# Interaction Notes

Note 89

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# Shields With Periodic Apertures

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Abstract

The field inside a dielectric cylinder coated with N identical perfectly conducting strips is calculated for two cases where (i) the electric field of the incident plane wave is polarized along the cylinder's axis, and (ii) the magnetic field of the incident plane wave is polarized along the cylinder's axis. Simple, and easily interpretable, expressions are obtained for the field inside the cylinder from the variational principle and the low-frequency approximations. These field expressions are related to a function depending on only one parameter which defines the "optical coverage" of the shield. This function is tabulated as well as plotted in this note.

## I. Introduction

This note reports the findings of our first attempt toward developing an adequate theory of a braided shield.

A braided shield is used mostly for providing shielding for a cable inside the shield. There are two theoretical aspects concerning a braided cable. One is that the shield is part of the cable, forming a transmission line with the cable as a core conductor. This aspect has been partially treated in the literature, and the most comprehensive treatment up to now is the forthcoming publication of Vance and Chang.(1) The other aspect is that the braid acts purely as a shield, providing shielding for a cable inside the shield against external disturbances. It is the latter aspect that we will consider in this note. The important quantity to be calculated is the "effective" field that the cable will "see" when the braided cable is exposed to an external electromagnetic wave. In calculating the "effective" field inside the shield, the presence of the cable will be assumed to have no effects and, hence, the cable can be removed in the calculation.

The model chosen for the present study is one limiting case of a braided shield, in which the pitch angle is zero and each belt of wires is taken as a perfectly conducting strip. Thus, we are actually considering the problem of a plane electromagnetic wave striking a dielectric cylinder coated with perfectly conducting strips (see Fig. 1), and we will calculate the transmitted field inside the cylinder as a function of the number and the width of the strips as well as the dielectric constant. The basic assumption in the calculation is that the wavelengths inside and outside the cylinder are much greater than the radius of the cylinder.

In section II we consider the case of E-polarization in which the electric field vector is parallel to the axis of the cylinder. A simple analytic expression is found for the transmitted field. The case of H-polarization is discussed in section III, the treatment of which parallels that of section II.

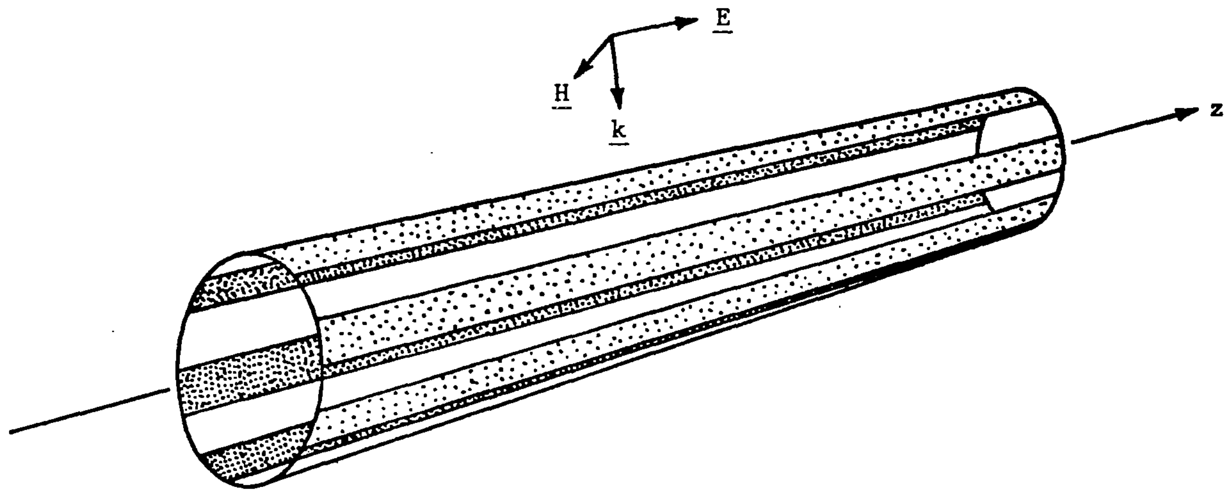


Figure 1A. Dielectric cylinder coated with perfectly conducting strips.

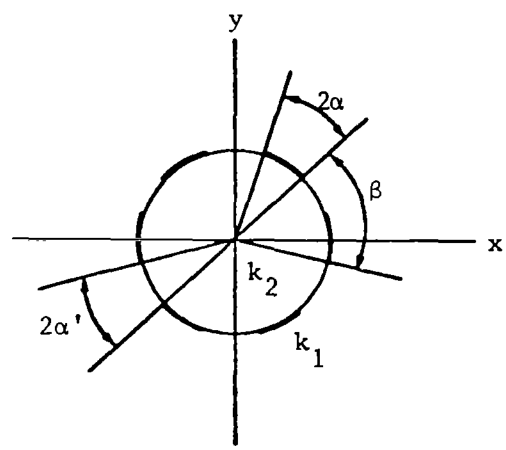


Figure 1B. Cross section of the coated cylinder.

## II. E-Polarization

Consider the situation depicted in Fig. 1 where a time-harmonic plane wave with the electric field vector given by (general incidence will be discussed on p. 15)

$$\underline{E}^{inc} = \underline{e}_z E_o e^{ik_1 x}$$

$$= \underline{e}_z E_o \sum_{m=0}^{\infty} \varepsilon_m i^m J_m(k_1\rho)\cos m\phi, \qquad \varepsilon_o = 1$$

$$\varepsilon_m = 2 \quad \text{if} \quad m > 0$$

strikes the dielectric cylinder coated with N identical, perfectly conducting strips of width $2\alpha$. We wish to calculate the electric field on the axis of the cylinder.

We write

$$E_z^{(1)} = E_z^{inc} + E_z^r + \sum_0^{\infty} a_m \frac{H_m^{(1)}(k_1\rho)}{H_m^{(1)}(k_1 a)} \cos m\phi, \qquad \rho > a$$

$$E_z^{(2)} = E_z^t + \sum_0^{\infty} b_m \frac{J_m(k_2\rho)}{J_m(k_2 a)} \cos m\phi, \qquad \rho < a \tag{1}$$

where $E_z^r$ and $E_z^t$ are, respectively, the reflected and transmitted fields in the absence of the metallic strips. Therefore, they satisfy the following boundary conditions:

$$E_z^{inc} + E_z^r = E_z^t \tag{2a}$$

$$\frac{\partial}{\partial\rho}(E_z^{inc} + E_z^r) = \frac{\partial}{\partial\rho} E_z^t \tag{2b}$$

for $\rho = a, \quad 0 < \phi \leq 2\pi.$

When $\rho = a$, we must have

$$E_z^{(1)} = E_z^{(2)}, \qquad \phi \in A \text{ (apertures)}$$

$$E_z^{(1)} = E_z^{(2)} = 0, \qquad \phi \in S \text{ (strips)} \tag{3a}$$

$$H_\phi^{(1)} = H_\phi^{(2)}, \qquad \phi \in A \tag{3b}$$

where $\phi \in A$ means that $\phi$ belongs to A. Applying (2a) and (3a) to (1) we get

$$a_m = b_m \tag{4}$$

$$\sum_0^\infty b_m \cos m\phi = -E_z^t, \qquad \phi \in S \tag{5}$$

where $E_z^t$ can be easily found to be

$$E_z^t = \frac{2E_o}{\pi k_1 a} \sum_0^\infty \frac{\epsilon_m i^{m+1} J_m(k_2\rho)\cos m\phi}{J_m(k_2a)H_m^{(1)'}(k_1a)-(k_2/k_1)J_m'(k_2a)H_m^{(1)}(k_1a)}$$

$$\approx E_o, \qquad \text{for} \quad k_1a \quad \text{and} \quad k_2a \ll 1 \tag{6}$$

Applying (2b) and (3b) to (1) we get, in conjunction with (4),

$$\sum_0^\infty \gamma_m b_m \cos m\phi = 0, \qquad \phi \in A \tag{7}$$

where

$$\gamma_m = \frac{k_1 H_m^{(1)'}(k_1a)}{H_m^{(1)}(k_1a)} - \frac{k_2 J_m'(k_2a)}{J_m(k_2a)}$$

$$\left.\begin{aligned} &\approx -\frac{2m}{a}, && m > 0 \\ &\approx \frac{1}{a \ln(k_1a)}, && m = 0 \end{aligned}\right\} \text{for} \quad k_1a \quad \text{and} \quad k_2a \ll 1 \tag{8}$$

Equations (5) and (7) constitute the formulation of our mixed boundary-value problem which we now proceed to solve. The quantity of interest is the electric field on the axis, which is, according to (1), given by

$$\begin{aligned} E_z^{(2)}(0) &= \frac{b_o}{J_o(k_2a)} + E_z^t(0) \\ &\approx b_o + E_z^t(0), \qquad \text{for} \quad k_1a \quad \text{and} \quad k_2a \ll 1 \end{aligned} \tag{9}$$

Starting from (7) we have

$$\sum_{0}^{\infty} \gamma_m b_m \cos m\phi = 0 \qquad \phi \in A$$
$$= -\, i\omega\mu J(\phi) \qquad \phi \in S \tag{10}$$

Here J is an unknown function on the strips and turns out to be just the induced current density. Equation (10) immediately gives

$$b_m = -\frac{i\omega\mu\varepsilon_m}{2\pi\gamma_m}\int_S J(\phi)\cos m\phi d\phi \tag{11}$$

Substitution of (11) in (5) yields

$$\frac{i\omega\mu}{2\pi\gamma_o}\int_S J(\phi)d\phi - \frac{i\omega\mu a}{2\pi}\int_S d\phi' J(\phi')\sum_{1}^{\infty} m^{-1}\cos m\phi \cos m\phi' = E_o, \qquad \phi \in S \tag{12}$$

where we have used the low-frequency approximations on $\gamma_m (m > 0)$ and $E_z^t$. Since we are dealing with strips symmetrically located on the dielectric cylinder (Fig. 1), we can add

$$\sum_{1}^{\infty} m^{-1} \sin m\phi \sin m\phi'$$

to the cosine series in (12) without affecting the solution $J(\phi)$. From Ref. 2 , we have

$$\sum_{1}^{\infty} m^{-1}(\cos m\phi \cos m\phi' + \sin m\phi \sin m\phi') = -\ln\left|2\sin\left(\frac{\phi-\phi'}{2}\right)\right| \tag{13}$$

Using (13) and (11) in (12) we get

$$\int_S \ln\left|2\sin\left(\frac{\phi-\phi'}{2}\right)\right| J(\phi')d\phi' = \frac{2\pi}{i\omega\mu a}(E_o + b_o), \qquad \phi \in S \tag{14}$$

Now we note that for low frequencies it is permissible to assume J to be the same on all the strips. This means that if N is the number of strips, $\beta$ the period, and $2\alpha$ the strip's width (see Fig. 1), then $J(\phi) = \cdots = J[\phi + (N-1)\beta]$. Thus, we can write

$$\int_S \ln\left|2\sin\left(\frac{\phi-\phi'}{2}\right)\right| J(\phi')d\phi' = \int_{-\alpha}^{\alpha} d\phi' J(\phi') \sum_{n=0}^{N-1} \ln\left|2\sin\left(\frac{\phi-\phi'-n\beta}{2}\right)\right|$$

$$= \ln 2 \int_{-\alpha}^{\alpha} J(\phi)d\phi + \int_{-\alpha}^{\alpha} \ln\left|\sin\frac{N}{2}(\phi-\phi')\right| J(\phi')d\phi', \qquad |\phi| < \alpha \tag{15}$$

The last step in (15) follows from formula 1.392 of Ref. 3. Using (15) in (14) together with (11) we get

$$\int_{-\alpha}^{\alpha} \ln\left|\sin\frac{N}{2}(\phi-\phi')\right| J(\phi')d\phi' = \frac{2\pi}{i\omega\mu a}\left[E_o + b_o(1+\gamma_o a \ln 2/N)\right], \qquad |\phi| < \alpha \tag{16}$$

Since the quantity of interest is $b_o$ which is proportional to $\int_{-\alpha}^{\alpha} J(\phi)d\phi$, one can obtain a reliable result via the variational principle without first solving (16). The variational expression of interest can be easily constructed from (16) and is given by

$$\frac{i\omega\mu a}{2\pi} \frac{\int_{-\alpha}^{\alpha} J(\phi)d\phi}{E_o+b_o(1+\gamma_o a \ln 2/N)} = \frac{\left[\int_{-\alpha}^{\alpha} J(\phi)d\phi\right]^2}{\int_{-\alpha}^{\alpha}\int_{-\alpha}^{\alpha} K(\phi,\phi')J(\phi)J(\phi')d\phi d\phi'} \tag{17}$$

where $K(\phi,\phi')$ is the kernel in (16). Using (11) and setting the right-hand side of (17) equal to $C^{-1}$ we have

$$-\frac{\gamma_o a}{N} \frac{b_o}{E_o+b_o(1+\gamma_o a \ln 2/N)} = \frac{1}{C}$$

Solving this for $b_o$ we get

$$b_o = -\frac{E_o}{1+(C+\ln 2)\gamma_o a/N}$$

Substituting this into (9) we obtain, for $k_1 a \ll 1$,

$$\frac{E_z^{(2)}(0)}{E_o} = \frac{f}{1+f}$$

$$f \equiv \frac{C+\ln 2}{N \ln(k_1 a)} = \frac{F}{N \ln(1/k_1 a)} \tag{18}$$

where we have used $\gamma_o a \approx 1/\ln(k_1 a)$ and, of course, $-F = C + \ln 2$.

To evaluate C we take J = constant as our trial function in (17). Then

$$C = \frac{1}{4\alpha^2} \int_{-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} \ln\left|\sin \frac{N}{2} (\phi - \phi')\right| d\phi d\phi'$$

$$= \frac{1}{N^2\alpha^2} \int_{-N\alpha/2}^{N\alpha/2} \int_{-N\alpha/2}^{N\alpha/2} \ln\left|\sin(x - x')\right| dx dx'$$

Let

$$u = x - x'$$

$$v = x + x'$$

$$\nu = N\alpha/\pi, \qquad 0 \le \nu \le 1$$

We can then easily reduce the double integral to a single one:

$$C = \frac{2}{(\nu\pi)^2} \int_0^{\nu\pi} (\nu\pi - u)\ln(\sin u)du$$

$$= 2 \int_0^1 (1 - x)\ln \sin(\nu\pi x)dx \tag{19}$$

With (19) we can show that $F(\nu)$ in (18) has the following properties:

$$F(1) = 0$$

$$F(\nu) \sim 3/2 - \ln(2\nu\pi), \qquad \text{as } \nu \to 0$$

Now it is obvious from (18) that (i) when $\nu = 1$ (i.e., 100% optical coverage) $E_z^{(2)}(0) = 0$, and (ii) when $\nu = 0$ (i.e., no metallic strips) $E_z^{(2)}(0) = E_o$. These are, of course, the expected results for these two limiting cases. Equation (18) also shows that for a given optical coverage (i.e., for a fixed $\nu$) one should make N large for better shielding effectiveness. This conclusion agrees with Kaden.[4] The function F, which is a function only of $\nu$, is graphed in Fig. 2 and also tabulated below.

**Table I**

| $\nu$ | F |
|---|---|
| .01 | 4.2627 |
| .02 | 3.5696 |
| .03 | 3.1643 |
| .04 | 2.8768 |
| .05 | 2.6539 |
| .06 | 2.4719 |
| .07 | 2.3181 |
| .08 | 2.1850 |
| .09 | 2.0676 |
| .10 | 1.9628 |
| .11 | 1.8681 |
| .12 | 1.7817 |
| .13 | 1.7023 |
| .14 | 1.6290 |
| .15 | 1.5608 |
| .16 | 1.4971 |
| .17 | 1.4374 |
| .18 | 1.3812 |
| .19 | 1.3282 |
| .20 | 1.2779 |
| .30 | .8864 |
| .40 | .6186 |
| .50 | .4216 |
| .60 | .2724 |
| .70 | .1590 |
| .80 | .0755 |
| .90 | .0196 |
| 1.00 | 0 |

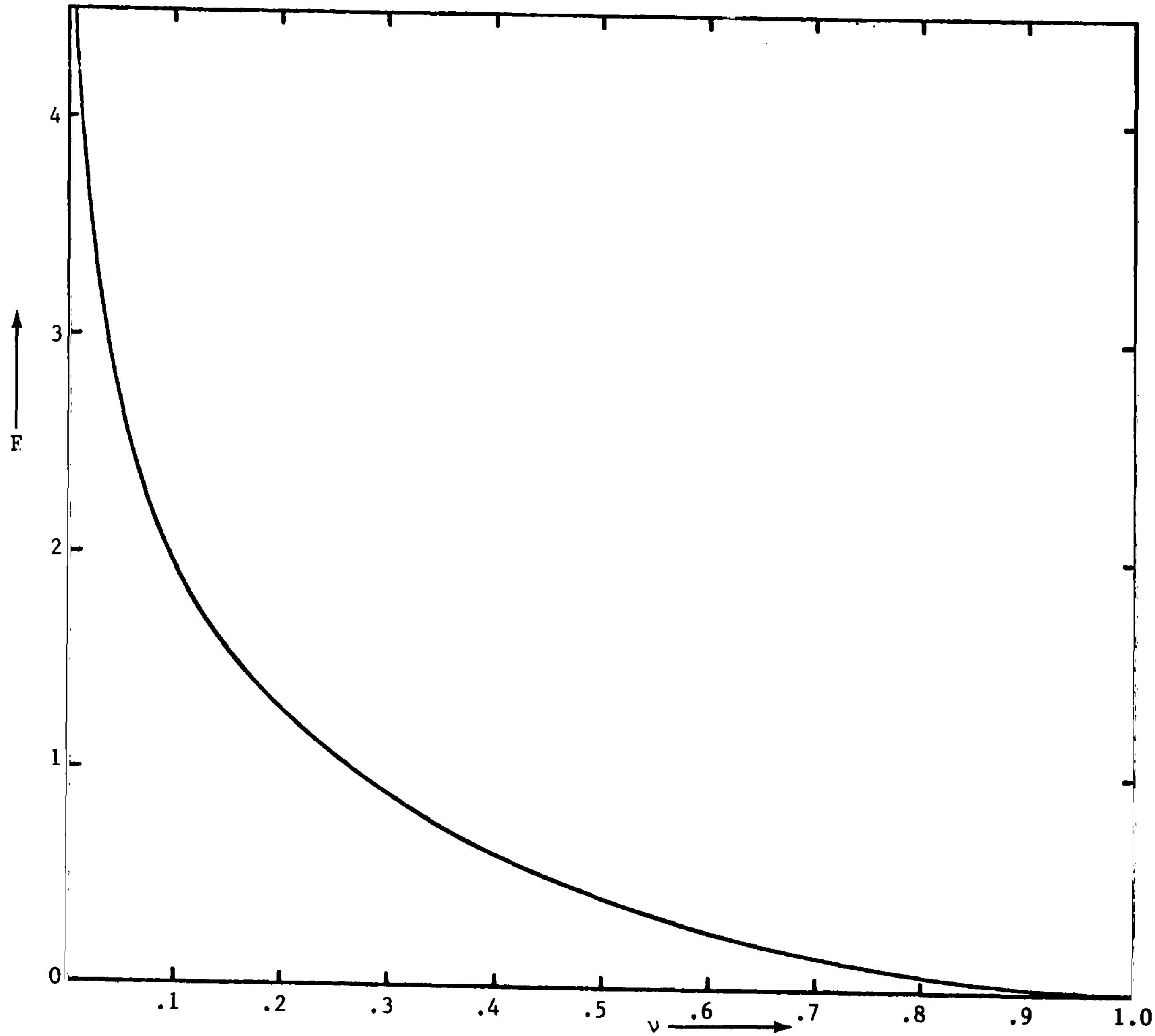


Figure 2.

Besides the quantity $E_z^{(2)}(0)/E_o$ given by (18), another interesting quantity to calculate is $E_z^{(2)}(0)/I$, I being the total induced current in the shield (i.e., in the metallic strips). Naturally, I is related to J by

$$I = \int_S J(\phi) a d\phi = Na \int_{-\alpha}^{\alpha} J(\phi) d\phi,$$

whereas $b_o$ is related to I by (11) as

$$b_o = -\frac{i\omega\mu}{2\pi\gamma_o} \int_S J(\phi) d\phi = -\frac{i\omega\mu}{2\pi\gamma_o a} I$$

Using these two expressions in the variational expression (17) we get, after some simple algebra,

$$\frac{E_o}{I} = \frac{i\omega\mu}{2\pi} \left[ \frac{C+\ln 2}{N} + \frac{1}{\gamma_o a} \right]$$

Thus,

$$b_o + E_o = \frac{i\omega\mu}{2\pi} \frac{C+\ln 2}{N} I = -\frac{i\omega\mu F(\nu)}{2\pi N} I$$

Substituting this into (9) we finally have, for $k_1 a << 1$,

$$\frac{E_z^{(2)}(0)}{I} = i\omega L_s$$

$$L_s \equiv -\frac{\mu}{2\pi N} F(\nu) \qquad (18')$$

$L_s$ has the dimension of henries per unit length and may be called the inductive (or magnetic) coupling coefficient.

## III. H-Polarization

When the magnetic field vector of the incident plane wave is parallel to the axis of the cylinder (see Fig. 1; general incidence will be discussed on p. 15), i.e.,

$$\underline{H}^{inc} = \underline{e}_z H_o e^{ik_1 x}$$

we start with the following field representations:

$$H_z^{(1)} = H_z^{inc} + H_z^r + \sum_0^\infty A_m \frac{H_m^{(1)}(k_1\rho)}{H_m^{(1)\prime}(k_1 a)} \cos m\phi, \qquad \rho > a$$

$$H_z^{(2)} = H_z^t + \sum_0^\infty B_m \frac{J_m(k_2\rho)}{J_m'(k_2 a)} \cos m\phi, \qquad \rho < a \tag{20}$$

where $H_z^r$ and $H_z^t$ are, respectively, the reflected and transmitted fields in the absence of the strips. Proceeding in the same manner as in section II, one can easily arrive at the following:

$$\sum_0^\infty B_m \cos m\phi = -\frac{i}{Z_2} E_\phi^t, \qquad \phi \in S \tag{21}$$

$$\sum_0^\infty \Gamma_m B_m \cos m\phi = 0, \qquad \phi \in A \tag{22}$$

where

$$\frac{E_\phi^t}{Z_2} = \frac{2H_o}{\pi k_1 a} \sum_0^\infty \frac{\varepsilon_m i^m J_m'(k_2\rho)\cos m\phi}{J_m(k_2 a)H_m^{(1)\prime}(k_1 a)-(k_1/k_2)J_m'(k_2 a)H_m^{(1)}(k_1 a)}$$

$$\approx \frac{ik_2 a}{2} H_o, \qquad \text{for } k_1 a \text{ and } k_2 a \ll 1 \tag{23}$$

$$\Gamma_m = (\varepsilon_1/\varepsilon_2)^{1/2} \frac{H_m^{(1)}(k_1 a)}{H_m^{(1)\prime}(k_1 a)} - \frac{J_m(k_2 a)}{J_m'(k_2 a)}$$

For $k_1a$ and $k_2a \ll 1$, we have

$$\Gamma_m \approx -(1 + \varepsilon_1/\varepsilon_2)\frac{k_2a}{m}, \qquad m > 0$$
$$\approx \frac{2}{k_2a}, \qquad m = 0 \tag{24}$$

The quantity that we are seeking is the magnetic field on the axis of the cylinder, which is given by (20) as

$$H_z^{(2)}(0) = \frac{B_o}{J_o'(k_2a)} + H_z^t(0)$$
$$\approx -\frac{2B_o}{k_2a} + H_o, \qquad \text{for } k_1a \text{ and } k_2a \ll 1 \tag{25}$$

To solve (21) and (22) for the $B_m$, we write

$$\sum_0^\infty B_m \cos m\phi = -\frac{i}{Z_2} E_\phi^t, \qquad \phi \in S$$
$$= \frac{i}{Z_2} E_\phi^s, \qquad \phi \in A \tag{26}$$

Here $E_\phi^s$ is an unknown function and turns out to be just the scattered electric field in the apertures.

We now proceed as in section II. First, we solve (26) for $B_m$ in terms of the unknown function $E_\phi^s$. Then, we substitute the result into (22) to obtain an integral equation for $E_\phi^s$. After making the low-frequency approximations we arrive at the following integral equation for $E_\phi^s$:

$$\int_A \ln\left|2\sin\left(\frac{\phi-\phi'}{2}\right)\right| E_\phi^s(\phi')d\phi' = \frac{2\pi i Z_2 B_o}{(1+\varepsilon_1/\varepsilon_2)(k_2a)^2}, \qquad \phi \in A \tag{27}$$

Assuming $E_\phi^s$ to be the same in all the apertures, as we have done for the current density on the strips in section II, one can easily deduce from (27) the following equation:

$$\int_{-\alpha'}^{\alpha'} \ln\left|\sin\frac{N}{2}(\phi-\phi')\right| E_\phi^s(\phi')d\phi' = \frac{2\pi i Z_2 B_o}{(1+\varepsilon_1/\varepsilon_2)(k_2 a)^2}, \qquad |\phi| < \alpha' \tag{28}$$

where $2\alpha'$ is the width of one aperture.

To find $B_o$ from (28), which is related to $\int_{-\alpha'}^{\alpha'} E_\phi^s d\phi$, we invoke the variational principle. Defining

$$\frac{1}{C'} = \frac{\left[\int_{-\alpha'}^{\alpha'} E_\phi^s(\phi)d\phi\right]^2}{\int_{-\alpha'}^{\alpha'}\int_{-\alpha'}^{\alpha'} K(\phi,\phi')E_\phi^s(\phi)E_\phi^s(\phi')d\phi d\phi'} \tag{29}$$

we find from (28) that

$$\frac{2B_o}{k_2 a} = \frac{(C'/N)(1+n^2)(k_1 a)^2}{1+(C'/N)(1+n^2)(k_1 a)^2}\frac{S}{2\pi}H_o \tag{30}$$

where $n^2 = \varepsilon_2/\varepsilon_1$, S = total arc length of the strips, and $K(\phi,\phi')$ is the kernel in (28). If we take $E_\phi^s$ = constant to be our trial function in (29), then $C' = C$ when $\alpha = \alpha'$ (where C has been defined in section II) and we have, after using (30) in (25),

$$\frac{H_z^{(2)}(0)}{H_o} = 1 - \frac{(1-\nu')f'}{1+f'}$$

$$f' = N^{-1}[F(\nu') - \ln 2](1 + n^2)(k_1 a)^2 \tag{31}$$

$$\nu' = \frac{N\alpha'}{\pi} = 1 - \nu$$

where F has been graphed and tabulated in section II. Several points regarding (31) will now be made in order. In deducing (31) we have made low-frequency approximations to the integral equation for $E_\phi^s$ in the apertures. By keeping only the dominant terms in the approximations we have arrived at (28). Then, using the variational principle and a constant for the trial function we have finally obtained (31). In conformity with the low-frequency approximations we should have expanded the result (31) in powers of $k_1 a$ and discarded terms of the same order as those being neglected in the process of deriving (28). However, equation (31), as it is, has the desirable feature that it tends to zero as $\nu'$ goes to zero. From (31) one can also see that $H_z^{(2)} = H_o$ when $\nu' = 1$, and that the shielding effectiveness increases with decreasing N for a given optical coverage.

So far we have restircted our consideration to normal incidence only. But our results can easily be extended to general incidence. Let $\theta_o$ be the angle between $\underline{k}_1$ and $\underline{e}_z$, and let $\psi_o$ be the angle between $\underline{E}^{inc}$ [or $\underline{H}^{inc}$] and its projection onto the plane formed by $\underline{k}_1$ and $\underline{e}_z$. Then we simply make the following changes in the equations: (1) replace $E_o$ [or $H_o$] by $E_o \cos \psi_o \sin \theta_o \exp(ik_1 z \cos \theta_o)$ [or by $H_o \cos \psi_o \sin \theta_o \exp(ik_1 z \cos \theta_o)$], and (2) replace $k_1$ and $k_2$ by $k_1 \sin \theta_o$ and $k_2 \sin \theta_o$, respectively.

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