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# SCATTERING OF ELECTROMAGNETIC RADIATION BY APERTURES

# III. AN ALTERNATIVE INTEGRAL EQUATION WITH ANALYTIC KERNELS FOR THE SLOTTED CYLINDER PROBLEM


by

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ABSTRACT

An alternative integral equation is derived for the problem of a plane electromagnetic wave incident on a perfectly conducting cylinder of infinite length containing a slot of arbitrary central angle. The electric vector of the incident plane wave lies along the axis of the slotted cylinder and the conductor is taken to be of infinitesimal thickness. The kernels of this integral equation for the diffraction problem are non-singular and the formal equation contains the electric field in the opening and the surface current on the conductor.

## FOREWORD

This report is the third in a series of reports concerning electromagnetic scattering by apertures in conducting screens and conducting enclosures. The first two reports of this series were published by the U.S. Naval Ordnance Laboratory at White Oak, Maryland, with the numbers NOLTR 70-58 and NOLTR 72-25. The work reported herein was partially sponsored by the Defense Nuclear Agency under Subtask EB-088.

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## 1. INTRODUCTION

The problem of scattering of electromagnetic radiation by a perfectly conducting cylinder of infinite length containing a slot parallel to its axis was not resolved until very recently. Progress had been made toward the solution of this diffraction problem, but only approximate solutions of greatly restricted validity had evolved. Presumably, Sommerfeld[1] published the first theoretical work, formulating the solution in terms of series expansions for the fields internal and external to the slotted cylinder. The application of boundary conditions did not lead to an explicit evaluation of the unknown expansion coefficients that appear in this formulation. To obtain an approximate solution, Sommerfeld assumed the slot arc length to be small compared with the wavelength. By applying the method of least squares, he obtained analytical expressions for the coefficients in this asymptotic limit. Sommerfeld furnished no numerical or experimental results. Morse and Feshbach[2] also used series expansions for the field. They, however, expressed the unknown series expansion coefficients in terms of an integral over the unknown electric field in the slot. The requirement that the azimuthal magnetic field must be continuous in the slot led to an integral equation for the unknown slot field. Morse and Feshbach approximately solved this integral equation by assuming that the distribution of the field over the slot, for the slotted cylinder, is proportional to the electrostatic slot field distribution for a slotted plane. An analytical result for this proportionality constant was obtained from the integral equation at the midpoint of the slot. No numerical or experimental results were furnished in this work either. Turner[3] derived an integral equation for the surface current on a half-cylindrical mirror and by a variational approximation obtained an expression for the total scattering cross-section per unit length of the mirror. Again, no experimental or numerical results were furnished in this special case.

Perhaps the most extensive earlier treatment of scattering by slotted cylinders was that of Macrakis. (The results of this study were presented as a PhD Dissertation to Harvard University in 1958 —"Backscattering Cross Section of Slotted Cylinders.") The backscattering cross section was of primary interest in this study. The approach utilized the method of Green's functions and yielded the same integral equation for the electric field in the slot as obtained by Morse and Feshbach. Macrackis assumed a static form for the slot field with unknown constants and carried through a variational calculation to obtain the backscattering cross section. Although the results are limited in usefulness to narrow slots, Macrackis did present analytical forms and numerical results. Probably more important, he published experimental results. He reported measurements of backscattering cross sections for a slotted cylinder and a half-cylindrical mirror for many wavelengths. Until very recently, the data of Macrackis had not been successfully interpreted and compared with theoretical results except at long wavelengths. These recent results will be included in a paper to be published by the present authors in the near future.

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[1] A. Sommerfeld, *Partial Differential Equations*, Academic Press, New York, 29-31, 1949.

[2] P.M. Morse, H. Feshbach, *Methods of Theoretical Physics*, Part II, 1387-1398, McGraw-Hill, New York, 1953.

[3] R. Turner, "Scattering of Plane Electromagnetic Radiation by an Infinite Cylindrical Mirror," Technical Report No. 161, Cruft Laboratory, Harvard University, Cambridge, Massachusetts, 1953.

In 1969, Barth[4] also considered the slotted cylinder scattering problem. His method is fundamentally similar to that of Morse and Feshbach; an essential difference, however, is that he assumed a finite series for the slot field. The corresponding finite number of series coefficients was evaluated by satisfying the integral equation for the slot field at an equal number of spatial points. This produces a finite solvable algebraic system at each wavelength of interest. Barth calculated numerically the distribution of the electric field in the slot at 500 megahertz (MHz) for a one-meter (1-m) radius cylinder with a slot that subtends an angle of 60 deg. In particular, Barth numerically obtained the electric field at the center of the slot in the Morse and Feshbach formulation. Thus, Barth did not publish calculations of the backscattering cross section or the scattered fields, it was inconvenient to compare his theoretical results with experiment. Nevertheless, there is a serious objection that can be raised concerning his formulation of the problem. It has been shown by Jones[5] that the normal component of the magnetic field at an edge possesses a singularity. Barth's assumed form of the slot electric field generates a normal component of the magnetic field that vanishes at the edge.

Earlier attempts to solve the problem of plane wave scattering by a slotted cylinder have been reviewed. We have recently investigated this problem and are presently preparing our findings for publication. This report presents the formulation of an alternative integral equation that has not been exploited heretofore—not even by the present authors.

## 2. INTEGRAL EQUATION FOR THE ELECTRIC FIELD IN THE SLOT

The target is a conducting infinite circular cylinder of radius $\rho_0$ whose axis is coincident with the z-axis and which contains an infinite slot parallel to the axis. This slot subtends the half angle $\phi_0$ at the cylinder axis as shown in figure 1. A monochromatic plane wave is assumed normally incident on the slotted cylinder. The incident wave vector, $\bar{k}_i$, is assumed parallel to the plane bisecting the slot and containing the z-axis. For simplicity, the x-axis is chosen to lie in this plane. Finally, the electric vector of the incident plane wave is assumed parallel to the cylinder axis.

After suppressing the harmonic time dependence $e^{-i\omega t}$, the incident fields are given by

$$\bar{E}_i(\bar{r}) = E_{iz}(\rho,\phi)\bar{e}_z = E_0 e^{-ik\rho\cos\phi}\bar{e}_z \ , \tag{1}$$

$$\bar{H}_i(\bar{r}) = H_{iy}(\rho,\phi)\bar{e}_y = H_0 e^{-ik\rho\cos\phi}\bar{e}_y \ , \tag{2}$$

where

$$k = 2\pi/\lambda = \omega/c \quad . \tag{3}$$

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[4] M.J. Barth, "Interior Fields of a Slotted Cylinder Irradiated with an Electromagnetic Pulse," Technical Report No. AFSWC-TR-69-9, Air Force Special Weapons Center, Kirtland Air Force Base, New Mexico, August 1969.

[5] D.S. Jones, *The Theory of Electromagnetism*, Pergamon Press, New York, 566-569, 1964.

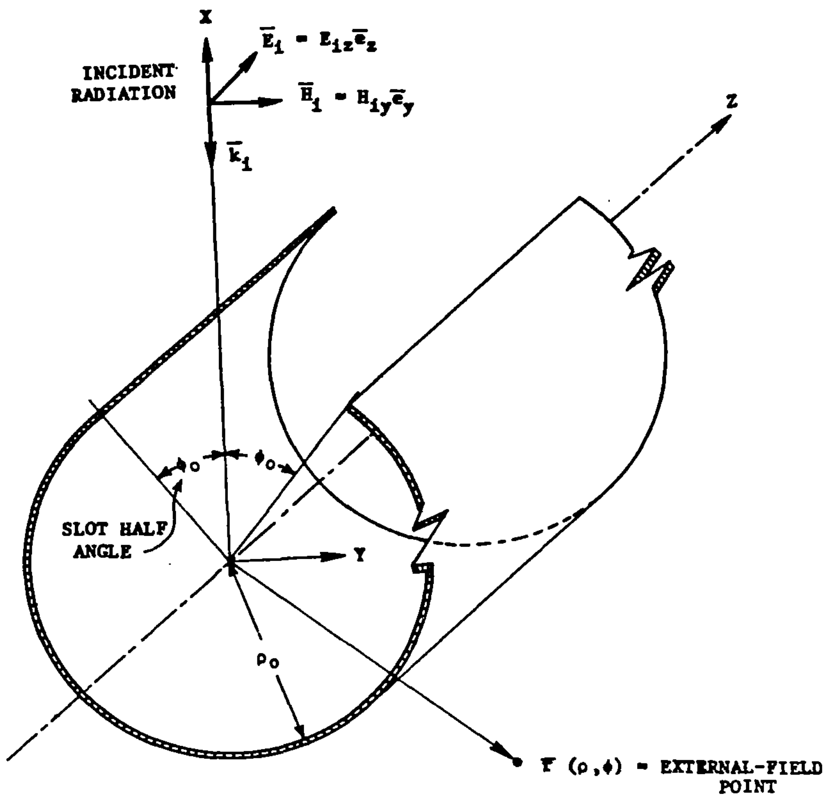


Figure 1. Linearly polarized plane electromagnetic wave incident on a slotted cylinder.

The amplitudes of the incident fields are related by

$$E_o = \sqrt{\mu_o/\varepsilon_o}H_o \quad , \tag{4}$$

where $\mu_o$ and $\varepsilon_o$ are, respectively, the free-space magnetic permeability and electric permittivity.

From the symmetry of the problem, it is clearly indicated that the electric field has only a z-component. This field satisfies the scalar wave equation at source free points,

$$\nabla^2 E_z(\bar{r}) + k^2 E_z(\bar{r}) = 0 \tag{5}$$

Furthermore, all the electromagnetic field components are independent of the z-coordinate. From the electric field, we obtain the magnetic field components via

$$H_\rho(\rho,\phi) = \frac{1}{i\omega\mu_o\rho}\frac{\partial E_z(\rho,\phi)}{\partial\phi} \tag{6}$$

$$H_\phi(\rho,\phi) = -\frac{1}{i\omega\mu_o}\frac{\partial E_z(\rho,\phi)}{\partial\rho} \tag{7}$$

Symmetry considerations also lead to the fact that the electric field is an even function of the coordinate $\phi$. Hereinafter, we shall append the superscript (i) to any quantity to indicate that it refers to the "interior" of the cylinder—that is, for $\rho<\rho_0$. Similarly, we use the superscript (e) to denote the "exterior" or $\rho>\rho_0$.

The form of the solution to the scalar-wave equation (eq. 5), for the interior region will be taken to be the series

$$E_z^{(i)}(\rho,\phi) = \sum_{m=0}^{\infty} \frac{J_m(k\rho)}{J_m(\eta)} A_m \cos m\phi \quad , \tag{8}$$

where the $A_m$ are expansion constants and $\eta$ is defined as

$$\eta = \frac{2\pi\rho_0}{\lambda} = k\rho_0 \quad , \tag{9}$$

that is, a convenient parameter describing the ratio of the slotted cylinder radius to the wavelength of the incident radiation.

A formal solution is constructed for the region exterior to the slotted cylinder, which is the sum of a well-known field that would be present in the absence of the slot[6], and, a field arising from the presence of the slot. Thus, we have

$$E_z^{(e)}(\rho,\phi) = E_0 \sum_{m=0}^{\infty} \varepsilon_m (-i)^m \left[J_m(k\rho) - \frac{J_m(\eta)}{H_m^{(1)}(\eta)} H_m^{(1)}(k\rho)\right] \cos m\phi \tag{10}$$

$$+ \sum_{m=0}^{\infty} \frac{H_m^{(1)}(k\rho)}{H_m^{(1)}(\eta)} C_m \cos m\phi \quad ,$$

where the coefficients $C_m$ are as yet undertermined, and

$$\varepsilon_m = \begin{cases} 1, & m = 0 \\ 2, & m = 1,2,3,\ldots \end{cases} \tag{11}$$

Now at $\rho=\rho_0$, the interior and exterior fields must be identical in the slot and both must vanish on the conducting cylinder itself. This requires that for all values of $\phi$,

$$\sum_{m=0}^{\infty} C_m \cos m\phi = \sum_{m=0}^{\infty} A_m \cos m\phi \quad ;$$

and consequently,

$$C_m = A_m \quad . \tag{12}$$

[6] W.R. Smythe, *Static and Dynamic Electricity*, 3rd Ed; McGraw-Hill, New York, 485, 1968.

The electric field in the slot at $\rho=\rho_0$ is denoted in this report by $E(\phi)$. The electric field at $\rho=\rho_0$ can be written as

$$E_z^{(i)}(\rho_0,\phi) = E_z^{(e)}(\rho_0,\phi) = \begin{cases} E(\phi), & 2\pi-\phi_0<\phi<\phi_0 \\ 0, & \phi_0\leq\phi\leq 2\pi-\phi_0 \end{cases} ; \qquad (13)$$

or, equivalently,

$$\sum_{m=0}^{\infty} A_m \cos m\phi = \begin{cases} E(\phi), & 2\pi-\phi_0<\phi<\phi_0 \\ 0, & \phi_0\leq\phi\leq 2\pi-\phi_0 \end{cases} \qquad (14)$$

Therefore, the expansion coefficients can be expressed in terms of the slot distribution of electric field—that is,

$$A_m = \frac{\varepsilon_m}{2\pi}\int_{-\phi_0}^{\phi_0} d\phi' E(\phi')\cos m\phi' \qquad (15)$$

Consequently, one way of explicitly solving the scattering problem is via a determination of the electric field in the slot.

An integral equation for the slot electric field will now be obtained from the physical requirement that the tangential magnetic field must be continuous in the aperture. That is,

$$H_\phi^{(i)}(\rho_0,\phi) = H_\phi^{(e)}(\rho_0,\phi) \quad 2\pi-\phi_0<\phi<\phi_0 , \qquad (16)$$

so that equations (7), (8), and (10) lead to

$$\sum_{m=0}^{\infty} \frac{A_m}{J_m(\eta)\, H_m^{(1)}(\eta)} \cos m\phi = E_0 \sum_{m=0}^{\infty} \frac{\varepsilon_m (-i)^m}{H_m^{(1)}(\eta)} \cos m\phi, \quad 2\pi-\phi_0<\phi<\phi_0 , \qquad (17)$$

where we have used the Wronskian relation

$$J_m'(\eta)\, H_m^{(1)}(\eta) - J_m(\eta)\, H_m^{(1)\prime}(\eta) = \frac{2}{\pi i \eta} . \qquad (18)$$

The surface current on an integral cylinder without a slot is denoted as $K_z^{(o)}(\phi)$ and is given by the well-known formula

$$K_z^{(o)}(\phi) = \frac{2}{\pi\eta}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_0 \sum_{m=0}^{\infty} \frac{\varepsilon_m (-i)^m}{H_m^{(1)}(\eta)} \cos m\phi \quad 0\leq\phi<2\pi . \qquad (19)$$

Equation (17) may now be written as

$$\sum_{m=o}^{\infty} \frac{A_m}{J_m(\eta) H_m^{(1)}(\eta)} \cos m\phi = \frac{\pi\eta}{2} \sqrt{\frac{\mu_o}{\varepsilon_o}}\, K_z^{(o)}(\phi), \quad -\phi_o<\phi<\phi_o \quad . \tag{20}$$

Substituting for the $A_m$ from equation (5) yields the integral equation for the electric field in the slot, namely,

$$\int_{-\phi_o}^{\phi_o} d\phi' E(\phi') \sum_{m=o}^{\infty} \frac{\varepsilon_m}{2\pi} \frac{\cos m(\phi-\phi')}{J_m(\eta) H_m^{(1)}(\eta)} = \frac{\pi\eta}{2} \sqrt{\frac{\mu_o}{\varepsilon_o}}\, K_z^{(o)}(\phi) \tag{21}$$

for $-\phi_o<\phi<\phi_o$. It is this particular integral equation—or rather the attempts to solve it—that constituted the historical view presented in the first part of this report.

Before continuing, two properties of the kernel of this integral equation should be pointed out. First, note that the kernel is singular at the zeros of the Bessel functions. This forces certain requirements to be placed on the slot electric field at the set of frequencies that generate the zeros of the $J_m$'s. The physics of this will be presented in a subsequent report in this series of publications. Another property of this kernel that should be mentioned is that it is a divergent infinite series. This is readily demonstrated by employing the large order asymptotic forms for $J_m(\eta)$ and $H_m^{(1)}(\eta)$

$$J_m(\eta) \rightarrow \frac{1}{\sqrt{2\pi m}} \left(\frac{e\eta}{2m}\right)^m$$

$$H_m^{(1)}(\eta) \rightarrow -i \sqrt{\frac{2}{\pi m}} \left(\frac{e\eta}{2m}\right)^m$$

as $m \rightarrow \infty$. Asymptotically, the product goes as

$$J_m(\eta)\; H_m^{(1)}(\eta) \rightarrow \frac{-i}{\pi m}$$

as $m \rightarrow \infty$ and, in turn, the series diverges.

The attempts of earlier workers to solve equation (21) were essentially all focused on assuming an approximate functional form for the electric field in the opening, which facilitates carrying out the integration analytically. This ostensibly generates a tractable approach. In the Rayleigh limit of approximation, the kernel reduces to a more manageable form that permits one to obtain —via Galerkin's method, for example—solutions for narrow slots. This long wavelength approximation to the problem will also be included in a subsequent report.

## 3. INTEGRAL EQUATION FOR THE SURFACE CURRENT ON THE CONDUCTOR

The diffraction problem has been cast into the following formulation, where the governing relation is an integral equation for the surface current on the slotted cylinder. Let $\bar{K}$ be the true surface current on the surface defined by $\rho=\rho_o$ so that

$$\bar{K}(\phi) = \bar{e}_\rho(\phi) \times [\bar{H}^{(e)}(\rho_o,\phi) - \bar{H}^{(i)}(\rho_o,\phi)] \quad , \tag{22}$$

where $\bar{e}_\rho(\phi)$ is an outward unit vector normal to the cylinder axis at an angle $\phi$ from the x-axis. This results in a surface current that has only a z-component which is given by

$$K_z(\phi) = H_\phi^{(e)}(\rho_o,\phi) - H^{(i)}(\rho_o,\phi) \quad ; \tag{23}$$

in turn, we find

$$K_z(\phi) = \frac{2}{\pi\eta}\sqrt{\frac{\varepsilon_o}{\mu_o}}\left\{E_o\sum_{m=o}^{\infty}\frac{\varepsilon_m(-i)^m}{H_m^{(1)}(\eta)}\cos m\phi - \sum_{m=o}^{\infty}\frac{A_m}{J_m(\eta)H_m^{(1)}(\eta)}\cos m\phi\right\} \tag{24}$$

The first term in equation (24) is the surface current for the cylinder in the absence of the slot; the second term represents the modification due to the presence of the opening.

Let us introduce the new simplifying notation

$$B_m \equiv E_o\frac{\varepsilon_m(-i)^m}{H_m^{(1)}(\eta)} - \frac{A_m}{J_m(\eta)H_m^{(1)}(\eta)} \quad \text{for } m = 0, 1, 2,... \tag{25}$$

which enables us to rewrite equation (24) more compactly as

$$K_z(\phi) = \frac{2}{\pi\eta}\sqrt{\frac{\varepsilon_o}{\mu_o}}\sum_{m=o}^{\infty} B_m\cos m\phi \quad . \tag{26}$$

Clearly, the surface current vanishes in the opening in the cylinder. Consequently, the surface current becomes

$$K_z(\phi) = \begin{cases} K(\phi), & \phi_o\le\phi\le 2\pi-\phi_o \\ 0 & , 2\pi-\phi_o<\phi<\phi_o \end{cases} \tag{27}$$

where we use the notation $K_z(\phi) \equiv K(\phi)$ on the conducting cylinder. Inverting equation (26), we obtain the formal expression for the expansion coefficients $B_m$: which, as is to be expected, are determined by the surface current on the conductor

$$\frac{2}{\pi\eta}\sqrt{\frac{\varepsilon_o}{\mu_o}}\, B_m = \frac{\varepsilon_m}{2\pi}\int_{\phi_o}^{2\pi-\phi_o} d\phi' K(\phi')\cos m\phi' \quad . \tag{28}$$

An integral equation is derived below for the surface current distribution. Multiply through equation (25) by $J_m(\eta)H_m^{(1)}(\eta)\cos m\phi$, sum over the index m, and rearrange to get

$$\sum_{m=0}^{\infty} B_m J_m(\eta) H_m^{(1)}(\eta)\cos m\phi = -\sum_{m=0}^{\infty} A_m \cos m\phi \tag{29}$$

$$+ E_0 \sum_{m=0}^{\infty} \varepsilon_m (-i)^m J_m(\eta)\cos m\phi \quad .$$

Using equation (28), the last result becomes the integral equation for surface current:

$$\frac{\eta}{4}\sqrt{\frac{\mu_0}{\varepsilon_0}} \int_{\phi_0}^{2\pi-\phi_0} d\phi' K(\phi')\; H_0^{(1)}\left[2\eta\left|\sin\left(\frac{\phi-\phi'}{2}\right)\right|\right] = E_{iz}(\rho_0,\phi);\phi_0<\phi<2\pi-\phi_0 \tag{30}$$

where we have used the identity[7]

$$\sum_{m=0}^{\infty} \varepsilon_m J_m(\eta) H_m^{(1)}(\eta)\cos m(\phi-\phi') = H_0^{(1)}\left[2\eta\left|\sin\frac{\phi-\phi'}{2}\right|\right] \quad . \tag{31}$$

Just as in the earlier situation with the integral equation for slot distribution of the electric field, no exact analytical solution for the surface current has been reported in the literature. We point out, however, that the kernel in equation (30) is a well-behaved function and readily lends itself to a direct numerical calculation for the problem. There arises, furthermore, a considerable simplification of equation (30) in the Rayleigh limit. This facilitates obtaining analytical results for narrow cylindrical strips. Such results have been obtained and will be discussed in a forthcoming report.

## 4. ALTERNATIVE INTEGRAL EQUATION FORMULATION

Several relations exist between the electric field in the opening and the surface current on the conducting cylinder. Examination of equation (25) reveals this to be quite evident. The first of these is embodied in equations (15) and (24), which together yield

$$K(\phi) = K_z^{(0)}(\phi) - \frac{2}{\pi\eta}\sqrt{\frac{\varepsilon_0}{\mu_0}} \int_{-\phi_0}^{\phi_0} d\phi' E(\phi') \sum_{m=0}^{\infty} \frac{\varepsilon_m}{2\pi} \frac{\cos m(\phi-\phi')}{J_m(\eta) H_m^{(1)}(\eta)} \tag{32}$$

for $\phi_0 \le \phi \le 2\pi-\phi_0$. Equation (32) provides a means of calculating the surface current on the conductor from knowledge of the electric field in the slot.

[7] I.S. Gradshteyn, I.M. Ryzhik, *Tables of Integrals, Series and Products*, Academic Press, New York, Formula 8.531, case 2, 979, 1965.

A second relation is obtained from equation (29) where the use of equations (14), (28), and (31) lead to

$$E(\phi) = E_{iz}(\rho_o,\phi) - \frac{\eta}{4}\sqrt{\frac{\mu_o}{\varepsilon_o}}\int_{\phi_o}^{2\pi-\phi_o} d\phi' K(\phi') H_o^{(1)}[2\eta|\sin\left(\frac{\phi-\phi'}{2}\right)|] \tag{33}$$

for $-\phi_o<\phi<+\phi_o$, which tells us how to find the slot electric field if we know the surface current distribution on the conductor.

A third and significant relation can also be derived. The derivation of this relation begins by multiplying equation (25) through by $(-i)^m J_m(\eta)$ $\cos m\phi$ and summing over m to obtain

$$\sum_{m=o}^{\infty} \frac{A_m}{H_m^{(1)}(\eta)} (-i)^m \cos m\phi + \sum_{m=o}^{\infty} B_m (-i)^m J_m(\eta) \cos m\phi = \tag{34}$$

$$\sum_{m=o}^{\infty} \frac{\varepsilon_m (-i)^m}{H_m^{(1)}(\eta)} [E_o(-i)^m J_m(\eta)] \cos m\phi \quad .$$

An integral representation of the Bessel function, i.e.

$$(-i)^m J_m(\eta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi' e^{-i\eta\cos\phi'} \cos m\phi' \quad ,$$

leads to

$$E_o(-i)^m J_m(\eta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi' \; E_{iz}(\rho_o,\phi')\cos m\phi' \quad . \tag{35}$$

Equation (35) and the integral representations of $A_m$ and $B_m$ allow us to write equation (34) as

$$\int_{-\phi_o}^{\phi_o} d\phi' E(\phi') \sum_{m=o}^{\infty} \frac{\varepsilon_m (-i)^m}{H_m^{(1)}(\eta)} \cos m\phi \cos m\phi'$$

$$+ \frac{\pi\eta}{2}\sqrt{\frac{\mu_o}{\varepsilon_o}} \int_{\phi_o}^{2\pi-\phi_o} d\phi' K(\phi') \sum_{m=o}^{\infty} \varepsilon_m (-i)^m J_m(\eta) \cos m\phi \cos m\phi'$$

$$= \int_{-\pi}^{\pi} d\phi' \; E_{iz}(\rho_o,\phi') \sum_{m=o}^{\infty} \frac{\varepsilon_m (-i)^m}{H_m^{(1)}(\eta)} \cos m\phi \cos m\phi' \quad ,$$

which can be written in the more succinct form

$$\int_{-\phi_0}^{\phi_0} E(\phi') \; K_z^{(o)}(\phi-\phi') + \int_{\phi_0}^{2\pi-\phi_0} d\phi' \; E_{iz}(\rho_0,\phi-\phi')K(\phi') \tag{36}$$

$$= \int_{-\pi}^{\pi} d\phi' E_{iz}(\rho_0,\phi') \; K_z^{(o)}(\phi-\phi') = \int_{-\pi}^{\pi} d\phi' E_{iz}(\rho_0,\phi-\phi')K_z^{(o)}(\phi') \; .$$

Integrals of this type were dubbed "reaction" integrals by Rumsey[8].

## 5. CONCLUSIONS

Equation (36) essentially represents a third formulation of the diffraction problem—meaning, of course, that one could choose to solve this integral equation rather than equation (21) or (30). It should be noted that the kernels of the integral operators in equation (36) are well behaved—that is, they have no singularities. Although such an attempt has not been reported, it would seem that a self-consistent numerical solution of equation (36), and consequently the diffraction problem itself, is quite feasible.

[8] V.H. Rumsey, Phys. Rev. *94*, 1483, 1954.