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AN ASYMPTOTIC SERIES FOR EARLY TIME RESPONSE IN EMP PROBLEMS

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ABSTRACT

The singularity expansion method is successful in evaluating the EMP response for large and moderate values of time. This paper suggests an asymptotic series suitable for early time, and thus complements the singularity expansion method. Once the solution in the frequency domain is known in certain forms of ascending series of frequency (including optical, diffracted, and creeping waves), asymptotic formulas are developed to "translate" it readily into the time domain. Accuracy of the asymptotic formulas is established by comparison with known exact solutions.

1. INTRODUCTION

In EMP (electromagnetic pulse) studies, a typical problem is the determination of the response of a scatterer due to a transient source. Usually, this problem is attacked first in the frequency domain (with variable s), and then an inverse Laplace transform is employed to obtain the solution in the time domain (with variable t). In the second step, the key lies in the successful evaluation of an integral typically given by

$$R(\vec{r}, t) = \frac{1}{2\pi i} \int_{\varepsilon-i\infty}^{\varepsilon+i\infty} V(s)\, G(\vec{r}, s)\, e^{st}\, ds \tag{1.1}$$

where $G(\vec{r}, s)$ is the frequency domain response for a unit source (Green's function), $V(s)$ is the frequency spectrum of the transient source, and $R(\vec{r}, t)$ is the transient solution. It should be remarked that in a more general case the source $V(s)$ may be also a function of $\vec{r}$, and then, the expression $R(\vec{r}, t)$ needs to be modified. Such a case will not be considered in this paper.

Generally, the evaluation of (1.1) by brute-forced numerical integration schemes is not efficient. A singularity expansion method was recently proposed by Baum.[1] It involves the deformation of the integration contour to the left-half s-plane in order to capture all of the singularities of $V(s)G(\vec{r}, s)$. Thus, the problem is converted to the finding of the singularities, and the determination of their contributions. In several test cases,[2, 3, 4] this method was found to be excellent for large and moderate values of t; accurate results can be obtained by including contributions from surprisingly few singularities which are located near the imaginary axis in the s-plane. However, for small t (early time), the necessity for considering nearly all of the singularities in the s-plane makes it inefficient. Thus, to complement the singularity expansion method, a scheme for evaluating (1.1) for early time is needed.

There is another reason for requiring a relatively simple way of evaluating (1.1) for early time. In many EMP problems, the source V(t) and response $R(\vec{r}, t)$ as a function of time typically have the forms as those sketched in Figure 1. After a propagation delay $(0 < t < t_o)$, the response is quickly built up toward a peak $(t = t_m)$, and then gradually tapers off by oscillation at the dominant natural frequency of the scatterer.[1, 2, 3, 4] In certain applications, the period of initial shock $(t_o < t \leq t_m)$ is of interest. Thus, if a rough estimation for this period is available, some tedious computations can be eliminated.

There is a third application of the prediction for the early time. In solving the H-integral equations in the time domain numerically, it is necessary to know the current distribution at a short interval after the arrival of the incident wave. Traditionally, the physical optics approximation $(\vec{J} = 2\hat{n} \times \vec{H}^{(i)})$ has been used. However, a more accurate prediction is desirable, since the solutions at later times are built up through iterations of the early-time result.

In this paper, we will describe a method for evaluating (1.1) for early time. This method is applicable when the frequency response $G(\vec{r}, s)$ can be expanded in either of the following two forms:*

$$\text{(I)} \qquad G(\vec{r}, s) = e^{-t_o s} \sum_{\nu} \frac{d_\nu}{s^\nu} \qquad (1.2)$$

$$\text{(II)} \qquad G(\vec{r}, s) = \exp - (t_o s + \beta s^{1/q}) \sum_{\nu} \frac{d_\nu}{s^\nu} \qquad (1.3)$$

where $\nu > -1$, $q > 1$, but neither of them needs to be an integer.

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* In terms of the angular frequency ω, $s = j\omega$ for $\exp(+j\omega t)$ time convention; and $s = -i\omega$ for $\exp(-i\omega t)$ time convention.

The above two forms cover a wide range of situations including contributions from optical waves [(1.2) with integer ν], edge-diffracted waves [(1.2) with fraction ν], and creeping waves [(1.3)].

This paper will be organized in the following manner. In Section 2 we will first present two formulas for inverse Laplace transforms. One of the formulas is well-known, while the derivation of the other will be discussed in some detail. Using these formulas we will develop an asymptotic series for small t for $R(\vec{r}, t)$ in (1.1) when $G(\vec{r}, t)$ is given by either (1.2) or (1.3). The following sections will be devoted to examples which illustrate the use of the asymptotic series. Finally, a conclusion is given in Section 6.

2. TWO FORMULAS FOR INVERSE LAPLACE TRANSFORM

In order to derive the asymptotic series for early time in Section 3, we shall need the two formulas which are discussed below. The first one reads

$$\frac{1}{2\pi i}\int_{\varepsilon-i\infty}^{\varepsilon+i\infty}\frac{1}{s^{\nu}}e^{\tau s}\,ds = \frac{1}{\Gamma(\nu)}\tau^{\nu-1}, \qquad \nu > -1 \text{ and } \varepsilon > 0 \tag{2.1}$$

where $\Gamma(\nu)$ is the Gamma function. This is a well-known inverse Laplace transform and can be found in standard textbooks. The second formula is more involved. The following integral for small τ will be evaluated:

$$U_{\nu}^{(q)}(\beta,\tau) = \frac{1}{2\pi i}\int_{\varepsilon-i\infty}^{\varepsilon+i\infty}\frac{1}{s^{\nu}}\exp\left(\tau s - \beta s^{1/q}\right)ds, \qquad q > 1 \text{ and real } \beta \tag{2.2}$$

The result is

$$U_{\nu}^{(q)}(\beta,\tau) \sim \frac{1}{\sqrt{2\pi}(q-1)^{1/2}}\, q^{\frac{2\sqrt{q}-1}{2(q-1)}}\, \beta^{\frac{q(1-2\nu)}{2(q-1)}}\, \tau^{\frac{2q(\nu-1)+1}{2(q-1)}}$$

$$\times\left\{1+\sum_{n=2,4,6}^{\infty}\frac{[q(1-\nu)-1][q(1-\nu)-2]\ldots[q(1-\nu)-n]}{\sqrt{\pi}\,(-2)^{-n/2}\,n!}\right.$$

$$\left.\times\,(q/\beta)^{\frac{nq}{2(q-1)}}\frac{\Gamma\left(\frac{n+1}{2}\right)\tau^{\frac{n}{2(q-1)}}}{[q(q-1)]^{n/2}}\right\}\times\exp\left\{\tau^{-\frac{1}{q-1}}\left[(\beta/q)^{\frac{q}{q-1}}-\beta(\beta/q)^{\frac{1}{q-1}}\right]\right\}, \tag{2.3}$$

which is an ascending power series of τ. The derivation of (2.3) is based on a saddle-point integration method, and will be briefly outlined below.

A change of the integration variable in (2.2) from s to z with

$$s = (z\tau^{-\frac{1}{q-1}})^q \tag{2.4}$$

results in

$$U_\nu^{(q)}(\beta, \tau) = \frac{q}{2\pi i} \tau^{\frac{q(\nu-1)}{q-1}} \int_P z^{q(1-\nu)-1} [\exp \tau^{-\frac{1}{q-1}}(z^q - \beta z)] \, dz \tag{2.5}$$

where contour P is shown in Figure 2. Since $q > 1$, the parameter $\tau^{-\frac{1}{q-1}}$ in (2.5) is large and the saddle-point integration method can be used. Apart from $\tau^{-\frac{1}{q-1}}$, the argument of the exponential function in (2.5) is

$$g(z) = z^q - \beta z \quad . \tag{2.6}$$

The saddle-point z_o is located at

$$z_o = \left(\frac{\beta}{q}\right)^{\frac{1}{q-1}} ; \tag{2.7}$$

the steepest descent path P_s is shown in Figure 2. Deforming the integration path P to P_s and expanding the integrand in (2.5) around z_o, i.e.,

$$z^{q(1-\nu)-1} = z_o^{q(1-\nu-1)}$$

$$\times \left[1 + \sum_{n=1}^{\infty} \frac{[q(1-\nu)-1][q(1-\nu)-2] \ldots [q(1-\nu)-n]}{n!} \left(\frac{z - z_o}{z_o}\right)^n\right] , \tag{2.8}$$

the result in (2.3) can be obtained.

In a later application, we need particularly $U_\nu^{(3)}(\beta, \tau)$. Its explicit form is given below:

$$U_\nu^{(3)}(\beta,\ \tau) \sim \frac{1}{2\sqrt{\pi}}\, 3^{\frac{6\nu-1}{4}}\, \beta^{\frac{3(1-2\nu)}{4}}\, \tau^{\frac{6\nu-5}{4}} \left[\exp\left(\frac{-2}{3\sqrt{3}}\, \beta^{3/2}\, \tau^{-1/2}\right)\right]$$

$$\times \left[1 - \frac{\sqrt{3}(3\nu - 2)(3\nu - 1)}{4\ \beta^{3/2}}\, \tau^{1/2} + \frac{3(3\nu - 2)(3\nu - 1)(3\nu + 0)(3\nu + 1)}{32\ \beta^{3}}\, \tau - \frac{\sqrt{3}(3\nu - 2)(3\nu - 1)\ \ldots\ (3\nu + 3)}{128\ \beta^{9/2}}\, \tau^{3/2} + 0(\tau^2)\right]. \tag{2.9}$$

The function $U_\nu^{(3)}(\beta,\ \tau)$ was studied earlier by Friedlander[5] using a slightly different method; his result was

$$\overline{U}_\nu^{(3)}(\beta,\ \tau) \sim \frac{1}{2\sqrt{\pi}}\, 3^{\frac{6\nu-1}{4}}\, \beta^{\frac{3(1-2\nu)}{4}}\, \tau^{\frac{6\nu-5}{4}} \left[\exp\left(\frac{-2}{3\sqrt{3}}\, \beta^{3/2}\, \tau^{-1/2}\right)\right]$$

$$\times \left[1 - \frac{\sqrt{3}(9\nu^2 - 6\nu + \frac{5}{12})}{4\ \beta^{3/2}}\, \tau^{1/2} + \ldots\right]. \tag{2.10}$$

The comparison of (2.10) with (2.9) reveals a small difference in the coefficients of $\tau^{1/2}$ in the last factor, which may be traced back to the fact that different approximations were used in expansions around the saddle point. In practical computations, this difference should not be significant.

3. ASYMPTOTIC SERIES

When $G(\vec{r}, s)$ is expressible in (1.2) or (1.3), our goal is to derive an asymptotic series for $R(\vec{r}, t)$ in (1.1) for $\tau = (t - t_o) \to 0$. Two types of source will be considered:

(A) Step source: $$V(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} \qquad (3.1)$$

(B) Exponential source: $$V(t) = \begin{cases} 0, & t < 0 \\ te^{-t/T}, & t > 0 \end{cases} \qquad (3.2)$$

Their frequency spectra are:

(A) $$V(s) = \frac{1}{s} \qquad (3.3)$$

(B) $$V(s) = \frac{1}{(s + \frac{1}{T})^2} = \frac{1}{s^2} \sum_{m=0}^{\infty} (-1)^m (m + 1) \left(\frac{1}{sT}\right)^m \quad . \qquad (3.4)$$

Practical sources in EMP studies can be well-approximated by the exponential source, which is sketched in Figure 1(a). The step source, on the other hand, is used widely in theoretical analysis.

When $G(\vec{r}, s)$ is represented by (1.2), an application of the formula (2.1) in (1.1) results immediately, for $(t - t_o) \to 0$,

(I-A) Step response:

$$R(\vec{r}, t) = \sum_{\nu} \frac{d_{\nu}}{\Gamma(\nu + 1)} (t - t_o)^{\nu} \qquad (3.5)$$

(I-B) Exponential response:

$$R(\vec{r}, t) = \sum_{\nu} \sum_{m=0}^{\infty} (m + 1)(-T)^{-m} \frac{d_\nu}{\Gamma(\nu + m + 2)} (t - t_o)^{\nu+m+1} \tag{3.6}$$

The property of convergence of (3.5) and (3.6) depends on that of the series in (1.2). Equation (1.2) is usually asymptotic. Then, under the worst circumstances, (3.5) and (3.6) should converge asymptotically as $(t - t_o) \to 0$.

When $G(\vec{r}, s)$ is represented by (1.3), we have, correspondingly,

(II-A) Step response:

$$R(\vec{r}, t) = \sum_{\nu} d_\nu \, U^{(q)}_{\nu+1}(\beta, t - t_o) , \tag{3.7}$$

(II-B) Exponential response:

$$R(\vec{r}, t) = \sum_{\nu} \sum_{m=0}^{\infty} (m + 1)(-T)^{-m} \, d_\nu \, U^{(q)}_{\nu+m+2}(\beta, t - t_o) \tag{3.8}$$

where $U^{(q)}_{\nu}(\beta, \tau)$ is given in (2.3).

In the next two sections, examples will be given to illustrate the use of the formulas in (3.5) through (3.8).

4. LINE SOURCE

We begin with an extremely simple problem, namely, the radiation from a two-dimensional line source with an electric current

$$J_z(\vec{\rho}, t) = V(t)\,\delta(x)\,\delta(y) \tag{4.1}$$

where V(t) is given by either (3.1) or (3.2). In the frequency domain (with $e^{-i\omega t}$ time variation), the Green's function takes the familiar form

$$G(\vec{\rho}, s) = \frac{i}{4} H_o^{(1)}(k\rho) = \frac{i}{4} H_o^{(1)}\left(is\frac{\rho}{c}\right) \tag{4.2}$$

where $k = \omega/c$ and c is the speed of light. The problem is to determine $R(\vec{\rho}, t)$ as defined in (1.1). When $R(\vec{\rho}, t)$ is known, the complete fields can be readily computed:

$$E_z(\vec{\rho}, t) = -\mu_o \frac{\partial}{\partial t} R(\vec{\rho}, t), \qquad H_\phi(\vec{\rho}, t) = -\frac{\partial}{\partial \rho} R(\vec{\rho}, t) \tag{4.3}$$

Therefore, in the following discussion we will concentrate on $R(\vec{\rho}, t)$.

The Green's function in (4.2) has the well-known asymptotic expansion:

$$G(\vec{\rho}, t) \sim \sqrt{\frac{1}{s\pi(s\rho/c)}}\, e^{-s(\rho/c)} \left[1 - \frac{1}{8(s\rho/c)} + \frac{9}{128(s\rho/c)^2} - \frac{225}{3072(s\rho/c)^3} + \ldots\right] \tag{4.4}$$

For a step source, the use of (1.2) and (3.5) yields*

$$R(\vec{\rho}, t) \sim \frac{\tau^{1/2}}{\sqrt{2\pi}} \left(1 - \frac{1}{12}\tau + \frac{3}{160}\tau^2 - \frac{15}{1344}\tau^3 + \ldots\right) \tag{4.5}$$

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* It is understood that $R(\vec{\rho}, t) = 0$ for $\tau < 0$.

where $\tau_1 = \frac{ct}{\rho} - 1$. For this simple case, the integral in (1.1) can be evaluated exactly with the following result[6]

$$R(\vec{\rho}, t) = \frac{1}{2\pi}\left[\ell n\left(\frac{ct}{\rho}\right) + \ell n\left(1 + \sqrt{1 - (\rho/ct)^2}\right)\right] \tag{4.6}$$

It is a simple matter to verify that for small values of t, the approximate evaluation of $R(\vec{\rho}, t)$ in (4.6) will result in (4.5).

Next, let us turn to the exponential source in (3.2). From (3.4), (4.4) and (3.5), the asymptotic solution of the response is immediately found to be

$$R(\vec{\rho}, t) \sim \frac{\sqrt{2}a}{3\pi}\left(\frac{\rho}{c}\right)\tau^{3/2}\left(1 + \sum_{n=1}^{\infty} h_n \tau^n\right) \tag{4.7}$$

where $h_1 = -\frac{2}{5}\left(\frac{2}{T'} + \frac{1}{8}\right)$,

$$h_2 = \frac{4}{35}\left(\frac{3}{T'^2} + \frac{1}{4}\frac{1}{T'} + \frac{9}{128}\right),$$

$$h_3 = -\frac{8}{315}\left(\frac{4}{T'^3} + \frac{3}{8}\frac{1}{T'^2} + \frac{18}{128}\frac{1}{T'} + \frac{225}{3072}\right),$$

. . . . .

and $T' = \frac{cT}{\rho}$.

To evaluate its accuracy, we may compare it with the exact solution, which is

$$R(\vec{\rho}, t) = \int_0^t t' \exp(t'/T)\, \mathrm{Re}\left\{\frac{1}{2\pi\left[(t' - t)^2 - \left(\frac{\rho}{c}\right)^2\right]^{1/2}}\right\} dt' \tag{4.8}$$

In Figures 3 and 4, the exact solution obtained by a numerical integration is shown, together with the asymptotic solutions based on the truncated version of (4.7). The symbol $R_N(\vec{\rho}, t)$ indicates that the result with only the first N terms in the series in (4.7) is retained. The following observations may be made:

(i) For very small τ (say $\tau \leq 0.5$), even R_1 gives very good results. For moderate τ (say $\tau \leq 3$), five or six terms may be needed.

(ii) The convergence rate of (4.7) depends on $T' = (cT/\rho)$ which appears in the denominators of the coefficients: the larger the T', the faster the convergence.

(iii) The series in (4.7) is alternating in sign. The exact value $R(\vec{\rho}, t)$ falls between $R_N(\vec{\rho}, t)$ and $R_{N+1}(\vec{\rho}, t)$ for every N. Thus, for an N-term series $R_N(\vec{\rho}, t)$, the truncation error has an upper bound given by $|R_{N+1} - R_N|$, which enables one to conveniently evaluate the accuracy of the series in (4.7).

5. CURRENT ON A CYLINDER

Consider an infinitely long, perfectly conducting cylinder with its axis in the z-direction and with radius a (Figure 5). For a time-harmonic plane wave incident in the direction of the negative x-axis, such that

$$\vec{E}^{(i)}(\vec{r}, s) = \hat{z}e^{-ikx} = \hat{z}e^{s(x/c)} \quad , \qquad (5.1)$$

the solution for the surface current on the cylinder is found asymptotically to be[7]

$$J_z(\phi, s) \sim J_z^{(op)}(\phi, s) + J_z^{(cp)}(\phi, s) \quad . \qquad (5.2)$$

It consists of two parts: the optics contribution $J_z^{(op)}$ and the creeping wave contribution $J_z^{(cp)}$. Let us first concentrate on $J_z^{(op)}$, namely,

$$J_z^{(op)}(\phi, s) = 2\sqrt{\frac{\varepsilon_o}{\mu_o}} \cos\phi \left[1 + \frac{1}{2\cos^3\phi \left(\frac{a}{c}\right)} \frac{1}{s} - \frac{1 + 3\sin^2\phi}{2\cos^6\phi \left(\frac{a}{c}\right)^2} \frac{1}{s^2} + \ldots\right]$$

$$e^{(a/c)s\cos\phi} \quad , \qquad (5.3)$$

for $0 \leq \phi < \frac{\pi}{2}$ and $\frac{3\pi}{2} < \phi \leq 2\pi$, and $J_z^{(op)} = 0$ for $\frac{3\pi}{2} > \phi > \frac{\pi}{2}$ (i.e., shadow). The use of (3.5) and (3.6) leads immediately to the response in the time domain,

(I-A) Step response

$$J_z^{(op)}(\phi, t) = 2\sqrt{\frac{\varepsilon_o}{\mu_o}} \cos\phi \left(1 + \frac{1}{2\cos^3\phi} \tau_1 - \frac{1 + 3\sin^2\phi}{4\cos^6\phi} \tau_1^2 + \ldots\right) \qquad (5.4)$$

(I-B) Exponential response

$$J_z^{(op)}(\phi, t) = 2 \left(\frac{a}{c}\right)\sqrt{\frac{\varepsilon_o}{\mu_o}} \cos\phi \left[\tau_1 + \left(\frac{1}{4\cos^3\phi} - \frac{1}{T'}\right)\tau_1^2\right.$$

$$\left. + \left(\frac{1}{2T'^2} - \frac{1}{6T'\cos^3\phi} - \frac{1 + 3\sin^2\phi}{12\cos^6\phi}\right)\tau_1^3 + \ldots\right] \quad (5.5)$$

for $0 \leq \phi < \frac{\pi}{2}$ and $\frac{3\pi}{2} < \phi \leq 2\pi$ where $T' = (cT/a)$ and $\tau_1 = (ct/a) + \cos\phi$. The fact that $J_z^{(op)} = 0$ for $\tau_1 < 0$ is understood.

Next consider the creeping-wave contribution, which in the frequency domain is asymptotically given by:

$$J_z^{(cp)}(\phi, s) \sim \sqrt{\frac{\varepsilon_o}{\mu_o}} \frac{1}{0.7012} \left(\frac{2c}{as}\right)^{1/3} \left[1 + 0.2338 \left(\frac{2c}{as}\right)^{2/3}\right]$$

$$\times \sum_{p=1,2} \exp -\Psi_p \left[\left(\frac{as}{c}\right) + 2.3381 \left(\frac{as}{2c}\right)^{1/3}\right] \quad (5.6)$$

In this case, the term $p = 1$ ($p = 2$) represents the creeping wave grazing at $\rho = a$, $\phi = \pi/2$ ($\rho = a$, $\phi = 3\pi/2$) and the angle Ψ_1 (Ψ_2) is the angle in radians measured counterclockwise (clockwise) from that point to the observation point (Figure 5). From (1.3), (3.7), (3.8), and (2.10) it may be shown that

(II-A) Step response

$$J^{(cp)}(\phi, t) \sim \sqrt{\frac{\varepsilon_o}{\mu_o}} \frac{(2c/a)^{1/3}}{0.7012} \sum_{p=1}^{2} \left[U_{4/3}^{(3)}(\beta_p, t - t_{op})\right.$$

$$\left. + 0.2338 \left(\frac{2c}{a}\right)^{2/3} U_2^{(3)}(\beta_p, t - t_{op})\right] \quad (5.7)$$

(II-B) Exponential response

$$J^{(cp)}(\phi, t) \sim \sqrt{\frac{\varepsilon_o}{\mu_o}} \frac{(2c/a)^{1/3}}{0.7012} \left\{ \sum_{p=1}^{2} \sum_{m=0}^{\infty} (m+1)(-T)^{-m} U^{(3)}_{m+7/3}(\beta_p, t - t_{op}) \right.$$

$$\left. + 0.2338\left(\frac{2c}{a}\right)^{2/3} \sum_{p=1}^{2} \sum_{m=0}^{\infty} (m+1)(-T)^{-m} U^{(3)}_{m+3}(\beta_p, t - t_{op}) \right\} \quad (5.8)$$

where $U^{(3)}_{\nu}(\beta, \tau)$ is given in (2.9).

In the illuminated region ($x > 0$), the magnitude of the creeping-wave contribution is relatively very small in the early time and can be neglected as compared to the optical wave. Now let us estimate the accuracy of formulas in (5.4) and (5.5). In Figure 6, (5.5) was computed with one, two, and three terms at the point ($\rho = a$, $\phi = 0$). For a two-term series, the upper bound of the error is estimated to be the difference between the two-term and the three-term series. Thus, the upper bound of the error is about 9% at $\tau' = 0.5$ and almost 100% at $\tau' = 1$. The result computed with three terms is believed to be more accurate.

As mentioned in the introduction, the result in Figure 6 may be used to estimate roughly the peak value of the current due to the exponential source specified by (3.2) and (5.1). From the information obtained in Section 4, the peak value of response occurs roughly at one time constant T after the arrival of the incident wave. In the present problem, the peak value of the current on the cylinder at $\phi = 0$ should occur roughly at $\tau' \simeq T' = 1$, which has a value of about 3 milliamperes. Since there is no exact solution available, the accuracy of the number "3 milliamperes" cannot be ascertained. However, for many applications in system design, this number "3 milliamperes" is valuable information, especially in view of the simple steps in arriving at the solution.

To demonstrate the angular variation of the transient current in the illuminated region, we display $J_z(\phi, t)$ for several different values of ϕ in Figure 7 (for step source) and Figure 8 (for exponential source). As may be expected, the currents assume maximum values at the specular point $\phi = 0$, and decrease toward the shadow boundary at $\phi = \pi/2$.

In the shadow region ($\pi/2 < \phi < 3\pi/2$), only the creeping waves as given in (5.7) and (5.8) exist. In Figures 9 and 10, the currents at the point ($\rho = a$, $\phi = \pi$) are plotted for step, and exponential sources, respectively. These results are computed from (5.7) and (5.8) by retaining terms only up to the fourth power of $\tau_1 = (ct/a) - (\pi/2)$. Because of the approximate nature of the formulas, we do not expect the results to be reliable for τ_1 beyond, e.g., 0.3. Generally the current due to creeping wave is built up exponentially in the early time, and the sudden jump at the arrival of the incident step source (Figure 7) no longer exists. The magnitude of the current due to the creeping wave is much smaller than the current in the illuminated region, as expected.

6. CONCLUSION

This paper proposes an asymptotic method for evaluating the early-time response in EMP problems, which can be used in combination with the singularity expansion method covering the entire range of time efficiently. Once the response in the frequency domain is known in the form of (1.2) or (1.3), its "translation" into the time domain is given by one of the formulas in (3.5) through (3.8). These formulas are asymptotic. However, in an example (Section 4) where the exact solution in the time domain is known, our formula can yield good results even when the normalized time ($\tau = ct/\rho$) is as large as unity (Figures 3 and 4). This is not surprising in view of the success enjoyed by other asymptotic methods in electromagnetic theory (e.g., geometrical theory of diffraction).

It is clear that not all EMP problems have a frequency response in the form of (1.2) or (1.3). A notable exception is the current on a thin wire, which is given by the form

$$G(z, s) = e^{-st_o} \sum_{\nu} \frac{d_\nu}{(\ell n\ as)^\nu} \quad . \tag{6.1}$$

A future task would be to develop a "translation" of this and other forms of frequency response commonly encountered in EMP problems.

A limitation of the present method lies in the fact that $G(\vec{r}, s)$ has to be known analytically in the form of (1.2) or (1.3). In many practical problems, $G(\vec{r}, s)$ may be given numerically. Then an immediate problem is to convert the numerical data to an expression in the form of (1.2) or (1.3) by curve fitting or other numerical processes. Until a systematic and efficient way is devised for this step, the early-time asymptotic series will be of limited use in such cases.

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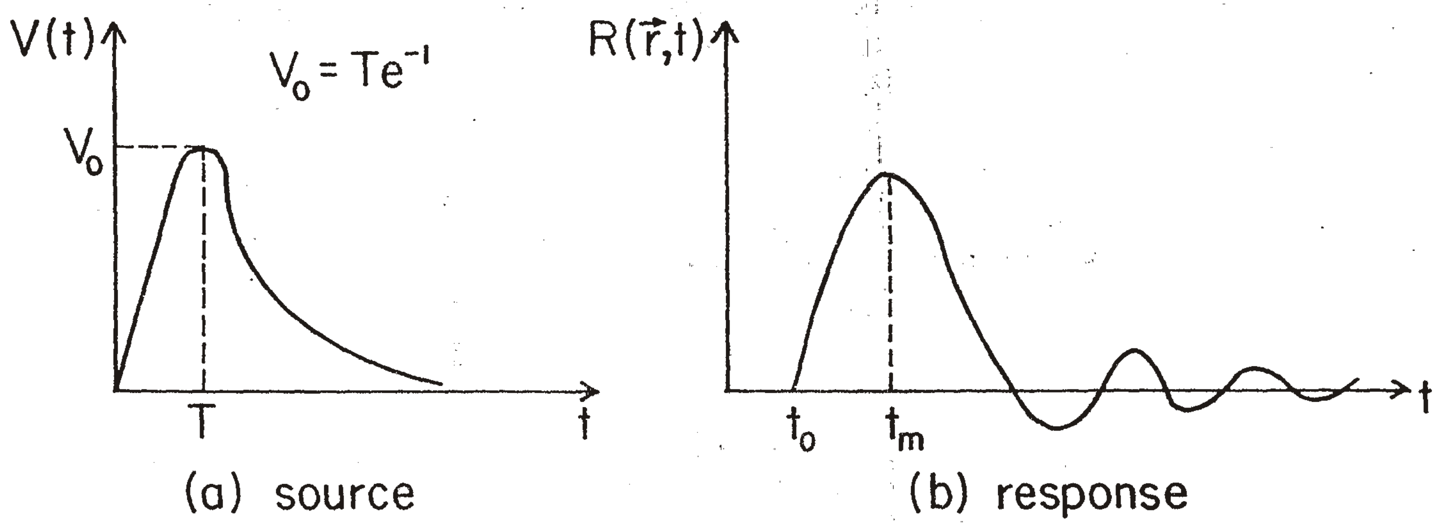

Figure 1. Typical EMP source and response

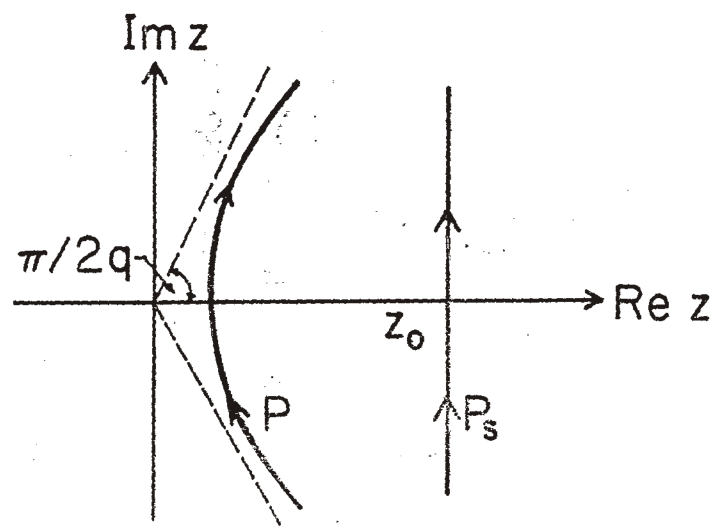

Figure 2. Contours in the complex z-plane

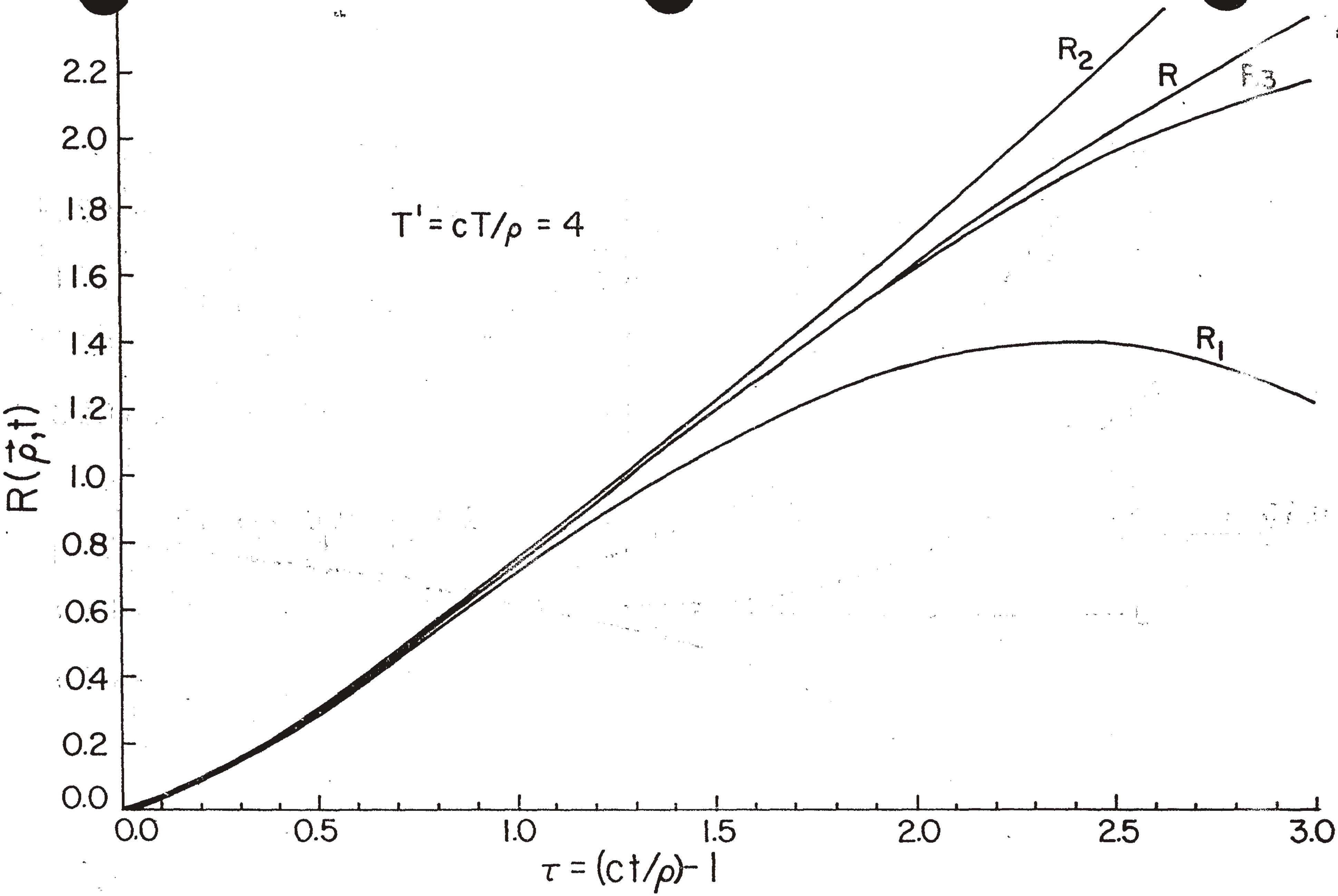

Figure 3. Early time response of a line source excited by an exponential current. R is the exact solution, and R_N is computed from Equation (4.7) with N terms in the series.

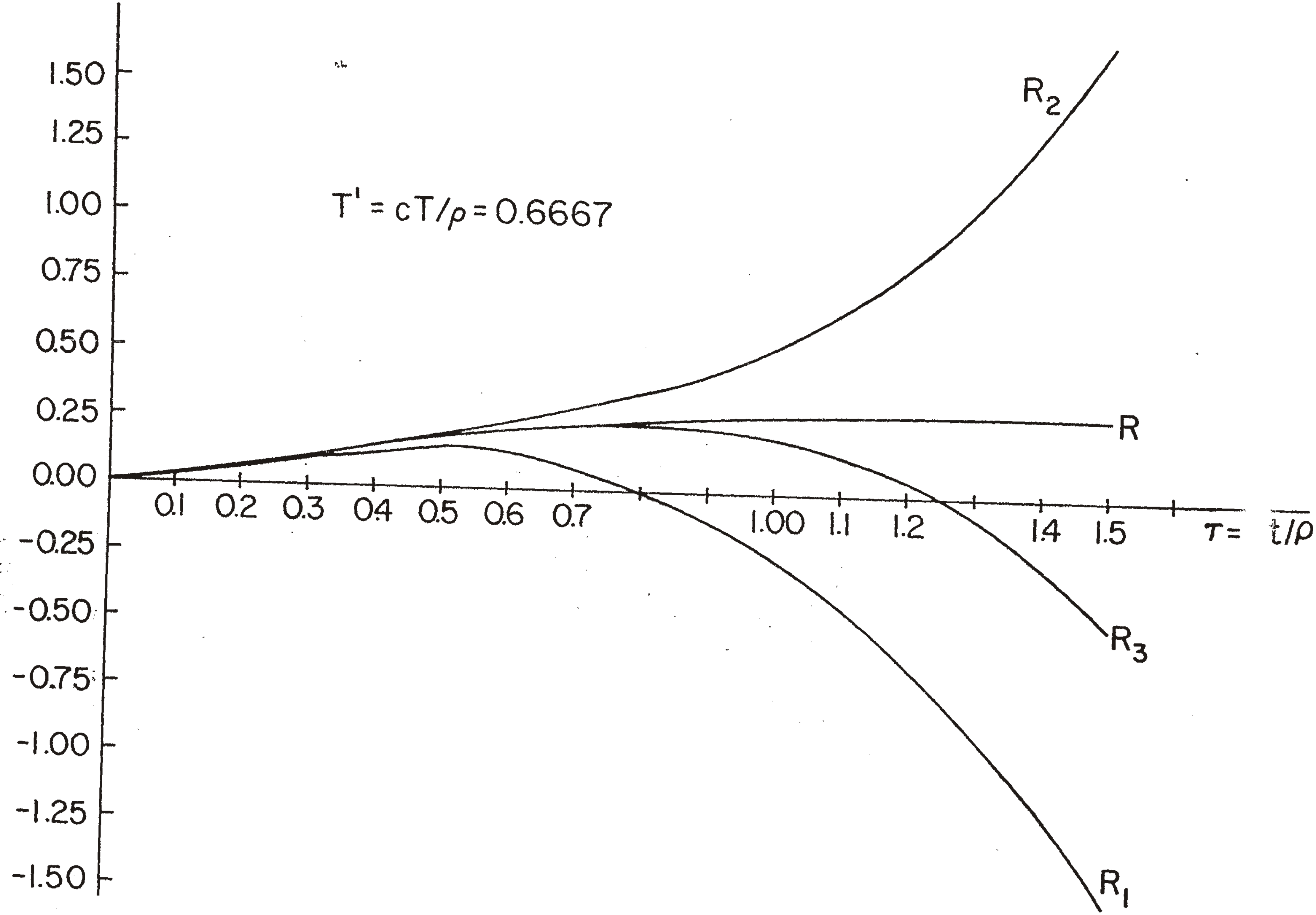

Figure 4. Early time response of a line source excited by an exponential current. R is the exact solution, and R_N is computed from Equation (4.7) with N terms in the series.

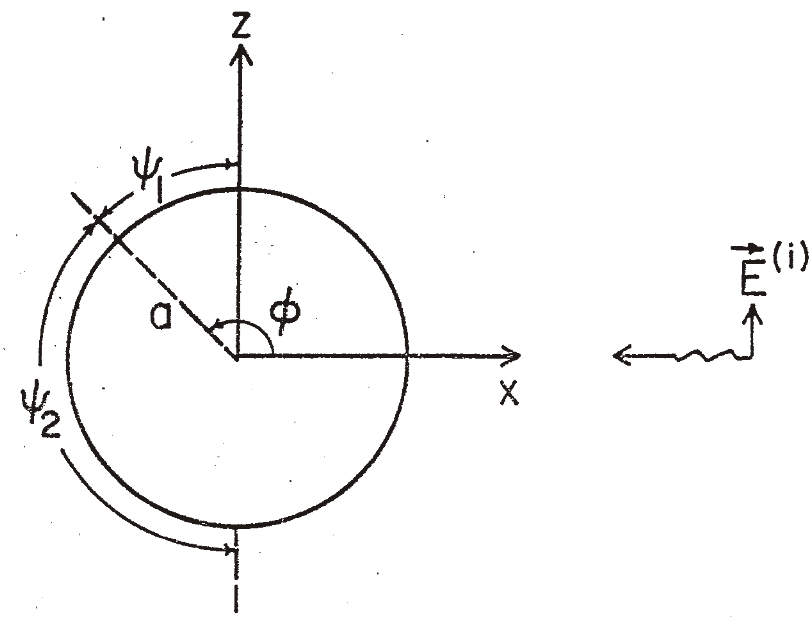

Figure 5. Cross section of an infinitely long cylinder

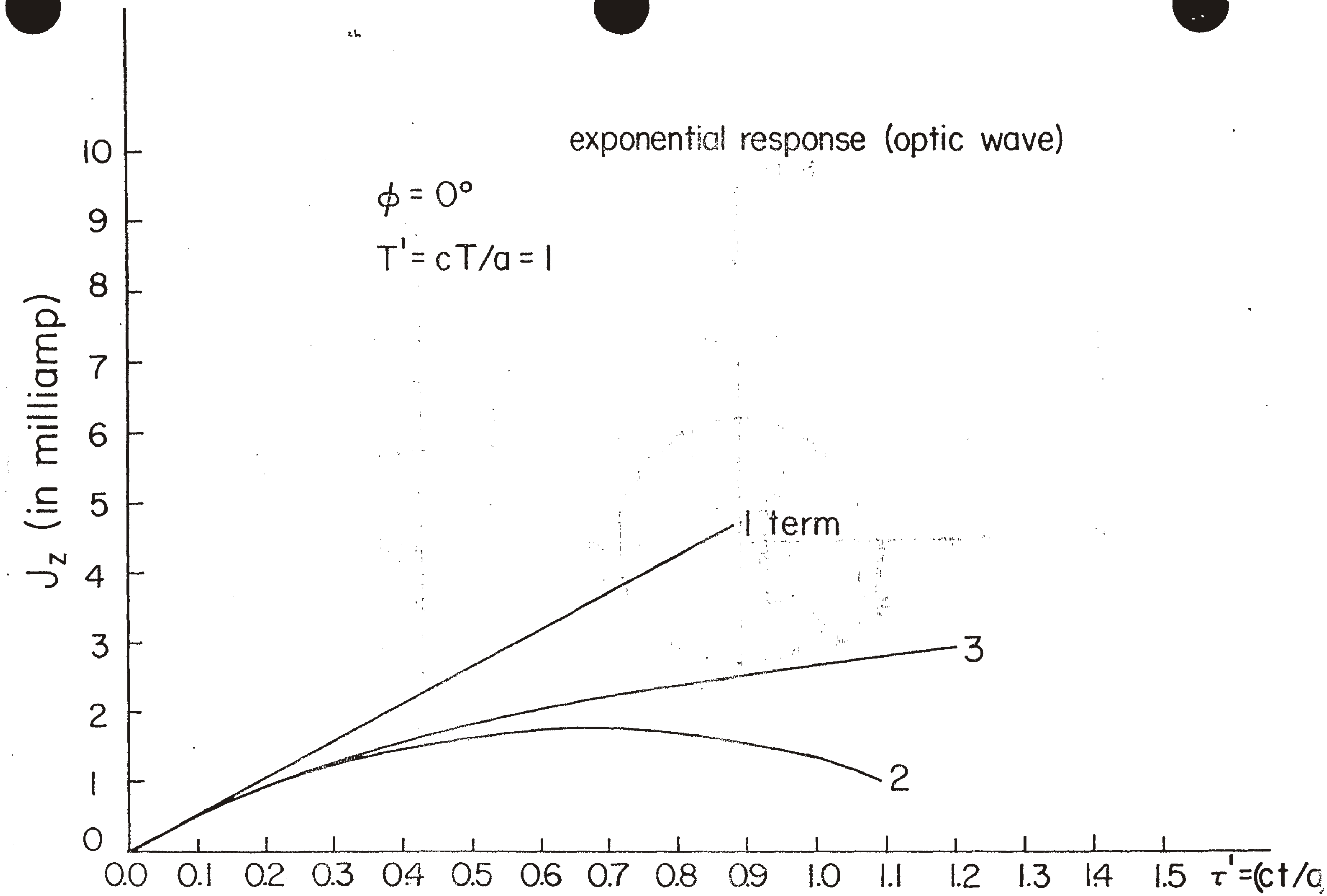

Figure 6. Current on a cylinder at $\phi = 0$ due to the incidence of an exponential source as computed from Equation (5.5) with one, two, or three terms

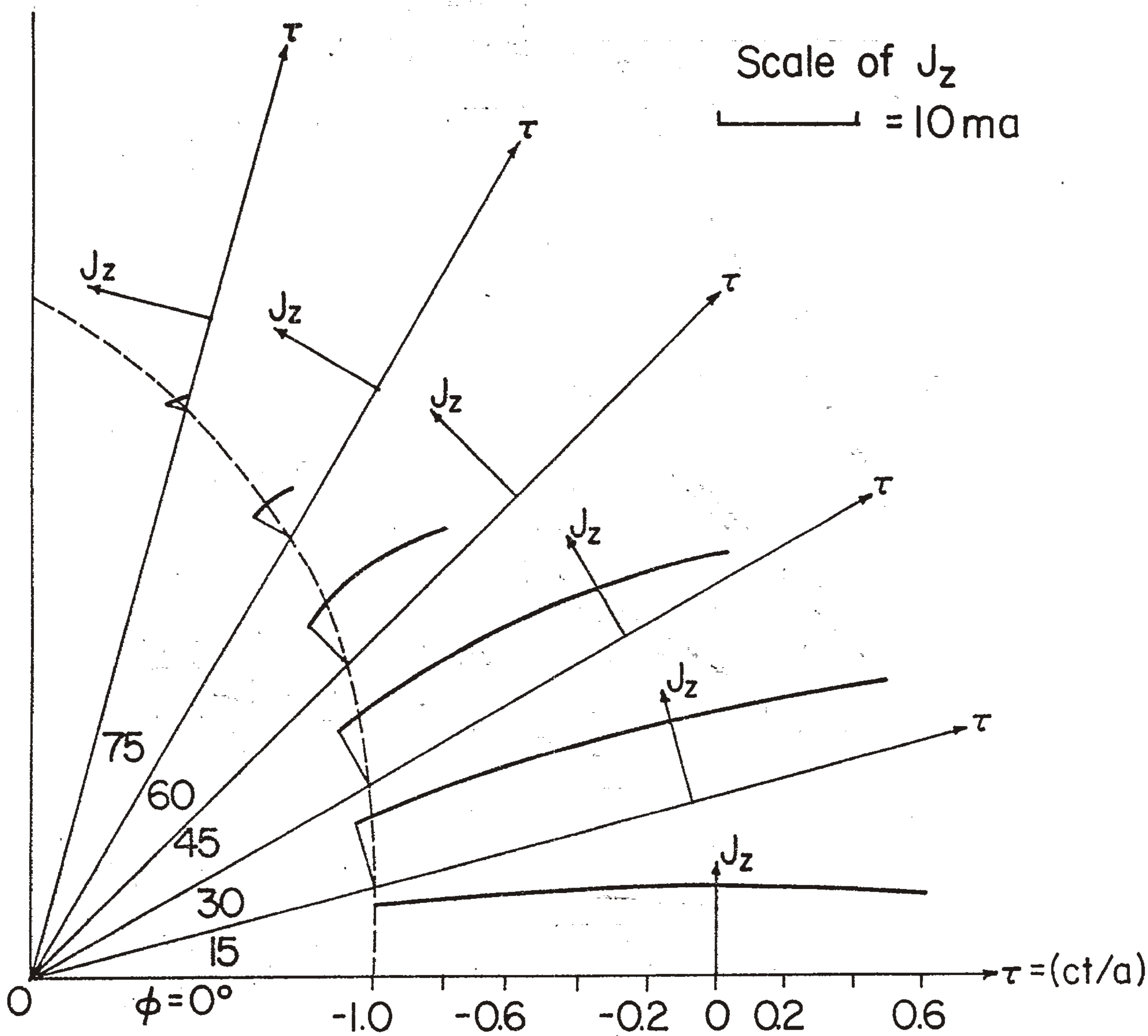

Figure 7. Early time response for the current on the cylinder due to a step source

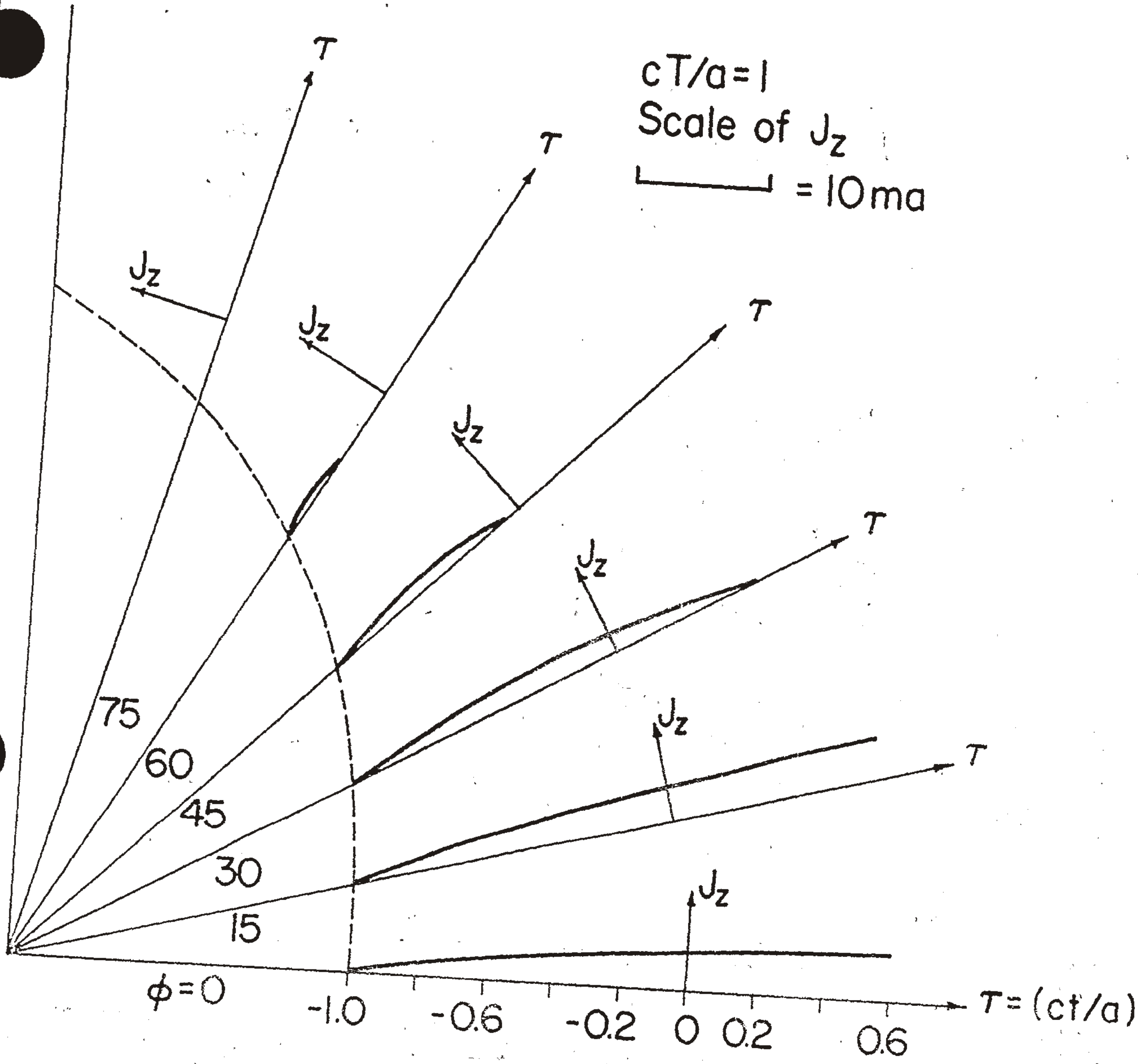

Figure 8. Early time response for the current on a cylinder due to an exponential source with T' = cT/a = 1

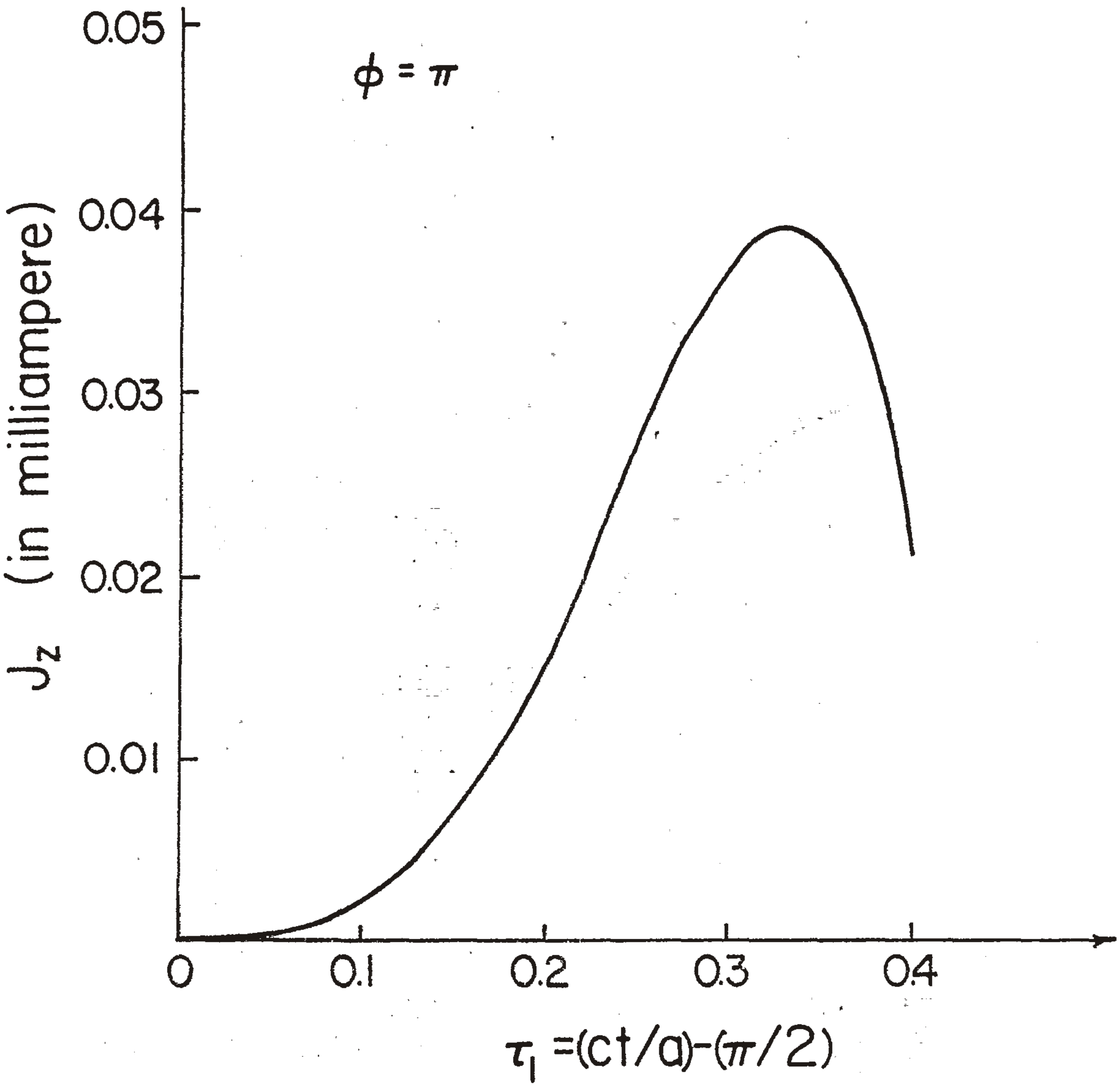

Figure 9. Early time response for the current on a cylinder at ($\rho = a$, $\phi = \pi$) due to a step source

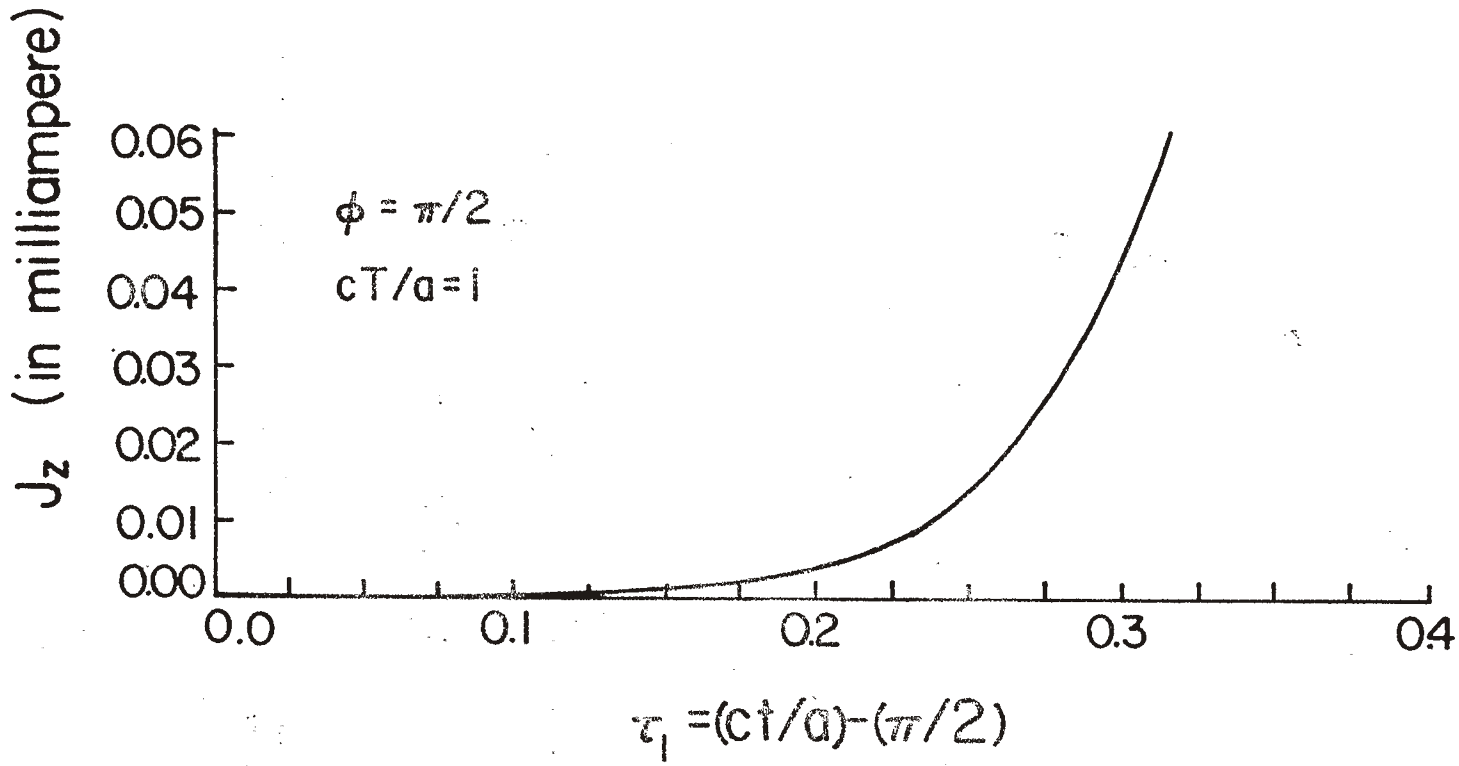

Figure 10. Early time response for the current on a cylinder at ($\rho = a$, $\phi = \pi$) due to an exponential source with $T' = cT/a = 1$