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ON THE EXCITATION OF A COAXIAL LINE THROUGH A SMALL APERTURE IN THE OUTER SHEATH

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ABSTRACT

A coaxial line having a small aperture in the outer-sheath and terminating in arbitrary impedances is considered to be illuminated by an incident plane wave. The excitation of currents in the termination impedances is shown to depend upon three independent factors. Simple analytical forms are given for equivalent voltage and current sources of the aperture excitation.

coaxial transmission lines, shielding

INTRODUCTION

The field that penetrates a small aperture in an electromagnetic shield is studied in a recent interaction note.[1] The problem is of interest since a time varying field penetrating the aperture may excite unwanted currents in the shielded region. In general this presents a formidable electromagnetic boundary value problem. However, various aspects of the problem may be studied by considering simple geometrical configurations. This paper is concerned with such a simplification, the excitation of a coaxial line through a small aperture in the sheath.

In an earlier interaction note small aperture diffraction theory is used to determine the penetration field.[1] This field is shown to be proportional to the current and charge densities that would exist in the region of the aperture if it were electrically shorted. Furthermore, it is shown that the penetration field has the same spacial dependence as static electric and magnetic dipoles except when the operating frequency is very near the frequency for a cavity mode of the shield interior.

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[1] Superscripts refer to the list of references at the end of this paper.

Recently the excitation of a coaxial line through a small aperture was studied by Harrison and King.[2] The resulting formulation involved an application of the reciprocal theorem. Their approximations are discussed in detail and compared with those of the formulation presented here.

The excitation of currents in a coaxial line through a small aperture in the sheath is shown to depend upon three independent factors: (1) the external currents and charges on the sheath, (2) the aperture configuration, and (3) the interior configuration of the coaxial line. Furthermore it is shown that equivalent voltage and current sources may be defined for the excitation of transmission line currents. Simple analytical forms are derived for these sources and a physical interpretation is given for the resulting expressions.

Sample numerical calculations are given to illustrate the application of the presented formulas.

ANALYSIS

A coaxial line with a small aperture cut in its sheath and impedances connected in series with the inner conductor at its ends is shown in Figure 1. Illuminating the exterior of the coaxial line is a plane wave as shown. Further the coaxial line is considered to be thin in order that the usual antenna theory may be applied to determine the exterior current and charge densities on the coaxial sheath. Under this condition the current on the sheath is essentially axial. The center of the aperture is located at (o,o,z_o) and the sheath extends from $z = 0$ to $z = s$.

The axial electric field that penetrates the aperture and drives the coaxial line may be obtained from an earlier interaction note.[1] It is

$$E_z(x,y,z) = - \frac{\eta k^2 M_o}{4\pi} \frac{e^{-jkr}}{(kr)^2} \left(-1 + j \frac{1}{kr}\right) ky + \frac{k^3 P_o}{4\pi\varepsilon} \frac{e^{-jkr}}{(kr)^3} \left[-1 + j \frac{3}{kr} + \frac{3}{(kr)^2}\right] k^2 (z-z_o) y \tag{1}$$

where

$$r = \sqrt{x^2 + y^2 + (z-z_o)^2} \tag{2}$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$\eta = \sqrt{\mu/\varepsilon}$$

Here P_o and M_o are the equivalent electric and magnetic dipole moments, respectively, of the aperture. For the aperture shown the electric dipole is directed along the y axis and the magnetic dipole along the x axis. The dipole moments depend upon two independent factors, the external charges and currents and the aperture configuration.

<u>Dipole Moments and the Impressed Field</u>

The effective (or equivalent) dipole moment of the field distribution in a small aperture are related to the field components that would exist at the position of the aperture if the aperture were shorted.[1,3] That is

$$\left.\begin{aligned} \vec{M}_o &= \alpha \cdot \vec{H}_o \\ \vec{P}_o &= \varepsilon\, \alpha \cdot \vec{E}_o \end{aligned}\right\} \qquad (3)$$

where α is a dyadic depending upon the aperture geometry and $(\vec{E}_o, \vec{H}_o)$ is the electromagnetic field at the position of the aperture if it were shorted.

Often it is more convenient to express the field components at the position of the aperture in terms of the surface current and charge densities. For the case of a coaxial cable with a small aperture, the total axial current on the exterior surface of the sheath may be readily obtained. If the cable is thin compared with wavelength then only the axial surface current need be considered. Hence

$$\left.\begin{aligned} \vec{M}_o &= \alpha_{xx} \frac{I_z^{ext}(z_o)}{2\pi b} \hat{x} \\ \vec{P}_o &= \alpha_{yy} \frac{\rho^{ext}(z_o)}{2\pi b} \hat{y} \end{aligned}\right\} \qquad (4)$$

where $I_z^{ext}(z_o)$ is the total external axial current at the $z = z_o$ cross section of the cable and $\rho^{ext}(z_o)$ is the charge per unit length along the external surface of the coaxial cable. Also α_{xx} and α_{yy} are the indicated elements of the dyadic α. From the equation of continuity of charge

$$\rho^{ext}(z_o) = \frac{j}{\omega} \frac{\partial}{\partial z} I_z^{ext}(z_o) \qquad (5)$$

The current distribution along a finite cylinder illuminated by an incident plane wave may be determined using standard techniques. For relatively short cylinders, $ks < 4\pi$, and broadside incidence the one term approximation of King[4] may be used

$$I_z^{ext}(z) = j \frac{4\pi}{\eta} \frac{E_z^{inc}}{k} \frac{\cos k(z-s/2) - \cos ks/2}{\Psi_{dU} \cos ks/2 - \Psi_U} \qquad (6)$$

where

$$\Psi_{dU} = (1-\cos ks/2)^{-1} \int_{-s/2}^{s/2} dz'(\cos kz' - \cos ks/2) \cdot [K(0,z') - K(s/2,z')]$$

$$\Psi_U = \int_{-s/2}^{s/2} dz'(\cos kz' - \cos ks/2)\ K(s/2,z')$$

$$K(z,z') = \exp[-jk\sqrt{(z-z')^2 + b^2}] \ / \ \sqrt{(z-z')^2 + b^2}$$

Of course the charge per unit length may be obtained by differentiating (6) according to (5).

Dipole Moments and the Aperture Configuration

The elements at the dyadic α depend entirely upon the aperture configuration. But analytical expressions for these elements have been obtained only for the elliptical aperture in an infinite plane.[1] The non zero elements are

$$\alpha_{xx} = -\frac{\pi}{3}\frac{\ell_1^3\ e^2}{K(e^2)-E(e^2)} \tag{7}$$

$$\alpha_{yy} = \frac{\pi}{3}\frac{\ell_1^3\ (1-e^2)}{E(e^2)} \tag{8}$$

$$\alpha_{zz} = -\frac{\pi}{3}\frac{\ell_1^3\ (1-e^2)e^2}{E(e^2)-(1-e^2)K(e^2)} \tag{9}$$

where $K(e^2)$ and $E(e^2)$ are complete elliptic integrals[5] of the first and second kinds, ℓ_1 is the length of the semimajor axis and e is the eccentricity of the ellipse*. In (7) and (9) the major axis is considered aligned with the x axis otherwise the expressions would be reversed with the major axis aligned with the z axis.

Dyadic elements for more general aperture shapes must be approximated.[1] And for elliptical apertures in curved surfaces (7)-(9) remain valid provided $\ell_1 \ll$ radius of curvature of the surface.

Interior Line Excitation

The aperture radiates into the interior of the coaxial line. That is, a distributed voltage is applied to the coaxial line. Along the inner conductor the differential applied voltage at the z cross section is[6]

$$dV_i(z) = -E_z(0,b,z)dz \tag{10}$$

And the differential voltage applied to the sheath of the coaxial line at the cross section z is

$$dV_0(z) = -\frac{1}{2\pi b}\left[\oint_C E_z(x,y,z)d\ell\right]dz \tag{11}$$

where C is the contour about the inner surface of the sheath at the cross section z. Applying a simple change of

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*The eccentricity is $e = (1+(\ell_1/\ell_2)^2)^{\frac{1}{2}}$

variable (11) becomes

$$dV_0(z) = -\frac{1}{\pi}\left[\int_0^{\pi} E_z(2b\sin\phi\cos\phi,\ 2b\sin^2\phi,\ z)\,d\phi\right]dz \tag{12}$$

The differential voltage driving the transmission line mode is

$$dV(z) = dV_i(z) - dV_0(z) \tag{13}$$

Using (1), (10) and (12) in (13) yields

$$dV(z) = \frac{\eta k^3 M_0}{4\pi}\left[g_2(z) - j\,g_3(z)\right]dz$$

$$-\frac{k^3 P_0}{4\pi\varepsilon}\left[g_3(z) - j\,3\,g_4(z) - 3\,g_5(z)\right]k(z-z_0)\,dz \tag{14}$$

where

$$g_n(z) = \frac{2kb}{\pi k^n}\int_0^{\pi}\left\{\frac{e^{-jk\sqrt{(z-z_0)^2+4b^2\sin^2\phi}}}{\left[(z-z_0)^2+4b^2\sin^2\phi\right]^{n/2}}\sin^2\phi\right.$$

$$\left.-\frac{1}{2}\,\frac{e^{-jk\sqrt{(z-z_0)^2+b^2}}}{\left[(z-z_0)^2+4b^2\sin^2\phi\right]^{n/2}}\right\}d\phi \tag{15}$$

It is noted that the $\{g_n(z)\}$ functions are sharply peaked about $z = z_0$. Whereas $g_2(z)$, $g_3(z)(z-z_0)$, and $g_4(z)(z-z_0)$ are all bounded, both $g_3(z)$ and $g_5(z)(z-z_0)$ possess logarithmic singularities at $z = z_0$.

For convenience, let the coaxial cable be represented by the two wire line terminated in impedances Z_s and Z_0, and driven from an arbitrarily located point in the line by the impedanceless generator developing voltage $dV(z)$ as given in (14). Evidently the differential generator current is given by

$$dI_g = \frac{dV(z)}{Z(s-z)+Z(z)} \tag{16}$$

where $Z(s-z)$, the impedance seen looking into the the transmission line above the cross section z, is

$$Z(s-z) = Z_c \frac{Z_s + Z_c \tanh \gamma(s-z)}{Z_c + Z_s \tanh \gamma(s-z)} \tag{17}$$

and $Z(z)$, the impedance seen looking into the transmission line below the cross section z, is

$$Z(z) = Z_c \frac{Z_0 + Z_c \tanh \gamma z}{Z_c + Z_0 \tanh \gamma z} \tag{18}$$

In the interest of brevity, the standard transmission line symbolism used here will not be defined.[7,8] Using standard transmission line considerations the load impedance currents are obtained as

$$dI(s) = dI_g \frac{Z_c}{Z_c \cosh \gamma(s-z) + Z_s \sinh \gamma(s-z)} \tag{19}$$

and

$$dI(0) = -dI_g \frac{Z_c}{Z_c \cosh \gamma z + Z_0 \sinh \gamma z} \tag{20}$$

Substituting (16) into (19) and (20) yields

$$dI(0) = - \frac{dV(z)}{D} [Z_c \cosh \gamma(s-z) + Z_s \sinh \gamma(s-z)] \tag{21}$$

and

$$dI(s) = \frac{dV(z)}{D} [Z_c \cosh \gamma z + Z_0 \sinh \gamma z] \tag{22}$$

where

$$D = Z_c(Z_c + Z_s) \cosh \gamma s + (Z_c^2 + Z_0 Z_s) \sinh \gamma s$$

The total current in the load impedances may be obtained by substituting (14) into (21) and (22) and integrating over the length of the cable. Thus

$$I(0) = - \frac{\eta k^3 M_0}{4\pi D} \int_0^s dz[Z_c \cosh \gamma(s-z) + Z_s \sinh \gamma(s-z)G_1(z) + \frac{k^3 P_0}{4\pi\varepsilon D} \int_0^s dz[Z_c \cosh \gamma(s-z) + Z_s \sinh \gamma(s-z)]G_2(z) \tag{23}$$

and

$$I(s) = \frac{\eta k^3 M_0}{4\pi D} \int_0^s dz[Z_c \cosh \gamma z + Z_0 \sinh \gamma z]G_1(z)$$

$$- \frac{k^3 P_0}{4\pi\epsilon D} \int_0^s dz[Z_c \cosh \gamma z + Z_0 \sinh \gamma z]G_2(z) \qquad (24)$$

where

$$G_1(z) = g_2(z) - j\, g_3(z)$$

$$G_2(z) = [g_3(z) - j\, 3g_4(z) - 3g_5(z)]k(z-z_0)$$

Just as for the $g_n(z)$ functions, $G_1(z)$ and $G_2(z)$ are sharply peaked about $z = z_0$ with both possessing logarithmic singularities. Therefore, it is to be expected that the integrals (23) and (24) may be well approximated by using a Taylor series expansion for the portions of the integrands multiplying $G_1(z)$ and $G_2(z)$, i.e.

$$\int_0^s dz\, f(z)\, G_{\substack{1\\2}}(z) = f(z_0) \int_0^s dz\, G_{\substack{1\\2}}(z) + f'(z_0) \int_0^s dz\, G_{\substack{1\\2}}(z)(z-z_0) + \ldots \qquad (25)$$

where the prime indicates the derivative with respect to the arguement of the function f. If the coaxial cable is thin, $(kb)^2 << 1$ and the aperture not too close to the ends of the cable, $z_0^2 >> b^2$ and $(s-z_o)^2 >> b^2$, then the leading terms of (26) and (27) become good approximations to the integrals. Thus

$$\int_0^s dz\ f(z)\ G_1(z) \simeq 2f(z_0) \int_{z_0}^{\infty} dz\ G_1(z) \qquad (26)$$

and because $G_2(z)$ is antisymmetric about $z = z_0$

$$\int_0^s dz\ f(z)\ G_2(z) \simeq 2f'(z_0) \int_{z_0}^{\infty} dz\ G_2(z)(z-z_0) \qquad (27)$$

Furthermore, it is easily exhibited that

$$\int_{z_0}^{\infty} dz\ G_1(z) \simeq j \int_{z_0}^{\infty} dz\ G_2(z)(z-z_0) \simeq + j \frac{1}{2k^2 b} \qquad (28)$$

Using (25)-(28) in (23) and (24) yields

$$I(0) \simeq - j \frac{\eta k^2 M_0}{4\pi k b \varepsilon} \frac{1}{D} [Z_c \cosh \gamma(s-z_0) + Z_s \sinh \gamma(s-z_0)$$

$$- \frac{k P_0}{4\pi k b} \frac{\gamma}{D} [Z_s \cosh \gamma(s-z_0) + Z_c \sinh \gamma(s-z_0)] \qquad (29)$$

and

$$I(s) \simeq j \frac{\eta k^2 M_0}{4\pi kb} \frac{1}{D} [Z_c \cosh \gamma z_0 + Z_0 \sinh \gamma z_0]$$

$$- \frac{kP_0}{4\pi\varepsilon kb} \frac{\gamma}{D} [Z_0 \cosh \gamma z_0 + Z_c \sinh \gamma z_0] \qquad (30)$$

Suppose now the transmission line is driven from an idealized voltage generator developing e_g an idealized current generator developing i_g both located at $z = z_0$ as shown in Figure 2. By using standard techniques the load currents are obtained as

$$I(0) = - \frac{e_g}{D} [Z_c \cosh \gamma(s-z_0) + Z_s \sinh \gamma(s-z_0)]$$

$$+ \frac{Z_c i_g}{D} [Z_s \cosh \gamma(s-z_0) + Z_c \sinh \gamma(s-z_0)] \qquad (31)$$

and

$$I(s) = \frac{e_g}{D} [Z_c \cosh \gamma z_0 + Z_0 \sinh \gamma z_0]$$

$$+ \frac{Z_c i_g}{D} [Z_0 \cosh \gamma z_0 + Z_c \sinh \gamma z_0] \qquad (32)$$

The equivalence of (29) and (31) as well as (30) and (32) is established when

$$e_g = j \frac{\eta k M_0}{4\pi b} = j\omega \frac{\mu M_o}{4\pi b} \qquad (33)$$

$$i_g = - \frac{\gamma P_0}{4\pi\varepsilon b} \frac{1}{Z_c} = - j\omega \frac{P_o}{4\pi b} \frac{\eta}{Z_c} \qquad (34)$$

In the foregoing it is tacitly assumed that the coaxial line is air-filled, i.e.

$$\gamma = jk = j\omega\sqrt{\mu\varepsilon} \qquad (35)$$

Therefore, it is shown that the excitation of currents in the coaxial line is equivalent to that produced by a pair of lumped voltage and current generators positioned as exhibited in Figure 2. For a physical interpretation of these results (4) is substituted into (33) and (35). This yields

$$e_g = j \omega m I_z^{ext} (z_0) \qquad (36)$$

$$i_g = - j \omega f_c \rho^{ext} (z_0) \qquad (37)$$

where m is a mutual inductance term with

$$m = \frac{\mu M_0}{8\pi^2 b^2} \tag{38}$$

and f_c is a measure of the electric flux density penetrating the aperture with

$$f_c = \frac{\eta P_0}{8\pi^2 b^2} \frac{1}{Z_c} \tag{39}$$

Further, it is noted that whereas f_c is dependent upon the dielectric constant of the material filling the coaxial line, m is independent of the material properties.

The polarity of the equivalent sources is defined as follows: a positive voltage implies the center conductor is at a higher potential than the sheath and a positive current implies a current in the positive z direction on the center conductor and a current in the negative z direction on the sheath.

ILLUSTRATIVE EXAMPLE

To demonstrate the application of the analysis consider an elliptical aperture in a 5062 coaxial cable (GR 874-A2), b = 0.244 in. = 0.0062 m. Let ℓ_1 = 0.00062 m, e = 0.894, s = 0.917 m, z_0 = 0.458 m, E_z^{inc}=1V/m, and f = 81.8 MHz so that λ = 1.832. For these data, the following are obtained:

$$\left.\begin{aligned}
\alpha_{xx} &= -\ell_1^3(0.814) = -1.94 \times 10^{-10}\ m^3 \\
\alpha_{yy} &= \ell_1^3(0.1777) = 0.424 \times 10^{-10}\ m^3 \\
\alpha_{zz} &= -\ell_1^3(0.2304) = -0.549 \times 10^{-10}\ m^3 \\
I_z^{ext}(z_0) &= (0.005049 - j\ 0.003617)A \\
M_0 &= -(2.514 - j\ 1.801) \times 10^{-11}\ Am^2 \\
P_0 &= 0 \\
e_g &= -(0.714 - j\ 0.512) \times 10^{-8}\ V \\
i_g &= 0
\end{aligned}\right\} \qquad (40)$$

Further the currents in matched load impedances, $Z_o = Z_s = Z_c$, are

$$\left.\begin{aligned} I(0) &= (+0.143 - j\,0.102) \times 10^{-9}\, e^{-\gamma z_o}\ \text{A} \\ I(s) &= (-0.143 + j\,0.102) \times 10^{-9}\, e^{-\gamma(s-z_o)}\ \text{A} \end{aligned}\right\} \qquad (41)$$

If the incident field differs from 1 V/m in amplitude, the resulting currents and voltage e_g may be obtained by multiplying the foregoing expressions by the field amplitude.

For a comparison with the results of a recent report by Harrison and King[2] the following data are considered: $f = 1$ MHz, $b = 1$m, $a = 0.001$m, $\ell_1 = 0.25$m, $e = 0.866$, $z_0 = 75$m, $s = 150$m, $E_z^{inc} = 1$V/m and $Z_c = 414.48$. Again the following results are obtained:

$$\left.\begin{aligned} \alpha_{xx} &= -\,0.01323\ \text{m}^3 \\ I_z^{ext}(z_0) &= (0.993 - j\,0.561)\ \text{A} \\ M_0 &= -\,(0.00211 - j\,0.00118)\ \text{A}\cdot\text{m}^2 \\ P_0 &= 0 \\ e_g &= -\,(0.00074 + j\,0.00132)\ \text{V} \\ i_g &= 0 \end{aligned}\right\} \qquad (42)$$

with matched load impedances at the cable ends

$$\left.\begin{aligned} I(0) &\simeq (1.59 - j\,0.89)\ \mu A \\ |I(0)| &\simeq 1.8\ \mu A \end{aligned}\right\} \qquad (43)$$

The corresponding value obtained by Harrison and King is 1.36 μA. This result is in satisfactory agreement with (43) inasmuch as totally different solution techniques were employed. The difference in the results must be attributed to the different approximations that are employed.

In the Harrison and King[2] treatment, a coaxial line with a small aperture in the sheath is considered to be driven from a generator in series with one of the two termination impedances and the far field radiated through the aperture is determined. This field is used to determine the current in a test dipole. Following the usual procedure with the use of the reciprocal theorem the test dipole is considered to be driven and the current in the termination impedance is obtained with the series generator shorted. In determining the dipole moments of the radiating aperture essentially the same approximations are introduced as are used in the preceding development. However to determine the radiated field from the aperture it is assumed to radiate as an aperture in an infinite plate. In an attempt to correct for including the infinite plate a correction factor is introduced to multiply the load impedance current. The result previously quoted from their paper included this correction factor.

CONCLUSION

Considering a coaxial line with a small aperture in the outer sheath and with an incident plane wave illumination, the currents produced in arbitrary termination impedances are obtained. The excitation of these currents is found to depend upon three independent factors: the external currents and charges on the sheath, the aperture configuration and the interior configuration of the coaxial line. Also the excitation of currents in the coaxial line is found to be equivalent to that of a pair of lumped voltage and current generators. Simple analytical expressions are derived for the output of these generators.

ACKNOWLEDGMENT

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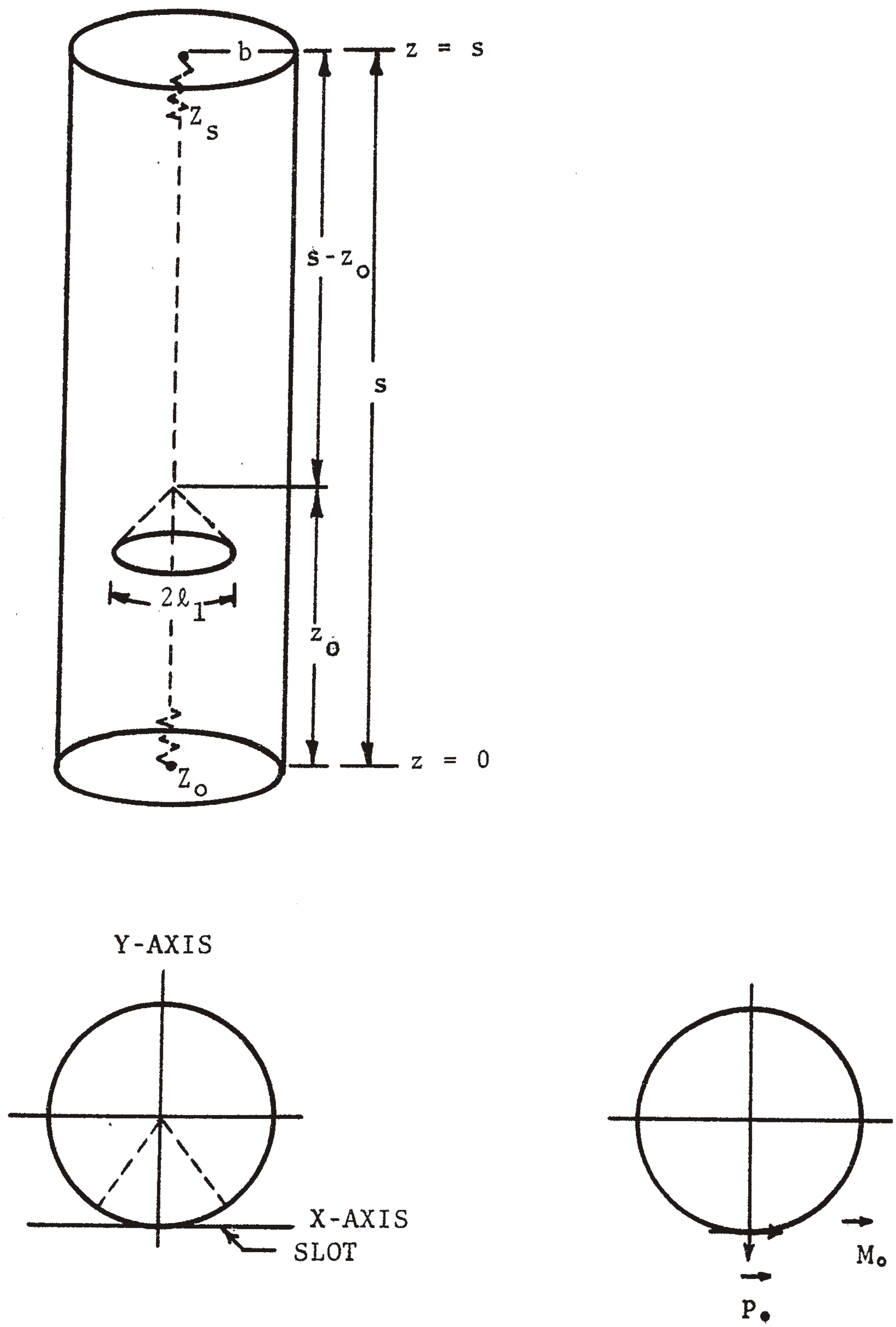

FIGURE 1: COAXIAL CABLE WITH AN ELEIPTICAL APERTURE IN THE SHEATH

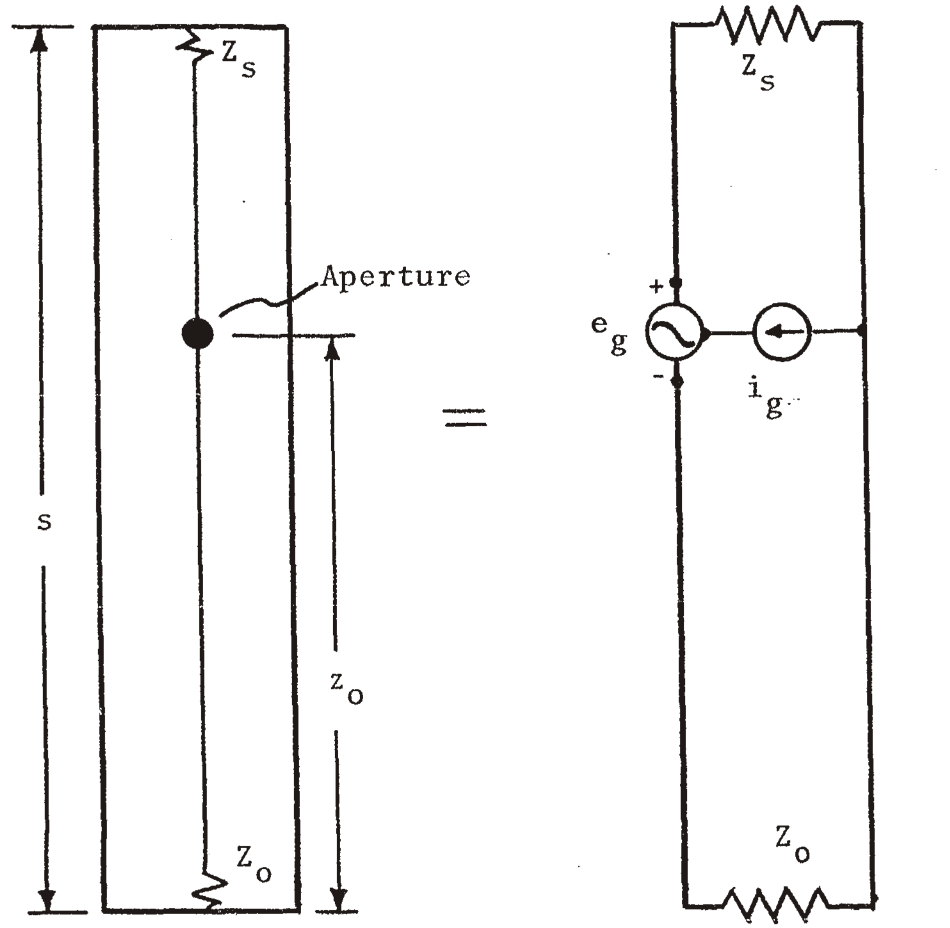

Figure 2: Equivalent Circuit of Determining the Excitation of a Coaxial Line Through a Small Aperture