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RESPONSE OF A SHORT MONOPOLE ANTENNA MOUNTED ON THE SKIN OF A FINITE-LENGTH CYLINDER

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ABSTRACT

The response to incident electromagnetic fields of a monopole probe mounted on the side of a cylinder is analyzed in this report. The analysis applies to monopoles and other antennas that are short with respect to a wavelength, with mounting positions on the side or on the end caps of the cylinder. Parametric curves for CW and transient response to a step incident field are given.

Key Word: Electromagnetic Scattering

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Introduction

The response to a plane-wave electromagnetic field of a short wire which protrudes through the skin of a missile is calculated from the model shown in Figure 1. Here, the missile is modeled as a perfectly conducting cylinder with half-length h and radius a, and the wire is modeled as a straight monopole probe of length h_1 and radius a_1. The axis of the probe is perpendicular to the surface of the cylinder and is terminated to the cylinder by an arbitrary resistive load. It is assumed that the length of the probe is small in comparison with the radius of the cylinder (i.e., $h_1 << a$) and that the incident electric field is parallel to the axis of the cylinder.

The probe is treated as a monopole which is excited by the electric fields that would be tangent to the probe if the probe were removed. Since the probe is very short, the low-frequency equivalent circuit for a monopole can be used for all frequencies of interest. Coupling from the probe to the cylinder is neglected.

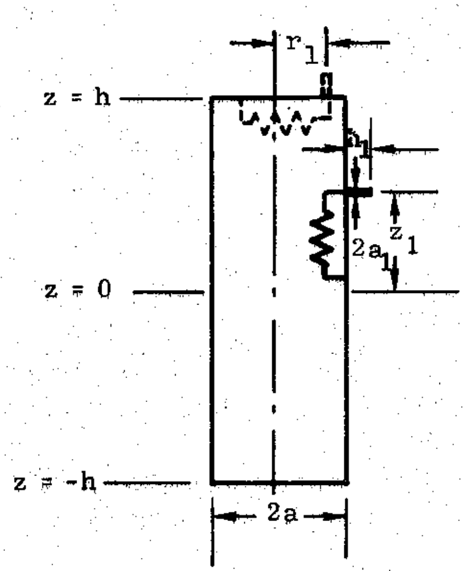

Figure 1. Model of a Missile With a Protruding Wire

Formulation

The low-frequency equivalent circuit of Figure 2 can be used to find the response of the monopole. Using v(0) = 0 and i(0) = 0, Figure 2 shows that

$$v(t) = \frac{1}{C}\int_0^t i(\tau)\, d\tau + Ri(t) \quad . \tag{1}$$

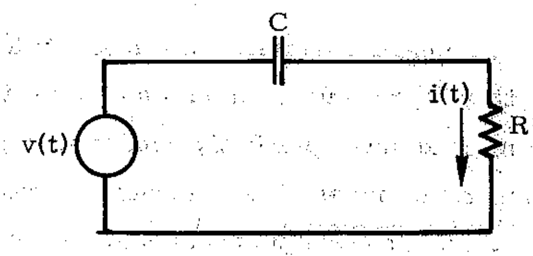

Figure 2. Low-Frequency Equivalent Circuit of a Monopole Antenna

Taking the Laplace transform of Equation 1 yields

$$I(s) = \frac{1}{R}\,\frac{1}{s + 1/RC}\left[sV(s)\right] \quad . \tag{2}$$

Here, I(s) and V(s) are the Laplace transforms of i(t) and v(t). Since v(0) = 0, inverse transformation of Equation 2 gives

$$i(t) = \frac{1}{R}\int_0^t e^{-(t-\tau)/RC}\left[\frac{d}{d\tau}\, v(\tau)\right] d\tau \quad . \tag{3}$$

According to the analysis in the reference,*

$$v(t) = v_s(t) + v_{inc}(t) = E_0 h_1 f_1(\Omega_1) h e_n(x/h, \Omega, tc/h)$$

$$+ E_0 h_1 f_1(\Omega_1) U(t, \text{probe location}) \tag{4}$$

*D. E. Merewether, Editor, Electromagnetic Pulse Handbook for Missiles and Aircraft in Flight, to be published.

and

$$C = 2h_1 f_2(\Omega_1) \quad . \tag{5}$$

Here, $v_s(t)$ is that part of v(t) due to the scattered field and $v_{inc}(t)$ is due to the incident field. Since the incident field is polarized in the z direction,

$$U(t, \text{probe location}) = \begin{cases} 1 & \text{if } t > 0 \text{ and the probe is on an end cap} \\ 0 & \text{otherwise} \quad . \end{cases}$$

The normalized scattered electric field perpendicular to the skin of the cylinder with the probe removed is e_n and the magnitude of the step electric field is E_0. If the probe is mounted on the side of the cylinder $x = z_1$ and if the probe is mounted on an end cap $x = r_1$, $\Omega = 2\ell n(2h/a)$ and $\Omega_1 = 2\ell n(2h_1/a_1)$. Plots of f_1 and f_2 are shown in Figure 3.

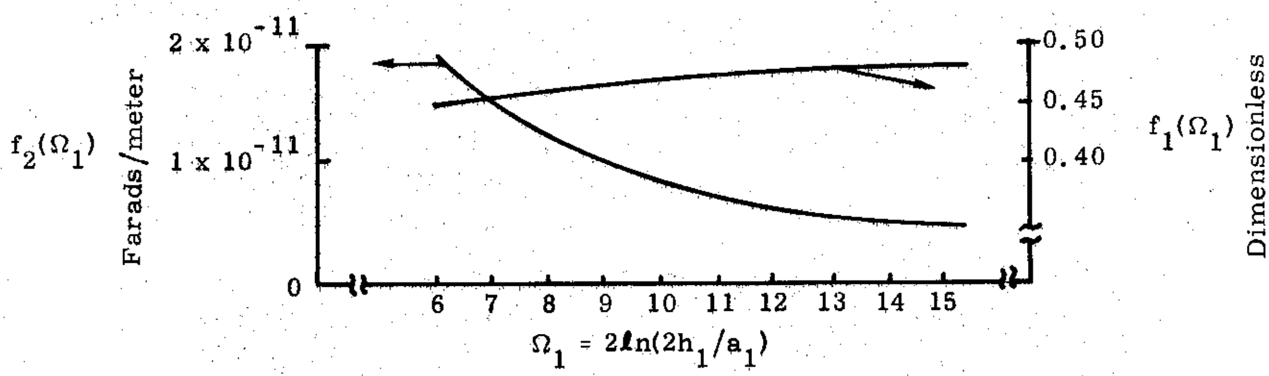

Figure 3. Graphs of $f_1(\Omega_1)$ and $f_2(\Omega_1)$

The remaining portion of this report deals mainly with the response due to $v_s(t)$. If the probe is located on an end cap, the response due to $v_{inc}(t)$ must be added to the response due to $v_s(t)$. (See the example problem under Results.)

By substituting Equations 4 and 6 into Equation 3, the current due to the scattered field, $i_s(t)$, can be written

$$i_s(t) = E_0 p_1 h_1^2 \int_0^{p_3/p_2} e^{-y} e_{nd}(x/h, \Omega, p_3 - p_2 y)\, dy \quad , \tag{6}$$

where

$$p_1 = 2cf_1(\Omega_1)f_2(\Omega_1) \quad , \tag{7}$$

$$p_2 = 2cf_2Rh_1/h \quad , \tag{8}$$

$$p_3 = tc/h \quad , \tag{9}$$

and

$$e_{nd}(x/h, \Omega, \gamma) = \frac{d}{d\gamma} e_n(x/h, \Omega, \gamma) \quad . \tag{10}$$

Equation 6 shows that the normalized current in R, $i_s(t)/(p_1h_1^2E_0)$, is a function of the parameters p_2, p_3, x/h, and Ω.

The transfer function for the current in R can be obtained by taking the Fourier transform of Equation 6 and multiplying the result by $j\omega/E_0$ to eliminate the effects of the step. The result is

$$I_s(\omega) = j(k_0h)p_1h_1^2 \int_0^\infty e^{-j(k_0h)\tau} \int_0^{\tau/p_2} e^{-y} e_{nd}(x/h, \Omega, \tau - p_2y)\, dy d\tau \quad . \tag{11}$$

Equation 11 shows that the normalized transfer function for the current in R, $I_s(\omega)/(p_1h_1^2)$, is a function of the parameters p_2, k_0h, x/h, and Ω. Here, $k_0 = \omega/c$.

Results

Equations 10 and 6 were evaluated by using numerical data for $e_n(x/h, \Omega, \gamma)$ from the reference.* The results are shown in Figures 4 and 5, which can be used to calculate the transient current due to a step incident field in the resistive load of the monopole probe. Since R is large when $p_2 \geq 10$, a good approximation to the transient open-circuit voltage can be found by multiplying the current (when $p_2 = 10$) in R by R. A good approximation to the short-circuit current can be found by using the curves where $p_2 = 0.01$.

*Ibid.

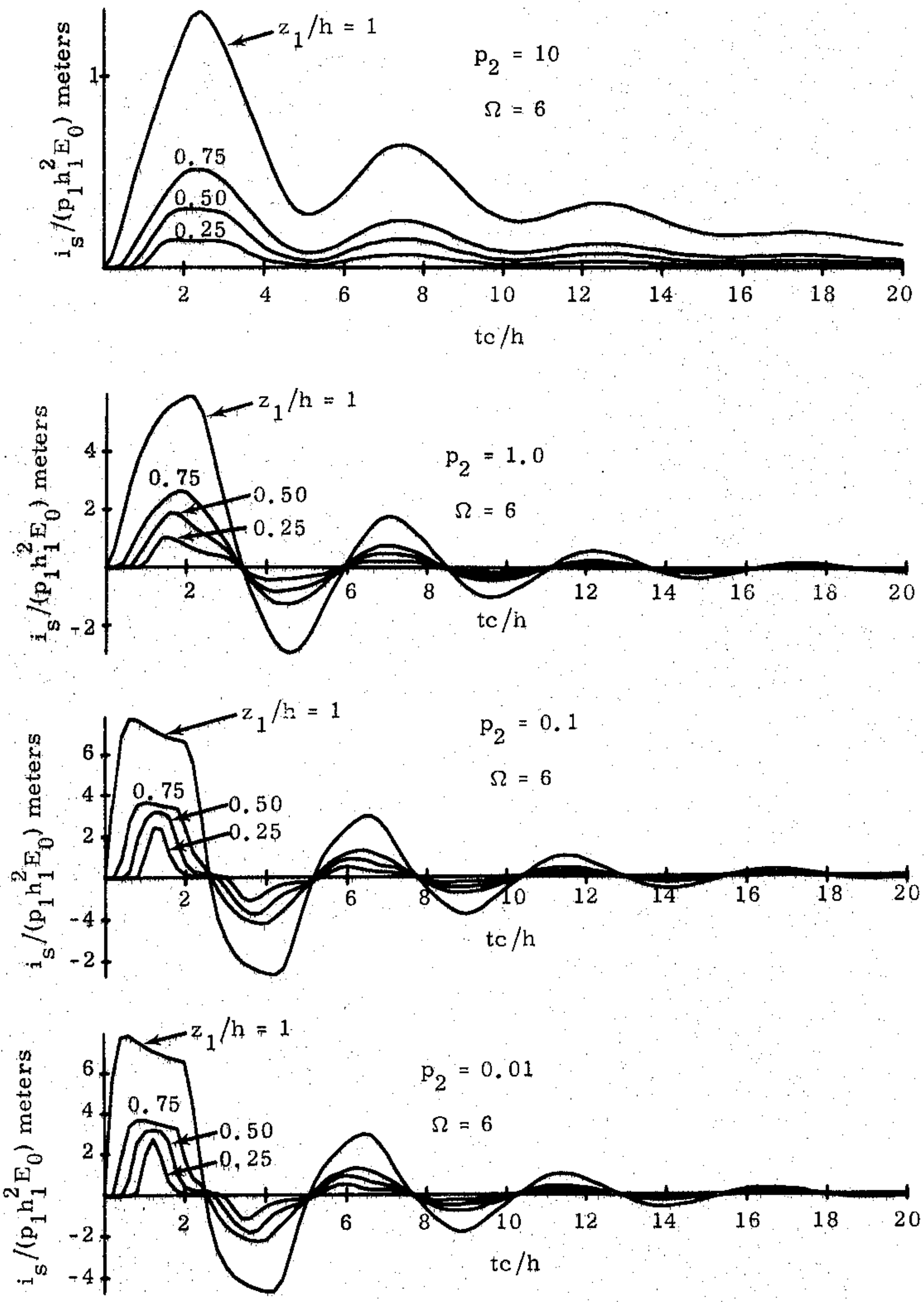

Figure 4. Normalized Transient Current in R Due to a Step Incident Field (probe on side of cylinder)

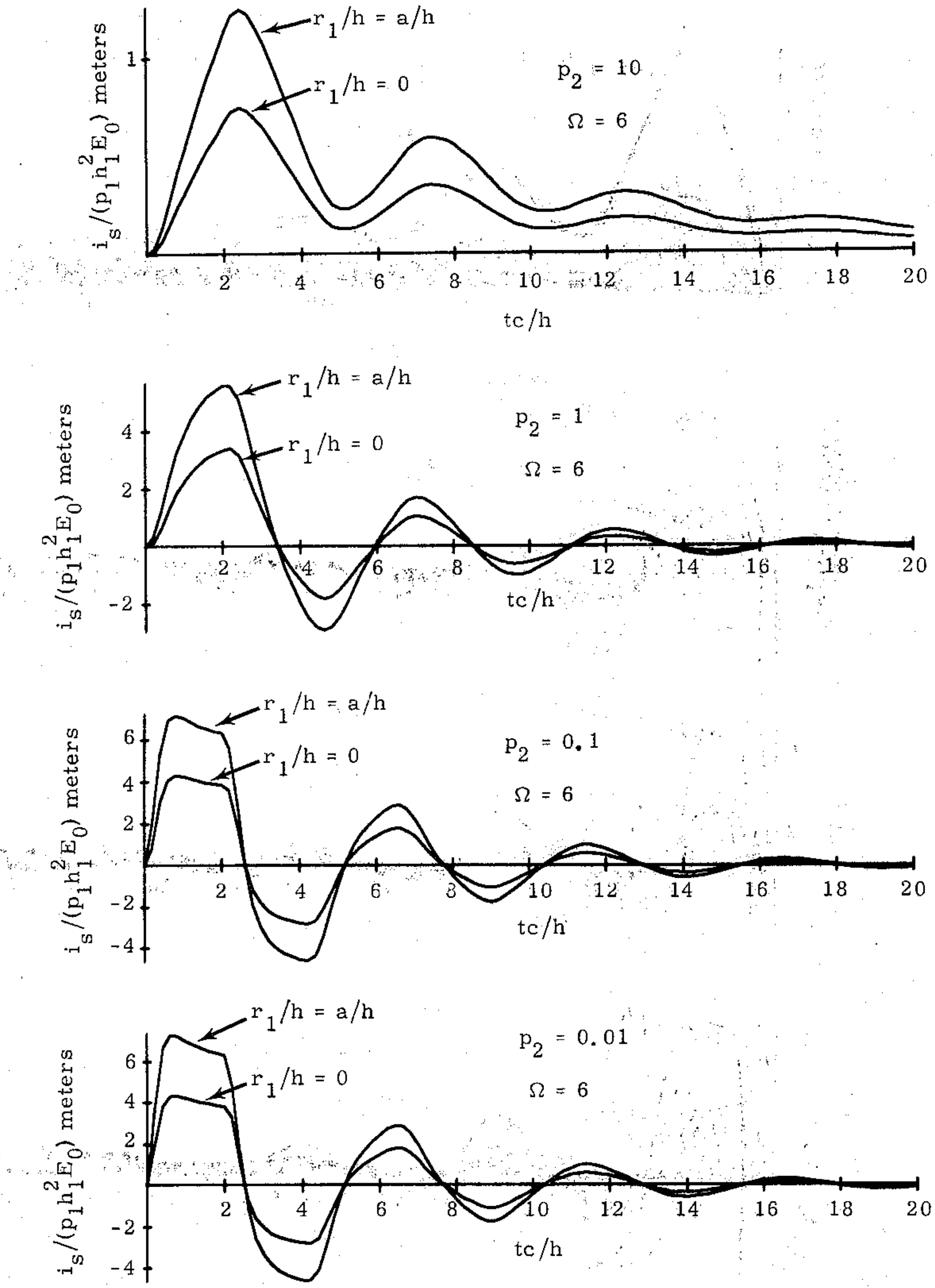

Figure 5. Normalized Transient Current in R Due to a Step Incident Field (probe on end cap)

Short-circuit current and open-circuit voltage transfer functions can be obtained by using the appropriate curves and equations of Figures 6 and 7.

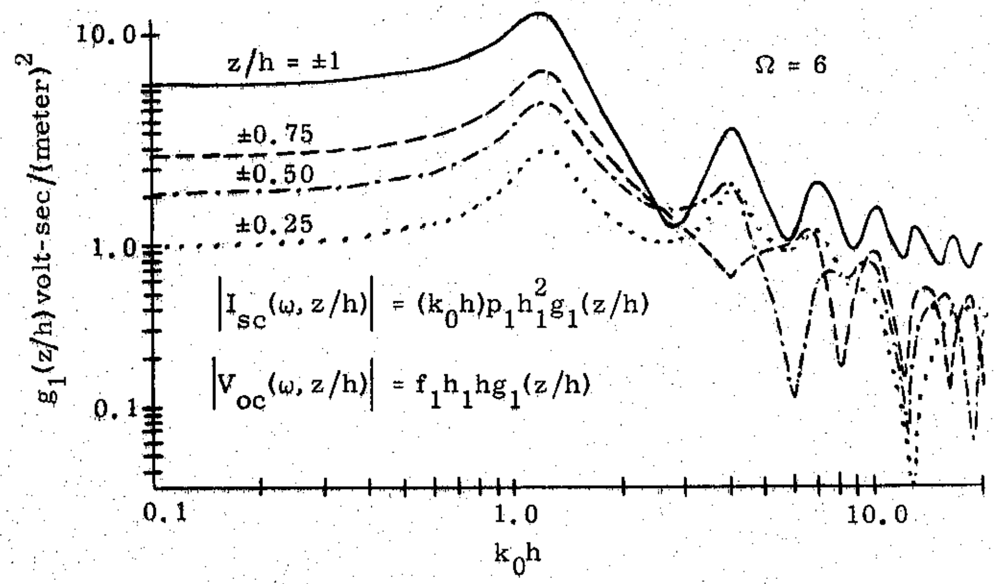

Figure 6. Short-Circuit Current and Open-Circuit Voltage Transfer Functions (probe on side of cylinder)

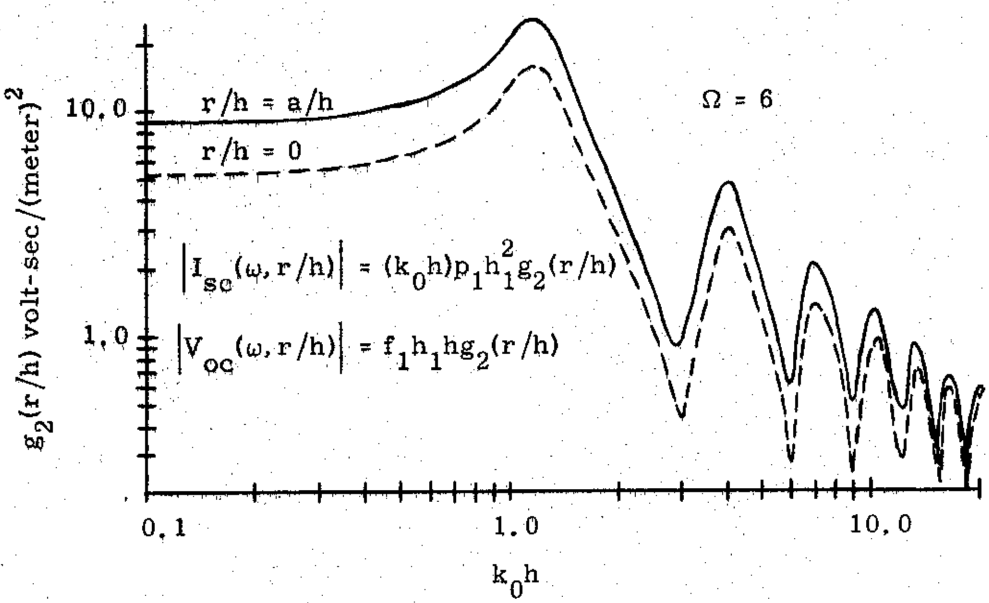

Figure 7. Short-Circuit Current and Open-Circuit Voltage Transfer Functions (probe on end cap)

The following example illustrates the use of Figures 3 through 7.

Problem: Find the current in R and voltage across R at t = 40 nanoseconds if a cylinder with an attached probe is illuminated by a 5 volt/meter step incident field. The incident field is parallel to the axis of the cylinder and h = 5 m, a = 0.5 m, h_1 = 5 cm, a_1 = 0.5 cm, and R = 8 x 10^3 ohms. Assume that the probe is on an end cap and near its edge; that is, $r_1 \simeq a$.

Solution: Calculate

$$\Omega = \Omega_1 = 2\ln\left[2 \times 5 \times 10^{-2}/(5 \times 10^{-3})\right] = 6$$

and

$$tc/h = 40 \times 10^{-9} \times 3 \times 10^{8}/5 = 2.4 \quad .$$

From Figure 3, $f_1(\Omega_1) = 0.45$ and $f_2(\Omega_1) = 2 \times 10^{-11}$ farads/meter. Then,

$$p_1 = 2 \times 3 \times 10^{8} \times 2 \times 10^{-11} \times 0.45 = 5.4 \times 10^{-3}/(\text{ohm-m}^2)$$

and

$$p_2 = 2 \times 3 \times 10^{8} \times 2 \times 10^{-11} \times 8 \times 10^{3} \times 5 \times 10^{-2}/5 \simeq 1 \quad .$$

From Figure 5 (second set of curves from the top),

$$i_s(40 \times 10^{-9})/(p_1 h_1^2 E_0) = 5.3 \text{ meters}$$

or

$$i_s(40 \times 10^{-9}) = 5.4 \times 10^{-3}(5 \times 10^{-2})^2 \times 5 \times 5.3$$

$$= 3.58 \times 10^{-4} \text{ amps}$$

and

$$v_s(40 \times 10^{-9}) = 8 \times 10^3 \times 3.58 \times 10^{-4} = 2.86 \text{ volts} \quad .$$

The response due to $v_{inc}(t)$ must be added to these results. From Equation 4 and Figure 2, the step response of the monopole is

$$i_{inc}(t) = \frac{E_0 h_1 f_1(\Omega_1)}{R} e^{-\frac{tc/h}{p_2}} \quad .$$

Substituting numerical values yields

$$i_{inc}(40 \times 10^{-9}) = \frac{5 \times 5 \times 10^{-2} \times 0.45}{8 \times 10^3} e^{-\frac{2.4}{1}} = 1.28 \times 10^{-6} \text{ amps} \quad ,$$

$$i(40 \times 10^{-9}) = i_s(40 \times 10^{-9}) + i_{inc}(40 \times 10^{-9}) = 3.59 \times 10^{-4} \text{ amps} \; ,$$

and

$$v(40 \times 10^{-9}) = Ri(40 \times 10^{-9}) = 2.87 \text{ volts} \quad .$$

Conclusions

An analysis and normalized curves of the CW and transient response to a step incident field of a short probe which is mounted on the surface of a finite-length cylinder have been presented. In this analysis the probe was assumed to be much shorter than the radius of the cylinder so that the value of the electric fields which exist at the surface of the cylinder can be used as the electric field which excites the probe along its entire length. The coupling from the probe to the cylinder was neglected. Accurate calculations of currents and voltages when the length of the probe is a significant portion of the radius of the cylinder would require a more complete analysis than that presented here.

This analysis need not be restricted to short monopole antennas. It can be applied to any small antenna that has a low-frequency equivalent circuit of the form shown in Figure 2.

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