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FIELD COUPLING PARAMETERS FOR A SINGLE ROUND WIRE CLOSE TO A GROUND PLANE OR TWO LARGE ROUND WIRES IN FREE SPACE

by

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ABSTRACT

The transverse electromagnetic (TEM) response of a conventional two-conductor transmission line to an external electromagnetic field is characterized by certain quantities termed magnetic- and electric-field coupling parameters. Values of these parameters for a line consisting of two small round conductors, or one small conductor above an infinite ground plane, have been determined previously. In this report we obtain simple expressions for the parameters when the round conductors are of arbitrary, but equal, radius. The error incurred in assuming the small-wire approximation is tabulated in the range $1.1 \le \rho \le 10$, where ρ is the height/radius ratio of the single conductor above ground, or the spacing/diameter ratio of the two equiradius conductors.

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1. INTRODUCTION

The response of a multiwire transmission line to an external electromagnetic field is characterized by certain quantities termed magnetic- and electric-field coupling parameters. A previous report discusses a procedure for determining both magnetic and electric coupling parameters from electrostatic considerations alone.[1] Generally, these parameters are not susceptible to determination by mathematical analysis.

For the case of any number of wires with radii small compared to their separation, the coupling parameters are readily calculated.[1-3] The lossless system of a single round conductor at arbitrary distance above a ground plane, or, equivalently, the case of two equiradius conductors at arbitrary spacing between conductors is also soluble; the solution is the subject of this report.

2. ANALYSIS

Figure 1 shows a round conductor (No. 1) near a ground plane. An additional "source plane," No. 2 conductor, is shown parallel to the ground plane at a large distance, D, that eventually will be made infinite thereby leaving the desired configuration. The external magnetic field, H_z^e, is transverse to the line, parallel to the ground plane, and independent of y. The external electric field is normal to the magnetic field and independent of y, but only the y-component, E_y^e, affects the transverse electromagnetic (TEM) response of the line.

Voltage and current at any distance, x, along the line are expressed as functions of continuous distribution of voltage and current sources along the line which are defined, respectively, as[4]

$$E^e(x) = j\,\omega L^e H_z^e(x) \quad \text{volts/meter}$$

$$H^e(x) = j\,\omega C^e E_y^e(x) \quad \text{henry/meter} \tag{1}$$

The quantities L^e (henrys) and C^e (farads) are the magnetic- and electric-field coupling parameters, respectively. A procedure for determining these parameters follows.

Apply a potential, V^s, to No. 2 conductor, and let No. 1 "float." This results in a charge, q_2, on No. 2 and a potential, V_1, on No. 1 where $q_1 = 0$. Introducing

$$k = V_1/V^s$$

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[1] Frankel, S., "Externally-Excited Transmission Line: Definition of Procedures for Determining Coupling Parameters," Frankel Associates report FA-180, December 1971.

[2] Harrison, C. W., "Generalized Theory of Impedance-Loaded Multiconductor Transmission Lines in an Incident Field," Sandia Lab. report SC-R-71 3303, July 1971.

[3] Taylor, C. D., R. S. Satterwhite, and C. W. Harrison, Jr., "The Response of a Terminated Two-Wire Transmission Line Excited by a Non-Uniform Electromagnetic Field," Sandia Lab. report SC-R-65-978A, November 1965.

[4] Frankel, S., "Response of a Multiconductor Transmission Line to Excitation by an Arbitrary Monochromatic Impressed Field Along the Line," Sandia Lab report SC-CR-71 5076, April 1971.

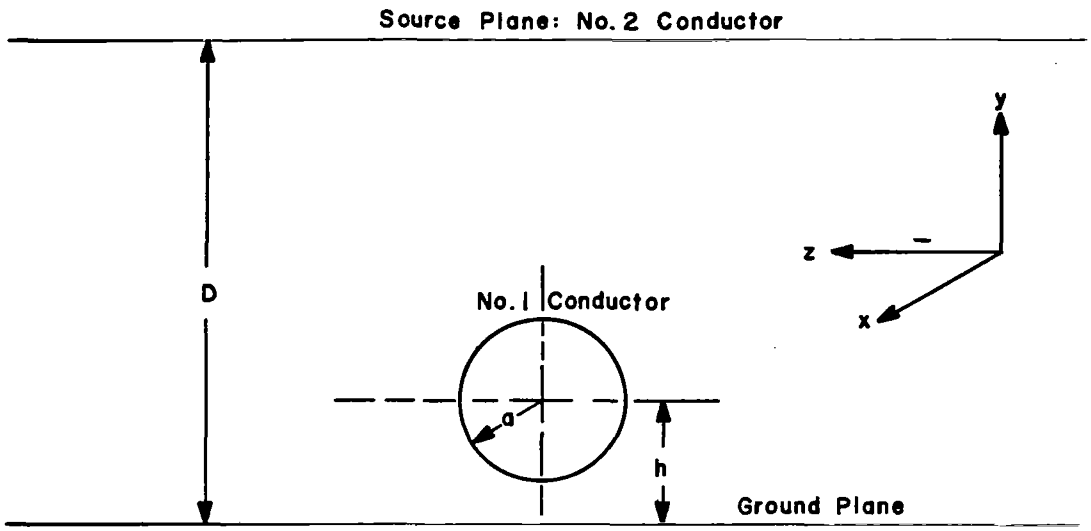

Figure 1. Round conductor of arbitrarily large radius above a ground plane.

allows the magnetic-field coupling parameter to be written as

$$L^e = \lim_{D\to\infty} (\mu \, k \, D) = \mu \lim_{D\to\infty} (k \, D) \tag{2}$$

where μ is the magnetic permeability of the medium. Similarly, the electric-field coupling parameter is given by

$$C^e = - \lim_{D\to\infty} (k \, D \, C_{11})$$

where C_{11} is the Maxwell coefficient of self-capacitance for conductor No. 1. Since C_{11}, by itself, has a finite limit as $D \to \infty$, the last result can be written as

$$C^e = - C_0 \lim_{D\to\infty} (k \, D) \tag{3}$$

where C_0 is the well-known capacitance per meter of line between the round conductor and ground, i.e.

$$C_0 = \lim_{D\to\infty} C_{11} = \frac{2\pi\epsilon}{\cosh^{-1}\left(\frac{h}{a}\right)} .$$

Thus, for both parameters, the problem reduces to the determination of

$$K = \lim_{D \to \infty} (k\, D) \tag{4}$$

which is fairly straightforward.

With potential V^s, charge q_2 on No. 2, and with $q_1 = 0$, the conductor potentials can be written as

$$V_1 = p_{12}\, q_2$$

$$V_2 = p_{22}\, q_2 = V^s$$

where p_{12} and p_{22} are Maxwell's potential coefficients. Thus,

$$k = (V_1/V^s)\,\big|_{q_1=0} = p_{12}/p_{22} = -\,C_{12}/C_{11} \tag{5}$$

where C_{11} and C_{12} are Maxwell's capacitance coefficients. As noted previously, $C_{11} \to C_0$ as $D \to \infty$. The coefficient C_{12} is determined by finding the charge induced on No. 2, q_2, by the potential V_1 on No. 1 when $V_2 = 0$:

$$C_{12} = (q_2/V_1)\,\big|_{V_2=0}. \tag{6}$$

In that which follows, C_{12} is determined using the assumption that D is large.

First, the ground plane is replaced by the images of the round conductor and the source plane as indicated in figure 2. Next, the field in the vicinity of the wire and its image is simulated by removing the wires and replacing their distributed surface charges ($\pm q_1$, say) by a pair of line charges separated by an appropriate distance,[5] d. The resulting configuration is that of a finite logarithmic dipole midway between parallel planes.

In figure 3(a) the dipole is shown at the origin of coordinates in the complex z-plane. By means of the transformation

$$w = \exp(\pi\, z/2\, D), \tag{7}$$

this configuration is transformed to a dipole above an infinite plane in the complex w-plane as shown in figure 3(b). In the z-plane the equation of the dipole axis is

$$z = j\, y, \quad -\,d/2 \le y \le d/2.$$

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[5] Smythe, W. R., Static and Dynamic Electricity, Chapter 4.

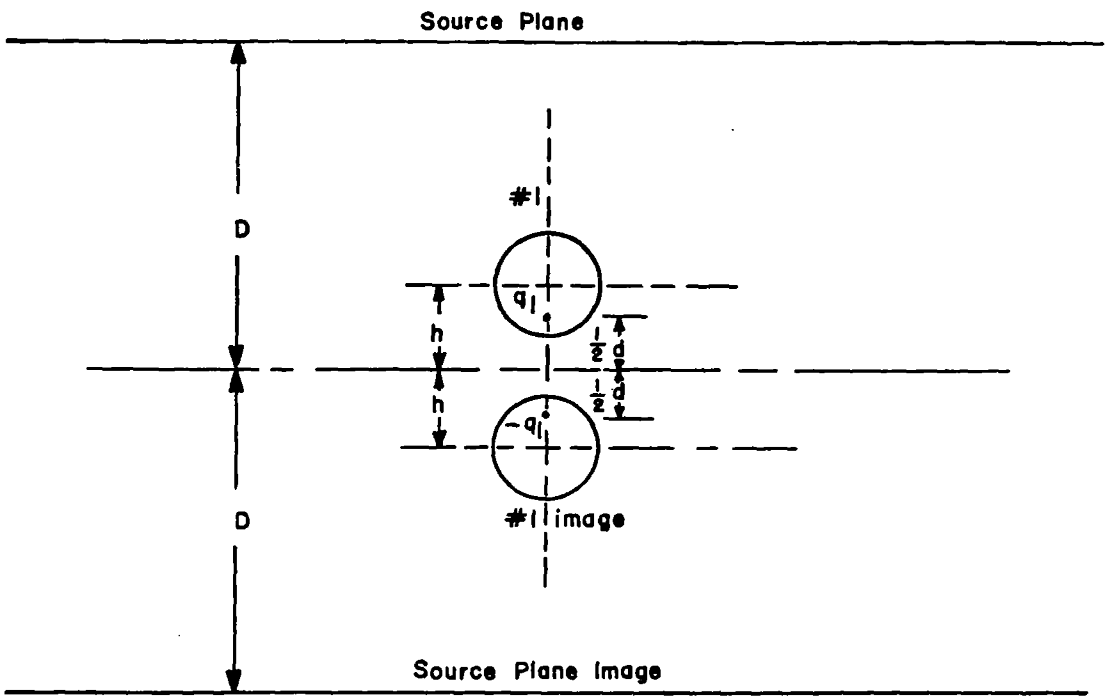

Figure 2. Ground plane replaced by images of conductor and source plane.

Substituting in equation (7),

$$w = \exp(j\,\pi\,y/2\,D), \quad -d/2 \le y \le d/2$$

As $D \to \infty$, this can be approximated over the length of the dipole, by

$$w \to 1 + j\,\pi\,y/2\,D, \quad -\frac{d}{2} \le y \le +\frac{d}{2}$$

Thus, in the w-plane the dipole becomes infinitesimal, of length $(\pi d/2D) \to 0$, while its axis is parallel to the ground plane situated at distance $u = 1$ from the plane. The latter, in turn, is replaced by the image of the dipole as in figure 4.

The potential of a point in the vicinity of an infinitesimal logarithmic dipole is

$$V_p = (m_w/2\,\pi\,\epsilon)\,(\cos\theta/r) \tag{8}$$

where m_w is the dipole moment in the w-plane:

$$m_w = (\pi\,d\,q_1/2\,D), \tag{9}$$

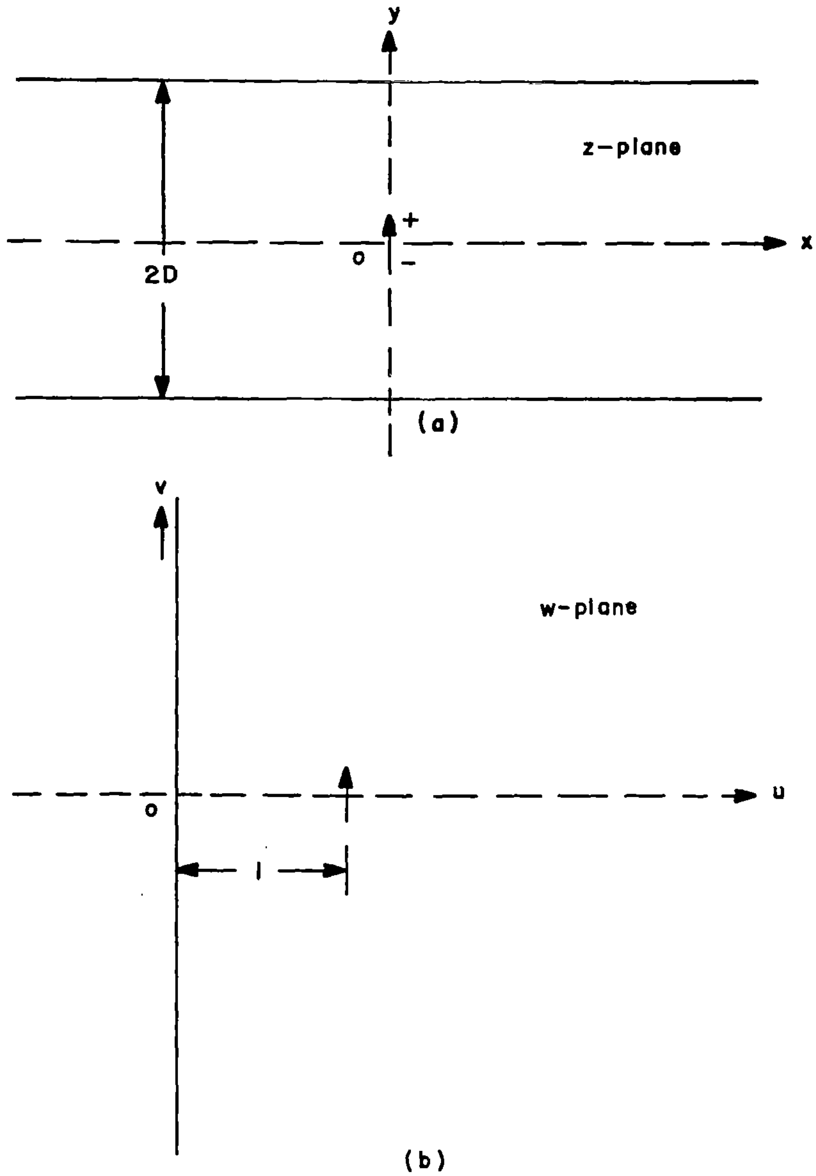

Figure 3. Transformation from a dipole between parallel planes to a dipole above a single plane.

r is the distance from the dipole to the field point, and θ is the angle between the dipole axis (zero reference in the sense of the dipole vector) and the radius vector to the field point. For a dipole located at u = 1 in the w-plane, equation (8) translates to

$$V_P' = \frac{m_w}{2\pi\epsilon} \frac{R \sin\phi}{1 + R^2 - 2R\cos\phi} \qquad (10)$$

where R and ϕ are the polar coordinates in the w-plane as indicated in figure 5. Similarly, for the dipole at u = − 1,

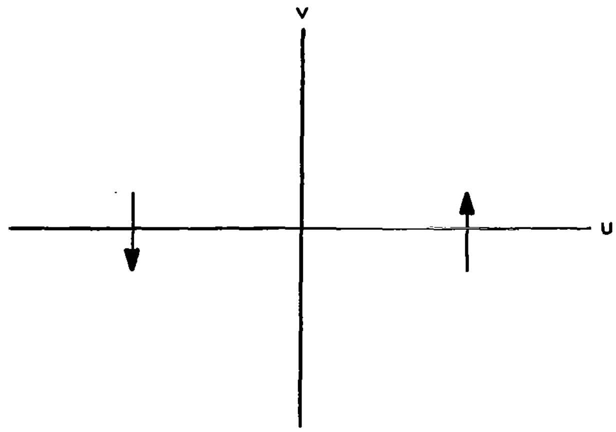

Figure 4. Ground plane replaced by dipole image.

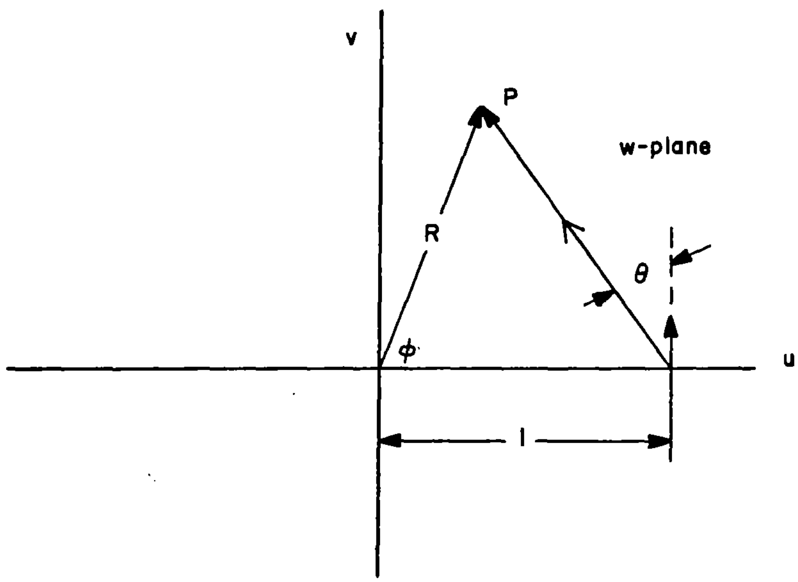

Figure 5. Potential of a dipole at u = 1 referred to the origin in the w-plane.

$$V_P'' = -\frac{m_w}{2\pi\epsilon}\,\frac{R\sin\phi}{1 + R^2 + 2R\cos\phi} \tag{11}$$

The total potential is, therefore,

$$\begin{aligned} V_P &= V_P' + V_P'' \\ &= \frac{m_w}{\pi\epsilon}\,\frac{R^2\sin 2\phi}{1 - 2R^2\cos 2\phi + R^4} \end{aligned} \tag{12}$$

Note that $V_P = 0$ for $\phi = \pm\pi/2$, i.e. on the v-axis.

The charge q_2 on the source plane (No. 2, fig. 1) is the same as the charge on the positive v-axis of figure 3(b). The surface charge density is

$$\begin{aligned}\sigma &= \frac{\epsilon}{R}\left(\frac{\partial V_P}{\partial \phi}\right)_{\phi=\pi/2} \\ &= -\frac{2\,m_w}{\pi}\,\frac{R}{(1+R^2)^2} \\ &= -\frac{2\,m_w}{\pi}\,\frac{v}{(1+v^2)^2}\end{aligned} \qquad (13)$$

The total charge on the positive v-axis is obtained by integrating equation (13):

$$\begin{aligned}q_2 &= -\frac{2\,m_w}{\pi}\int_0^\infty \frac{v\,d\,v}{(1+v^2)^2} \\ &= -\frac{m_w}{\pi} \\ &= -\frac{d}{2\,D}\,q_1\end{aligned} \qquad (14)$$

The following relations are available from elementary electrostatics,[6] for $D \to \infty$:

$$d = 2\,h\,\frac{\Gamma^{-1}-\Gamma}{\Gamma^{-1}+\Gamma}$$

where

$$\Gamma = \exp\,[-\,(2\,\pi\,\epsilon\,V_1)/q_1] = \exp\,[-\,(2\,\pi\,\epsilon)/C_0] = e^{-\mu}$$

for

$$\mu = \frac{2\,\pi\,\epsilon}{C_0} = \cosh^{-1}\rho;\ \rho = \frac{h}{a}.$$

Thus, d becomes

$$\begin{aligned}d &= 2\,h\,\tanh\,[\cosh^{-1}\rho] \\ &= 2\,h\,\frac{\sqrt{\rho^2-1}}{\rho}.\end{aligned} \qquad (15)$$

[6]Frankel, S., Applications of Multiconductor Transmission Line Theory, Lecture Notes for Seminar held at HDL Woodbridge Facility, Oct. 1971, pp. 4-7 ff.

Since

$$q_1 \to C_0 V_1$$

equation (14) becomes

$$q_2 \to -\frac{d}{2D} C_0 V_1$$

Therefore equation (6) yields

$$C_{12} \to -\frac{d}{2D} C_0$$

and equation (5) becomes

$$k \to -\frac{d}{2D}\frac{C_0}{C_{11}} \to \frac{d}{2D}$$

such that equation (4) may be written as

$$K = \lim_{D\to\infty} (k\, D) = d/2 \qquad (16)$$

$$= h\frac{\sqrt{\rho^2 - 1}}{\rho}$$

Finally, equations (2) and (3) yield

$$L^e = \mu\, h\frac{\sqrt{\rho^2 - 1}}{\rho}$$

$$C^e = - C_0\, h\frac{\sqrt{\rho^2 - 1}}{\rho} \qquad (17)$$

$$= -\frac{2\pi\epsilon h}{\cosh^{-1}\rho}\frac{\sqrt{\rho^2 - 1}}{\rho}.$$

For two equi-radius conductors in free space, whose axes are in a plane normal to the magnetic field,

$$L^e = \mu\, D_c\frac{\sqrt{\rho^2 - 1}}{\rho}$$

$$C^e = -\frac{\pi\epsilon D_c}{\cosh^{-1}\rho}\frac{\sqrt{\rho^2 - 1}}{\rho} \qquad (18)$$

Table I. Error in Approximation for Various Values of ρ.

ρ	F	E(%)
1.1	0.417	140
1.2	0.553	81.0
1.3	0.640	56.1
1.5	0.748	33.8
1.7	0.810	23.3
2.0	0.866	15.2
2.5	0.917	9.0
3.0	0.943	6.0
5.0	0.980	2.0
7.0	0.990	1.0
10.0	0.995	0.5

where

$\rho = D_c/d_c$

D_c = distance between conductor centers

d_c = conductor diameter = 2a

If the magnetic field makes an angle, ψ, with the normal to the plane of the wire axes, then equations (18) are multiplied by cos ψ:

$$L^e = \mu D_c \frac{\sqrt{\rho^2 - 1}}{\rho} \cos \psi$$

$$C^e = - \frac{\pi \epsilon D_c}{\cosh^{-1} \rho} \frac{\sqrt{\rho^2 - 1}}{\rho} \cos \psi \qquad (19)$$

3. DISCUSSION OF RESULTS

The proximity factor

$$F = \frac{\sqrt{\rho^2 - 1}}{\rho} \qquad (20)$$

has the following asymptotic properties:

$$F \sim \sqrt{2 (\rho - 1)} \quad \text{as} \quad \rho \to 1 \qquad (21)$$

and

$$F \sim 1 - \frac{1}{2 \rho^2} \quad \text{as} \quad \rho \to \infty. \qquad (22)$$

The "small wire" approximation assumes $\rho \to \infty$, $F = 1$. The error in the approximation, E, is given in Table I for various values of ρ.

Insofar as coupling to the external field is concerned, the effective height of the wire above ground, or the effective spacing between wires in free space, is diminished by the factor F. In terms of the magnetic field coupling to the wires in free space, it appears clear that when the conductors are sufficiently separated the impressed field is diverted equally on both sides of each wire. However, as the wires are brought closer together, interaction between wires begins to occur and the field tends to become diverted more and more around the whole conductor system. In the limit, of course, as the wires make contact, the field becomes completely diverted and the coupling goes to zero.

Interestingly, the electric field parameter does not go to zero as $\rho \to 1$ but, rather,

$$\lim_{\rho \to 1} C^e = -\pi \epsilon D_c = -\pi \epsilon d_c$$

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