WL-EMP-IN-130

INTERACTION NOTES
Note 130
23 February 1972

TEM Response of a Multiwire Transmission Line (Cable) to an Externally Impressed Electromagnetic Field: Recipe for Analysis

Sidney Frankel
Sidney Frankel Associates
Menlo Park, Calif. 94025

Abstract

This paper summarizes, in handbook form, the definitions, formulas, and pertinent parameters required to determine the terminal response of a TEM multi-wire line to an arbitrary electromagnetic field impressed continuously along the line, or, in particular, as a space impulse at any point along the line, including its terminals. The line model is based on a set of simplifying assumptions which are explicated. Examples of determination of terminal matrices in terms of physical impedances are discussed. The response of a two-conductor line above ground is analyzed in detail as guidance in handling a typical problem.

transmission lines, calculations, effects of radiation

CONTENTS

Page

Glossary . . . . iii-iv

Assumptions 1

Definitions 1-13

1. The Line 1

2. Coordinate System: Magnetic Field 2

3. Electric Field 5

4. Location of Line in Coordinate System. Schematic Designations 5

5. Voltages and Currents 5

6. Matrices of Impedance, Admittance, Capacitance, Potential, and Inductance Coefficients 8

7. Termination Admittance Matrices 9

8. Determination of $\underline{Y}^d$ 11

9. Impressed-field Coupling Parameters 11

10. Normalized Load Admittance Matrices 12

11. Miscellaneous Notations 13

Computation of Dynamic Quantities 15-18

12. Voltages and Current at any Point Along the Line and at the Terminals 13

12. Special Cases 14

Line and Field Parameters 15-18

14. Definition and Determination of Maxwell's Coefficients of Capacitance and Potential 15

15. Definition and Determination of Inductance Coefficients 17

16. Definition and Determination of Impressed Field Coupling Parameters L_i^e, C_i^e 18

CONTENTS (CONT)

Page

APPENDICES

A. Determination of $\underline{Y}^i$ or $\underline{Y}^o$: Typical Elementary Examples 22

B. Response of a Two-conductor Line Above Ground to a Uniform Transverse Magnetic Field (Plane Wave Incident in the Transverse Plane, Electric Vector Normal to Plane of Incidence) 27

FIGURES

1. Multiwire Line Configurations . . . . 3&4
2. Schematic, N-Line Excited by External Electromagnetic Field . 6
3. N-port Driving-point Admittance 10
4. Model of a Multiwire Line Immersed in Magnetostatic and Electrostatic Fields 19
5. Examples of Termination Networks and Their Corresponding Termination Matrices . . . . 23&24
6. Configuration Illustrating the Termination of Fig 5(g) 26
7. Schematic, Two-wire Line Above Ground (N = 2). 28

REFERENCES 36

ACKNOWLEDGEMENT 37

GLOSSARY

Symbol	Defined in Section
$\underline{C}$	6
$\underline{C}^e$	9
$\underline{E}^e(x)$	9
$E_y^e\ (x)$	3
$\underline{H}^e\ (x)$	9
$H_z^e\ (x)$	2
$\underline{I}\ (x)$, $\underline{I}^i$, $\underline{I}^o$	5
$\underline{I}$	6
$\underline{K}\ (\ell)$	11
$\underline{L}$	6
$\underline{L}^e$	9
N-line	1
$\underline{P}$	6
$\underline{P}^i$, $\underline{P}^o$	10
$\underline{S}$	11
$\underline{U}\ (x)$	11
$\underline{V}\ (x)$, $\underline{V}^i$, $\underline{V}^o$	5
v	6
$W\ (x)$	11
x, y, z	2
$\underline{Y}$	6
$\underline{Y}^i$, $\underline{Y}^o$	7

GLOSSARY (Con't.)

Symbol	Defined in Section
$\underline{Z}$	6
β	11
ε	6
Θ, Θ'	11
μ	6

ASSUMPTIONS

1. All conductors (including shields and ground planes, if any) are lossless, of arbitrary cross-section invariant in the direction of propagation.

2. The dielectric is lossless, homogeneous, and isotropic. (See Appendix B).

3. The impressed fields* are independent of position, in any transverse plane, over the region occupied by the conductor system (including shields and ground planes) (see Appendix B).

4. If a ground plane is present, the impressed magnetic field is parallel to that plane. In any case the magnetic field is assumed to lie in the line's transverse plane.

5. The impressed electric field may have any direction normal to the magnetic field.

Most of these restrictions may be removed at the expense of varying degrees of increased complexity of analysis.

DEFINITIONS

1. The line. The cable (including shields and ground plane) consists of (N + 1) conductors, one of which is taken at reference or "ground" potential, while the potentials of the remaining N conductors are referred

---

* That is, the impressed fields that would exist in the region of the cable, and in the space between cable and ground plane (if any), in the absence of the cable system.

to it. This system of (N + 1) conductors is called an N-line. If a ground plane is present it is usually taken as the reference conductor; if not, then the cable consists of (N + 1) conductors, any one of which may be taken as reference. If the cable has an overall shield continuously connected to ground, that shield is usually taken as reference. If it lacks an overall shield (and there is no ground plane), but one or more of the cable conductors has an individual shield, any such shield may conveniently be taken as reference. If there are no individual shields, and no ground plane is present, any one of the conductors may conveniently be chosen as reference. Figure 1 shows some examples.

When a ground plane is present, the distance between cable and ground may be of importance to the accuracy of numerical calculations. Accuracy or even solvability, may also be affected when the line terminations are such as to reduce the number of independent line currents below the rank (i.e., the N-number) of the line. For instance, for the problem discussed in Appendix B, we have at every point of the line

$$I_1 + I_2 = 0$$

so that although the line is a 2-line, there is only one independent current. Under some conditions this could lead to computational difficulties.

2. Coordinate System: Magnetic Field. In Figure 2, the rectangular

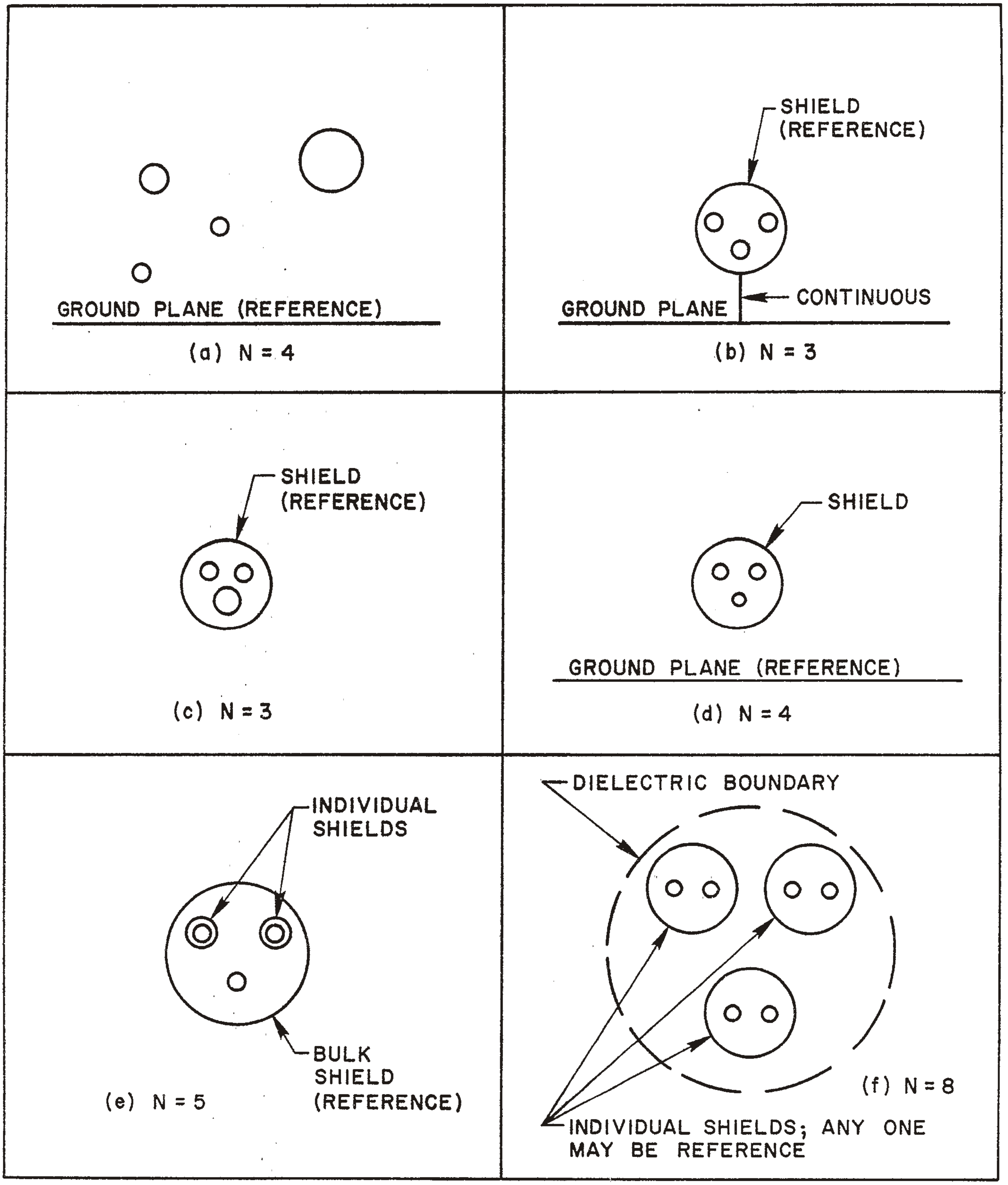

Fig. 1. Multiwire line configurations.

(g) Any conductor may be taken as reference; however, symmetry conditions suggest it may be convenient to use the middle one. N = 2.

(h) Conventional two-wire line far from ground. N = 1

Fig. 1. (Concluded)

coordinate system is chosen as follows:

1. The z-axis is parallel to the impressed magnetic field, designated H_z^e. Thus the z-axis is in the line's transverse plane.

2. The x-axis is parallel to the direction of TEM propagation.

3. The y-axis completes the right-hand co-ordinate system.

3. Electric field. Since the impressed electric field is normal to H_z^e, it has components E_x^e and E_y^e. However, only E_y^e affects the TEM response of the line; E_x^e is involved in the scattering behavior of the line. Consequently Figure 2 displays only the components H_z^e and E_y^e of impressed fields.

4. Location of line in coordinate system. Schematic designations. In Figure 2 the left end of the line is at x = o. Line and termination parameters, and line dynamic quantities associated with this end carry the superscript "i" (for "input"). The right end of the line is at $x = \ell$. Parameters and quantities associated with this end carry the superscript "o" (for output). The upper line segment labelled "N" represents the N conductors which are not at reference potential. The line segment labelled "G" represents the reference conductor.

5. Voltages and currents. Label from 1 to N, in any convenient sequence, the N independent conductors not used as reference. Use subscripts to associate particular quantities with a given conductor. For instance,

V_j^i = input potential, jth conductor

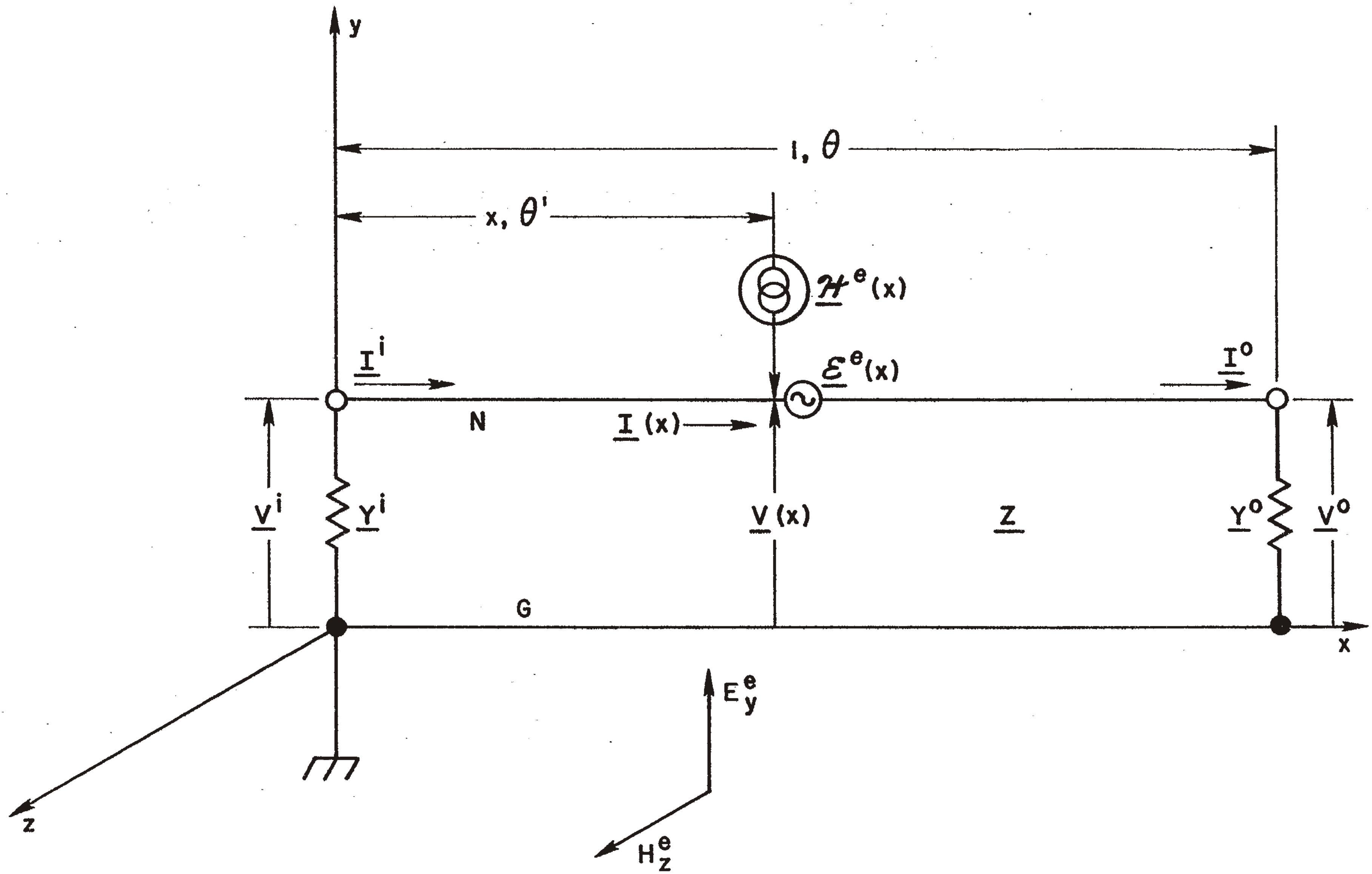

Fig. 2. Schematic, N-line excited by external electromagnetic field.

I_k^o = output current, k<u>th</u> conductor

(Note that the sign convention on currents is positive in the direction of increasing x).

Write

$\underline{V}^i$ = column vector of input potentials

$$= \begin{bmatrix} V_1^i \\ V_2^i \\ \cdot \\ \cdot \\ \cdot \\ V_N^i \end{bmatrix}$$

$\underline{V}^o$ = column vector of output potentials

$$= \begin{bmatrix} V_1^o \\ V_2^o \\ \cdot \\ \cdot \\ \cdot \\ V_N^o \end{bmatrix}$$

$\underline{I}^i$ = column vector of input currents

$$= \begin{bmatrix} I_1^i \\ I_2^i \\ \cdot \\ \cdot \\ \cdot \\ I_N^i \end{bmatrix}$$

$\underline{I}^o$ = column vector of output currents

$$\underline{I}^{o} = \begin{bmatrix} I_1^o \\ I_2^o \\ \cdot \\ \cdot \\ \cdot \\ I_N^o \end{bmatrix}$$

At any point, x, along the line, write

$$\underline{V}(x) = \text{voltage vector at } x$$

$$= \begin{bmatrix} V_1(x) \\ V_2(x) \\ \cdot \\ \cdot \\ \cdot \\ V_N(x) \end{bmatrix}$$

$$\underline{I}(x) = \text{current vector at } x$$

$$= \begin{bmatrix} I_1(x) \\ I_2(x) \\ \cdot \\ \cdot \\ \cdot \\ I_N(x) \end{bmatrix}$$

6. <u>Matrices of impedance, admittance, capacitance, potential, and inductance coefficients</u>. Associated with the conductor system and depending only on the system cross-section geometry and the electromagnetic properties of the dielectric, define the line <u>impedance coefficient matrix</u>

$$\underline{Z} = (Z_{ij}), \; i, j = 1, \ldots, N$$

and the line <u>admittance-coefficient matrix</u>

$$\underline{Y} = \underline{Z}^{-1} = (Y_{ij}), \; i,j = 1,\ldots, N$$

These may be determined electrostatically as follows:

$$\underline{Y} = v\,\underline{C} = v\,[C_{ij}],\ i,j = 1,\ldots,N$$

C_{ij} are the Maxwell's coefficients of capacitance for the line (See Section 14)

$$v = \text{velocity of TEM propagation} = \sqrt{\mu\varepsilon}$$

$$\mu = \text{magnetic permeability, Hy./meter}$$

$$\varepsilon = \text{dielectric permittivity, Fd./meter}$$

$$\underline{Z} = v^{-1}\,\underline{C}^{-1} = v^{-1}\,\underline{P} = v^{-1}\,[p_{ij}],\ i,j = 1,\ldots,N$$

p_{ij} are the Maxwell's coefficients of potential for the line (Section 14).

Although not necessary for assessing the line response, we have, also

$$\underline{Z} = v\,\underline{L} = v\,[L_{ij}],\ i,j = 1,\ldots,N$$

L_{ij} are the line inductance coefficients (Section 15)

Clearly,

$$\underline{Z}\,\underline{Y} = v^2\,\underline{L}\,\underline{C} = I$$

$$I = N \times N \text{ unit matrix}$$

This result yields sufficient independent equations to determine the L_{ij}, given the C_{ij}.*

7. Termination Admittance Matrices. Figure 3 shows an N-port driving-point admittance. The (N + 1) st terminal, normally connected to the reference conductor of the N-line, is indicated by a ground symbol. Suppose a potential, v_k, is applied to the k^{th} terminal, while all other terminals are grounded. Then current flows into each terminal, proportional to v_k. The coefficients of proportionality are elements of the termination matrix provided by connecting the N-port to the line.

---

* provided $\underline{C}$ is non-singular

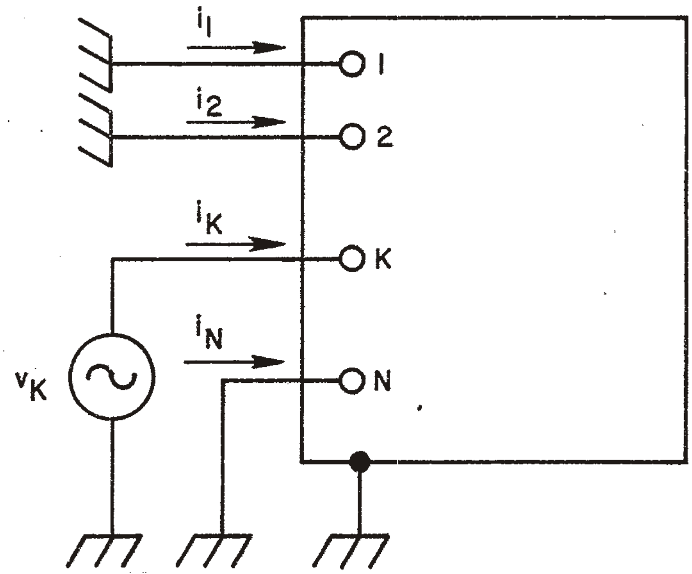

Fig 3. N-port driving-point admittance.

Let $\underline{Y}^d$ represent either $\underline{Y}^i$ or $\underline{Y}^o$: $\underline{Y}^d = [Y^d_{jk}]$, $j,k = 1,\ldots, N$ where Y^d_{jk} is the coefficient of proportionality in

$$i_j = Y^d_{jk}\, v_k \quad (v_i = o,\ i \neq k).$$

By reciprocity,

$$Y^d_{kj} = Y^d_{jk} \qquad , j, k = 1,\ldots, N$$

and by superposition, if the potentials applied to the terminals are arbitrary,

$$i_j = \sum_{k=1}^{N} Y^d_{jk}\, v_k \qquad , j = 1,\ldots, N$$

or, in matrix motation,

$$\underline{i} = \underline{Y}^d\, \underline{v}.$$

8. Determination of $\underline{Y}^d$. The defining procedure for determining the elements of the termination matrices is given in the preceding section, where $\underline{Y}^i$ and $\underline{Y}^o$ are given the generic symbol $\underline{Y}^d$. Appendix A gives some simple examples, including terminations with singular matrices and one termination for which the definition, and therefore the matrix specification, fails. An artifice for circumventing this difficulty is suggested.

9. Impressed-field coupling parameters.

a. Define

$$\underline{E}^e(x) = j\omega\, \underline{L}^e\, H^e_z(x)$$

where $\omega = 2\pi f$. $\underline{L}^e$ is the magnetic field coupling vector*

$$\underline{L}^e = [L^e_i] \quad , i = 1,\ldots, N \quad (\text{Hy.})$$

---

* that is, a column matrix, not to be confused with field vector quantities, such as $H^e_z\, \overline{k}$, where $\overline{k}$ is the z-direction unit vector.

L_i^e is the magnetic field coupling parameter for the $i^{\underline{th}}$ conductor. The quantity $(-L_i^e H_z^e)$ is that portion of the flux per meter of line passing between the $i^{\underline{th}}$ conductor and the reference conductor due to the external field H_z^e. $\underline{E}^e(x)$ has units of electric field intensity (volts/meter), and appears in the line equations as a series voltage-generator vector* per meter of line (1).** Fundamentally, L_i^e is measured by determining the magnetic flux passing between the $i^{\underline{th}}$ conductor and reference when no current flows in any conductor, that is, with all conductors "floating" with respect to reference (See Section 16).

b. Define

$$\underline{H}^e(x) = -j\omega \underline{C}^e E_y^e(x)$$

where $\underline{C}^e$ is the electric field coupling vector*

$$\underline{C}^e = [C_i^e] \quad , i = 1,\ldots, N \text{ (Fd.)}$$

C_i^e is the electric field coupling parameter. The quantity $(-C_i^e E_y^e)$ is that portion of the electric charge per meter bound on the $i^{\underline{th}}$ conductor by the external field E_y^e. $\underline{H}^e(x)$ has units of magnetic field intensity (amp/meter), and appears in the line equations as a shunt-current generator vector* per meter of line (1).** Fundamentally, C_i^e is measured by determining the charge bound on the $i^{\underline{th}}$ conductor when all conductors are at reference potential, i.e. "grounded". (Section 16)

10. Normalized load admittance matrices. The load admittance matrices

---

* see footnote, p. 11.
** cf. Fig. 2

$\underline{Y}^i$ and $\underline{Y}^o$, are normalized by pre-multiplication with the line impedance matrix, $\underline{Z}$:

$$\underline{P}^i = \underline{Z}\,\underline{Y}^i$$

$$\underline{P}^o = \underline{Z}\,\underline{Y}^o$$

11. <u>Miscellaneous Notations.</u>

$$\beta = \frac{\omega}{V}$$

$$\theta' = \beta x$$

$$\theta = \beta \ell$$

$$\underline{S} = (\underline{P}^i + \underline{P}^o)\cos\theta + j(\underline{I} + \underline{P}^o\,\underline{P}^i)\sin\theta$$

$$\underline{U}(x) = \int_0^x (\underline{E}^e(\xi)\cos[\beta(x-\xi)] - j\underline{Z}\underline{H}^e(\xi)\sin[\beta(x-\xi)])\,d\xi$$

$$\underline{W}(x) = \underline{Y}\int_0^x (\underline{Z}\underline{H}^e(\xi)\cos[\beta(x-\xi)] - j\underline{E}^e(\xi)\sin[\beta(x-\xi)])\,d\xi$$

$$\underline{K}(\ell) = \underline{Z}\,[\underline{W}(\ell) - \underline{Y}^o\underline{U}(\ell)]$$

<u>COMPUTATION OF DYNAMIC QUANTITIES</u>

12. <u>Voltages and Current at any Point Along the Line and at the Terminals.</u> [1]

$$\underline{V}(x) = (\underline{I}\cos\theta' + j\,\underline{P}^i\sin\theta')\,\underline{S}^{-1}\underline{K}(\ell) + \underline{U}(x)$$

$$\underline{V}^i = \underline{S}^{-1}\,\underline{K}(\ell)$$

$$\underline{V}^o = (\underline{I}\cos\theta + j\,\underline{P}^i\sin\theta)\,\underline{S}^{-1}\,\underline{K}(\ell) + \underline{U}(\ell)$$

$$\underline{I}(x) = -\underline{Y}\,(\underline{P}^i\cos\theta' + j\underline{I}\sin\theta')\,\underline{S}^{-1}K(\ell) + \underline{W}(x)$$

$$\underline{I}^i = -\,\underline{Y}^i\,\underline{S}^{-1}\,\underline{K}(\ell) + W(\ell)$$

$$\underline{I}^o = -\,(\underline{Y}^i\cos\theta + j\,\underline{Y}\sin\theta)\,\underline{S}^{-1}\,\underline{K}(\ell) + W(\ell)$$

Note that these equations require that $\underline{S}$ be non-singular.

13. <u>Special Cases</u>:

a. $E_y^e \equiv 0$

Forms of equations in section 12 remain the same but $\underline{U}$, $\underline{W}$, and $\underline{K}$ have the special values

$$\underline{U}\ (x) = \int_0^x \underline{E}^e(\xi)\ \cos[\beta(x - \xi)]\ d\xi$$

$$\underline{W}\ (x) = -\ j\ \underline{Y} \int_0^x \underline{E}^e(\xi)\ \sin\ [\beta(x - \xi)]\ d\xi$$

$$K\ (\ell) = -\int_0^\ell (\underline{P}^o\ \cos\ [\beta(\ell - \xi)] + j\ \underline{I}\ \sin\ [\beta(\ell - \xi)])\ \underline{E}^e(\xi)\ d\xi$$

b. $E_y^e = o$; H_z^e = constant, independent of x.

Again, the forms of equations Section 12 are unaltered, but $\underline{U}$, $\underline{W}$, and $\underline{K}$ take the special values

$$\underline{U}\ (x) = \frac{\underline{\underline{E}}^e}{\beta} \sin \beta x = \frac{\underline{\underline{E}}^e}{\beta} \sin\theta'$$

$$\underline{W}\ (x) = \frac{\underline{\underline{Y}}\ \underline{\underline{E}}^e}{j\beta}\ (1 - \cos\beta x)$$

$$= \frac{\underline{\underline{Y}}\ \underline{\underline{E}}^e}{j\beta}\ (1 - \cos\theta')$$

$$\underline{K}\ (\ell) = \frac{1}{j\beta}\ [\underline{I}(1 - \cos\theta) - j\underline{P}^o\ \sin\theta]\ \underline{E}^e$$

where

$$\underline{E}^e = j\omega\ \underline{L}^e\ H_z^e$$

c. <u>Elementary Example</u>: An example of a two-wire line above ground ($N = 2$) excited by a uniform magnetic field, H_z^e, is discussed in Appendix B. The results show that the potential across the input terminals of the line is proportional to the difference in coupling parameters of the two conductors, to the velocity of propagation, and, of course, to the impressed magnetic intensity. This potential is a complicated function of the line electrical length and the load admittances* normalized to the odd-mode characteristic admittance of the line.** When the line is a half-wavelength long, the latter function becomes simply the reciprocal of the normalized common load admittance.

On the other hand, the potentials of each of the conductors to ground constitute a different story. The analysis shows that these potentials go to infinity like $\csc\theta$ as $\theta \to \pi$. In reality, of course, the line has some losses, due to finite conductor and dielectric conductivity, as well as to a small amount of radiation. If α represents the attenuation constant of the line, then the potential buildup is limited only by the quantity cosech $(\alpha \ell)$.

Another example, the response of a three-wire system, far from ground, is given in Reference 3.

LINE AND FIELD PARAMETERS

14. Definition and Determination of Maxwell's Coefficients of Capacitance and Potential. Maxwell's coefficients are defined in terms of electrostatic concepts. Let a set of constant potentials, $\underline{V}$, be applied to the conductors of an N-line:

$$\underline{V} = [V_i] \quad , \quad i = 1, \ldots, N$$

---

* assumed equal at the two terminals.

** The odd-mode characteristic admittance is the admittance of the semi-infinite line excited at one end in such a way that equal currents flow in opposite directions in the two conductors.

Let $\underline{q} = [q_j]$, $j = 1,\ldots, N$ be the resulting electrostatic charges on the N conductors above ground. The quantity

$$\sum_{j=1}^{N} q_j$$

is generally different from, but may be equal to zero. However, if the reference [(N + 1) st] conductor is included we have

$$\sum_{j=1}^{N+1} q_j = 0$$

always.

The coefficients of capacitance are defined by

$$\underline{q} = \underline{C}\,\underline{V},$$

$$\underline{C} = [C_{ij}], \quad i,j = 1,\ldots, N$$

For measurement or analysis this definition implies

$$C_{ij} = \left.\frac{q_j}{V_i}\right|_{V_k = o,\ k \neq i}$$

For various properties of the C_{ij} see Reference 4. For methods of determining capacitance see References 4,5,6.

As stated in Section 6 the coefficients of potential are determined from

$$\underline{P} = [p_{ij}] = \underline{C}^{-1}$$

provided $\underline{C}$ is non-singular. Singular $\underline{C}$ indicates an improperly stipulated model. However, $\underline{C}$ can be sufficiently close to singularity so that inversion yields large error. In that case the model may need to be modified.

15. Definition and Determination of Inductance Coefficients, L_{ij}.

The inductance coefficients are defined in terms of magnetostatic concepts. Let a set of steady currents, $\underline{I}$, flow in the line conductors:

$$\underline{I} = [I_i] \quad , i = 1, \ldots, N$$

The quantity

$$\sum_{i=1}^{N} I_i$$

is generally different from, but may equal, zero. However, if the reference conductor is included,

$$\sum_{i=1}^{N+1} I_i = o$$

always.

As a result of the currents, I_i, magnetic flux, ϕ_j, per meter of line passes between the $j^{\underline{th}}$ conductor and the reference conductor, such that

$$\phi_j = \sum_{i=1}^{N} L_{ij} I_i \quad , j = 1, \ldots, N$$

which defines the inductance coefficients, L_{ij}. If $\underline{\phi} = [\phi_j]$, $j = 1, \ldots, N$ then

$$\underline{\phi} = \underline{L}\ \underline{I}$$

For measurement or analysis this definition implies

$$L_{ij} = \left.\frac{\phi_j}{I_i}\right|_{I_k = o,\ k \neq i}$$

However, as indicated in Section 6, the L_{ij} may be computed from electrostatic parameters according to

$$\underline{L} = \frac{1}{v^2}\,\underline{C}^{-1} = \underline{P}\,/v^2$$

16. <u>Definition and Determination of Impressed Field Coupling Parameters L_i^e, C_i^e</u>

The impressed field coupling parameters, L_i^e and C_i^e, have already been defined in Section 9. A method for determining both sets of parameters through electrostatic concepts is discussed in Reference 2. Results for the special case of small round wires are given in that reference.

The system of conductors to be studied is conceived to lie between parallel planes so oriented that, at large distances from the conductor system, lines corresponding to the impressed magnetic intensity are parallel to the planes. Consult Figure 4 which shows a system of three conductors above ground. In this case, the ground plane takes the place of one of the parallel planes mentioned above. In the absence of a ground plane the parallel planes are chosen so that the conductor system is approximately halfway between them. The planes are chosen sufficiently far from the conductor system so that dynamic reaction from the conductor system does not affect the field at the planes. Parallelism of magnetic lines at the edges of the planes is maintained by introducing magnetic boundaries, either analytically, if the parameters are to be computed, or by the use of "guard planes" if they are to be measured. In Figure 4, solid field lines are magnetic lines generated by low-frequency currents I and -I flowing in the planes.

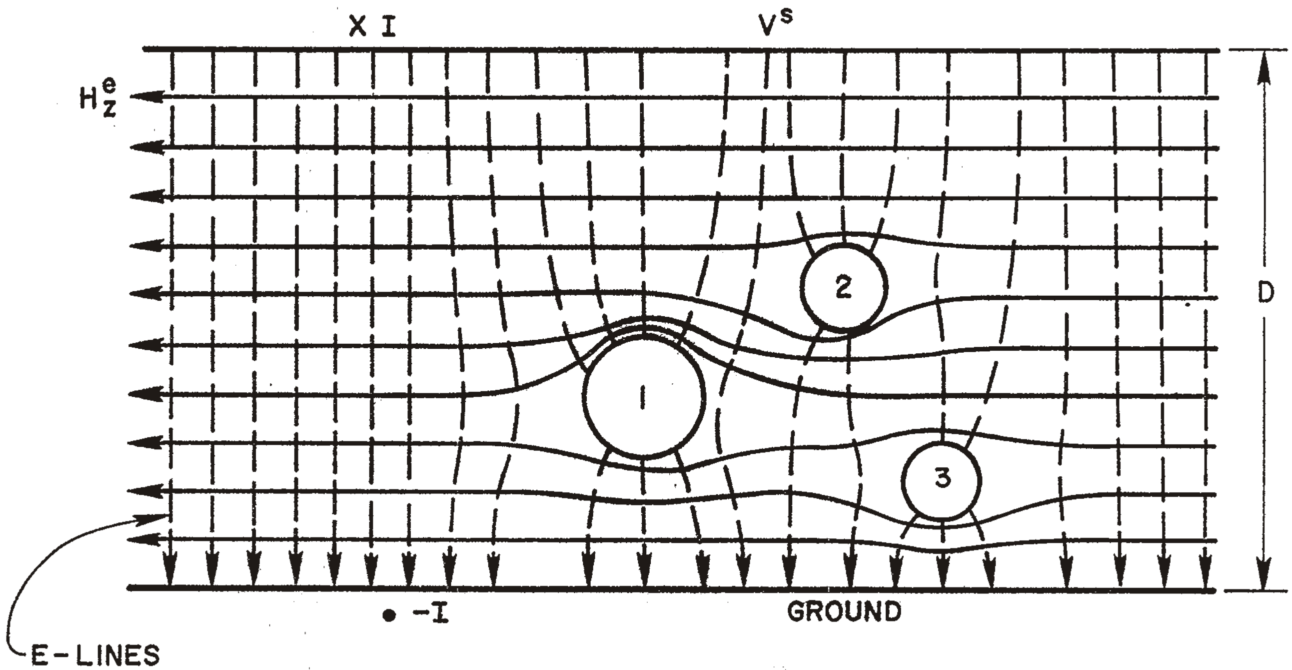

Fig. 4. Model of a multiwire line immersed in magnetostatic and electrostatic fields.

The dashed lines represent electric field lines generated by potentials $\pm V^s$ on the parallel planes, if no ground is present, or potentials V^s and zero if one of the planes is a ground plane.

In order to determine the inductive coupling parameters electrostatically, potentials $\pm V^s$, or V^s and zero are applied to the parallel planes. The conductors of the actual system (other than the reference conductor) are permitted to "float", so that they carry zero charge. The potentials of the various conductors are then determined. Write

$$k_i = \frac{V_i}{V^s}$$

where V_i is the potential of the $i^{\underline{th}}$ conductor. Then it is shown in reference 2 that, for a system with a ground plane

$$L_i^e = \lim_{D\to\infty} (\mu k_i D)$$

where D is the distance between parallel planes. When no ground plane is present,

$$L_i^e = \lim_{D\to\infty} [\mu(k_i - k_o)D]$$

where

$$k_o = \frac{V_o}{V^s}$$

V_o = potential attained by the reference conductor and D is half the distance between parallel planes.

The same coefficients, k_i, in conjunction with the capacitance coefficients, C_{ij}, for the line under investigation, may be used to compute the

C_i^e.* Thus, when a ground plane is the reference conductor,

$$C_i^e = - \lim_{D\to\infty} [D \sum_{j=1}^{N} k_j \; C_{ij}]$$

For a system of (N + 1) conductors without a ground plane,

$$C_i^e = - \lim_{D\to\infty} [D \sum_{j=1}^{N} (k_j - k_o) \; C_{ij}]$$

Consult reference 2 for sample calculations.

* This is in spite of the fact that the C_i^e are defined fundamentally in terms of grounded, rather than floating conductors. It is shown in Reference 2 that the floating configuration may be used instead, thus obviating the need for solving an additional electrostatic problem.

APPENDIX A

Determination of $\underline{Y}^i$ or $\underline{Y}^o$: Typical Elementary Examples

Figure 5 shows a series of examples of elementary possible terminations and their admittance matrices, $\underline{Y}^d$.

(a) is the conventional termination for a two-conductor line far from ground plane (N = 1).

(b) is the conventional termination for a two-conductor line near ground (N = 2). Note that the termination matrix is singular.

(c) is a general termination for a two-conductor line near ground. Note that its matrix is singular if $Y_1 = Y_2 = o$, (which reduces it to case (b)); <u>or</u>, if $Y_3 = o$ <u>and</u> either Y_1 or Y_2 is zero. Thus, if necessary, the singularity in (b) can be removed temporarily by introducing a small Y_1, <u>or</u> Y_2, eventually permitting it to tend to zero. Analysis of case (b) for an externally-impressed, uniform magnetic field shows that difficulty occurs only when the line length is a multiple of half wavelengths, and then only in determining conductor potentials with respect to ground, which become infinite. This is not really a breakdown in analysis, since such a result is to be expected in the idealized system assumed. Actually, even for this extreme case, the potential difference <u>across</u> the line remains finite for finite resistive terminations. See Appendix B.

Case (d) is a common one presenting no difficulties. At (e) we have a termination that could represent a pair of conventional two-conductor

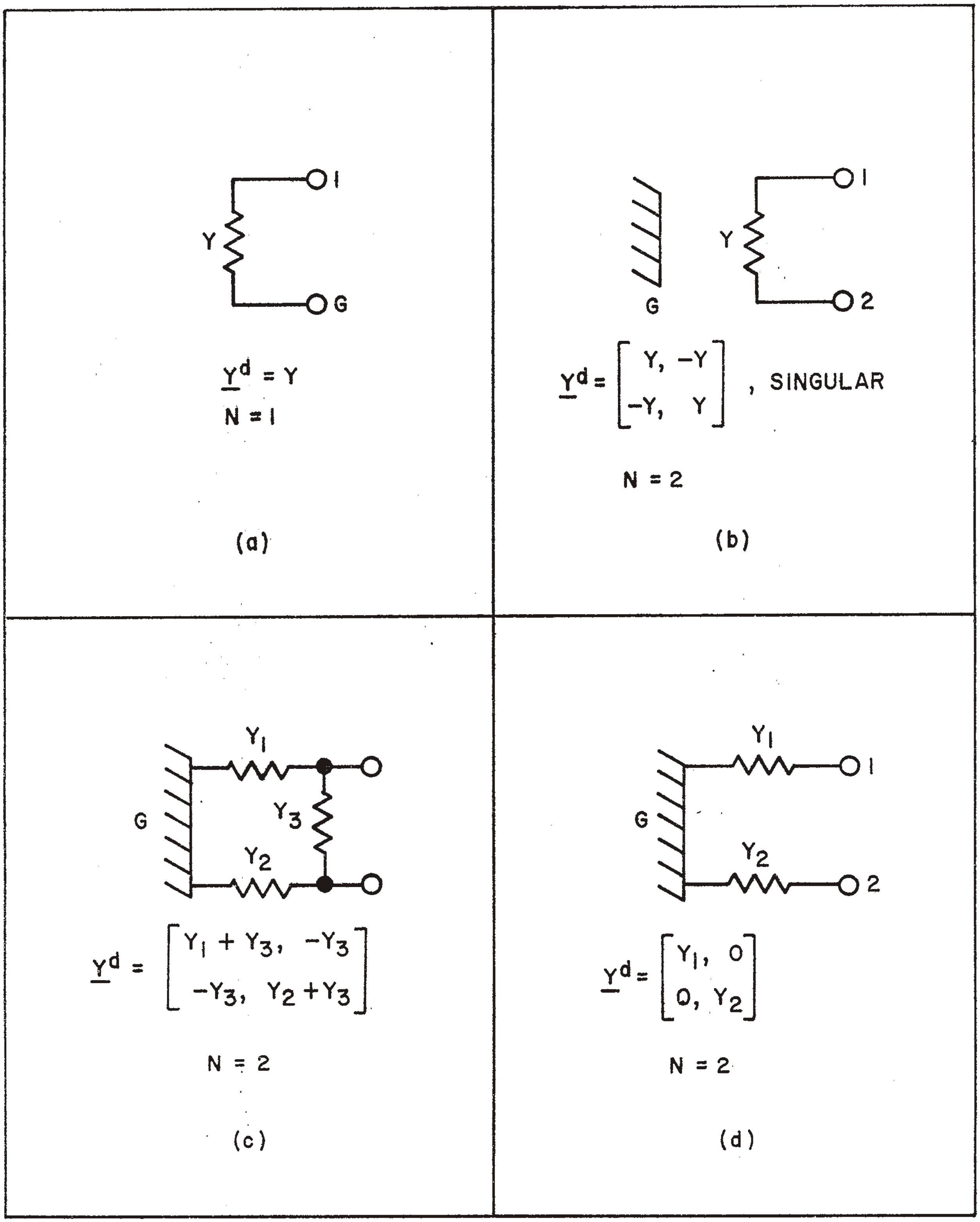

Fig. 5. Examples of termination networks and their corresponding termination matrices. (Concluded on next page).

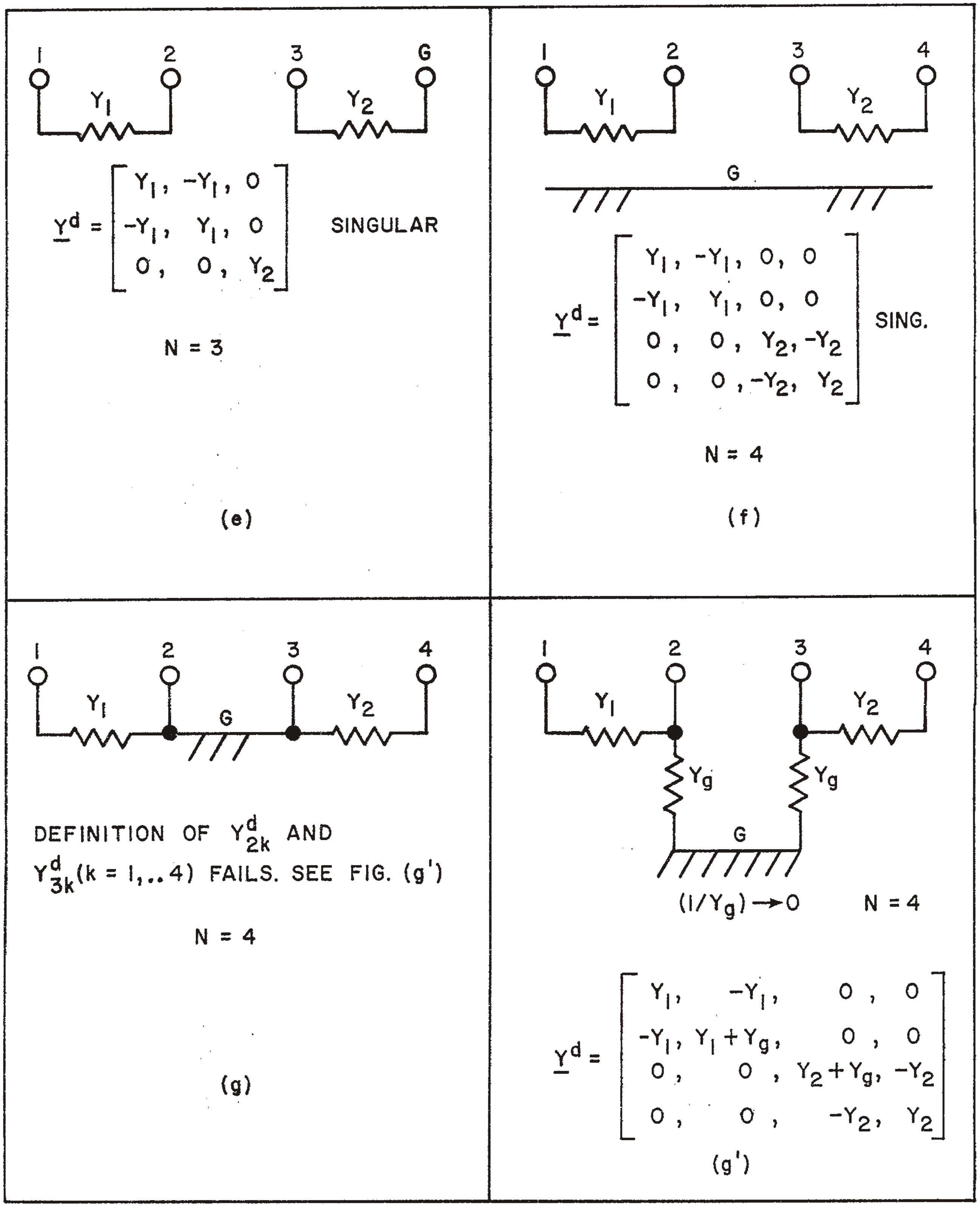

Fig. 5. (Concluded)

lines, far from ground, coupled to each other. One of the conductors is taken as reference, so that N = 3. Again the termination matrix is singular; remarks similar to those concerning case (b) should apply, that is, potentials across each line (1-2 and 3-4) should be finite for resistive terminations, but potentials between conductors of different lines (1-3 or 2-4) can become infinite for line lengths of half-wavelength multiples.

Case (f) is case(e) with the conductor sufficiently near ground so that N = 4.

Case (g) represents a breakdown in concept. The definition of Y^d_{jk} for terminals 2 and 3 fails, since a voltage cannot be applied to these terminals ungrounded, in order to measure resulting currents for applying the definition. A suggested way around this difficulty (not fully tested by this writer) is to introduce large finite admittances between terminals 2 and 3, and ground, as in (g′).

A possible configuration giving rise to the situation in Figure 5(g) is shown in Figure 6. (left-hand end). The difficulty would not be resolved if the shields were also grounded at the right-hand end, but left separated in between. However, if the shields are in continuous contact along the whole line length, they may then be treated as a single reference conductor, the line becomes a 2-line, and the appropriate terminal diagram is Figure 5(d).

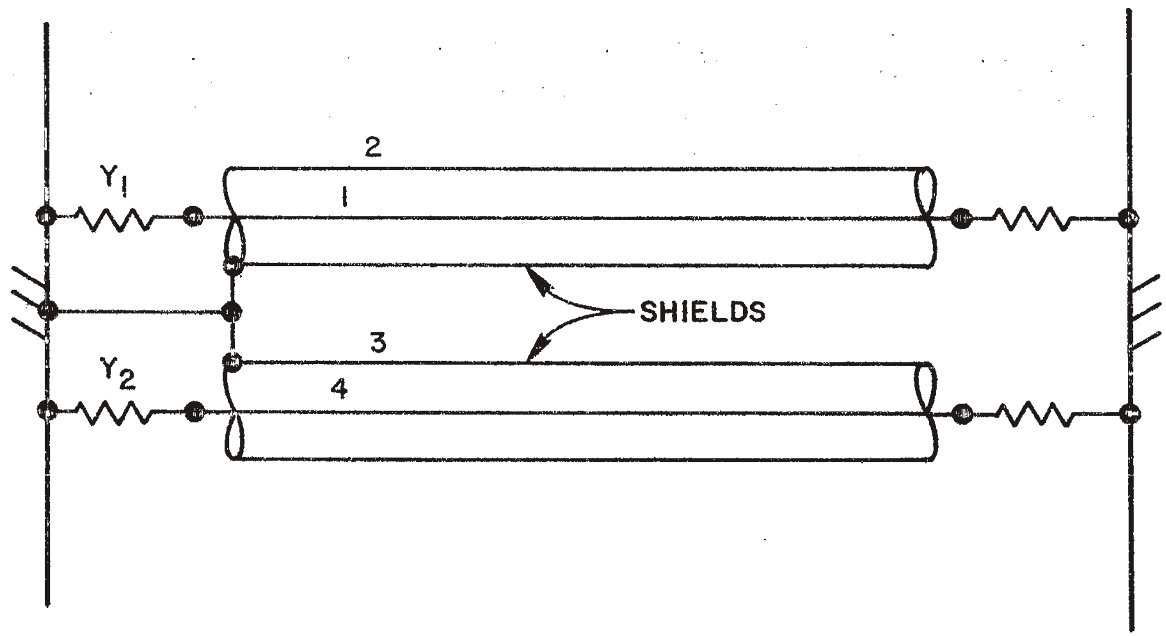

Fig. 6. Configuration illustrating the termination of Fig. 5(g).

APPENDIX B

Response of a Two-conductor Line Above Ground to a Uniform Transverse Magnetic Field (Plane Wave Incident in the Transverse Plane, Electric Vector Normal to Plane of Incidence)

In this appendix a simple example illustrating application of the information in Sections 1-12 is discussed.

Consult Figure 7, which shows a line consisting of two conductors above ground, with equal terminating conductances, G. By Section 1 this is a 2-line (N = 2). The line impedance matrix is (Section 6)

$$\underline{Z} = \begin{pmatrix} Z_{11}, & Z_{12} \\ Z_{21}, & Z_{22} \end{pmatrix}$$

The termination admittance matrices are (Section 7 and Figure 5(b), Appendix A)

$$\underline{Y}^i = \underline{Y}^o = G\begin{pmatrix} 1, & -1 \\ -1, & 1 \end{pmatrix}$$

The normalized load-admittance matrices are (Section10)

$$\underline{P}^o = \underline{P}^i = \underline{ZY}^o = \underline{ZY}^i$$

$$= G\begin{pmatrix} Z_{11}, & Z_{12} \\ Z_{21}, & Z_{22} \end{pmatrix} \begin{pmatrix} 1, & -1 \\ -1, & 1 \end{pmatrix}$$

$$= G\begin{pmatrix} (Z_{11} - Z_{12}), & -(Z_{11} - Z_{12}) \\ -(Z_{22} - Z_{21}), & (Z_{22} - Z_{21}) \end{pmatrix}$$

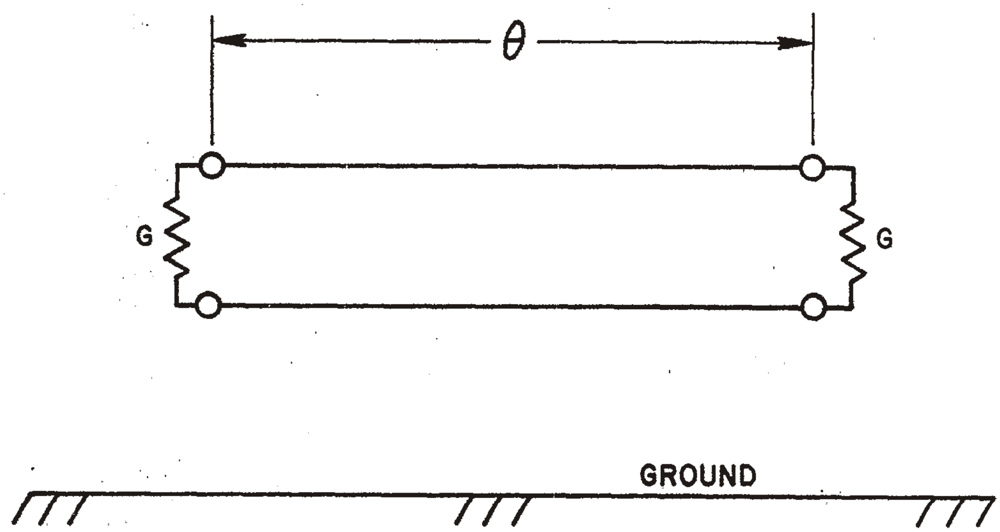

Fig. 7. Schematic, Two-wire line above ground N = 2.

Define

Z^0_{01} = odd-mode characteristic impedance, conductor No. 1, with respect to ground

$= (Z_{11} - Z_{12})$

Z^0_{02} = odd-mode characteristic impedance, conductor No. 2, with respect to ground

$= (Z_{22} - Z_{21})$

so that

$$\underline{P}^i = \underline{P}^0 = G \begin{pmatrix} Z^0_{01}, & -Z^0_{01} \\ -Z^0_{02}, & Z^0_{02} \end{pmatrix}$$

We have (Section 11)

$$\underline{S} = 2\,\underline{P}^i \cos\theta + j[\underline{I} + (\underline{P}^i)^2] \sin\theta$$

where

$$(\underline{P}^i)^2 = G^2 \begin{pmatrix} Z^0_{01}, & -Z^0_{01} \\ -Z^0_{02}, & Z^0_{02} \end{pmatrix} \begin{pmatrix} Z^0_{01}, & -Z^0_{01} \\ -Z^0_{02}, & Z^0_{02} \end{pmatrix}$$

$$= G^2 \begin{pmatrix} [(Z^0_{01})^2 + Z^0_{01}\ Z^0_{02}], & -[(Z^0_{01})^2 + Z^0_{01}\ Z^0_{02}] \\ -[(Z^0_{02})^2 + Z^0_{01}\ Z^0_{02}], & [(Z^0_{02})^2 + Z^0_{01}\ Z^0_{02}] \end{pmatrix}$$

Define (see footnote **, p.15, Section 13)

Z^0_0 = odd-mode characteristic impedance of the line

$= Z^0_{01} + Z^0_{02}$

Then

$$(\underline{P}^i)^2 = G^2 \begin{pmatrix} Z^0_{01} \cdot Z^0_0, & -Z^0_{01} \cdot Z^0_0 \\ -Z^0_{02} \cdot Z^0_0, & Z^0_{02} \cdot Z^0_0 \end{pmatrix}$$

and we have

$$\underline{S} = \begin{pmatrix} S_{11}, & S_{12} \\ S_{21}, & S_{22} \end{pmatrix}$$

$$= 2\ G\cos\theta \begin{pmatrix} Z^0_{01}, & -Z^0_{01} \\ -Z^0_{02}, & Z^0_{02} \end{pmatrix} + j\sin\theta \begin{pmatrix} (1 + G^2\ Z^0_{01}\ Z^0_0), & -Z^0_{01}\ Z^0_0\ G^2 \\ -Z^0_{02}\ Z^0_0\ G^2, & (1 + G^2\ Z^0_{02}\ Z^0_0) \end{pmatrix}$$

Thus,

$$S_{11} = 2G\ Z^0_{01}\cos\theta + j(1 + G^2 Z^0_{01}\ Z^0_0)\sin\theta$$

$$S_{12} = -2G\ Z^0_{01}\cos\theta - jG^2 Z^0_{01} Z^0_0 \sin\theta$$

$$S_{21} = -2G\ Z^0_{02}\cos\theta - jG^2 Z^0_{02} Z^0_0 \sin\theta$$

$$S_{22} = 2G\ Z^0_{02}\cos\theta + j(1 + G^2\ Z^0_{02}\ Z^0_0)\sin\theta$$

Next,

$$D_s\ \underline{S}^{-1} = \begin{pmatrix} S_{22}, & -S_{12} \\ -S_{21}, & S_{11} \end{pmatrix}$$

where

$$D_s = S_{11}\ S_{22} - S_{12}\ S_{21} = \text{det of } \underline{S}.$$

Straightforward substitution and reduction yields

$$D_s = j\sin\theta\ \{2G\ Z^0_0\cos\theta + j[1 + G^2(Z^0_0)^2]\sin\theta\}$$

For uniform H^e_z, and $E^e_y = 0$*, we have (Section 13)

$$\underline{K}(\ell) = \frac{1}{j\beta}\left[\underline{I}\ (1 - \cos\theta) - j\ \underline{P}^0 \sin\theta\right] \underline{E}^e$$

We compute only the input voltage, $\underline{V}^i$. We have (Section 12)

---

* A plane wave, with Poynting vector in the line's transverse plane, and with electric field normal to the plane of incidence, has incident and reflected components which combine to yield this type of field, provided the cross-section dimensions of the whole system are much less than a wavelength. See subsection of this appendix entitled "Effects of Assumptions".

$$\underline{V}^i = \underline{S}^{-1} \; \underline{K}(\ell)$$

$$= D_s^{-1} \begin{pmatrix} S_{22}, & -S_{12} \\ -S_{21}, & S_{11} \end{pmatrix} \begin{pmatrix} K_1(\ell) \\ K_2(\ell) \end{pmatrix}$$

$$= D_s^{-1} \begin{pmatrix} S_{22} \; K_1(\ell) - S_{12} \; K_2(\ell) \\ -S_{21} \; K_1(\ell) + S_{11} \; K_2(\ell) \end{pmatrix}$$

where

$$K_1(\ell) = \frac{1}{j\beta} \{(1 - \cos\theta) \; E_1^e - jG \; Z_{01}^0 \; (E_1^e - E_2^e) \sin\theta\}$$

$$K_2(\ell) = \frac{1}{j\beta} \{ \; jG \; Z_{02}^0 \; (E_1^e - E_2^e) \sin\theta + (1-\cos\theta) \; E_2^e\}$$

By substitution and reduction V_1^i becomes

$$V_1^i = \frac{1}{j\beta D_s} \{[2G \cos\theta \; (1-\cos\theta) \; (Z_{02} \; E_1^e + Z_{01}^0 \; E_1^e) + G \; Z_{01}^0 \; (E_1^e - E_2^e) \sin^2\theta]$$

$$+ \; j \sin\theta \; (1-\cos\theta) \; [E_1^e + G^2 \; Z_0^0 \; (Z_{02}^0 \; E_1^e + Z_{01}^0 \; E_2^e)] \; \}$$

$$V_2^i = \frac{1}{j\beta D_s} \{ \; [2G \cos\theta \; (1-\cos\theta)(Z_{02}^0 \; E_1^e + Z_{01}^0 \; E_2^e) - G \; Z_{02}^0 \; (E_1^e - E_2^e) \sin^2\theta]$$

$$+ \; j \sin\theta \; (1-\cos\theta) \; [E_2^e + G^2 \; Z_0^0 \; (Z_{02}^0 \; E_1^e + Z_{01}^0 \; E_2^e)] \; \}$$

where

$$D_s = j \sin\theta \; \{2G \; Z_0^0 \cos\theta + j \; [1 + G^2 \; (Z_0^0)^2] \sin\theta\}$$

Evidently the factor, $\sin\theta$, in D_s suggests sharp resonances at $\theta = m\pi$, m an integer, unless the numerators for V_1^2 and V_2^i also go to zero at least like $\sin\theta$ at $\theta = m\pi$. As $\theta \rightarrow m\pi$, we have

$$D_s \rightarrow (-1)^m \; j2G \; Z_0^0 \sin\theta$$

For m odd, the numerators in V_1^i and V_2^i both tend to

$$-4G \; (Z_{02}^0 \; E_1^e + Z_{01}^0 \; E_2^e)$$

which, in general, is different from zero, indicating that indeed, both potentials are resonant. The potentials appear indeterminate for m even, but further analysis shows that, in fact they become zero. For m odd, the sharpness of resonance is limited by losses which have not been taken into account.

Also, for m odd, although V_1^i and V_2^i tend to infinity separately, the difference $V_1^i - V_2^i$ is finite; in fact,

$$V_1^i - V_2^i = \frac{(1-\cos\theta) - jG\, Z_0^0 \sin\theta}{j\beta\{2GZ_0^0\cos\theta + j[1 + G^2(Z_0^0)^2]\sin\theta\}} \quad (E_1^e - E_2^e)$$

which, for instance, tends to

$$\frac{E_2^e - E_1^e}{j\beta G Z_0^0}$$

as $\theta \to \pi$:

(See Sections 9 and 16 on determination of E_1^e, E_2^e)

Effects of Assumptions

Before concluding this appendix it may be well to emphasize some of the assumptions on which the results are based. First, as has already been pointed out, losses have been ignored. These, of course, have a profound effect in reducing the largest potentials resulting from external field excitation. Second, the field has been assumed uniform at the conductor system, including the space between conductor group (e.g., cable) and ground. For higher frequencies (say up to 10^8 Hz), this assumption places considerable restriction on cable height above ground and/or grazing angle (complement of incidence angle) for a plane wave producing the field. Thus let

H_o = amplitude of magnetic intensity

ψ = angle of incidence on ground plane

h = height of cable above ground

(assumed at least five times cable diameter)

such that the relative variation in magnetic intensity between ground and cable is

$$\alpha = \frac{H_z^e(0) - H_z^e(h)}{H_z^e(o)}$$

$$= 2\sin^2\left(\frac{1}{2}\beta h \cos\psi\right)$$

$$= 2\sin^2\left(\pi \frac{h}{\lambda}\cos\psi\right)$$

where λ is the wavelength of the impressed wave.

If the assumption of constant field is to be reasonable, then $\alpha \ll 1$, i.e.,

$$2\sin^2\left(\pi\frac{h}{\lambda}\cos\psi\right) \approx 2\pi^2\left(\frac{h}{\lambda}\right)^2\cos^2\psi \ll 1$$

or

$$\frac{h}{\lambda}\cos\psi \ll \frac{1}{\pi\sqrt{2}} = 0.225$$

For various angles of incidence we have

ψ (deg)	$\cos\psi$	$\frac{h}{\lambda}$ much less than
0	1	0.225
30	0.866	0.260
45	0.707	0.318
60	0.500	0.450
90	0	∞

Finally, we must consider the possibility that a group of conductors above ground may be assembled in the form of an unshielded cable, so that the assumption of homogeneity of the dielectric is violated. Under the assumption of a lossless system, such an arrangement affects primarily the capacitance coefficients of the system (Sections 6 and 14).

Consider an unshielded cable of N conductors at height, h, above ground. Initially, let h be large. According to Section 14, the coefficient, C_{ij}, is determined by placing a potential, V_i, on the $i^{\underline{th}}$ conductor, setting all other conductors at ground potential, measuring the charge on the $j^{\underline{th}}$ conductor, and computing

$$C_{ij} = \left.\frac{q_j}{V_i}\right|_{V_k = o,\ k \neq i}$$

But, regardless of the cable dielectric permittivity, when all conductors except the $i^{\underline{th}}$ are grounded, and h is large, almost all electric field lines leaving the $i^{\underline{th}}$ conductor will end on the remaining conductors in the cable, and the variation, with h, of the distribution of these lines among the various conductors will be small. Thus, for large h, the C_{ij} are independent of h. The higher the permittivity relative to air, the smaller h may be without significant effect on the values of the C_{ij}.

However, it is also true that the larger h and/or the relative permittivity, the more nearly singular the capacitance matrix becomes, and the more prone to error on inversion.

As h tends to zero, the precise balance between capacitance and inductance coefficients, given by (Section 6)

$$\underline{L}\,\underline{C} = v^{-2}\,\underline{I} = \mu\,\varepsilon\,\underline{I}$$

ceases to exist, and cable behavior can no longer be analyzed in terms of $\underline{C}$ alone. In fact, transmission is no longer in the TEM mode. Separate investigation is required to determine the seriousness of this deviation.

REFERENCES

1. Response of a Multiconductor Transmission Line to Excitation by an Arbitrary Monochromatic Impressed Field Along the Line. Sandia Laboratories Report SC-CR-71 5076, April 1971, Interaction Note 80.

2. Externally-excited Transmission Line: Definition of Procedures for Determining Coupling Parameters. Harry Diamond Laboratories Report HDL-TM-72-11, May 1972, Interaction Note 121.

3. Sample Calculation: Response of Multiwire Line to External Field Along the Line. Harry Diamond Laboratories Technical Memo for File: FA-179, 26 November 1971.

4. Ramo, S., and J. R. Whinnery, Fields and Waves in Modern Radio. Wiley, New York, 1944.

5. Smythe, W. R. Static and Dynamic Electricity, Chapter 4, McGraw-Hill, New York.

ACKNOWLEDGEMENT

The work reported herein was sponsored by Harry Diamond Laboratories under the Defense Nuclear Agency Subtask EB-088.