_{SC}(t)$ is:

$$I_{SC}(t) = 1/2\left(I_{earth}(t) + I_{air}(t)\right) \quad , \tag{23}$$

computed from (19) and (22) by assigning region 3 of Figure 3.2 the appropriate electrical properties. As we shall show shortly, the crude approximation (23) does remarkably well.

The result (23) is identical to that obtained from the calculation of integral (22) using an equivalent surface cable impedance of the form:

$$Z_{SC}(\omega) = \frac{2Z_{earth}(\omega)\; Z_{air}(\omega)}{Z_{earth}(\omega) + Z_{air}(\omega)} \quad . \tag{24}$$

The equivalent transmission line circuit element representing (24) is just the parallel combination of impedances Z_{earth} and Z_{air}. Of course, our analysis strictly applies only to an infinite transmission line in which the current on the line has no z dependence; nevertheless, the results should be approximately applicable to a line of finite length.

Note that we can obtain from (24) the estimate of Whitson and Vance by assuming $Z_{air} \gg Z_{earth}$, whence $Z_{SC} \approx 2Z_{earth}$.

The Marston-Graham estimate, $Z_{SC} \approx Z_{earth}$, also follows from (24) under the assumption $Z_{air} \approx Z_{earth}$. Let us now test these various approximations by way of comparing the resulting current predictions with the finite difference solution.

3.3 Numerical Comparison

As a test case we chose cable dimensions (Figure 2.1):

$$r_1 = 3 \times 10^{-2} \text{ m}$$

$$r_2 = r_1 + 2.25 \times 10^{-2} \text{ m}.$$

The reflecting outer boundary r_3 in the finite difference solution was fixed at 1.15 m, allowing a clear time $\sim 10^{-8}$ sec. Choosing a larger r_3 would have involved additional computer time with little gain in useful information.

The infinite air medium was assumed to be free space with

$$\varepsilon_0 = \frac{10^{-9}}{36\pi} \text{ f/m}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ h/m} \quad .$$

For the sheath and infinite earth media we have taken:

$$\mu_{sheath} = \mu_{earth} = \mu_0$$

$$\varepsilon_{sheath} = 2.75\ \varepsilon_0$$

$$\varepsilon_{earth} = 10\ \varepsilon_0$$

$$\sigma_{sheath} = 0$$

$$\sigma_{earth} = 2 \times 10^{-2},\ .1,\ 1 \text{ mhos/m} \quad .$$

Figures 3.3, 3.4 and 3.5 display the calculated transient surface cable currents for various earth conductivities. The labels "earth", "air" and "earth-air average" refer to the current components and resultant value of the analytic estimate (23). The "earth" currents are the consequences of the Marston-Graham (MG) estimate. The Whitson-Vance (WV) current estimates would clearly be 1/2 of the MG values.

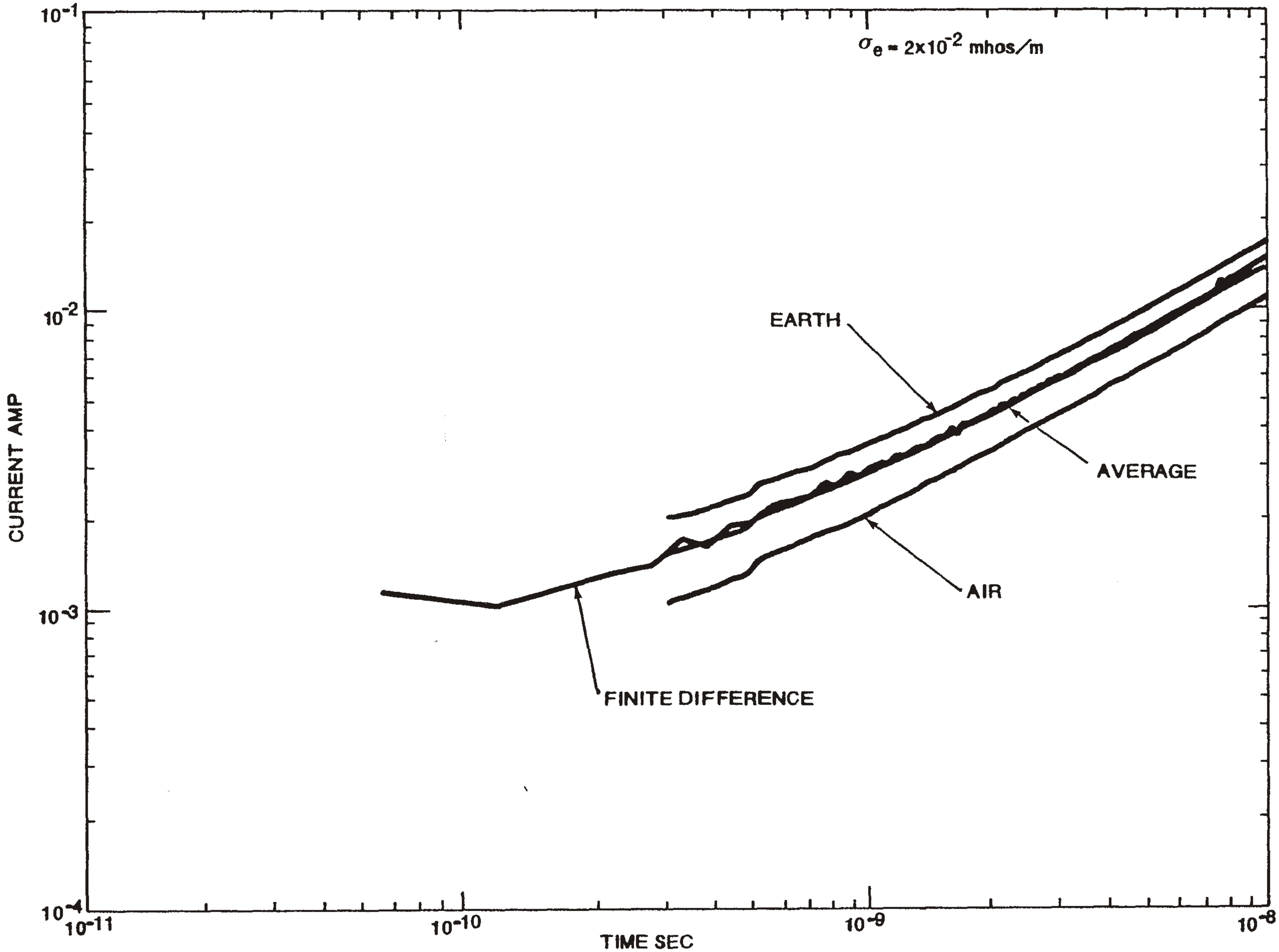

Figure 3.3 Current Versus Time

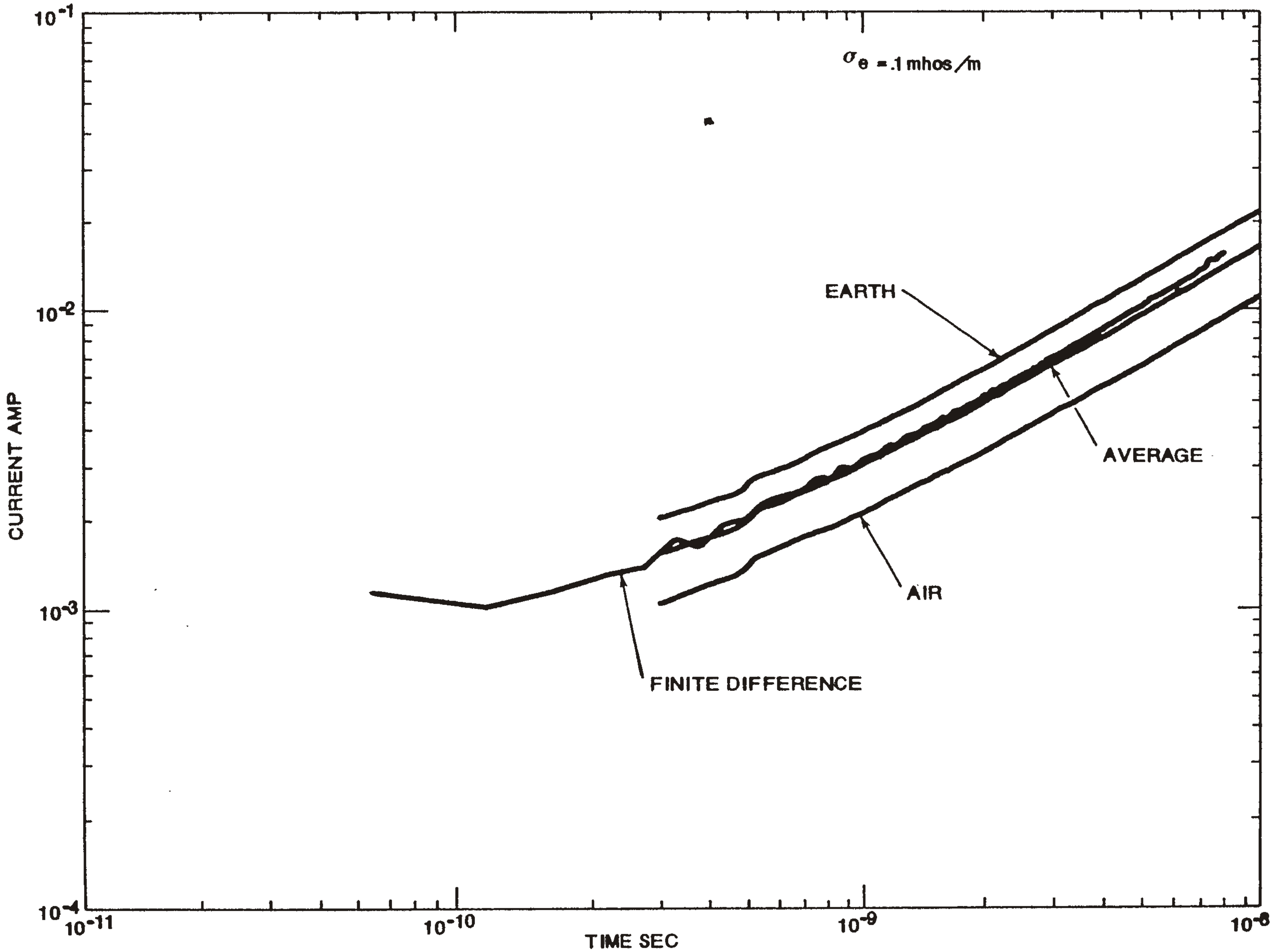

Figure 3.4 Current Versus Time

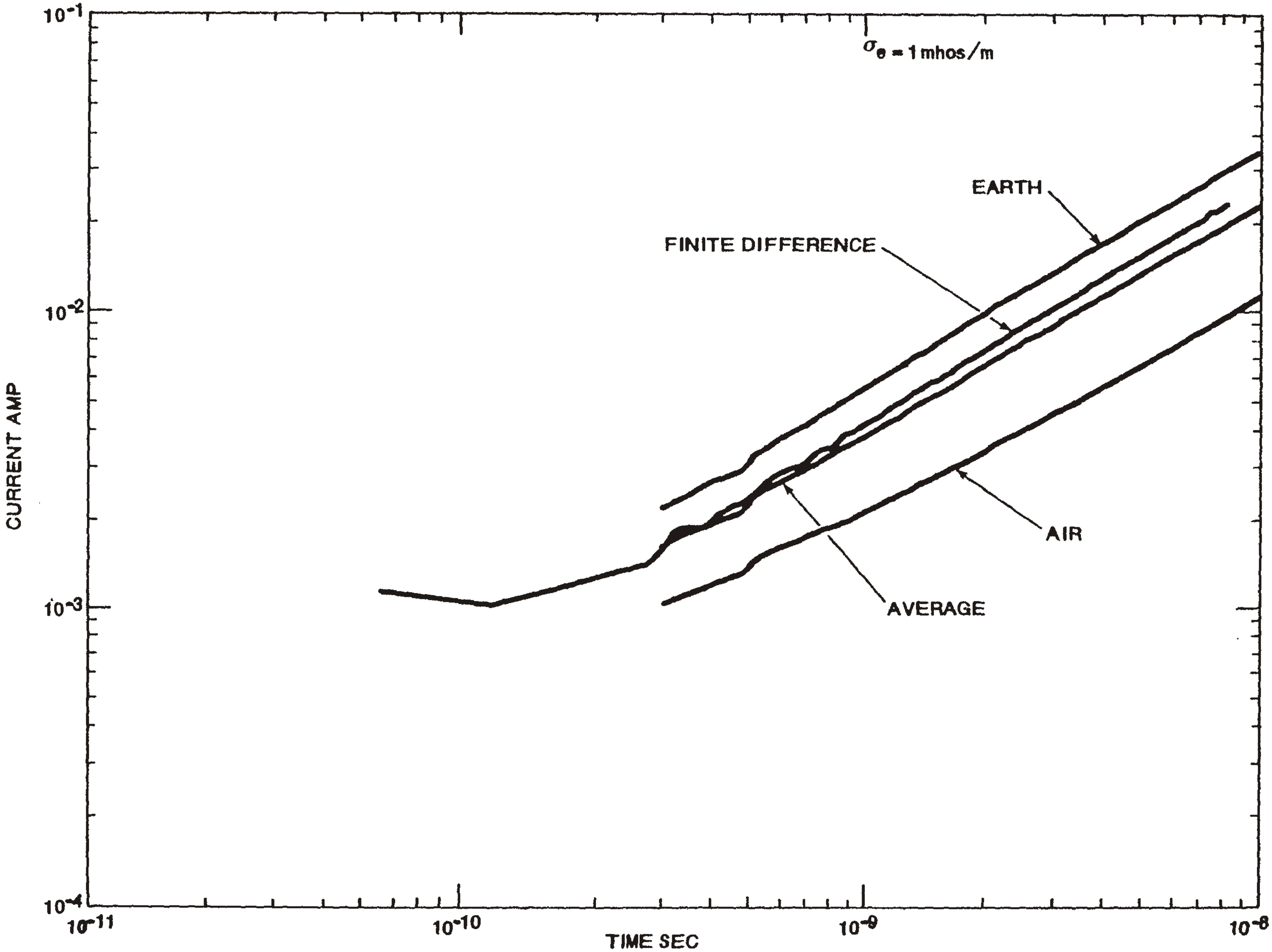

Figure 3.5 Current Versus Time

At least within the limitations of our two-dimensional SC model, it is evident that the simple average analytic estimate yields a quite reasonable engineering approximation to the more accurate SC transient solutions. The MG and WV predictions are, of course, also reasonable, respectively overestimating and underestimating the finite difference result by factors of $\sim 3/2$.

Note that the deviation of the average current from the finite difference solution grows slightly with increasing time as the earth conductivity is increased. This result, as well as the overall reasonableness of the simple analytic estimates, is illuminated by examining the surface current densities on the cable as displayed in Figures 3.6, 3.7 and 3.8. The finite difference values are simply the average magnetic field $H_\phi^{(2)}(r=r_1)$ over both the top and bottom semicircles of the cable. The "earth", "air" and "average" current density values are just the analytic model currents previously discussed divided by $2\pi r_1$.

The magnetic field (from time zero) requires an interval of about $\pi r_1/c \sim 3 \times 10^{-10}$ sec to circulate around one semicircle; following roughly this time, we see that the individual top and bottom (finite difference) current densities may be distinguished. The bottom density is naturally largest (since the air has no conductivity) and this quantity is rather well approximated by the analytic "earth" estimate. The top current density in the finite difference solution is evidently more sensitive to increased earth conductivity and grows significantly faster than the "air" estimate would suggest. Thus most of the error in the analytic model is due to the air segment, but since the top cable current is ~1/2 the bottom current, the net effect on the average current estimate is small.

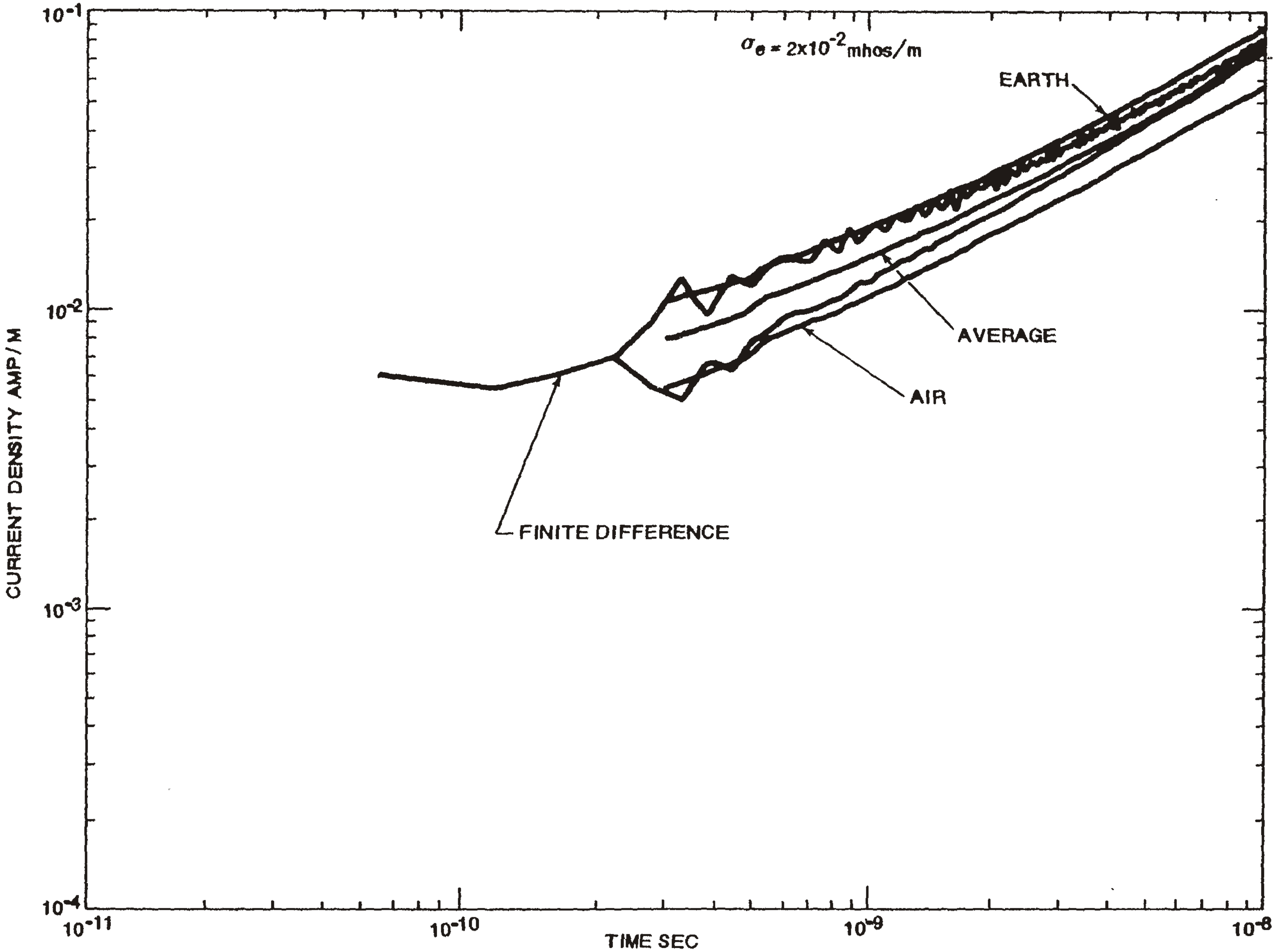

Figure 3.6 Current Density Versus Time

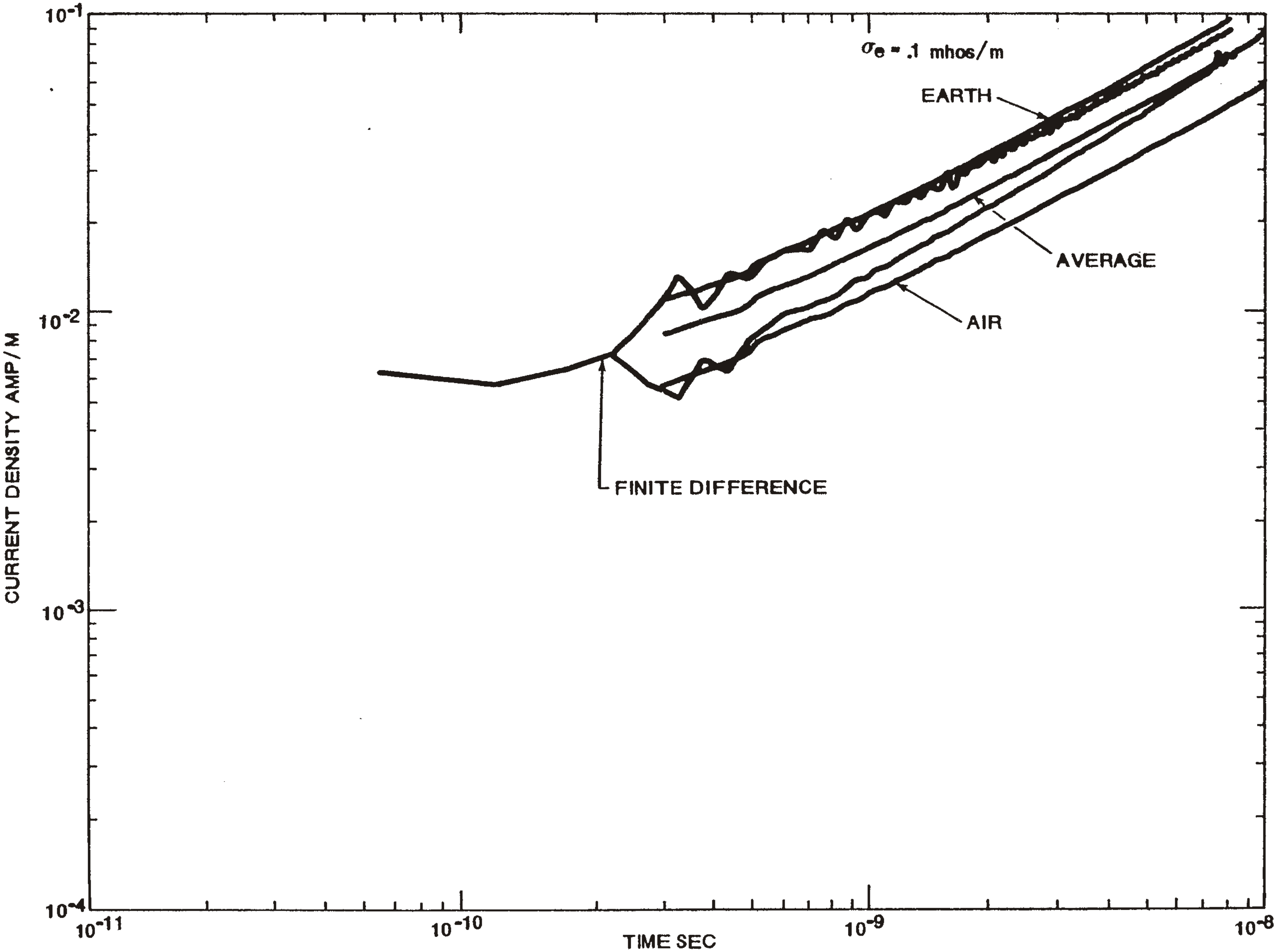

Figure 3.7 Current Density Versus Time

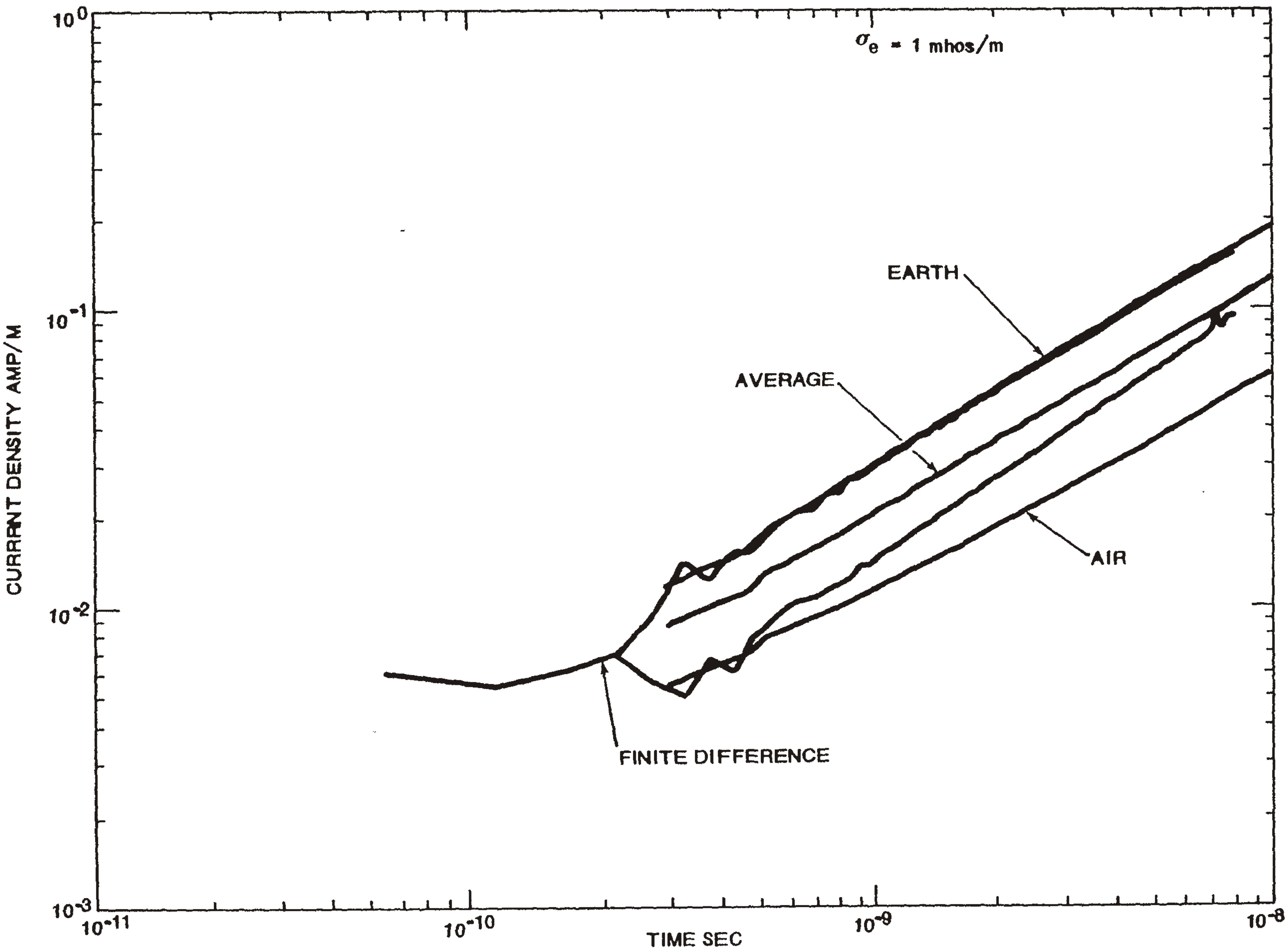

Figure 3.8 Current Density Versus Time

Section 4

CONCLUSIONS

If one assumes a spatially uniform incident pulse, then the transient response of an (infinite) insulated wire symmetrically embedded at an idealized earth-air interface can be easily obtained from a straightforward finite difference solution of the Maxwell equations. In the absence of the earth-air interface, the latter problem can be exactly solved analytically. A simple average of the analytic transient solutions computed for cables embedded in both earth and air media yields a satisfactory engineering approximation to the finite difference result. Thus, the effective surface cable longitudinal impedance may be quite reasonably estimated from the exact analytic impedance expressions:

$$Z_{SC}(\omega) \approx \frac{2\, Z_{air}(\omega)\, Z_{earth}(\omega)}{Z_{air}(\omega) + Z_{earth}(\omega)} .$$

To within factors ~3/2 of the computed transient currents, the latter analytic estimate justifies the similar suggestions of previous authors. Our results do not, however, validate their actual surface cable current calculations.

APPENDIX

IMPEDANCE CALCULATIONS

The boundary value problem appropriate to Figure 3.2, given a constant applied field E_a (axially uniform) at the surface $r=r_1$, is:

$$\frac{\partial^2 E_z^{(j)}}{\partial r^2} + \frac{1}{r}\frac{\partial E_z^{(j)}}{\partial r} + k_j^2 E_Z^{(j)} = 0 \; ; \quad j = 1,2 \quad ,$$

subject to boundary conditions

$$E_z^{(2)}(r=r_1) = -E_a$$

$$E_z^{(3)}(r=r_3) = 0 \quad .$$

Adopting an $e^{i\omega t}$ convention, the propagation constants for regions 1 and 2 are:

$$k_j^2 = \mu_j \varepsilon_j \omega^2 - i\mu_j \sigma_j \omega \; , \; I_m(k_j) < 0.$$

The wave equation solutions for each region are linear combinations of zero order Bessel, $J_o(k_j r)$ and Neumann, $Y_o(k_j r)$ functions as defined in [6]. The transverse magnetic field is

$$H_\phi^{(j)} = \frac{-i}{\omega\mu_j} \frac{\partial E_z^{(j)}}{\partial r} \quad .$$

From the solution to the above problem, the longitudinal impedance of the cable is

$$Z = \frac{-E_z^{(2)} \; (k_2 r_1)}{2\pi r_1 \; H_\phi^{(2)}(k_2 r_1)} \quad . \qquad (1)$$

Explicitly we find

$$Z = \frac{i\omega\mu_2}{2\pi r_1 k_2}\left[\frac{Y_o(k_2r_1) + C \quad J_o(k_2r_1)}{Y_1(k_2r_1) + C \quad J_1(k_2r_1)}\right] \tag{2}$$

where the coefficient C is

$$C = -\frac{Y_1(k_2r_2)\ F_o + \dfrac{\mu_2k_3}{\mu_3k_2}\ Y_o(k_2r_2)\ F_1}{J_1(k_2r_2)\ F_o + \dfrac{\mu_2k_3}{\mu_3k_2}\ J_o(k_2r_2)\ F_1} \tag{3}$$

The F coefficients in (3) are

$$F_o = J_o(k_3r_2)\ Y_o(k_3r_3) - J_o(k_3r_3)\ Y_o(k_3r_2) \tag{4}$$

$$F_1 = J_o(k_3r_3)\ Y_1(k_3r_2) - J_1(k_3r_2)\ Y_o(k_3r_3) \tag{5}$$

In the long wavelength approximation ($kr \ll 1$), Z reduces to

$$Z_{\ell.w.} \sim \frac{i\omega}{2\pi}\left[\mu_3 \ell n\ (r_3/r_2) + \mu_2\ \ell n\ (r_2/r_1)\right] \tag{6}$$

The result (6) demonstrates that for the finite outer (r_3) radius case, the geometry dominates the impedance and ε_j, σ_j play no essential role. The transient response of the Figure 3.2 cable to a unit step is therefore basically linear in time. Likewise, the finite difference transient solution of Section 3 is essentially a ramp response for times greater than the clear time. In such a case the approximation (6) would be adequate for the "split-media" cable with a perfectly conducting finite outer shield.

The surface cable formulation of greatest interest contains infinite media. The relevant impedance is again (2) and taking the limit in the coefficient (3) (using asymptotic Bessel expansions) we find:

$$C_\infty = \lim_{r_3 \to \infty} (C) = - \frac{Y_o(k_2 r_2) - \frac{\mu_3 k_2}{\mu_2 k_3} \frac{H_o^{(2)}(k_3 r_2)}{H_1^{(2)}(k_3 r_2)} Y_1(k_2 r_2)}{J_o(k_2 r_2) - \frac{\mu_3 k_2}{\mu_2 k_3} \frac{H_o^{(2)}(k_3 r_2)}{H_1^{(2)}(k_3 r_2)} J_1(k_2 r_2)} \tag{7}$$

Coefficient (7) contains Hankel functions of the <u>second</u> kind. The long wavelength approximation to the latter impedance is

$$Z_{\ell.\,w.} \sim \frac{i\omega\mu_2}{2\pi} \ell n\,(r_2/r_1) - \frac{i\omega\mu_3}{2\pi} \ell n\left(\frac{\gamma k_3 r_2}{-2i}\right) , \tag{8}$$

where γ ~1.781. Approximation (8) is closely related to a corresponding expression utilized by Whitson and Vance [1], but these authors had incorrectly employed Hankel functions of the <u>first</u> kind in their ($e^{i\omega t}$ convention) model formulation.

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