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**EXTERNALLY EXCITED TRANSMISSION LINE:
DEFINITION OF PROCEDURES FOR DETERMINING
COUPLING PARAMETERS**

by
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ABSTRACT

Magnetic- and electric-coupling parameters between a multi-conductor transmission line and an external electromagnetic field are defined. An electrostatic method for determining both types of parameters is proposed. Both types of parameters require determination of the same set of constants. While the magnetic-coupling parameters are directly proportional to these constants (one constant for each conductor), the electric parameters are proportional to a linear combination of the constants, the coefficients of the linear forms being certain capacitance coefficients of the line. Since the latter must be known, in any case, in order to complete the solution of the coupling problem, the added complication in determining the electric-coupling parameters is not of great significance.

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1. INTRODUCTION

An unshielded multiwire cable or open-wire transmission line may be excited by application of an external electromagnetic field. Such a field could be generated by a stroke of lightning, by a nuclear explosion, by a high-powered radar or radio communication source, or by other similar means. This report defines procedures for determining the parameters coupling the external field to the line.

The discussion is based upon and is an expansion of the analysis contained in an earlier report.¹ A fairly general class of applied fields is postulated, the main restriction being that the magnetic intensity component lie in the transverse plane of the line.

2. PHYSICAL MODEL

Figure 1 pictures the physical situation schematically. The line segment marked N actually represents a collection of N conductors of arbitrary but invariant cross section, assumed lossless. The line segment marked G represents an (N + 1)-th (lossless) conductor taken as the potential (i.e., voltage) reference. Actually, G may be a ground plane; in that case, the impressed magnetic field is assumed parallel to that plane. On the other hand, the cable may be so far removed from extended surfaces, with no terminal connection to such surfaces, that G actually represents one of the cable conductors. In that case the cable consists of a total of (N + 1) conductors, any one of which may be taken as the reference G. For instance, the cross section of a three-conductor line is shown in figure 2 wherein the center conductor represents G of figure 1, and in which, evidently,² N = 2. In this class of situations the coordinate system is chosen so that the z-axis is parallel to the magnetic intensity, H_z^e .

The external electric intensity is assumed normal to the magnetic intensity, and both are assumed invariant in direction with respect to x. More general situations are possible but are not considered here. The electric intensity, although normal to H_z^e , is not necessarily parallel to the y-axis. However, only the y^z -component, E_y^e , affects the TEM response of the line. For instance, in the special case of immediate interest, the line is excited by a plane wave with Poynting vector in the line's transverse plane, and E-vector normal to the plane of incidence. Thus $E_y^e = 0$, and only H_z^e is significant in exciting the TEM mode.

Identify the N conductors not used as potential reference by the integers 1, ..., N. Use subscripts k (k = 1, ..., N) to identify any current or voltage associated with the k-th conductor. Then \underline{V}^i and \underline{V}^o are the line voltage vectors at the left and right ends, respectively.

¹ Frankel, Sidney, "Response of a Multiconductor Transmission Line to Excitation by an Arbitrary Monochromatic Impressed Field along the line," Sandia Laboratories, Report SC-CR-71 5076, Albuquerque, New Mexico, April 1971.

² Frankel, Sidney, "Sample Calculation, Response of Multiwire Line to External Field along the Line," Sidney Frankel and Associates, Technical Memorandum, No. FA-179, Menlo Park, California, November 1971.

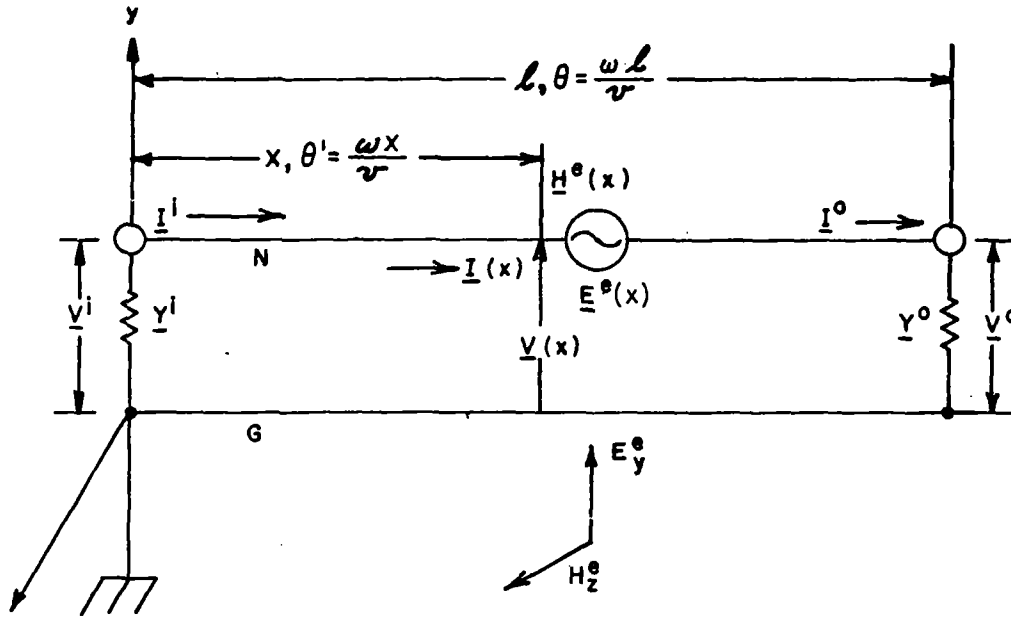


Figure 1. Schematic diagram of N -conductor line excited by external electromagnetic field.

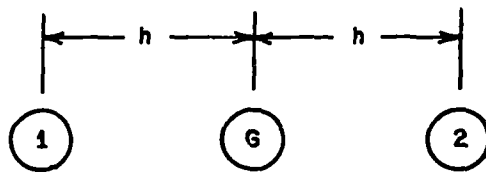


Figure 2. Line consisting of two conductors plus reference: $N = 2$.

$$\underline{V}^i = \begin{bmatrix} V_1^i \\ \cdot \\ \cdot \\ V_N^i \end{bmatrix}$$

$$\underline{V}^o = \begin{bmatrix} V_1^o \\ \cdot \\ \cdot \\ V_N^o \end{bmatrix}$$
(1)

Similarly, \underline{I}^i and \underline{I}^o are the input- and output-line current vectors with sign conventions as indicated in figure 1:

$$\underline{I}^i = \begin{bmatrix} I_1^i \\ \cdot \\ \cdot \\ I_N^i \end{bmatrix}$$

$$\underline{I}^o = \begin{bmatrix} I_1^o \\ \cdot \\ \cdot \\ I_N^o \end{bmatrix}$$
(2)

The quantities \underline{Y}^i and \underline{Y}^o are $N \times N$ termination matrices defined by

$$\underline{I}^i = -\underline{Y}^i \underline{V}^i; \quad \underline{Y}^i = [Y_{ij}^i]$$

$$\underline{I}^o = \underline{Y}^o \underline{V}^o; \quad \underline{Y}^o = [Y_{ij}^o]$$
(3)

$i, j = 1, \dots, N$

\underline{Y}^i and \underline{Y}^o may be determined from the actual terminal networks by methods discussed in earlier work.³

Since the left end of the line is at the origin in figure 1, x measures distance along the line from the left end, expressed in radians as θ' . The total length of the line is l meters or θ radians. At x , the voltage and current vectors are, respectively,

$$\underline{V}(x) = \begin{bmatrix} V_1(x) \\ \cdot \\ \cdot \\ V_N(x) \end{bmatrix}$$

$$\underline{I}(x) = \begin{bmatrix} I_1(x) \\ \cdot \\ \cdot \\ I_N(x) \end{bmatrix}$$
(4)

³ Frankel, Sidney, "Response of a Multiconductor Cable to Excitation by an Arbitrary, Single-Frequency, Constant-Impedance Source," Sandia Laboratories, Report SC-CR-71 5058, Albuquerque, New Mexico, April 1971.

\underline{Z} is the $N \times N$ line impedance matrix:

$$\underline{Z} = [Z_{ij}], \quad i, j = 1, \dots, N \quad (5)$$

The Z_{ij} may be computed from the Maxwell potential coefficients P_{ij}

$$Z_{ij} = P_{ij}/v \quad (6)$$

where v is the wave velocity of propagation on the line:

$$v = (\mu\epsilon)^{-1/2} \quad (7)$$

in which μ and ϵ are the permeability and permittivity of the dielectric medium, assumed homogeneous and isotropic (MKS units used throughout). In addition, we have the line admittance matrix:

$$\underline{Y} = [Y_{ij}] = \underline{Z}^{-1} = v\underline{C} \quad (8)$$

where

$$\underline{C} = [C_{ij}], \quad i, j = 1, \dots, N \quad (9)$$

the C_{ij} being Maxwell's coefficients of capacitance.

3. LINE-VOLTAGE EQUATIONS:

EFFECT OF EXTERNAL MAGNETIC FIELD IN TRANSVERSE PLANE

The differential equations of the line have been derived¹ using a simple extension of the method for conventional, end-excited lines. The basic laws invoked are Faraday's law of induction and the law of continuity; thus, the potential of the i -th conductor becomes:

$$\frac{dV_i}{dx} + \sum_{j=1}^N (j\omega L_{ji}) I_j - j\omega L_i^e H_z^e = 0 \quad (10)$$

This equation states that the rate of increase of potential along the i -th conductor is balanced by induced voltages due to currents on all conductors and due to the applied field H_z^e . The quantity $(L_{ji} I_j)$ is that portion of the magnetic flux per meter of line passing between the i -th conductor and reference conductor due to the current I_j flowing in the j -th conductor ($j = 1, \dots, N$). The quantity $-L_i^e H_z^e$ is that portion of the flux per meter of line passing between the i -th conductor and reference conductor due to the external field H_z^e .

Determination of the magnetic-field coupling parameter, L_i^e ($i = 1, \dots, N$), is a primary concern of this report.

The inductance coefficient matrix,

$$\underline{L} = [L_{ij}]$$

is related to the capacitance matrix by

$$\underline{L} \underline{C} = v^{-2} \underline{I}$$

where \underline{I} is the $N \times N$ unit matrix.

¹ Frankel, Sidney, "Response of a Multiconductor Transmission Line to Excitation by an Arbitrary Monochromatic Impressed Field along the Line," Sandia Laboratories, Report SC-CR-71 5076 (appx A), Albuquerque, New Mexico, April 1971.

The quantity $(j\omega L_i^e H_z^e)$ is given¹ the single-letter designation E_i^e , since it has the dimensions of a voltage gradient (V/m). In the line equation, it may be interpreted as a continuous emf source in series with the i -th conductor.

Clearly, the line potential is affected by H_z^e alone only if all currents I_j ($j = 1, \dots, N$) are zero. That is, if the conductors are terminated in open circuits,

$$\left. \frac{dV_i}{dx} \right|_{I_j=0} - j\omega L_i^e H_z^e = 0 \quad (11)$$

Then, solving equation (11) for L_i^e yields, in principle, a definition for determining that parameter:

$$L_i^e = \frac{1}{j\omega} \frac{\left. \frac{dV_i}{dx} \right|_{I_j=0}}{H_z^e} \quad (12)$$

Equation (12) requires the following: (a) leaving all conductors of the system floating with respect to reference, apply a transverse magnetic field H_z^e , which is constant over a small length of line Δx and of angular frequency ω , (b) measure the resulting voltage difference ΔV at the two ends of this small distance, and (c) calculate L_i^e from equation (12), using $\Delta V/\Delta x$ in place of dV_i/dx .

Practically, a much simpler approach is available. Consider, first, how the above experiment might be performed. Imagine a convenient scale model of the line to be set up between large parallel conducting planes carrying currents that produce the magnetic field H_z^e as in figure 3. If the actual line has a ground plane, then it replaces one of the two parallel conductors. The exciting plane(s) is at distance D from the conductor system, sufficiently large that any reaction field due to circulating currents on the open-circuited line conductors is negligible at the plane conductor surface. Furthermore, the planes are sufficiently wide that edge fringing does not affect the field applied to the conductor system. Similarly, the length of the system in the direction of propagation on the line (i.e., normal to the cross section shown) should be great enough so that measurements could be made well away from terminal-end effects. The two plane conductors could then be excited at some convenient frequency with the plane conductors shorted at the "load" end, but with the frequency low enough so that the current, I , is uniform along the line.

A simpler approach, however, is suggested by exciting the plane conductors at one end and terminating the other end in an appropriate network to prevent reflection. The dielectric then transmits a TEM field, with both transverse magnetic and transverse electric fields, which are everywhere simply related by the wave impedance n . Since well-known theory states that line-inductance coefficients may be computed from the electrostatic capacitance coefficients, the suggested

¹ Frankel, Sidney, "Response of a Multiconductor Transmission Line to Excitation by an Arbitrary Monochromatic Impressed Field along the Line," Sandia Laboratories, Report SC-CR-71 5076, Albuquerque, New Mexico, April 1971.

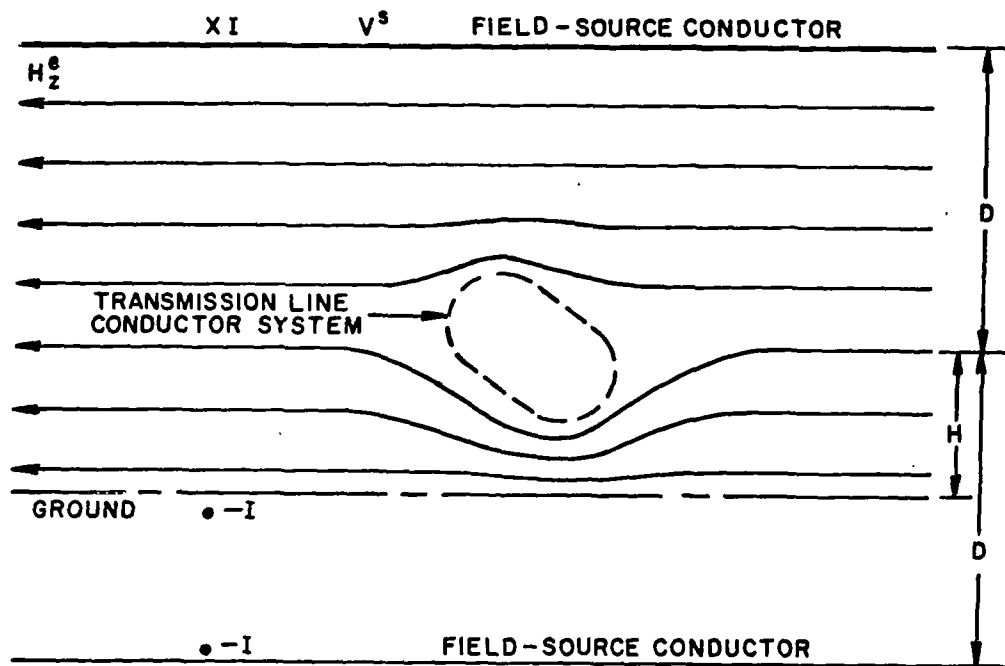


Figure 3. Experiment to excite conductor system with external magnetic field. Source of magnetic field is current on upper plane conductor, with return either in the ground plane or, if none is present, the lower field-source conductor.

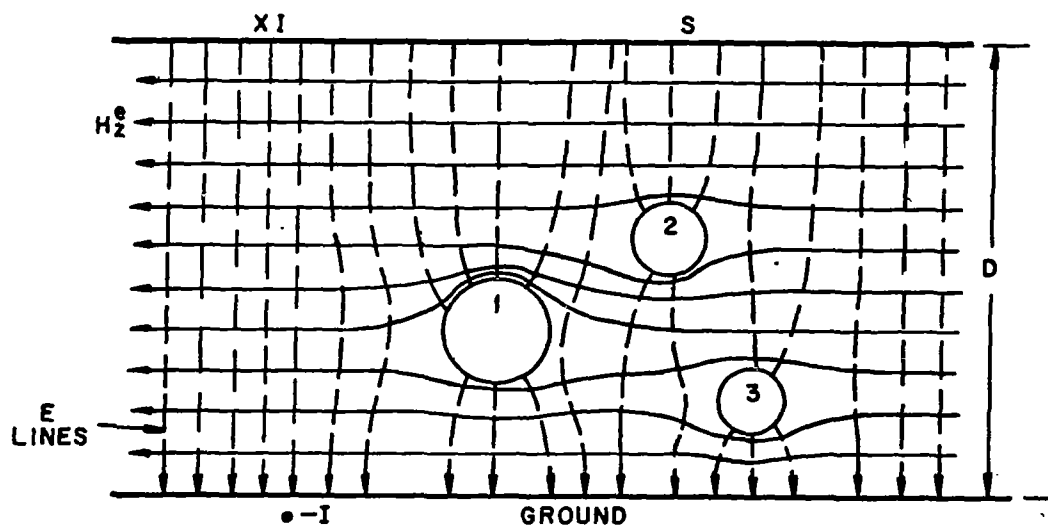


Figure 4. Model of a multiwire line immersed in a TEM field: $N = 3$.

arrangement implies that the magnetic-coupling parameter L_i^e can be determined electrostatically. This will now be proven.

4. ELECTROSTATIC ANALOG

To assist in concrete visualization, a typical system consisting of three conductors above ground is represented in figure 4. A forward wave of current I per meter of width, flows in the source plane S and returns in the ground plane G . The potential of the source plane is V^S ; the floating conductors illustrated in figure 4 (1,2,3) assume potentials V_1, V_2, V_3 , respectively. Lines of magnetic intensity are shown solid, those of electric intensity are dashed. Since these systems of lines are orthogonal, the solid lines represent, also, electrostatic equipotentials, while the dashed lines represent, also, magnetostatic equipotentials.

The total magnetic flux passing between the i -th conductor and ground, per meter of line, is given by

$$\phi_i = \mu \int_{S_i} \vec{H} \cdot \vec{n} \, dS_i$$

where S_i can conveniently be taken as the cylindrical surface generated by sweeping a one-meter-long axial line segment from the i -th conductor to ground along an electric-field line as indicated in figure 5. In that case, \vec{H} is everywhere normal to the surface and we have

$$\phi_i = \mu \int_{C_i} H \, ds$$

where ds is an element of distance along C_i . But, from elementary TEM theory,

$$H = E/\eta = E\sqrt{\epsilon/\mu}$$

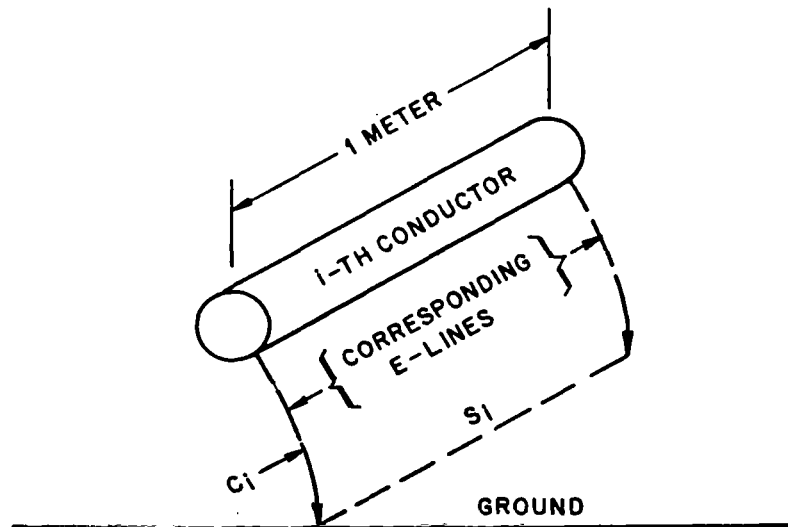


Figure 5. Generation of curve C_i bounding surface S_i by sweeping along an E-line.

Therefore

$$\phi_i = \sqrt{\mu\epsilon} \int_{C_i} E ds = -\frac{V_i}{v} \quad (13)$$

where V_i is the potential of the i -th conductor. But since

$$\phi_i = -L_i^e H_z^e$$

equation (13) yields

$$L_i^e = \frac{V_i}{v H_z^e} \quad (14)$$

V_i is proportional to V^s :

$$V_i = k_i V^s \quad (15)$$

where k_i depends on the geometry of the system and is determined as the solution of an electrostatic problem.

Far from the conductor system under investigation, where the field is uniform,

$$V^s = ED = n H_z^e D \quad (16)$$

where D is the distance between source and ground planes. Then equations (15) and (16) in equation (14) yield, for large D ,

$$L_i^e = \frac{1}{v H_z^e} (k_i n H_z^e D) = \mu k_i D$$

that is,

$$L_i^e = \lim_{D \rightarrow \infty} (\mu k_i D) \quad (17)$$

Determination of L_i^e is thus reduced to the electrostatic problem of evaluating k_i , where

$$k_i = V_i / V^s \quad (18)$$

If we are dealing with a system of conductors in the absence of a ground plane, the conductors are assumed to be located about midway between a pair of source planes charged to potentials V^s and $-V^s$, respectively, and separated by a distance $2D$ as in figure 3. One of the conductors of the group is taken as reference. Suppose it assumes a potential, V_0 , and write

$$k_0 = V^s / V_0$$

Then it is straightforward to show that

$$L_i^e = \lim_{D \rightarrow \infty} [\mu (k_i - k_0) D]$$

5. EXAMPLES FOR L_i^e

Case I: Small round wire above a ground plane

Let the wire be at height h above ground. With D and h sufficiently large compared with the wire radius, the potential at the surface of the wire is the same as that of the point at the wire

center without the wire. (Further information is available through HDL, Woodbridge Facility--pp. 5-17, eq. (45) of lecture notes for HDL Seminar, October 1971.) The value of this potential is clearly

$$V = \frac{h}{D} V^s$$

That is,

$$k = \frac{h}{D} \quad (19)$$

Substituting for k_i in equation (17),

$$L^e = \mu h \quad (20)$$

Case II: A number of small round wires above a ground plane, radii much smaller than spacings and distances to ground plane

In this case, there is no interaction among conductors and we have, by extension of equation (20),

$$L_i^e = \mu h_i, \quad i = 1, \dots, N \quad (21)$$

Case III: A system of three small round wires as in figure 2; magnetic field normal to plane of wires

In this case, the appropriate configuration for analysis/measurement is shown in figure 6. We have at any height y above the lower source plane,

$$V = -V^s + \frac{2V^s}{2D} y = \frac{y - D}{D} V^s$$

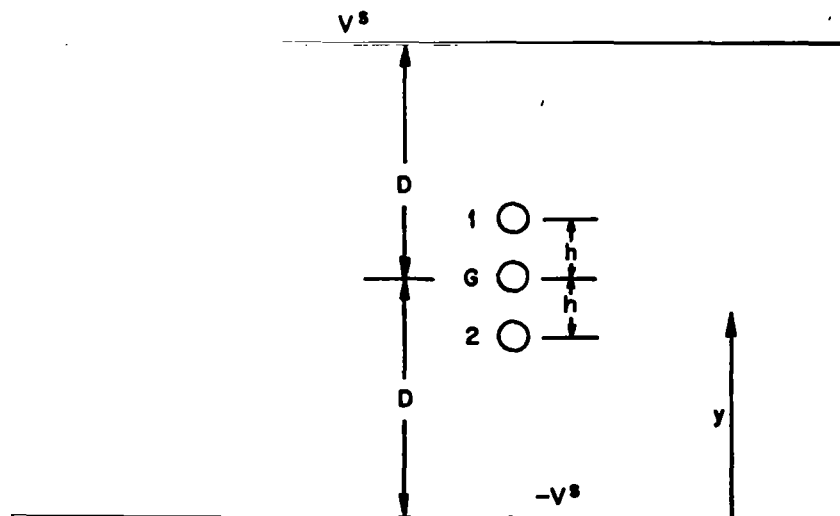


Figure 6. Determination of magnetic coupling parameter; magnetic field normal to plane of wires: $N = 2$.

We have, therefore, successively

$$V_1 = \frac{(D+h) - D}{D} v^s = \frac{h}{D} v^s$$

$$V_0 = \frac{D - D}{D} v^s = 0$$

$$V_2 = \frac{(D-h) - D}{D} v^s = -\frac{h}{D} v^s = -V_1$$

whence

$$k_1 = \frac{V_1 - V_0}{v^s} = \frac{h}{D}$$

$$k_2 = \frac{V_2 - V_0}{v^s} = -\frac{h}{D} \quad (22)$$

and equation (17) then yields

$$L_1^e = \mu h = -L_2^e \quad (23)$$

Note that this checks the values used in an earlier report.²

If the impressed magnetic intensity makes an angle ψ with the plane of the wires, then resolution into normal and tangential components shows immediately that equation (23) need only be modified by the factor $\cos \psi$ multiplying (μh) .

6. LINE CURRENT EQUATIONS: EFFECT OF EXTERNAL ELECTRIC FIELD

The second set of line differential equations is based on the law of current continuity. For the i -th conductor, we have

$$\frac{dI_i}{dx} + \sum_{j=1}^N (j\omega C_{ij} V_j) - j\omega C_i^e E_y^e = 0 \quad (24)$$

For consistency of notation, the sign of C_i^e in this report is the reverse of that reported earlier.¹

This equation states that the rate of increase of current along the i -th conductor is balanced by the time rate of change in amount of charge bound on the conductor due to the potentials on all conductors plus the time rate of change in amount of charge bound on the i -th conductor by the external field E_y^e . The quantity $C_{ij} V_j$ is that portion of the charge per meter bound on the i -th conductor due to potential V_j on the j -th conductor. The quantity $-C_i^e E_y^e$ is that portion of the charge per meter bound on the i -th conductor by the external field E_y^e . C_i^e is the electric field coupling parameter.

The quantity $(j\omega C_i^e E_y^e)$ is given the single-letter designation H_i^e ,

² Frankel, Sidney, "Sample Calculation, Response of Multiwire Line to External Field along the Line," Sidney Frankel and Associates, Technical Memorandum, No. FA-179, Menlo Park, California, November 1971.

¹ Frankel, Sidney, "Response of a Multiconductor Transmission Line to Excitation by an Arbitrary Monochromatic Impressed Field along the Line," Sandia Laboratories, Report SC-CR-71 5076, Albuquerque, New Mexico, April 1971.

since it has the dimensions of a current gradient, or magnetic intensity (A/m). In the line equations, it may be interpreted as a continuous current source impressed on the i -th conductor.

Clearly, the current is affected by E_y^e alone only if all potentials, V_j , ($j = 1, \dots, N$), are zero. Thus, the coefficient C_i^e may be determined by a model entirely similar to that for determining L_i^e , except that, instead of floating all conductors, they must be placed at reference, or ground, potential. In fact, the C_i^e may be determined from the same open-circuit model as the L_i^e , plus a knowledge of the line-capacitance matrix C , which is an array of quantities independent of any external excitation.

The procedure is derived as follows: (a) first, with E_y^e present, let all conductors "float" with respect to ground, so that they carry zero net charges, but assume potentials V_j^e ($j = 1, \dots, N$); (b) next, suppose all conductors grounded. The effect is the same as applying to them a set of potentials, $-V_j^e$, where such a set must result in a set of charges

$$q_i^e = \sum_{j=1}^N C_{ij} (-V_j^e) = - \sum_{j=1}^N C_{ij} V_j^e$$

However, as in the case of the L_i^e determination,

$$V_j^e = k_j V^s$$

where k_j has the same meaning as in equation (15). But, as argued previously,

$$V^s = -E_y^e D$$

where D is defined in figure 3. Thus,

$$\begin{aligned} q_i^e &= - \sum_{j=1}^N C_{ij} k_j (-E_y^e D) \\ &= DE_y^e \sum_{j=1}^N k_j C_{ij} \end{aligned} \quad (25)$$

Then, for large D ,

$$C_i^e = -D \sum_{j=1}^N k_j C_{ij}$$

that is,

$$C_i^e = - \lim_{D \rightarrow \infty} \left(D \sum_{j=1}^N k_j C_{ij} \right) \quad (26)$$

and

$$H_i^e = - \lim_{D \rightarrow \infty} \left(j \omega DE_y^e \sum_{j=1}^N k_j C_{ij} \right) \quad (27)$$

7. EXAMPLES FOR C_i^e

It is clear that, except for the complication involved in including the C_{ij} as linear sums in equations (24) and (25), determination of the C_i^e requires, essentially, determination of the k_j , and is, therefore, fundamentally the same problem as that of the L_i^e , though a little more complicated.

Case I: Small round wire at height h above a ground plane

Since, by equation (19),

$$k = h/D$$

we get, from equation (26),

$$C_i^e = -D \left(\frac{h}{D} \right) C_{ij} = -hC_{11}$$

If the wire radius is a ,

$$C_{11} = \frac{2\pi\epsilon}{\ln(2h/a)}$$

so that

$$C_i^e = - \frac{2\pi\epsilon h}{\ln(2h/a)} \quad (28)$$

Case II: A number of small wires of radius a_j above a ground plane, widely separated

We have

$$k_j = h_j/D$$

so that

$$C_i^e = - \sum_{j=1}^N h_j C_{ij} \quad (29)$$

In this case the C_{ij} are complicated, the complexity increasing rapidly with N . However, the potential coefficients p_{ij} are relatively simple. We have (Lecture notes for HDL Seminar, October 1971, pp. 4-14--available through HDL, Woodbridge Facility)

$$p_{ij} = \frac{1}{2\pi\epsilon} \ln \frac{r'_{ij}}{r_{ij}} \quad (30)$$

where, for $j \neq i$, ($i = 1, \dots, N$)

- r_{ij} = distance between centers, i -th and j -th conductors
- r'_{ij} = distance from center of i -th conductor to center of image of j -th conductor
- = distance from center of j -th conductor to center of image of i -th conductor

For $j = i$, ($i = 1, \dots, N$), we have

- r_{ii} = radius of i -th conductor

r'_{ii} = distance between i -th conductor and its image
 = twice the height of the i -th conductor above ground
 = $2h_i$

Writing

$$\underline{C}^e = [C_i^e], \text{ a } 1 \times N \text{ column vector}$$

$$\underline{h} = [h_i], \text{ a } 1 \times N \text{ column vector}$$

we get, from equation (29)

$$\underline{C}^e = -\underline{C} \underline{h} = -\underline{P}^{-1} \underline{h} \quad (31)$$

where

$$\underline{P} = [p_{ij}] = \underline{C}^{-1} \quad (32)$$

Equation (31) easily reduces to equation (28) for $N = 1$.

Case III: System of three small round wires as in figure 2;
electric field parallel to plane of wires

From equation (22),

$$k_1 = -k_2 = h/D \quad (33)$$

By equation (26),

$$\begin{aligned} C_1^e &= -D [k_1 C_{11} + k_2 C_{12}] \\ &= -D \left[\frac{h}{D} C_{11} - \frac{h}{D} C_{12} \right] \\ &= -h(C_{11} - C_{12}) \end{aligned} \quad (34)$$

$$\begin{aligned} C_2^e &= -D [k_1 C_{12} + k_2 C_{22}] \\ &= -D \left[\frac{h}{D} C_{12} - \frac{h}{D} C_{22} \right] \\ &= h(C_{22} - C_{12}) = -C_1^e \end{aligned} \quad (35)$$

since $C_{11} = C_{22}$. Thus, we find

$$\begin{aligned} P_{11} = P_{22} &= \frac{1}{\pi \epsilon} \ln \frac{h}{a} \\ P_{12} &= \frac{1}{2\pi \epsilon} \ln \frac{h}{2a} \end{aligned} \quad (36)$$

where matrix inversion yields

$$C_{11} = C_{22} = \frac{P_{11}}{P_{11}P_{22} - P_{12}^2} = \frac{P_{11}}{P_{11}^2 - P_{12}^2} \quad (37)$$

and

$$C_{12} = -\frac{P_{12}}{P_{11}^2 - P_{12}^2} \quad (38)$$

Then, from equations (37) and (38)

$$C_{11} - C_{12} = \frac{P_{11} + P_{12}}{P_{11}^2 - P_{12}^2} = \frac{1}{P_{11} - P_{12}}$$

$$C_{11} - C_{12} = \frac{2\pi\epsilon}{\ln(2h/a)}$$

so that, finally, equations (34) and (35) yield

$$C_1^e = -C_2^e = -\frac{2\pi\epsilon h}{\ln(2h/a)} \quad (39)$$

which should be compared with equation (28).

8. DISCUSSION

The foregoing analysis has yielded definitions sufficient, in principle, for determination of the external-field-coupling parameters L_i^e and C_i^e ($i = 1, \dots, N$) for an N-line (that is, a line of N conductors plus reference-potential conductor), and for the consequent calculation of the equivalent forcing-function parameters E_i^e and H_i^e . These may then be used in the appropriate formulas for calculating the response of a multi-wire line.¹

Some simple examples of coupling-parameter determination have been given when the line consists of wires whose radii are small compared with the distances between them or their distances to nearby ground planes. In general, however, such analytic determinations are not possible, and recourse must be had either to modeled or analog experiments, or to numerical analysis of the appropriate electrostatic problem.

Even for the case of small wires, no indication has been given of how "small" is small enough. It would appear that as a next analytical task, it should be useful to study the proximity effect for two not-so-small conductors.

An additional question, the answer to which is needed for any non-analytical determination of the coupling parameters, involves the magnitude of the source distance D (fig. 3 and 4). Too small a value of D leads to excessive error due to reaction of the line on the source plane; too large a value leads to excessively cumbersome experiments or excessive computer storage requirements.

9. CONCLUSIONS

We have shown that both magnetic- and electric-field coupling parameters can be determined by solution of the same electrostatic problem for certain potential ratios, k_i . Determination of the electric-field parameter is a little more complicated in that it requires linear combinations of the k_i , in which the coefficients of the linear forms are certain capacitance coefficients of the line. However, since these need to be determined in any case in order finally to evaluate the response of the line, the additional complexity is not of great significance.

¹ Frankel, Sidney, "Response of a Multiconductor Transmission Line to Excitation by an Arbitrary Monochromatic Impressed Field along the Line," Sandia Laboratories, Report SC-CR-71 5076 (appx A), Albuquerque, New Mexico, April 1971.

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13. ABSTRACT <p>Magnetic- and electric-coupling parameters between a multi-conductor transmission line and an external electromagnetic field are defined. An electrostatic method for determining both types of parameters is proposed. Both types of parameters require determination of the same set of constants. While the magnetic-coupling parameters are directly proportional to these constants (one constant for each conductor), the electric parameters are proportional to a linear combination of the constants, the coefficients of the linear forms being certain capacitance coefficients of the line. Since the latter must be known, in any case, in order to complete the solution of the coupling problem, the added complication in determining the electric-coupling parameters is not of great significance.</p>		

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