IEEE Trans. EMC, Aug. 1975, pp 159-169

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Interaction Notes

Note 132

January 1973

Application of Modal Analysis to Braided-Shield Cables

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Abstract

Transmission-line equations are derived for a braided-shield cable by modal analysis. The parameters of the braided shield appear in the coefficients as well as in the source terms of the equations. The source terms also depend on the currents and charges on the outer surface of the shield with all the shield's apertures short-circuited.

I. Introduction

In the theoretical study of braided-shield cables the main goal is to derive a set of transmission-line equations from which one can calculate the voltage and current induced by external sources in a load impedance, which may represent a piece of electronic equipment connected to one such cable. The coefficients of the equations should contain, among other things, the parameters of the braided shield, and the source terms should involve the currents and charges on the outer surface of the shield with all the shield's apertures short-circuited.

As of now there exist several articles directly related to the study of braided-shield cables [1]-[6]. The common approach employed in these references, except [2], is to incorporate the concepts used in statics into the conventional transmission-line theory. The approach in this note is different in that we start with Maxwell's field equations and work our way towards the frequency range where the transmission-line theory applies. Therefore, this note is written with the field theorist in mind and, hopefully, will get him interested in braided-shield cables.

A real braided-wire shield will be modeled by a perfectly conducting thin shell with many small apertures in it. Other aspects of the shield, such as finite wire conductivity, contact resistances between wires, dielectric effects, etc., can be taken into account in some approximate manner as discussed in [6] and will not be treated here. From the viewpoint of boundary-value problems the tangential electric field in the apertures is all that is required for the determination of the fields everywhere and, hence, the field theorist will pay particular attention to obtaining this aperture electric field. Although an integral equation can, in principle, be set up for this field, it will require an inordinate amount of computer time to obtain any meaningful numerical results from this equation. Besides, the wavelengths in cable applications are always such that one does not really need detailed information on the aperture electric field. Thus, this integral-equation approach, although rigorous, is impractical. If one makes the observation that all practical braided shields for cables are actually very good shields, then the problem of calculating the electromagnetic interaction of a braided-shield cable with external sources can be divided into three separate problems, namely, (a) the exterior scattering problem of determining the induced currents and charges on the outer surface of the shield with all shield's apertures short-circuited, (b) the problem of calculating the transmission coefficients of the apertures or, equivalently, the static electric and magnetic polarizabilities α_e and $\underline{\underline{\alpha}}_m$ in the low-frequency approximation, and (c) the interior problem of determining the magnitudes and also the propagation characteristic of the cable voltage and current.

In this note we treat exclusively problem (c), assuming problems (a) and (b) to have been solved. In fact, a large amount of results is available for problems (a) and (b) in the literature. To develop from Maxwell's equations a set of transmission-line equations that will involve the polarizabilities α_e and $\underline{\underline{\alpha}}_m$ and also the currents and charges on the outer surface of the shield, it is most expedient to use the modal analysis [7], [8]. Throughout the analysis we will focus our attention on the dominant mode and calculate the perturbation of the shield's apertures on the propagation characteristics of that mode. We will also see that in the modal analysis the two source terms (the series voltage source and the shunt current source) appear naturally in the transmission-line equations.

In Section II we briefly review the modal analysis and single out the dominant mode for discussion. We then derive equations for the cable voltage and current, which are directly related to the mode voltage and current of the dominant mode. The source terms are expressed in terms of the "virtual" electric and magnetic current densities which are introduced to replace the effects of the aperture discontinuities in the outer conductor of the coaxial cable. These "virtual" electric and magnetic current densities can be approximated, respectively, by point electric and magnetic dipoles for small apertures. Detailed calculations are given for a single aperture.

In Section III we relate the "virtual" current densities to the tangential electric and magnetic fields in the aperture. It turns out that the dominant mode would not couple to the tangential aperture magnetic field, i.e., the "virtual" electric current density. However, the tangential aperture electric field, when taking the size of the aperture into account, will give rise to a magnetic dipole in addition to an electric dipole and all multipoles. We then demonstrate that problem (c) can be reformulated with the magnetic dipole (and perhaps all the multipoles) as the series voltage sources and the electric dipole (and perhaps all the multipoles) as the shunt current sources in the transmission-line equations.

Section IV is devoted to the calculations of the equivalent lumped circuit elements of a single aperture in the outer conductor of a coaxial cable, and in Section V transmission-line equations are derived for a braided-shield cable.

II. A Coaxial Cable With an Aperture in the Outer Conductor

Figure 1 depicts a coaxial cable with a small aperture in its sheath. For simplicity, all conductors of the cable are assumed to be perfectly conducting and the cable is taken to be infinitely long. In this section we will calculate the effects of the aperture on the waveguide modes, especially the TEM mode, within the coaxial cable and also the coupling of electromagnetic energy through the aperture from the exterior region to the interior region of the cable. In a later section we will extend this analysis to a coaxial cable with many apertures in its sheath, e.g., a braided-shield cable.

To treat a waveguide problem with aperture discontinuities as shown in Figure 1 it is most expedient to use the modal analysis [7]. However, it must be borne in mind that only within a uniform (i.e., discontinuity-free) guide do the guide modes form a complete orthonormal set. The situation in Figure 1 can be remedied by invoking the Schelkunoff field equivalence theorems as has been done by Marcuvitz and Schwinger [8], and others [9]. These theorems enable one to duplicate the effects of any geometric discontinuities, such as the aperture in Figure 1, on the fields within the waveguide by some appropriate distributions of electric and magnetic current densities, $\underline{J}$ and $\underline{J}^*$, over the discontinuities. Thus, the original problem can be reformulated as a Kirchhoff problem of calculating the fields produced by $\underline{J}$ and $\underline{J}^*$ inside a discontinuity-free coaxial waveguide. Of course, $\underline{J}$ and $\underline{J}^*$ are unknown and, generally speaking, they have to be obtained by solving some appropriate boundary-value problem. Later on in this note their distributions will be approximated by point dipoles with undetermined dipole strengths.

Given a Kirchhoff problem within a discontinuity-free coaxial waveguide we can apply modal analysis to solve Maxwell's equations. As will be seen in the following, this analysis is by far the simplest to derive the transmission-line equations for the TEM mode in a waveguide with aperture discontinuities.

Let us start with Maxwell's equations with $\underline{J}$ and $\underline{J}^*$:

$$\begin{aligned} \nabla\times\underline{E} &= i\omega\mu\underline{H} - \underline{J}^* \\ \nabla\times\underline{H} &= -i\omega\varepsilon\underline{E} + \underline{J} \end{aligned} \qquad (1)$$

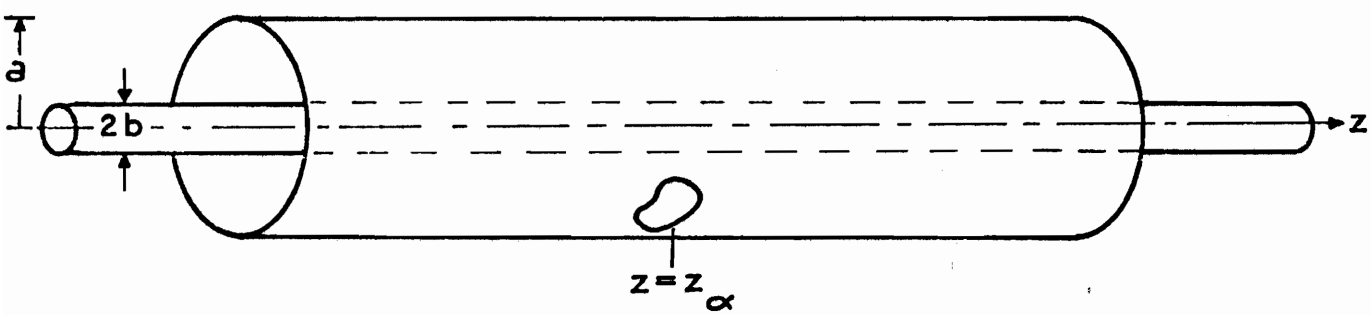

Figure 1. A small hole in the shield of a coaxial cable.

where the time-harmonic factor $e^{-i\omega t}$ has been used and suppressed, and where $\underline{J}$ and $\underline{J}^*$ are related to the electric polarization $\underline{P}$ and magnetic polarization $\underline{M}$ by

$$\underline{J} = -i\omega\underline{P}, \qquad \underline{J}^* = -i\omega\mu\underline{M} \tag{2}$$

Vector multiplication of (1) by the unit vector $\underline{e}_z$ gives

$$\begin{aligned} \underline{e}_z\times(\nabla\times\underline{E}) &= \nabla_t E_z - \frac{\partial}{\partial z}\underline{E}_t = -i\omega\mu\underline{H}_t\times\underline{e}_z + \underline{J}^*\times\underline{e}_z \\ \underline{e}_z\times(\nabla\times\underline{H}) &= \nabla_t H_z - \frac{\partial}{\partial z}\underline{H}_t = -i\omega\varepsilon\underline{e}_z\times\underline{E}_t + \underline{e}_z\times\underline{J} \end{aligned} \tag{3}$$

Scalar multiplication of (1) by $\underline{e}_z$ gives

$$\begin{aligned} \underline{e}_z\cdot(\nabla\times\underline{E}) &= -\nabla_t\cdot(\underline{e}_z\times\underline{E}_t) = i\omega\mu H_z - J_z^* \\ \underline{e}_z\cdot(\nabla\times\underline{H}) &= \nabla_t\cdot(\underline{H}_t\times\underline{e}_z) = -i\omega\varepsilon E_z + J_z \end{aligned} \tag{4}$$

Here $\underline{E} = \underline{e}_z E_z + \underline{E}_t$, $\nabla = \underline{e}_z(\partial/\partial z) + \nabla_t$, etc. Elimination of E_z and H_z from (3) by means of (4) gives

$$\begin{aligned} -\frac{\partial}{\partial z}\underline{E}_t &= -i\omega\mu\underline{H}_t\times\underline{e}_z + \frac{1}{i\omega\varepsilon}\nabla_t\nabla_t\cdot(\underline{H}_t\times\underline{e}_z) + \underline{J}^*\times\underline{e}_z - \frac{\nabla_t J_z}{i\omega\varepsilon} \\ -\frac{\partial}{\partial z}\underline{H}_t &= -i\omega\varepsilon\underline{e}_z\times\underline{E}_t + \frac{1}{i\omega\mu}\nabla_t\nabla_t\cdot(\underline{e}_z\times\underline{E}_t) + \underline{e}_z\times\underline{J} - \frac{\nabla_t J_z^*}{i\omega\mu} \end{aligned} \tag{5}$$

These equations are identical to equations (1) and have been proven most expedient in analyzing various waveguide problems [7].

We now expand the transverse fields $\underline{E}_t$ and $\underline{H}_t$ in terms of a complete set of orthonormal, vector functions $\underline{e}_n(x,y)$ and $\underline{h}_n(x,y)$, where x and y are the transverse coordinates of the coaxial waveguide. Some of the properties of these functions that are pertinent for the present discussion are given in Appendix A. Thus,

$$\underline{E}_t(x,y,z) = \sum_n V_n(z)\underline{e}_n(x,y)$$

$$\underline{H}_t(x,y,z) = \sum_n I_n(z)\underline{h}_n(x,y) \tag{6}$$

where n sums all TE and TM modes. Note that the TEM mode is included in the TM modes.

Substituting (6) into (5) and using the orthogonality condition (A.7) and equations (A.8) to (A.11) one gets

$$\frac{dV_n}{dz} = i\kappa_n Z_n I_n - v_n$$

$$\frac{dI_n}{dz} = i\kappa_n Y_n V_n - i_n \tag{7}$$

where for a TE mode

$$\kappa'_n = \sqrt{k^2 - k'^2_{cn}}, \qquad Z'_n = \frac{1}{Y'_n} = \frac{\kappa'_n}{\omega\varepsilon} ; \tag{8}$$

for a TM mode

$$\kappa''_n = \sqrt{k^2 - k''_{cn}}, \qquad Z''_n = \frac{1}{Y''_n} = \frac{\omega\mu}{\kappa''_n} ; \tag{9}$$

and

$$v_n(z) = \iint_S \left(\underline{J}_t^* \cdot \underline{h}_n - \frac{\underline{e}_n \cdot \nabla_t J_z}{i\omega\varepsilon} \right) dxdy$$

$$= \iint_S (\underline{J}_t^* \cdot \underline{h}_n - Z_n J_z e_{nz}) dxdy \tag{10a}$$

$$i_n(z) = \iint_S \left(\underline{J}_t \cdot \underline{e}_n - \frac{\underline{h}_n \cdot \nabla_t J_z^*}{i\omega\mu} \right) dxdy$$

$$= \iint_S (\underline{J}_t \cdot \underline{e}_n - Y_n J_z^* h_{nz}) dxdy \tag{10b}$$

Here, the integration is over the waveguide cross section S. These equations, (7)-(10), are not new and can be found, for example, in [8].

Here and henceforth, we will restrict our discussion mostly to the TEM mode, occasionally referring to the higher-order modes when discussing the mode couplings. Transforming the mode voltage V_n and mode current I_n of the TEM mode to the line voltage V and line current I, etc. according to the transformation rules in Appendix A, one easily gets from (7)

$$\frac{dV}{dz} = i\omega LI + \frac{1}{2\pi} \int_o^{2\pi} \int_b^a J_\phi^* d\rho d\phi$$

(11)

$$\frac{dI}{dz} = i\omega CV + \frac{Z_w}{Z_c} \frac{1}{2\pi} \int_o^{2\pi} \int_b^a J_\rho d\rho d\phi$$

where $L = (\mu/2\pi)\ln(a/b)$, $C = 2\pi\varepsilon/\ln(a/b)$, $Z_w = \sqrt{\mu/\varepsilon}$, and $Z_c = \sqrt{L/C}$. The sign convention of V and I is shown in Figure 2 together with the cylindrical coordinates (ρ,ϕ,z).

Equations (11) are exact for any number of apertures. The effects of the apertures on the TEM mode are expressed by the integrals of J_ϕ^* and J_ρ. These current densities are unknown quantities and are determined by solving some appropriate boundary-value problem which would involve, among other things, the external field and all the waveguide modes within the coaxial cable. Such a boundary-value problem is, in general, difficult to solve and will not be treated in this note. As stated in the Introduction, however, the relevant boundary-value problems one needs to solve are quite tractable for all practical braided-shield cables. The point to be made here is that the integrals in (11) take into account the mutual interactions among all the waveguide modes and the coupling of external energy into the interior of the cable's shield.

In the case we are considering, the apertures are all located in the outer

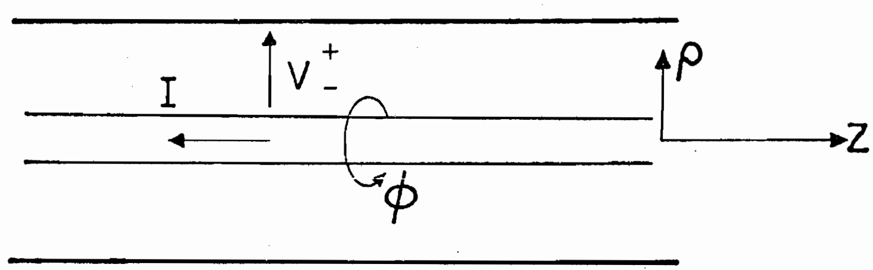

Figure 2. Sign convention of V and I.

conductor (i.e., the shield) of the coaxial cable. Hence,

$$J^*_\phi = K^*_\phi(a,z,\phi)\delta(\rho - a_-)$$
$$\tag{12}$$
$$J_\rho = K_\rho(a,z,\phi)\delta(\rho - a_-)$$

where $a_- = a - \varepsilon$, ε being a small positive number. Equations (11) become

$$\frac{dV}{dz} = i\omega LI + \frac{1}{2\pi}\int_o^{2\pi} K^*_\phi d\phi$$
$$\tag{13}$$
$$\frac{dI}{dz} = i\omega CV + \frac{Z_w}{Z_c}\frac{1}{2\pi}\int_o^{2\pi} K_\rho d\phi$$

Equations (13) are as far as we can get from Maxwell's equations without making any approximations. To proceed further we will use the small-aperture approximation [10], which is justifiable in braided-cable calculations. For a small aperture located at $\phi = \phi_\alpha$, $z = z_\alpha$ in the cable's shield we have, from (2) and (13),

$$\underline{K}^* = -i\omega\mu\underline{m}\,\frac{\delta(\phi-\phi_\alpha)}{\rho}\,\delta(z - z_\alpha)$$
$$\tag{14}$$
$$\underline{K} = -i\omega\underline{p}\,\frac{\delta(\phi-\phi_\alpha)}{\rho}\,\delta(z - z_\alpha)$$

where the dipoles $\underline{m}$ and $\underline{p}$ are obtained by comparing the transmission field far from the aperture (which may be gotten by solving some appropriate boundary-value problem [10]) with the far field radiated by $\underline{m}$ and $\underline{p}$ in the presence of an infinite conducting screen.*

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*In some references, like [6], a subscript "eff", a shorthand for "effective", is attached to $\underline{m}$ and $\underline{p}$. However, this subscript is deleted throughout this note for reason of simplicity. If one compares the free-space dipole field formulas with

Insertion of (14) in (13) gives

$$\frac{dV}{dz} = i\omega LI - \frac{i\omega\mu}{2\pi a} \underline{m}\cdot\underline{e}_\phi \delta(z - z_\alpha)$$

(15)

$$\frac{dI}{dz} = i\omega CV - \frac{i\omega Z_w}{2\pi a Z_c} \underline{p}\cdot\underline{e}_\rho \delta(z - z_\alpha).$$

The terms proportional to $\underline{m}$ and $\underline{p}$ have been derived previously by employing different methods [2], [4], [6]. By comparison it is easy to see that modal analysis is the simplest of all the methods that lead to equations (15).

Let us see how to generalize (15) to a cable of non-circular cross sections or to a cable where the inner conductor is not coaxial with the shield. From (7) and (10) it is not difficult to generalize (15) to be

$$\frac{dV}{dz} = i\omega LI + V_{eq}\delta(z - z_\alpha)$$

(16)

$$\frac{dI}{dz} = i\omega CV + I_{eq}\delta(z - z_\alpha)$$

where

$$V_{eq} = i\omega\mu(N)^{1/2}\ \underline{m}\cdot\underline{h}_o$$

(17)

$$I_{eq} = i\omega(N)^{-1/2}\ \underline{p}\cdot\underline{e}_o,$$

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the transmission field to obtain the appropriate dipole strengths (let them be denoted by $\underline{m}_{eq}$ and $\underline{p}_{eq}$), then of course, $\underline{m}_{eq} = 2\underline{m}$ and $\underline{p}_{eq} = 2\underline{p}$ [9]. So, if $\underline{m}_{eq}$ and $\underline{p}_{eq}$ are used in (14) and subsequent equations, a factor of 1/2 will have to be carried through. For reason of simplicity and for reason that the equivalent dipoles for an aperture have different signs for the scattered field in the illuminated and shadow regions, we will use $\underline{m}$ and $\underline{p}$ instead of $\underline{m}_{eq}$ and $\underline{p}_{eq}$. This usage also agrees with that in [7].

$\underline{e}_o$ and $\underline{h}_o$ are the normalized transverse electric and magnetic fields of the TEM mode and N is the normalization constant equal to the ratio of the line characteristic impedance ($=\sqrt{L/C}$) to the TEM mode impedance (see Appendix A). Of course, (17) is evaluated at the position of the aperture. Since $\underline{e}_o$ and $\underline{h}_o$ can be obtained from the consideration of only one two-dimensional static problem, a large class of cross-sectional geometries can be solved analytically by the method of conformal mapping.

The approach described above may also apply to multi-wire cables.

III. Further Considerations of the Source Terms

Before applying equations (15) to the calculation of the equivalent lumped circuit elements of a single aperture in the cable's shield and eventually to a braided-shield cable, let us discuss briefly the electric and magnetic volume current densities, $\underline{J}$ and $\underline{J}^*$, that appear in the above equations. For the purpose of deriving (15) it is most expedient to think of $\underline{J}$ and $\underline{J}^*$ as arising respectively from an electric dipole and a magnetic dipole located close to the inside wall of the cable's outer conductor and at the position where the aperture was. The dipole strengths are proportional to the external field as well as the field of the TEM mode inside the cable, and they can be calculated by the method described in [10] and [6]. An alternative interpretation of $\underline{J}$ and $\underline{J}^*$ is to use the tangential electric and magnetic fields in the aperture:

$$\underline{J} = -\underline{n}\times\underline{H}_a\delta(\rho - a) \qquad \underline{J}^* = \underline{n}\times\underline{E}_a\delta(\rho - a) \tag{18}$$

where $\underline{n}$ is the outward unit normal to the aperture and equal to $\underline{e}_\rho$ in Figure 2, ~~$\underline{r}_a$ is the position vector describing the aperture~~. Actually, one only needs the magnetic current density $\underline{J}^*$ to describe the effect of an aperture discontinuity in a waveguide. This can be easily seen if one recalls that the tangential aperture electric field is all that is required for the determination of fields everywhere. Thus, the source terms given by equations (10) reduce to

$$v_n(z) = \iint_S \underline{J}_t^*\cdot\underline{h}_n dxdy$$

$$i_n(z) = -Y_n \iint_S J_z^* h_{nz} dxdy \tag{19}$$

For the TEM mode, $h_{oz} \equiv 0$ and hence $i_o \equiv 0$. Consequently, the source term for the second equation of (13) is also zero and equations (13) become

$$\frac{dV}{dz} = i\omega LI + \frac{1}{2\pi}\int_o^{2\pi} K_\phi^* d\phi$$

$$\frac{dI}{dz} = i\omega CV \tag{20}$$

Solving these equations and keeping only the outgoing wave we get

$$I(z) = \frac{1}{2Z_c}\frac{1}{2\pi a}\iint_A e^{ik|z-z'|}K_\phi^* dS', \qquad (dS' = a d\phi' dz')$$

$$= \frac{e^{ikz}}{2Z_c}\frac{1}{2\pi a}\iint_A e^{-ikz'}K_\phi^* dS', \qquad z > z'$$

$$\simeq \frac{e^{ikz}}{2Z_c}\frac{1}{2\pi a}\iint_A (1 - ikz')K_\phi^* dS'$$

$$= -\frac{e^{ikz}}{2}\left[\frac{i\omega\mu}{2\pi a Z_c}\,\underline{m}\cdot\underline{e}_\phi + \frac{i\omega Z_w}{2\pi a Z_c}\,\underline{p}\cdot\underline{e}_\rho\right] \tag{21}$$

where A is the aperture and where we have used the usual definitions:

$$\iint_A \underline{K}^* dS = -i\omega\mu\underline{m} \tag{22a}$$

$$\frac{\varepsilon}{2}\iint_A \underline{r}_s\times\underline{K}^* dS = -p \tag{22b}$$

($\underline{r}_s$ lies in the aperture). Here, care must be exercised to choose the coordinate origin for the definition of $\underline{p}$. From (21) we obtain V:

$$V = \frac{1}{i\omega C}\frac{dI}{dz}$$

$$\simeq -\frac{e^{ikz}}{2}\left[\frac{i\omega\mu}{2\pi a}\,\underline{m}\cdot\underline{e}_\phi + \frac{i\omega Z_w}{2\pi a}\,\underline{p}\cdot\underline{e}_\rho\right] \tag{23}$$

Equations (21) and (23) agree with the solutions of (15). As expected, the tangential aperture electric field alone gives rise to all the source terms in the transmission-line equations (15).

It is not difficult to see from (22) that $\underline{m}$ is related to normal aperture magnetic field and $\underline{p}$ is related to the electrostatic potential, ϕ, in the aperture.

Noting that (∇_s is a surface operator lying in the aperture)

(i) $\nabla_s \cdot \underline{K}^* = i\omega\mu \underline{n} \cdot \underline{H}$

(ii) $\underline{E} = -\nabla\phi$, (static limit)

(iii) $\phi = 0$ at the aperture contour C

(iv) the component of $\underline{K}^*$ tangent to C = 0

one can easily show from the definitions (22) that

$$\underline{m} = \frac{1}{-i\omega\mu} \iint_A \underline{K}^* dS = \frac{1}{i\omega\mu} \iint_A \underline{r}_s \nabla_s \cdot \underline{K}^* dS$$

$$= \iint_A \underline{r}_s \underline{n} \cdot \underline{H} dS \tag{24}$$

$$\underline{p} = -\frac{\varepsilon}{2} \iint_A \underline{r}_s \times \underline{K}^* dS = \frac{\varepsilon}{2} \iint_A \underline{r}_s \times (\underline{n} \times \nabla_s \phi) dS$$

$$= \frac{\varepsilon}{2} \underline{n} \iint_A \underline{r}_s \cdot \nabla_s \phi dS = -\frac{\varepsilon}{2} \underline{n} \iint_A \phi \nabla_s \cdot \underline{r}_s dS$$

$$= -\varepsilon \underline{n} \iint_A \phi dS \tag{25}$$

Thus, in the low-frequency approximation $\underline{m}$ is proportional to the normal component of the static aperture magnetic field, whereas $\underline{p}$ is proportional to the static aperture potential in agreement with [6].

IV. Lumped Circuit Elements of an Aperture

In this section we will obtain a lumped network representation for a small aperture in the shield of a coaxial cable and give explicit calculations for the case where the aperture is a long slit. In the next section we will extend our calculations to a braided-shield cable.

The lumped network elements will be expressed in terms of the electric polarizability, α_e, and the dyadic magnetic polarizability, $\underline{\underline{\alpha}}_m$, of the aperture, where α_e and $\underline{\underline{\alpha}}_m$ have been computed and measured for various aperture shapes [1], [6], [11]. These network elements can be easily found from equations (15). Following customary procedures we define

$$\underline{p} = \varepsilon\alpha_e \underline{E}_o$$

$$\underline{m} = \underline{\underline{\alpha}}_m \cdot \underline{H}_o \qquad (26)$$

where $\underline{E}_o$ and $\underline{H}_o$ are the fields of the TEM mode at the position of the aperture when the aperture is closed and they are given by equations (A.14). Here and henceforth, one should keep in mind that in the "illuminated" side of the screen $\underline{p}$ is always anti-parallel to the electric field of the incident wave at the aperture, whereas in the "shadow" side of the screen $\underline{p}$ is always parallel to the electric field of the incident wave at the aperture. The corresponding relationship between the component of $\underline{m}$ along the incident magnetic field and the incident magnetic field itself is just opposite [12].

Now substituting (2b) and (A.14) into (15) we get

$$\frac{dV}{dz} = i\omega[L + L_a\delta(z - z_\alpha)]I$$

$$\frac{dI}{dz} = i\omega[C - C_a\delta(z - z_\alpha)]V \qquad (27)$$

where

$$L_a = \frac{\mu\alpha_m}{4\pi^2 a^2}$$

$$C_a = \frac{\varepsilon\alpha_e Z_w^2}{4\pi^2 a^2 Z_c^2} \qquad (28)$$

and α_m denotes the $\phi\phi$-component of $\underline{\underline{\alpha}}_m$. The equivalent networks of the transmission line and the aperture are shown in Figures 3a and 3b. Figure 3b is a symmetric "Tee" network for the aperture and is equivalent to Figure 3a when terms of the order of $\omega^2 L_a C_a$ and higher are neglected compared to unity. Equations (27) show that a longitudinal aperture can be represented by a positive series inductance and a negative shunt capacitance, as one would expect on physical grounds.

As an example we now apply (27) to calculating the characteristic impedance of a coax with a narrow, infinitely long slit in the outer conductor (Figure 4). First, we find α_e from (25) with the static potential distribution ϕ in the aperture given in [13]:

$$p = -\varepsilon \int_{-d}^{d} \phi(x)dx = \varepsilon E_o \int_o^d \sqrt{d^2 - x^2}\, dx = \frac{\varepsilon E_o d^2 \pi}{4}$$

from which

$$\alpha_e = \frac{\pi d^2}{4} .$$

Substitution into (28) gives

$$C_a = \frac{\varepsilon}{\pi} \left(\frac{dZ_w}{4aZ_c}\right)^2 \text{ farads/meter} \qquad (29)$$

Since $\alpha_m = \alpha_e$ for an infinite slit, we have from (28)

$$L_a = \frac{\mu}{\pi} \left(\frac{d}{4a}\right)^2 \text{ henries/meter} \qquad (30)$$

The relative changes in capacitance and inductance are

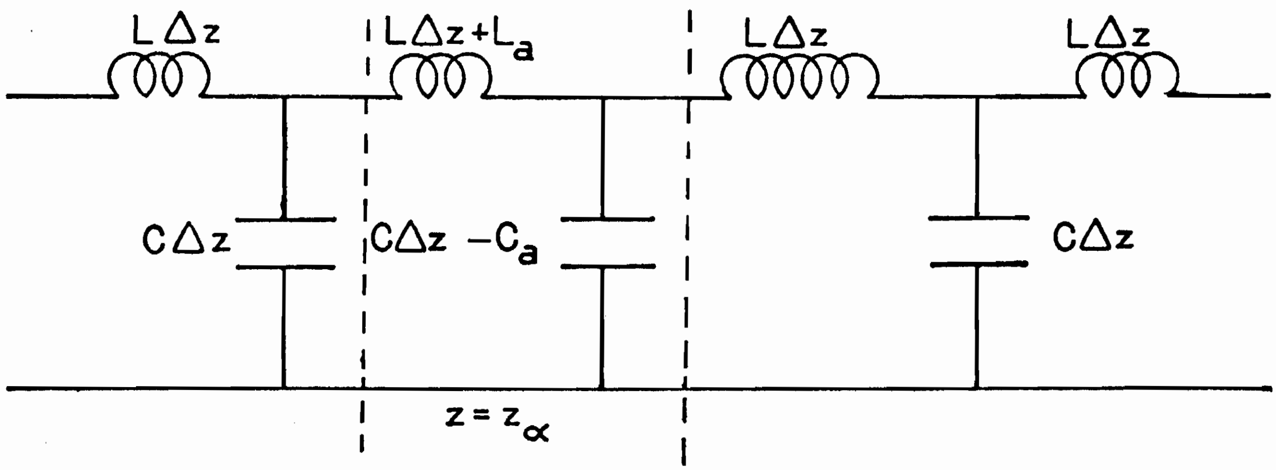


Figure 3a. Equivalent network of an aperture imbedded in the network of a coax.

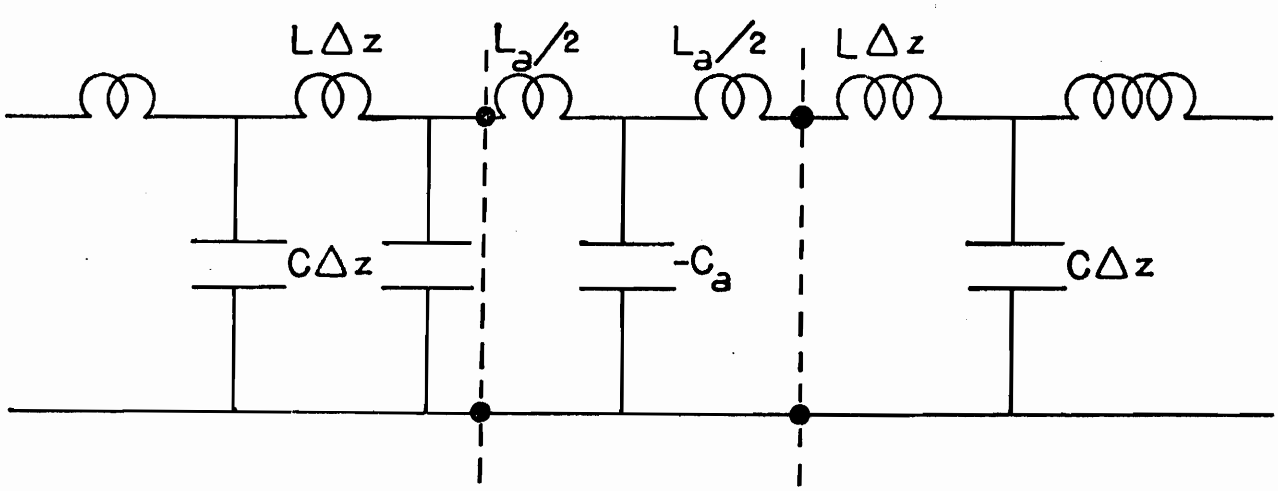


Figure 3b. Equivalent network of an aperture in a coax.

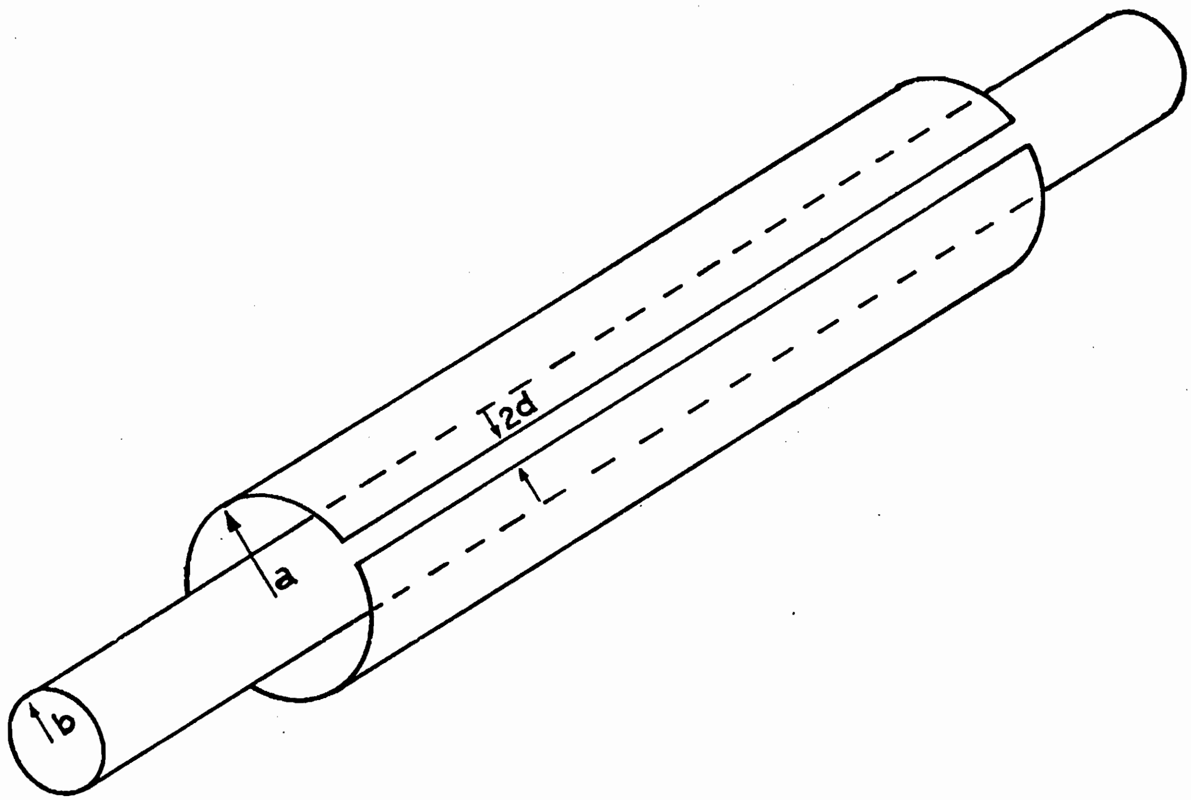


Figure 4. A narrow, infinitely long slit in the shield of a coaxial cable.

$$\frac{\Delta C}{C} = -\frac{C_a}{C} = -\frac{\ln(a/b)}{2\pi^2}\left(\frac{d}{4a}\right)^2 \frac{Z_w^2}{Z_c^2} = -\frac{1}{\pi}\left(\frac{d}{4a}\right)^2 \frac{Z_w}{Z_c}$$

$$\frac{\Delta L}{L} = \frac{L_a}{L} = \frac{2}{\ln(a/b)}\left(\frac{d}{4a}\right)^2 = \frac{1}{\pi}\left(\frac{d}{4a}\right)^2 \frac{Z_w}{Z_c} \tag{31}$$

and the characteristic impedance is increased by

$$\frac{\Delta Z_c}{Z_c} = \frac{\Delta L}{L} = -\frac{\Delta C}{C} = \frac{1}{\pi}\left(\frac{d}{4a}\right)^2 \frac{Z_w}{Z_c} \tag{32}$$

Equations (29)-(32) agree with those given by Kaden [14]. For N slits, $\alpha_e(N,d)$ can be obtained by either conformal mapping [6] or the integral equation method [15], and α_m is still numerically equal to α_e. Then the relative increase in Z_c is simply given by

$$\frac{\Delta Z_c}{Z_c} = \frac{\Delta L}{L} = -\frac{\Delta C}{C} = \frac{N\alpha_e(N,d)}{4\pi^2 a^2} \cdot \frac{Z_w}{Z_c} \tag{33}$$

V. Braided-Shield Cables

Before considering the electromagnetic interaction of a braided-shield cable with an external source we return momentarily to equations (15) which apply to a coax with a small aperture at $z = z_\alpha$ in the outer conductor. As has been pointed out before, $\underline{p}$ is proportional to the normal components of the interior ($\underline{E}^{int}$) and exterior ($\underline{E}^{ext}$) electric fields and $\underline{m}$ is proportional to the tangential components of the interior ($\underline{H}^{int}$) and exterior ($\underline{H}^{ext}$) magnetic fields. These field components are evaluated at the position of the aperture when the aperture is closed. Thus, when the coax is immersed in an external field we have

$$\underline{p} = (\epsilon\alpha_e\underline{E})^{int} + (\epsilon\alpha_e\underline{E})^{ext}$$

$$\underline{m} = (\underline{\underline{\alpha}}_m\cdot\underline{H})^{int} + (\underline{\underline{\alpha}}_m\cdot\underline{H})^{ext} \tag{34}$$

where $(\epsilon\alpha_e\underline{E})^{int} \equiv \epsilon^{int}\alpha_e^{int}\underline{E}^{int}$, etc. If the wall of the outer conductor is infinitely thin, then

$$\underline{\underline{\alpha}}_m^{int} = \underline{\underline{\alpha}}_m^{ext} = \underline{\underline{\alpha}}_m \tag{35a}$$

$$(\epsilon\alpha_e)^{int} = (\epsilon\alpha_e)^{ext} = \frac{2\epsilon_r}{\epsilon_r+1}\,\epsilon^{int}\alpha_e \tag{35b}$$

where $\epsilon_r = \epsilon^{ext}/\epsilon^{int}$ and α_e is for the case where ϵ^{int} is set equal to ϵ^{ext}. Equation (35b) can be easily deduced by considering the electrostatic problem of a perfectly conducting sheet with a small aperture and sandwiched between two half-space dielectrics with different dielectric constants. For simplicity, we will consider the case $\epsilon^{int} = \epsilon^{ext} = \epsilon$. Equations (34) can be rewritten as

$$\underline{p} = \epsilon\alpha_e(\underline{E}^{int} + \underline{E}^{ext})$$

$$\underline{m} = \underline{\underline{\alpha}}_m\cdot(\underline{H}^{int} + \underline{H}^{ext}) \tag{36}$$

Of Course, $\underline{E}^{int}$ and $\underline{H}^{int}$ are just $\underline{E}_o$ and $\underline{H}_o$, the fields of the TEM mode as used in (26). Substituting (36) into (15) and using Figure 5 for the definition of the signs of various quantities we obtain

$$\frac{dV}{dz} = i\omega L[1 + \Delta_L \delta(z - z_\alpha)]I + V_{eq}\delta(z - z_\alpha)$$

$$\frac{dI}{dz} = i\omega C[1 - \Delta_C \delta(z - z_\alpha)]V + I_{eq}\delta(z - z_\alpha) \tag{37}$$

where

$$\Delta_L = \frac{L_a}{L}, \qquad \Delta_C = \frac{C_a}{C}$$

$$V_{eq} = \frac{i\omega\mu\alpha_m}{2\pi a} H_\phi^{ext} = i\omega L_a I^{ext} \tag{38}$$

$$I_{eq} = -\frac{i\omega\epsilon\alpha_e Z_w}{2\pi a Z_c} E_\rho^{ext} = -i\omega(C_a/C)Q^{ext}$$

and L_a and C_a are given by (28). The total axial current I^{ext} and the total linear charge density Q^{ext} on the outside wall of the outer conductor satisfy the continuity equation and are obtained when all the aperture is short-circuited. Equations (37) are rederived in Appendix B from the coupled transmission-line analysis which utilizes the results given in [7] as the starting point.

We will now generalize (37) to a braided-wire shield cable whose physical configuration is shown in Figures 6a and 6b. The braided-wire shield is to be modeled by a perfectly conducting thin shell with many small holes in it. We will first assume that the distribution of the holes is periodic along the line with period d and then we will generalize the result to the case where there are, on the average, n holes per unit length along the line. We now have a series of delta functions in (37) with $z_{\alpha+1} - z_\alpha = z_\alpha - z_{\alpha-1} = d$, etc. Thus,

$$\sum_\alpha \delta(z - z_\alpha) = \sum_{\alpha=-\infty}^{\infty} \delta(z - \alpha d)$$

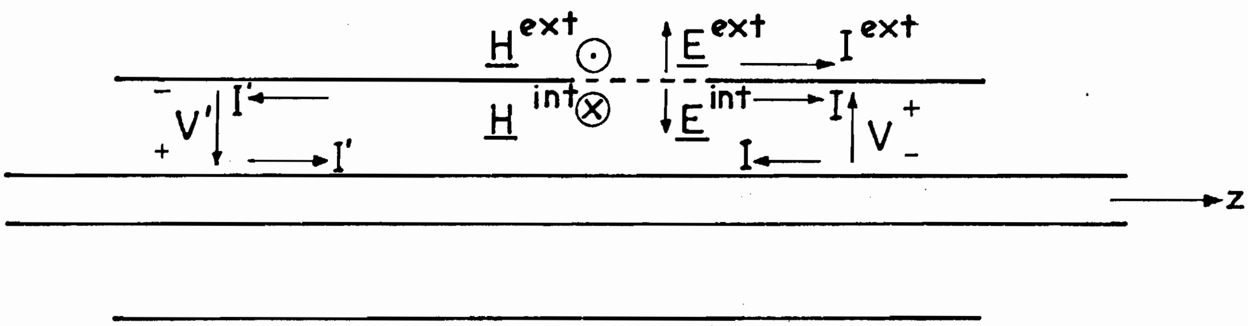


Figure 5. Definition of signs of various quantities (V' and I' are used in [6]).

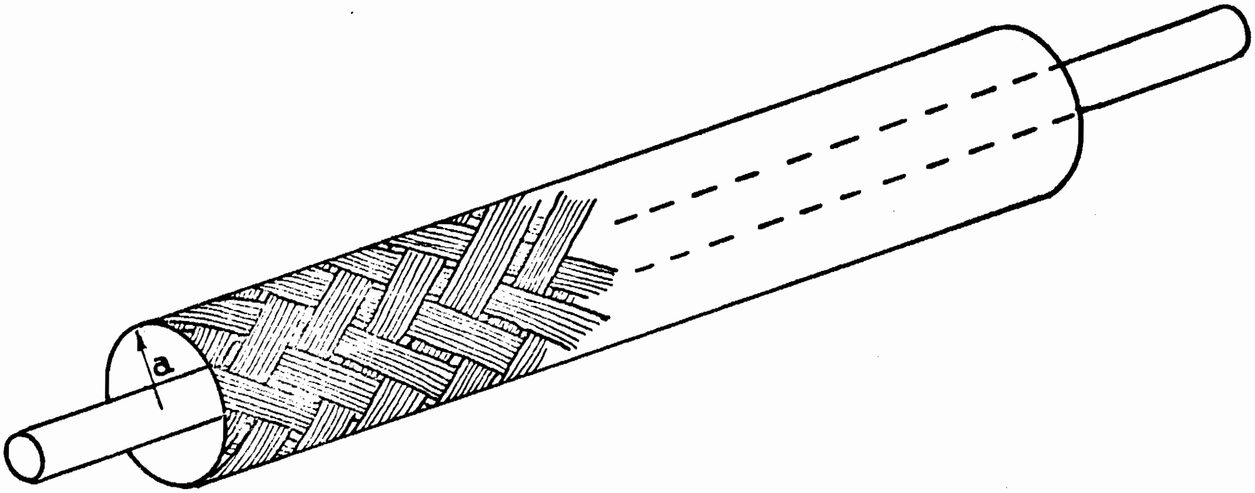


Figure 6a. Physical configuration of a braided-wire shield cable.

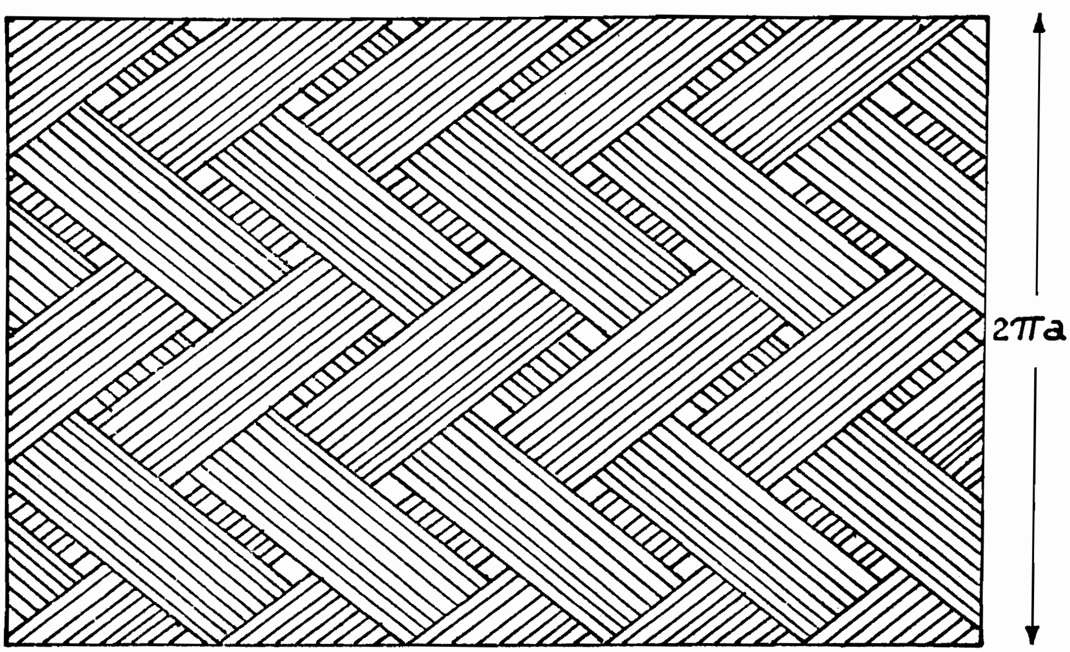


Figure 6b. Braid pattern developed on a plane [4].

The sum can be transformed into a cosine series by either the Poisson summation formula or expanding $\delta(z)$ in a cosine Fourier series in the interval $-d/2$ to $d/2$. Hence,

$$\sum_{\alpha=-\infty}^{\infty} \delta(z - \alpha d) = \frac{1}{d} + \frac{2}{d} \sum_{k=1}^{\infty} \cos\left(\frac{2k\pi z}{d}\right) \tag{39}$$

Since our interest is mainly in the effects of the apertures on the TEM mode, it is permissible to retain only the average-value term $1/d$ in (39). In this connection one can refer to Appendix C for proof by means of Floquet's theorem. For one aperture per period d equations (37) become

$$\frac{dV}{dz} = i\omega L(1 + \Delta_L/d)I + i\omega(L_a/d)I^{ext}$$

$$\frac{dI}{dz} = i\omega C(1 - \Delta_C/d)V - i\omega C_a/(dC)Q^{ext} \tag{40}$$

If there are n apertures per unit length along the line we can, to the same accuracy as in deriving (40), simply replace $1/d$ in (40) by n. From the practical viewpoint it is more convenient to express n in terms of the optical coverage ν of the shield (the case $\nu = 1$ means that there are no apertures). Elementary consideration immediately gives

$$n = (1 - \nu)\frac{2\pi a}{A_{av}} \tag{41}$$

where A_{av} is the average area of an aperture. Finally, we have, for n apertures per unit length,

$$\frac{dV}{dz} = i\omega L(1 + \delta_m)I + i\omega L\delta_m I^{ext}$$

$$\frac{dI}{dz} = i\omega C(1 - \delta_e)V - i\omega\delta_e Q^{ext} \tag{42}$$

where the magnetic (inductive) coupling parameter δ_m and the electric (capacitive) coupling parameter δ_e are defined by

$$\delta_m = \frac{n\alpha_m Z_w}{4\pi^2 a^2 Z_c} \qquad (43)$$

$$\delta_e = (\alpha_e/\alpha_m)\delta_m$$

Except for the sign difference in the source terms which arise from using different sign convention for I and V (see Figure 5), equation (42) are the same as those derived in [6] where L_s and S_s are used as the coupling parameters and are related to the dimensionless parameters δ_m and δ_e as follows:

$$L_s = -L\delta_m \qquad (44)$$

$$S_s = -\delta_e/C$$

Tables and curves pertinent to L_s and S_s are also given in that reference for a braided shield modeled by a perfectly conducting shell with periodic distribution of diamond-shaped apertures.

Although we have been successful in characterizing, under certain reasonable assumptions, a braided-shield cable by only two important coupling parameters δ_m and δ_e, theoretical determinations of these parameters are by no means easy for a real braided-shield cable. As can be seen in [6], to make the calculations of these parameters tractable one has to assume, among other things, that the apertures are of the same shape and that the shield is planar. Therefore, it would be interesting to see how these parameters can be determined experimentally. Figure 7a is the schematic sketch of an experiment for determining δ_m. The cable under test is electrically very short and hence there is no significant voltage drop along the entire length of the cable. From the

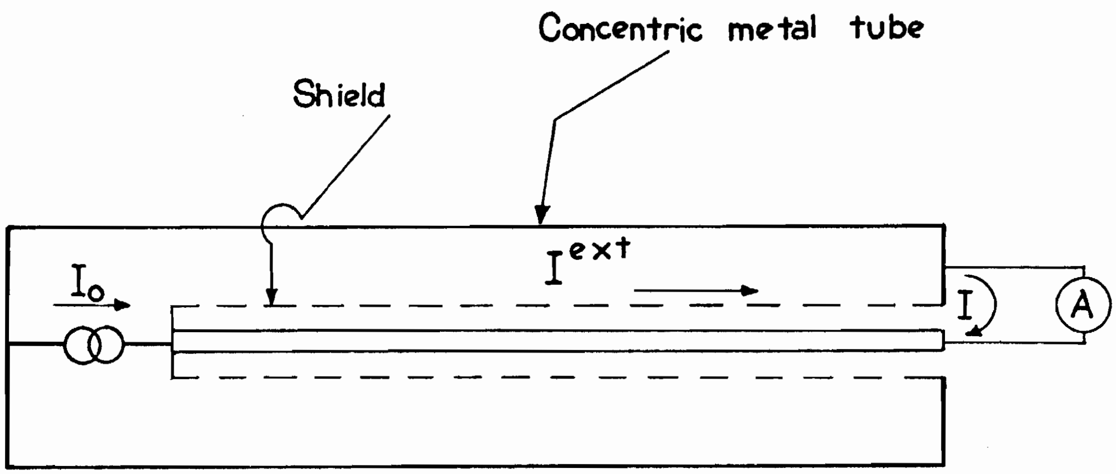


Figure 7a. Schematic diagram for determining δ_m experimentally.

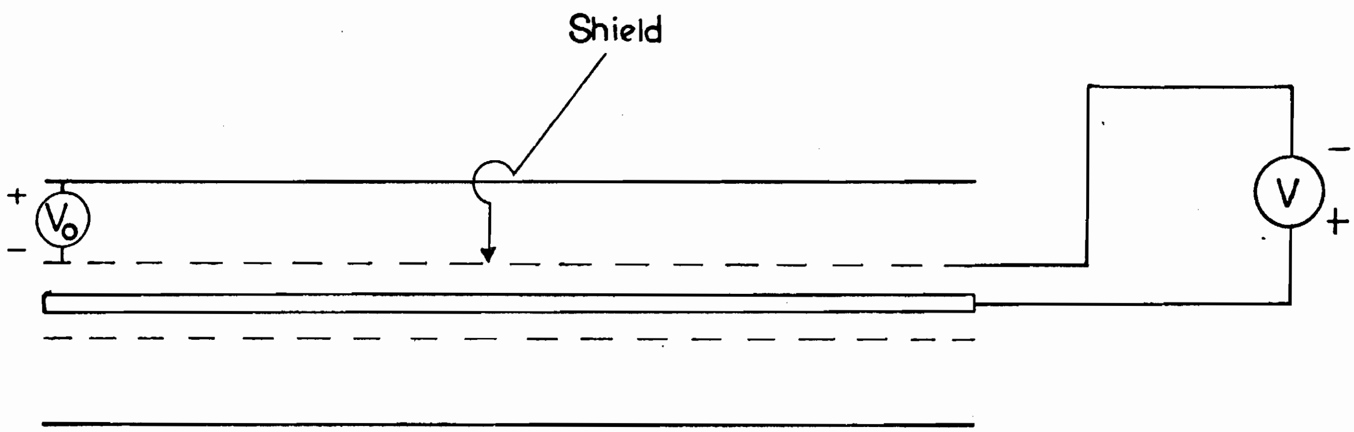


Figure 7b. Schematic diagram for determining δ_e experimentally.

first equation of (42) one can easily get

$$\delta_m = \frac{(I/I_o)}{1-(I/I_o)} \tag{45}$$

since $I^{ext} = I_o$ as shown in Figure 7a. From the measurements of I/I_o, I_o being the current driving the shield, one can obtain δ_m from relation (45). Figure 7b is the schematic sketch of an experiment for determining δ_e. Note that Figure 7b is the dual of Figure 7a. The relation between δ_e and the measured value of V/V_o can easily be obtained from the second equation of (42) by noting that $Q^{ext} = C_o V_o$:

$$\delta_e = \frac{(V/V_o)}{(C_o/C)-(V/V_o)} \tag{46}$$

where V_o is the voltage driving the shield and an outer conductor coaxial with the shield, and C_o is the corresponding capacitance when the apertures in the shield are all closed. It would be quite valuable to obtain measured values for some practical braided shields and check the measurements against theoretical calculations reported in [6].

Figure 7 can be thought of as illustrating two elementary tests for magnetic and electric coupling through a braided shield as long as the radian wavelength is large compared to the sample length. There are many sophistications that one can incorporate in such test fixtures.

Figure 8 shows a test configuration for determining both δ_e and δ_m. For a given cable length to be tested the balanced drive scheme gives better separation between charge per unit length and current. There is a common signal generator (or two separate ones if preferred) used to drive the test fixture in the two separate test modes by means of switching some connections. Note that this source should be designed for balanced output, at least for driving in the current mode. At either or both ends of the test cable the current and/or voltage driving the cable shield and that resulting inside the cable can be monitored. When driving the cable shield in the current mode the voltage (and thus charge) on the cable shield with respect to the outer shield can be observed to be small. When driving in the charge mode the current on the cable shield can be observed to be small. Note that both ends

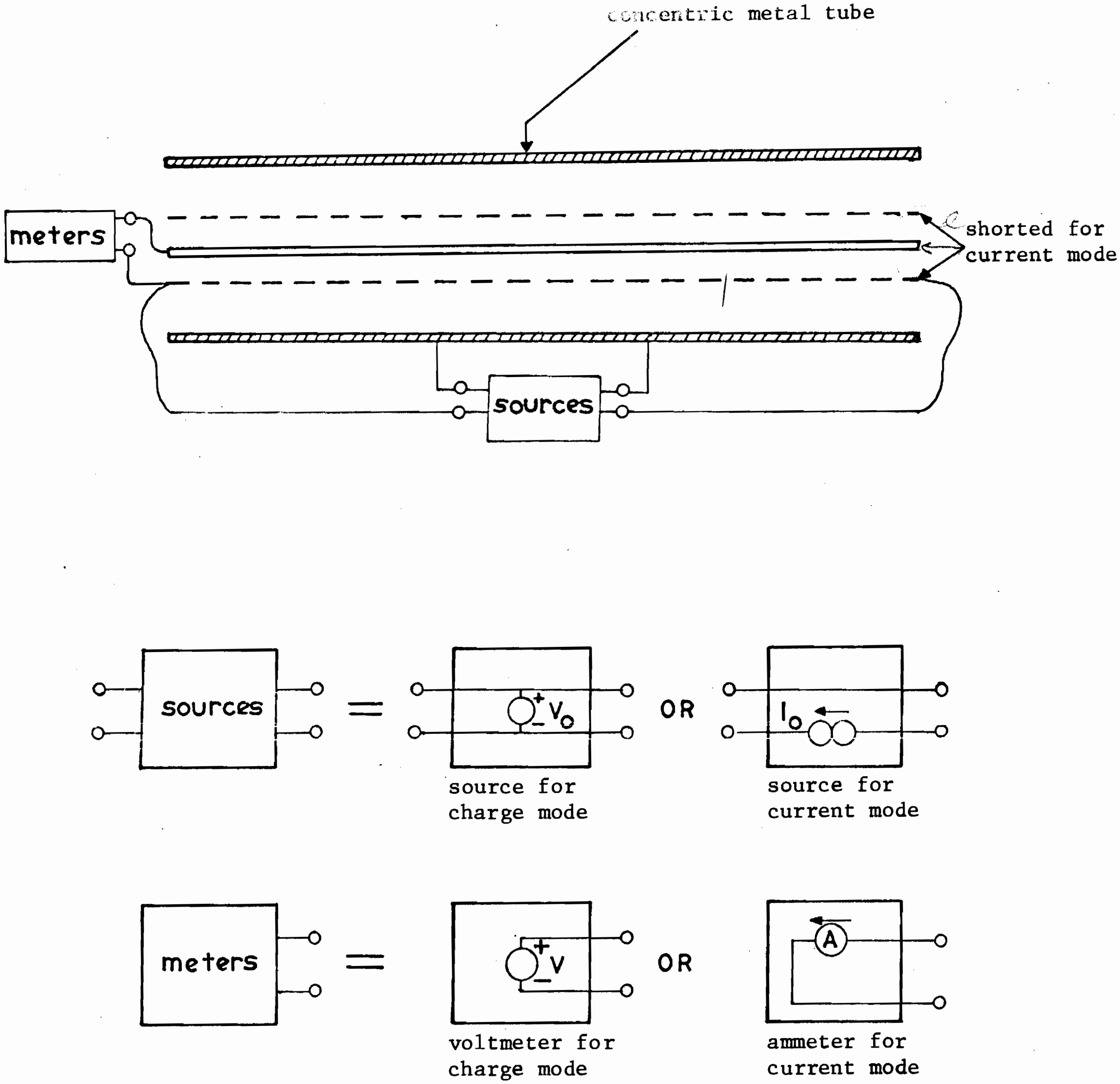


Figure 8. Schematic diagram for determining both δ_e and δ_m experimentally.

of the center conductor of the coax under test should be open circuited for the δ_e measurement and both ends should be short circuited for the δ_m measurement.

There are other variations on this general configuration. For example one might ~~measure~~ connect the source current and voltage at the center of the cable shield. Similarly one might measure the resulting internal current and voltage at the center of the test cable. There are various design details (such as the design for shielding the ends of the test cable) that are beyond the scope of this note. Perhaps some future notes can consider the detailed design of such test fixtures for testing cable shielding. Note that for the present discussion we are only considering aperture coupling through the cable shield. There are other effects associated with finite but nonzero conductivities of the cable components that change the form of the results somewhat.

Appendix A

Some Properties of Cylindrical Waveguide Modes

We will list, for the purpose of easy reference, the properties of the vector mode functions, $\underline{e}_n$ and $\underline{h}_n$, that are needed for the derivations of the formulas in the text. For an exhaustive study of these modes one can refer to References [7] and [16].

For a TM mode one has,

$$\underline{e}'_n = -\nabla_t \Phi_n(x,y)$$

$$\underline{h}'_n = \underline{e}_z \times \underline{e}'_n \tag{A.1}$$

where Φ_n satisfies, in the waveguide cross section S,

$$(\nabla_t^2 + k'^2_{cn})\Phi_n = 0 \tag{A.2}$$

subject to the boundary condition

$$\Phi_n = 0 \tag{A.3}$$

on the periphery of S.

For a TE mode one has

$$\underline{h}''_n = -\nabla_t \psi_n(x,y)$$

$$\underline{e}''_n = \underline{h}''_n \times \underline{e}_z \tag{A.4}$$

where ψ_n satisfies

$$(\nabla_t^2 + k''^2_{cn})\psi_n = 0, \qquad \text{in S} \tag{A.5}$$

subject to the boundary condition that the normal derivative

$$\frac{\partial \psi_n}{\partial \nu} = 0 \tag{A.6}$$

on the periphery of S, $\underline{\nu}$ being normal to the periphery of S. The vector mode functions are orthonormal in the sense that

$$\iint_S \underline{e}'_m \cdot \underline{e}'_n dS = \iint_S \underline{e}''_m \cdot \underline{e}''_n dS = \delta_{mn}$$

$$\iint_S \underline{e}'_m \cdot \underline{e}''_n dS = 0 \tag{A.7}$$

where δ_{mn} is the Kronecker delta. The eigenvalues k'_{cn} and k''_{cn} are usually called the modal cutoff wave numbers and depend solely on the geometry of the cross section of the waveguide.

To derive equation (7) from equation (5) in Section II we need several relations which will now be worked out. For $\underline{E}'_{tn} = V'_n(z)\underline{e}'_n(x,y)$ and $\underline{H}'_{tn} = I'_n(z)\underline{h}'_n(x,y)$, we have

$$\nabla_t \nabla_t \cdot (\underline{H}'_{tn} \times \underline{e}_z) = I'_n \nabla_t \nabla_t \cdot (\underline{h}'_n \times \underline{e}_z) = I'_n \nabla_t \nabla_t \cdot \underline{e}'_n$$

$$= -I'_n \nabla_t \nabla_t^2 \Phi_n = I'_n k'^2_{cn} \nabla_t \Phi_n = -I'_n k'^2_{cn} \underline{e}'_n \tag{A.8}$$

$$\nabla_t \nabla_t \cdot (\underline{e}_z \times \underline{E}'_{tn}) = V'_n \nabla_t \nabla_t \cdot (\underline{e}_z \times \underline{e}'_n)$$

$$= -V'_n \nabla_t \nabla_t \cdot (\underline{e}_z \times \nabla_t \Phi_n) = 0 \tag{A.9}$$

For $\underline{E}''_{tn} = V''_n(z)\underline{e}''_n(x,y)$ and $\underline{H}''_{tn} = I''_n(z)\underline{h}''_n(x,y)$, we have

$$\nabla_t \nabla_t \cdot (\underline{H}''_{tn} \times \underline{e}_z) = I''_n \nabla_t \nabla_t \cdot (\underline{h}''_n \times \underline{e}_z)$$

$$= -I''_n \nabla_t \nabla_t \cdot (\nabla_t \psi_n \times \underline{e}_z) = 0 \qquad \text{(A.10)}$$

$$\nabla_t \nabla_t \cdot (\underline{E}''_{tn} \times \underline{e}_z) = V''_n \nabla_t \nabla_t \cdot (\underline{e}_z \times \underline{e}''_n) = V''_n \nabla_t \nabla_t \cdot \underline{h}''_n$$

$$= -V''_n \nabla_t \nabla_t^2 \psi_n = V''_n k''^2_{cn} \nabla_t \psi_n = -V''_n k''^2_{cn} \underline{h}''_n \qquad \text{(A.11)}$$

In the text we deal exclusively with the dominant mode of a coaxial waveguide, i.e., the TEM mode. For this mode we have [7]

$$\underline{E}_{to} = V_o \underline{e}_o = -\underline{e}_\rho \frac{V_o}{2\pi\rho\sqrt{N}}$$

$$\text{(A.12)}$$

$$\underline{H}_{to} = I_o \underline{h}_o = -\underline{e}_\phi \frac{I_o}{2\pi\rho\sqrt{N}}$$

where

$$N = \frac{1}{2\pi} \ln\left(\frac{a}{b}\right)$$

for a coaxial line with inner radius b and outer radius a. Instead of the mode voltage V_o and mode current I_o one is more interested in the line voltage V and line current I in the transmission-line equations (7). To do this one simply makes the transformation

$$V_o \rightarrow \frac{V}{\sqrt{N}}$$

$$I_o \rightarrow \sqrt{N}\, I \qquad \text{(A.13)}$$

$$Z_o \rightarrow \frac{Z_c}{N}$$

in equation (7) for n = 0, where Z_c is the characteristic impedance of the line. The transformation (A.13) indeed preserves the power definition

$$\mathrm{Re}(VI^*) = \mathrm{Re}(V_o I_o^*)$$

With this transformation (A.12) becomes

$$\underline{E}_{to} = -\underline{e}_\rho \frac{V}{\rho \ln(a/b)}$$

$$\underline{H}_{to} = -\underline{e}_\phi \frac{I}{2\pi\rho} \qquad \text{(A.14)}$$

which clearly show that V and I have the usual meaning of voltage and current in the transmission-line theory.

Appendix B
Coupled Transmission-Line Equations

In this appendix we wish to rederive the basic equations (37) obtained from modal analysis considerations from the coupled transmission-line analysis which utilizes the results in [7] as the starting point. The notations that will be used in the following are the same as those of Marcuvitz [7], Vance [4], and Dairiki [17]. They all use the time convention $e^{j\omega t}$.

Consider a section of length d of two concentric coaxial guides coupled by a small aperture in the common wall of zero thickness (see Fig. B1, which is taken from [7]). The elements of the equivalent circuit of the aperture are given by

$$\frac{Y_o'}{Y_o} = \frac{\ln(R_2/R_1)}{\ln(R_3/R_1)}$$

$$\frac{B_a}{Y_o} = \frac{\omega P}{4\pi c R_2^2 \ln(R_2/R_1)}\left(1 + \frac{Y_o'}{Y_o}\right) \tag{B.1}$$

$$\frac{B_a}{Y_o} = \frac{2\pi c R_2^2 \ln(R_2/R_1)}{\omega M} \tag{B.2}$$

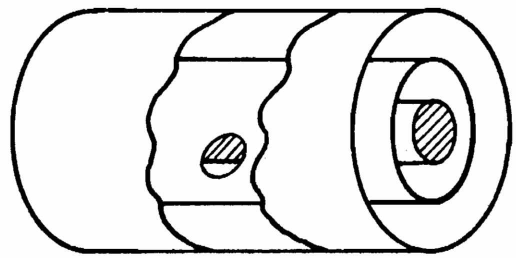
$$\frac{B_c}{Y_o} = \frac{\omega P}{2\pi c R_2^2 \ln(R_3/R_2)} \tag{B.3}$$

$$\frac{B_d}{Y_o} = \frac{B_a}{Y_o}\frac{Y_o'}{Y_o} \tag{B.4}$$

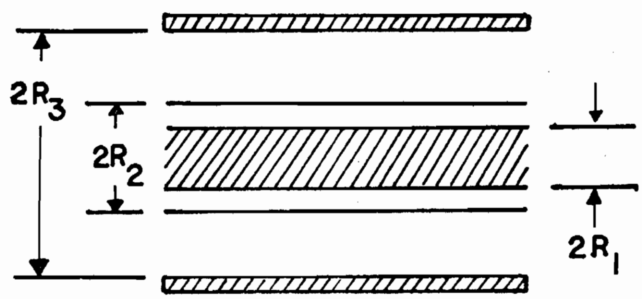
where

c = propagation speed of the TEM modes in both coaxial guides

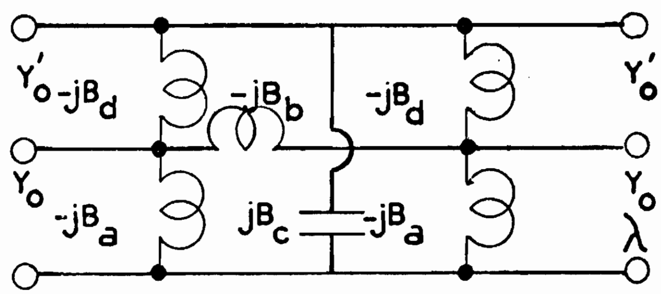
P = "effective" electric polarizability = α_e



(a) general view



(b) longitudinal view



(c) equivalent circuit

Figure B1. Aperture in double concentric coaxial line.

M = "effective" magnetic polarizability = α_m

Y_o = characteristic admittance of inner coax

Y'_o = characteristic admittance of outer coax

Let us now imagine the equivalent circuit of the aperture to be embedded in the circuits of two uniform coaxial guides. For simplicity, let there be one aperture per period d, which is assumed to be less than the wavelength but large enough so that the interactions between neighboring apertures are via the TEM mode only. Then, one can easily write down the coupled transmission-line equations from Kirchhoff's laws. Before doing so, we let $R_3/R_2 \to \infty$ while keeping the charge per unit length on the walls of the outer coax constant. Then, equations (B.1) to (B.4) become, as $R_3/R_2 \to \infty$,

$$\frac{B_a}{Y_o} = \frac{\omega P}{4\pi c R_2^2 \ln(R_2/R_1)} \tag{B.1)'$$

$$\frac{B_b}{Y_o} = \frac{2\pi c R_2^2 \ln(R_2/R_1)}{\omega M} \tag{B.2)'$$

$$\frac{B_c}{Y_o} = \frac{\omega P}{2\pi c R_2^2 \ln(R_3/R_2)} \to 0 \tag{B.3)'$$

$$\frac{B_d}{Y_o} = 0 \tag{B.4)'$$

The circuit representation of a section of length d is depicted in Fig. B2. In obtaining Fig. B2 we have discarded terms of order of $(B_a/\omega dC)^2$ so that the two B_a's in Fig. B1 just add.*

*This can be seen by using successive Pi-to-Tee transformations.

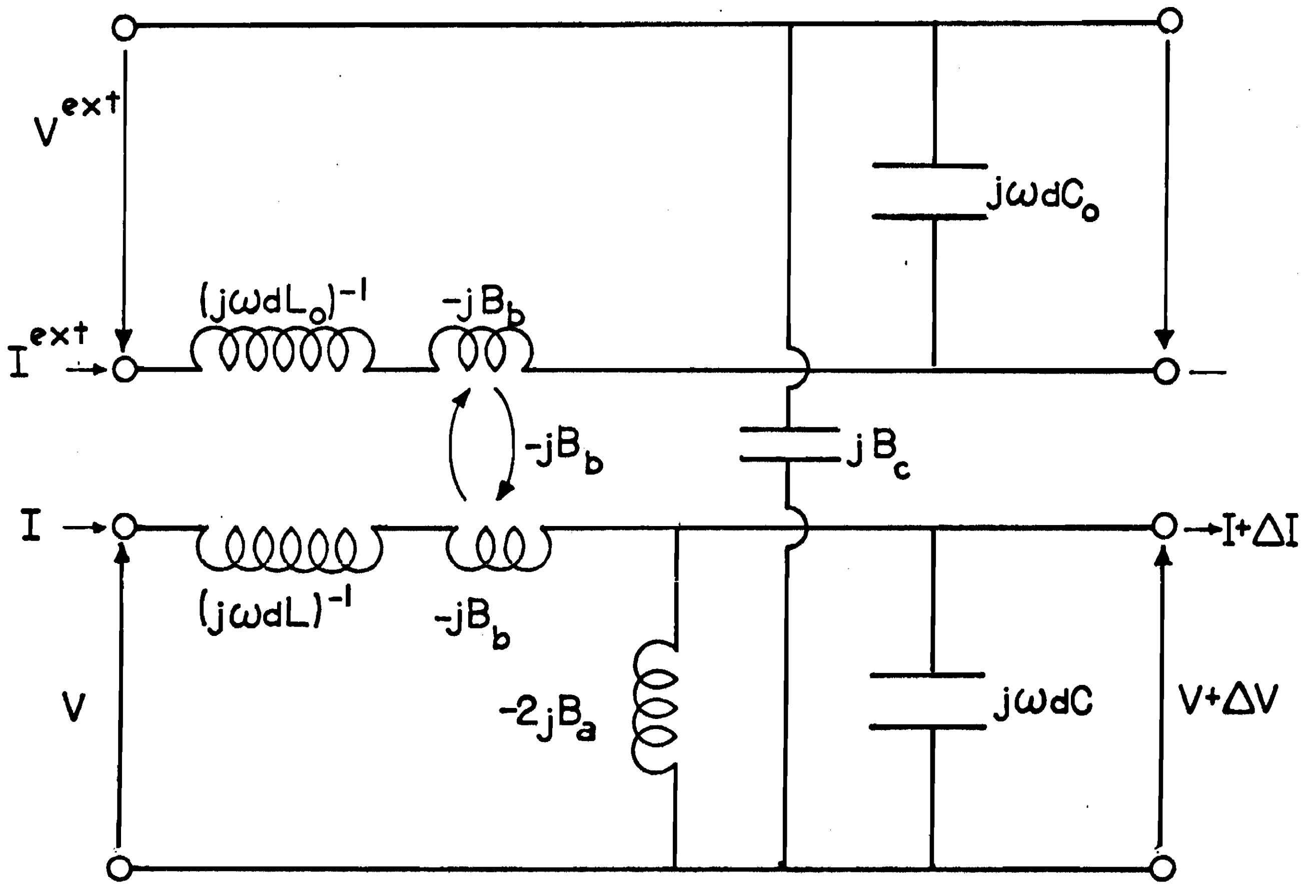


Figure B2. Coupled transmission lines.

Applying Kirchhoff's laws to Fig. B2 we have

$$-\Delta V = j\omega dLI + \frac{I+I^{ext}}{-jB_b} \qquad (B.5)$$

$$-\Delta I = j\omega dCV - 2jB_a V - jB_c V^{ext} \qquad (B.6)$$

Using equations (B.1)' to (B.3)' and letting

$$\frac{\Delta V}{d} = \frac{dV}{dz}$$

$$\frac{\Delta I}{d} = \frac{dI}{dz}$$

$$\frac{2\pi\varepsilon}{\ln(R_3/R_2)} V^{ext} = Q^{ext}$$

we have

$$\frac{dV}{dz} = -j\omega L\left(1 + \frac{MZ_w}{4\pi^2 R_2^2 Z_o d}\right)I - j\omega \frac{MZ_w}{4\pi^2 R_2^2 Z_o d} I^{ext} \qquad (B.7)$$

$$\frac{dI}{dz} = -j\omega C\left(1 - \frac{PZ_w}{4\pi^2 R_2^2 Z_o d}\right)V + j\omega \frac{PZ_w}{4\pi^2 R_2^2 Z_o d} Q^{ext} \qquad (B.8)$$

which are identical to equations (37) for the case where there is one aperture per period d. If there are n apertures per section of length d, one then replaces d^{-1} by n in (B.7) and (B.8) and arrives at equations (42). This replacement is valid only if the interactions of the apertures are predominately via the TEM mode.

Appendix C
Periodic Distributions of Apertures

Let there be one small aperture in the outer conductor of a coax per period d along the line. Let d be such that the aperture - aperture coupling is predominantly via the TEM mode. Hence, we have from equation (37)

$$\frac{dV}{dz} = i\omega L\left[1 + \Delta_L \sum_{\alpha=-\infty}^{\infty} \delta(z - \alpha d)\right] I$$

$$\frac{dI}{dz} = i\omega C\left[1 - \Delta_C \sum_{\alpha=-\infty}^{\infty} \delta(z - \alpha d)\right] V \tag{C.1}$$

Here, we have taken the source terms to be zero and will study the homogeneous equations in the following. Let us restrict our consideration to a wave propagating in the +z direction. According to Floquet's theorem [16] we can write

$$V(z) = \sum_{-\infty}^{\infty} V_n e^{i(\kappa d + 2n\pi)z/d}$$

$$I(z) = \sum_{-\infty}^{\infty} I_n e^{i(\kappa d + 2n\pi)z/d} \tag{C.2}$$

where κ is the propagation constant of the dominant mode and is yet to be determined. The infinite sum of delta functions can be written as, by means of the Poisson summation formula,

$$\sum_{\alpha=-\infty}^{\infty} \delta(z - \alpha d) = \frac{1}{d} \sum_{m=-\infty}^{\infty} e^{i2m\pi z/d} \tag{C.3}$$

Substitution of (C.2) and (C.3) into (C.1) and identifying terms of like exponentials give

$$(\kappa + \frac{2\pi p}{d})V_p = \omega L(1 + \frac{\Delta_L}{d})I_p + \frac{\omega L \Delta_L}{d} \sum_{m\neq 0} I_{p-m}$$

$$(\kappa + \frac{2\pi p}{d})I_p = \omega C(1 - \frac{\Delta_C}{d})V_p - \frac{\omega C \Delta_C}{d} \sum_{m\neq 0} V_{p-m} \tag{C.4}$$

where p = 0, ±1, ±2, $\cdots$. For nontrivial solutions, i.e., $V_p \neq 0$ and $I_p \neq 0$, the determinant of (C.4) has to be zero, which gives an equation for determining κ. If the modes are only weakly coupled among themselves. Then for the dominant mode (p = 0) we have

$$\kappa V_o = \omega L\left(1 + \frac{\Delta_L}{d}\right) I_o$$

$$\kappa I_o = \omega C\left(1 - \frac{\Delta_C}{d}\right) V_o \tag{C.5}$$

from which we obtain

$$\kappa^2 = \omega^2 LC\left(1 + \frac{\Delta_L}{d}\right)\left(1 - \frac{\Delta_C}{d}\right) \tag{C.6}$$

which agrees with the propagation constant governing equations (40).

As is evident from (C.4), the condition that $\Delta_L/d \ll 1$ and $\Delta_C/d \ll 1$ implies that the coupling among modes is weak, and vice versa. Then, one may agrue that Δ_L/d and Δ_C/d in (C.5) should be discarded. However, there are cases where one knows a priori that higher-order modes may not be excited at all or, if excited, can be neglected entirely as far as measurements are concerned. A coax with long slits in its outer conductor and a cable with a good braided shield are such examples. Thus, by retaining Δ_L/d and Δ_C/d in (C.5) and (40) one gets a set of equations that would describe more accurately the behavior of the dominant mode within such a cable.

Acknowledgment

The authors would like to thank Dr. Lennart Marin for several useful and enlightening discussions, and Dr. Phil Castillo for his interest in the work.

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