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SC-CR-71 5141

FINAL REPORT

RESPONSE OF A MULTICONDUCTOR CABLE TO AN EXTERNAL ELECTROMAGNETIC FIELD

by

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for

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conductors, cables, effects of EMP

August 1971

Prepared Under Contract No. DO-AF(29-601)-64-4457

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FIGURES

SYMBOL GLOSSARY AND INDEX

a: Eqs. (4)

$a_{\pm}$: Eqs. (16)

a_{ik}: Eq. (51)

a_{kk}: Eq. (48)

b: Eqs. (4)

$b_{\pm}$: Eqs. (16)

$\underline{C}$: Line capacitance-coefficient matrix; Eq. (9)

$C_{ij}(i,j = 1,\ldots,N)$: N-line capacitance coefficient between i^{th} and j^{th} conductors; element of i^{th} row, j^{th} column of $\underline{C}$.

f: Frequency, Hz

$\underline{I}^i$, $\underline{I}^o$: Eqs. (3)

$\underline{I}^i_{\pm}$, $\underline{I}^o_{\pm}$: Fig. 10. See, also, accompanying text.

$I^i_k(k = \pm 1,\ldots,\pm N)$: Input current, k^{th} conductor; Eqs. (3)

$I^o_k(k = \pm 1,\ldots,\pm N)$: Output current, k^{th} conductor; Eqs. (3)

$\underline{\mathcal{I}}$: Unit matrix; Eq. (18)

$\underline{\mathcal{I}_c}$: Unit N-vector; Eq. (14)

I_T^o: Bulk output current

$$I_T^o = \sum_{k=1}^{N} I_k^o$$

$j = \sqrt{-1}$

$k_\pm$: Eq. (20)

L, $L_\pm$: Eqs. (22)

ℓ: Line length, meters

$M_\pm$, $N_\pm$: Eqs. (17)

N-line: A line consisting of N conductors plus a reference conductor; i.e., (N + 1) conductors in all. Section 2.1

$P_\pm$: Eqs. (17)

$\underline{V}^e$: Eq. (13)

$\underline{V}^g$: Thévenin generator open-circuit emf; Eq. 12

$\underline{V}^i$, $\underline{V}^o$: Eqs. (3)

$\underline{V}_\pm^i$, $\underline{V}_\pm^o$: Fig. 10. See, also, accompanying text.

$\underline{V}_o$: Fig. 9. See, also, accompanying text, Section 2.3.

V_G: Shield voltage source

V_k^g: k^{th} component of $\underline{V}^g (k = 1,\ldots,N)$

SYMBOL GLOSSARY AND INDEX (CONT'D)

V_k^i, $(k = \pm 1, \ldots, \pm N)$: Input voltage, k^{th} conductor; Eqs. (3)

V_k^o, $(k = \pm 1, \ldots, \pm N)$: Output voltage, k^{th} conductor; Eqs. (3)

v: Speed of propagation, meters/second

$\underline{Y}$: Line admittance matrix; Section 2.1

Y_{ij}: N-line admittance coefficient; element of the i^{th} row, j^{th} column of $\underline{Y}(i,j = 1,\ldots,N)$

$\underline{Y}^i$: Canonical passive termination matrix, "i" end of line; Eq. (11)

$\underline{Y}^o$: Canonical passive termination matrix, "o" end of line; cf. $\underline{Y}^i$

Y_{ij}^i: Element of the i^{th} row, j^{th} column of $\underline{Y}^i$

Y_{ij}^o: Element of the i^{th} row, j^{th} column of $\underline{Y}^o$

$Y_{\pm}^o$: Fig. 10. See, also, accompanying text.

Y_m^c: Common-mode characteristic admittance of m^{th} conductor with respect to ground; Eq. (26)

$\underline{Y}^c$: Common-mode admittance vector; Eq. (27)

Y_o^c: Common-mode characteristic admittance of whole line; Eq. (30)

$Y_{\pm}^{oT}$: Eq. (31) and accompanying text

$\underline{Z}$: Line impedance matrix; Eq. (6)

Z_{ij}: N-line impedance coefficient between i^{th} and j^{th} conductor ($i,j = 1,\ldots,N$); element of the i^{th} row, j^{th} column of $\underline{Z}$.

$\gamma(k_-, k_+; a_+)$: Eq. (34)

δ_k^j: Kronecker delta; Eq. (48a)

ε: Dielectric permittivity, F/meter

θ: Electrical length of line or cable, radians; Eqs. (5)

$\theta_\pm$: Fig. 10. See, also, accompanying text. cf. "θ"

$\underline{\Lambda}$: Eq. (19). See, also, Section 2.5.2.

μ: Magnetic permeability, H/meter

π: 3.14159

σ_i: Eq. (52)

ω: Angular frequency, radians/second

$$\omega = 2\pi f$$

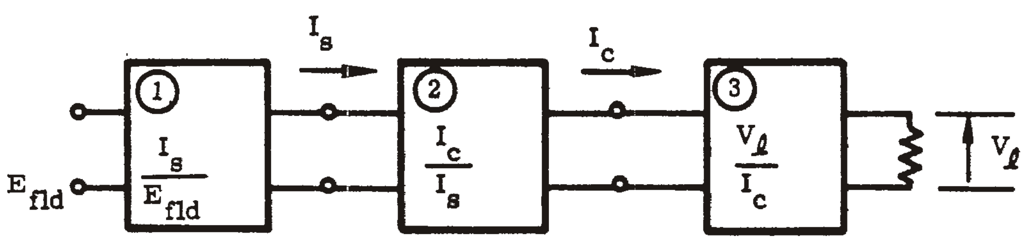
1. INTRODUCTION

This report summarizes results of a study of the transmission properties of multiconductor cables excited in various related ways, and coordinates the analyses and data presented in the pertinent interim reports.[1-4;25] The investigation supplements one part of an overall study undertaken by Sandia Laboratories,* with the purpose of developing rational procedures for analyzing the effects of an electromagnetic pulse on missiles and aircraft in flight.[5] According to the approach adopted by Sandia (op. cit.), the effect of such a pulse on some electrical device or circuit is to be analyzed by conceiving the physical environment as a sequence of transducers (Fig. 1). Such a sequence could consist of (1) the external surface of the missile or aircraft, (2) some coupling means between body surface and the conductors of a cable (e.g., a radiation slot on the missile surface), and (3) the cable itself, connected to the affected device or circuit. The assignment for the study reported here was to develop the relations among the dynamic quantities (currents and voltages) of the third element in this transducer sequence.

The essence of the work statement included in the applicable contract (No. 11-1756, Aug. 7, 1970) reads as follows:

*Under Contract No. DO AF(29-601)-64-4457

$$V_\ell = \frac{I_s}{E_{fld}} \times \frac{I_c}{I_s} \times \frac{V_\ell}{I_c} \times E_{fld}$$



(1) external surface of missile or aircraft

(2) coupling means

(3) multiconductor cable

Fig.1. Example of transfer-function factoring

"Proposed Research

The analysis of simple circuits should provide much of the desired information. Consider wires interconnecting two packages of electronic equipment, completely enclosed in a metal sheath except at one point where the sheath is broken. Some interfering voltage is assumed to be impressed across the gap in the shield.

The analysis of this circuit is to reveal:

A. The internal input impedance (contribution due to components inside the cable shield) across the broken shield at the breakpoint as a function of location, cable length, number of wires, the load impedances, and the frequency of the excitation.

B. The voltages across the various load resistors as a function of the bulk current, and the dependence of this voltage distribution on break location, cable length, number of wires, load impedances and excitation frequency are to be established.

It is recognized that wire diameters, sheath diameter, insulating materials and cable wrap will affect the mutual capacitances and inductances within the cable. The effect of varying these parameters is to be established as a part of this study. However, average or typical values may be used in (A) and (B) above.

The second situation to be considered is similar to the above except that the cable in this case shall be assumed to be unshielded and excited by a local source at some location along the cable.

For this circuit is required:

A. The input impedance defined as the ratio of source voltage to bulk current. The dependence of the input impedance on source location, cable length, number of wires, excitation frequency and the load impedances are to be evaluated.

B. The distribution of voltages on the terminating load impedances should be related to the bulk current entering the package. The dependence of this relation on source location, cable length, number of wires, excitation frequency and the load impedances are also to be evaluated.

C. The high frequency loss in current (ratio of bulk current at the end of the cable to bulk current at the source) is to be evaluated.

Another problem involves a shielded and unshielded internal coupling involving balanced and unbalanced sources. For both the shielded and the unshielded circuits, the following information is required:

A. Can the voltages across the various loads on the receiving end still be related to the bulk current entering the receiving package?

B. Assuming that the relation is true for some values of the termination and the cable length, indicate the dependence of the relation on cable length, number of wires, excitation frequency and the various load impedances.

Both balanced and unbalanced sources are to be considered as indicated above. Here again, typical values of cable parameters are to be used to focus attention on the part of the problem of interest."

Briefly stated, the assignment can be divided into two main activities:

(1) Establish appropriate mathematical models describing the dynamic response of the cables to the specified forces.

(2) Interrogate the models for the information requested in the contract statement.

Subject to certain restrictions, to be discussed in the next section, the first activity was accomplished to the extent that, in the end, a single general model, satisfying all contract-stipulated situations as special cases, was developed. The model yields explicit expressions for voltages and currents at all accessible terminals of all conductors of a cable in terms of the pertinent line - and terminal parameters. By implication it also yields an explicit expression for the bulk current at any point, since this is simply the sum of the currents on the individual conductors.*

However, these expressions are generally too complicated for interrogation by hand analysis. Except for the simplest cases,[2] the process must be computerized. Thus, the second of the activities listed above has, for the most part, been handled by Sandia, using the CDC-6600 computer.** The results of this activity are reported by Sandia elsewhere.[6]

A further qualification on the extent of the results obtained must be made. The formulations are functions of a set of internal line parameters - the line admittance coefficients.*** Useful analytical solutions for these coefficients are available when the conductor diameters are small compared to their spacings and their distances from the shield,[1] or when the conductor diameters are so large that they almost touch each other and and the shield.[4] Intermediate situations require solution of two-dimensional field problems by machine computation or by

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*Excluding the current in the shield or, more generally, the ground return.

**Sandia personnel participated as follows: Mr. George Steigerwald programmed the general solution while Dr. David Merewether and Mr. James Spence planned and analyzed the data runs.

***Or, alternatively, their inverses, the impedance coefficients.[7]

analog experiments. Schedule limitations permitted only a preliminary statement of this problem.[24]

2. TECHNICAL DISCUSSION - GENERAL REMARKS

In the interim reports previously submitted, the assignments specified in the contract work statement were given designations as follows:[3]

- Type I: Broken Shield Problem[1]
- Type II: Exposed Line Externally Excited[3]
- Type III: End-Excited Shielded Cable[2]
- Type IV: End-Excited Exposed Line[2]

Except possibly for the internal immittance parameters there is no essential difference between types III and IV, and they will usually be discussed together.

Some general remarks are in order at this point:

1. Although all of the tasks call for single-frequency analysis, signals in general, and EMP signals in particular are polychromatic. Consequently, the results obtained in this study must be applied to Fourier-analyzed forcing signals and synthesized over the spectrum for a total response.

2. Similarly, Problem Type II, in which the forcing function is a space impulse, or Green's function, at an arbitrary point along the line, is, by itself, an artificial problem, in that it is difficult to see how such a situation would arise. Actually,

it is to be considered as an elementary forcing function which, when specified at every point of the line, can be integrated to yield an arbitrary, physically real, distributed forcing function.

3. In spite of the fact that the conductors of a cable may be twisted (without, however, greatly modifying their immittance coefficients vis-a-vis straight conductors), and in spite of the fact that the cable cross-section may consist of more than one dielectric of somewhat different permittivities, it was felt that taking these factors into account would introduce serious complexities in the analysis not justified by the phenomenological (rather than precise design) nature of the study. Assuming that the cable dielectric is uniform and the line immittances are constant, and adding the additional assumption that the conductor losses are small, permits us to study the phenomena in the cable as transverse electromagnetic (TEM).* In fact, by agreement, the analysis was confined to lossless lines, although from a mathematical standpoint, extension to the case where dielectric losses predominate is elementary. (See Section 2.7.1.)

*Of course, we are also assuming that we are dealing only with the transmission mode. All conductors are assumed spaced from one another much less than a wavelength and the total current at any cross-section (shield - or ground current included) is zero.

4. The work statement requires, in the case of Problem Types III and IV, that consideration be given to situations involving balanced and unbalanced sources. These cases were not given special treatment since they really represent conditions on the line terminations, easily inserted in the general solution. Hence, the effects of these terminations must be ascertained by appeal to the computer.

2.1 Physical Model of a Multiconductor TEM Line: Canonical Equations

Physically, a multiconductor TEM line generally consists of a number of lossless conductors of equal length and of arbitrary cross-section, embedded in a homogeneous isotropic dielectric. The geometry of the system is invariant in the direction of propagation.

For a specified set of terminal conditions at a single frequency the electromagnetic field structure surrounding the conductors consists, generally, of two waves traveling at equal speeds in opposite directions along the line. The relative field distributions of these waves differ, generally, from each other (except in the case of the conventional two-conductor line), depending on the terminal conditions. Given a system of (N + 1) conductors, one conductor (e.g., a cable shield or a ground plane) is taken as a reference conductor, and the potentials of the remaining N conductors are referred to it. Conventionally, we refer to these latter conductors as the "N conductors above ground." The total current in (N + 1) conductors at any cross-section of the line is zero, while the total current in the N conductors above ground may be, but generally is not, equal to zero.

A line so described will be called an N-line. Thus, the seven conductor cable* discussed later in this report is a 7-line. The conventional two-wire line is a 1-line. A balanced line above ground is a 2-line, etc. The canonical equations of such a line may be written in various forms, the choice depending on the nature of the problem. Two of these forms, one the inverse of the other, may be written

$$\begin{bmatrix} \underline{V}^i \\ \underline{V}^o \end{bmatrix} = \begin{bmatrix} a, & b \\ -b, & -a \end{bmatrix} \begin{bmatrix} \underline{Z}\ \underline{I}^i \\ \underline{Z}\ \underline{I}^o \end{bmatrix} \qquad (1)$$

or

$$\begin{bmatrix} \underline{I}^i \\ \underline{I}^o \end{bmatrix} = \begin{bmatrix} a, & b \\ -b, & -a \end{bmatrix} \begin{bmatrix} \underline{Y}\ \underline{V}^i \\ \underline{Y}\ \underline{V}^o \end{bmatrix} \qquad (2)$$

The superscript, i, (for "input" represents quantities at one end of the line, while the superscript, o, (for "output") represents quantities at the other end. If we number the N conductors above ground from 1 to N, the quantities $\underline{V}^i$, $\underline{V}^o$,

*Seven conductors within a conducting shield, i.e., eight conductors in all.

$\underline{I}^i$, $\underline{I}^o$ are N-vectors representing the voltages and currents at the line terminals, as follows:

$$\left.\begin{array}{l} \underline{V}^i = \begin{bmatrix} V_1^i \\ \cdot \\ \cdot \\ \cdot \\ V_N^i \end{bmatrix} ; \quad \underline{V}^o = \begin{bmatrix} V_1^o \\ \cdot \\ \cdot \\ \cdot \\ V_N^o \end{bmatrix} \\ \\ \underline{I}^i = \begin{bmatrix} I_1^i \\ \cdot \\ \cdot \\ \cdot \\ I_N^i \end{bmatrix} ; \quad \underline{I}^o = \begin{bmatrix} I_1^o \\ \cdot \\ \cdot \\ \cdot \\ I_N^o \end{bmatrix} \end{array}\right\} \tag{3}$$

where V_k^i, V_k^o, I_k^i, I_k^o represent quantities associated with the k^{th} conductor ($k = 1, \ldots, N$). a and b are functions of the electrical length of the line:

$$\left.\begin{array}{l} a = -j \cot \theta \\ \\ b = j \csc \theta \end{array}\right\} \tag{4}$$

where, as usual,

$$\left.\begin{array}{l} \theta = \frac{\omega \ell}{v} \\ \\ \omega = 2\pi f \\ \\ v = (\mu\varepsilon)^{-\frac{1}{2}} \end{array}\right\} \tag{5}$$

ℓ is the line length, f is the operating frequency, v the velocity of propagation, μ and ϵ the permeability and permittivity of the propagation medium, respectively.*

The matrix, $\underline{Z}$, is the matrix of the line impedance coefficients:

$$\underline{Z} = \begin{bmatrix} Z_{11}, & Z_{12}, & \cdots, & Z_{1N} \\ Z_{21}, & Z_{22}, & \cdots, & Z_{2N} \\ \cdot & \cdot & \cdot & \cdot \\ Z_{N1}, & Z_{N2}, & \cdots, & Z_{NN} \end{bmatrix} \quad (6)$$

while

$$\underline{Y} = \underline{Z}^{-1} = \begin{bmatrix} Y_{11}, & Y_{12}, & \cdots, & Y_{1N} \\ Y_{21}, & Y_{22}, & \cdots, & Y_{2N} \\ \cdot & \cdot & \cdot & \cdot \\ Y_{N1}, & Y_{N2}, & \cdots, & Y_{NN} \end{bmatrix} \quad (7)$$

$\underline{Y}$ is readily determined when the Maxwell electrostatic capacitance coefficients of the line are known.** We have

$$\underline{Y} = v\underline{C} \quad (8)$$

*MKS units used throughout, unless specified otherwise.

**See Ref. 8, Section 6.24 ff; Ref. 9, Chapter IV; Ref. 10 Section 2.14 ff.

and

$$\underline{C} = \begin{bmatrix} C_{11}, & C_{12}, & . & . & . & , C_{1N} \\ C_{21}, & C_{22}, & . & . & . & , C_{2N} \\ . & . & . & . & . & . \\ C_{N1}, & C_{N2}, & . & . & . & , C_{NN} \end{bmatrix} \qquad (9)$$

where the C_{ij} $(i, j = 1, \ldots, N)$ are the Maxwell capacitance coefficients.

The sign convention for the currents is into the line at the "i" end and out of the line at the "o" end.

The two matrix equations obtained by expanding Equations (1) are

$$\left.\begin{aligned} \underline{V}^{i} &= a\ \underline{Z}\ \underline{I}^{i} + b\ \underline{Z}\ \underline{I}^{o} \\ \underline{V}^{o} &= -b\ \underline{Z}\ \underline{I}^{i} - a\ \underline{Z}\ \underline{I}^{o} \end{aligned}\right\} \qquad (10)$$

Naturally, these equations hold for the special case of the 1-line, for which all quantities become scalars. In particular, $\underline{Z}$ becomes Z_{11} and is identified as the characteristic impedance of the line.

These ideas suggest a simple way of representing an N-line schematically, namely, as a generalization of a 1-line. Thus, we get the diagram of Fig. 2, which is the same as the usual representation of a 1-line, except that (a) the upper line

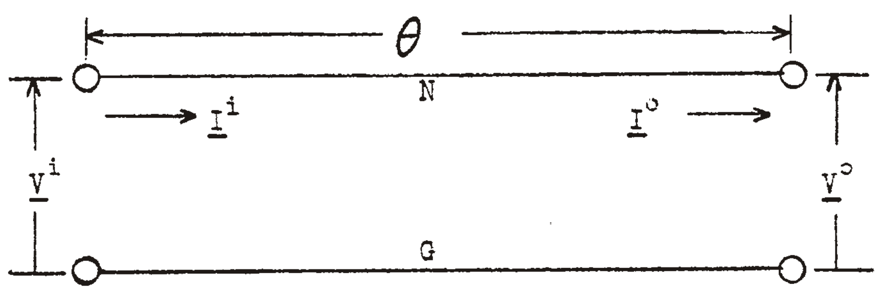

Fig.2. Schematic representation of an N-line.

segment (N) represents the N conductors above ground while the lower segment (G) represents the reference conductor, and (b) all dynamic quantities are N-vectors in place of the scalar voltages and currents of the 1-line.

2.2 Terminal Conditions

The terminations of an N-line in general contain both sources and sinks of energy. The terminal structures themselves may consist of networks of lumped and/or distributed elements. In the present study the only situations requiring special attention involve lumped (passive or active) elements.

2.2.1 Canonical Passive Termination; Thévenin Load

Consider, first, the case of a passive terminating network or load. Some of the nodes of such a network are connected to the line terminals while others are not. For example, in Fig. 3, node "a" is not connected to any line terminal.

Whatever the internal network configuration, it is clear that if one were to disconnect the termination from the line and measure all the possible admittances among the (N + 1) terminals in pairs, they would number $\frac{1}{2}N(N + 1)$, that is, the number of combinations of (N + 1) things taken two at a time. An equivalent network (as seen from the line) could then be constructed by joining the terminals in pairs with admittances derived from the measurements (Fig. 4). Again, there would be

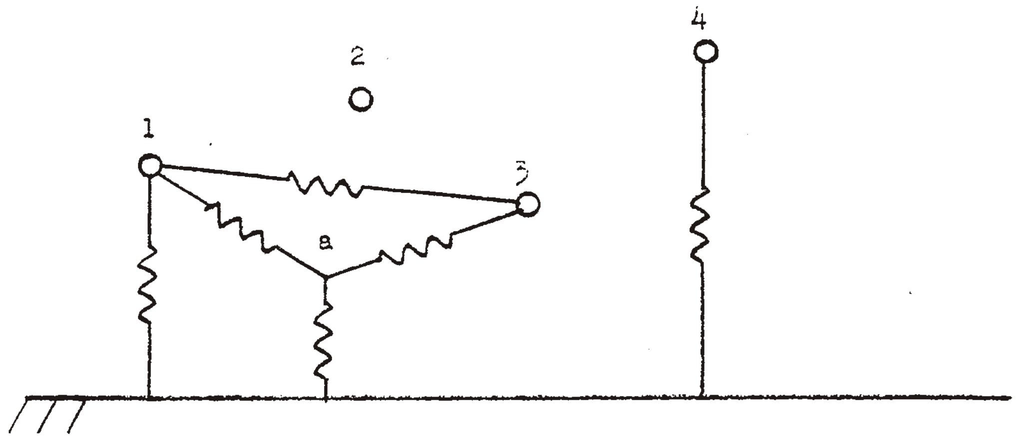

Fig.3. Example of N-line terminating network; N = 4.

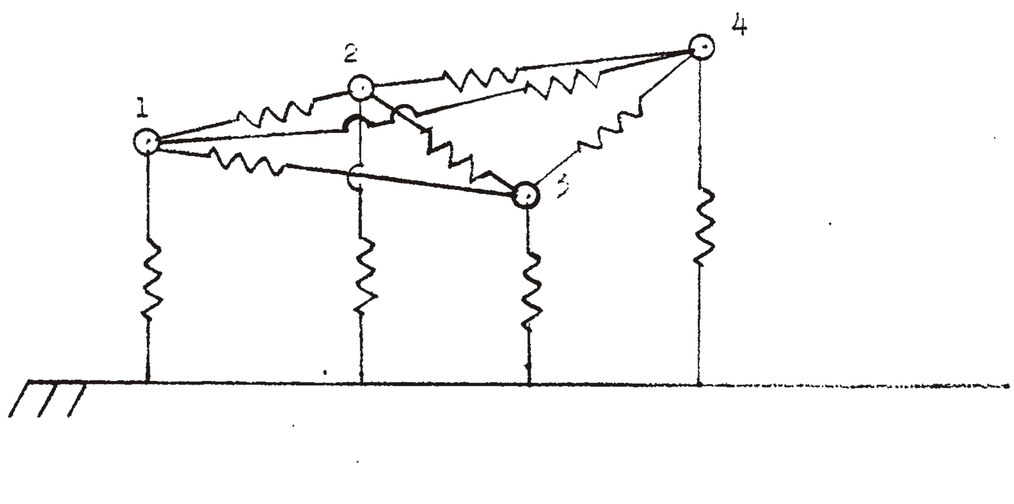

Fig.4. Equivalent termination as seen from line.

$\frac{1}{2}N(N + 1)$ such admittances,* with their values uniquely determined by the measurements.**

From the standpoint of analysis of the dynamic response of the line, the equivalent network represented by the example of Fig. 4 is sufficient. An adequate representation of the termination admittance matrix could, for instance, be

$$\underline{Y}^i = \begin{bmatrix} Y^i_{11}, & Y^i_{12}, & \cdots, & Y^i_{1N} \\ Y^i_{21}, & Y^i_{22}, & \cdots, & Y^i_{2N} \\ \cdot & \cdot & \cdot & \cdot \\ Y^i_{N1}, & Y^i_{N2}, & \cdots, & Y^i_{NN} \end{bmatrix} \quad (11)$$

where $Y^i_{ij}(i,j = 1, \ldots, N)$ is the admittance defined as the ratio of the current flowing out of the i^{th} terminal into the "output" load to the potential V^i_j at the j^{th} terminal, when all output potentials except V^i_j are zero.

On the other hand, if the conditions of the problem require, for instance, that the potential at "a" in Fig. 3 be determined, then one is faced with a somewhat generalized terminal-value problem. An example of such a problem, occurring in an earlier Sandia study, is discussed in Ref. 11.

*This is true in general. However, some values may tend to zero in the limit, others to infinity. Such cases are handled, where necessary in computation, by replacing zero or infinity by a very small, different from zero, number or a very large, finite one.

**See Ref. 1, Section 2.

A form like Equation (11) is taken as the canonical form for termination matrices in this study. For a reason which will be apparent presently, it is also referred to as a Thévenin load.

2.2.2 Thévenin Source

In the second interim report[2] we presented a simple generalization of the standard Thévenin's theorem.* Consider an N-source.** Let the open-circuit terminal potentials of the source be V_k^g ($k = 1, \ldots, N$). Set equal to zero all the emfs internal to the source (so that the V_k^g are all zero) and measure the $\frac{1}{2}N(N + 1)$ admittances between all terminal pairs of the source. Let $\underline{Y}^i$ (Equation (11)) be the unique termination matrix determined from these measurements. Then the equivalent Thévenin source is, as shown in Fig. 5., simply a vector of impedanceless generators

$$\underline{V}^g = \begin{bmatrix} V_1^g \\ \cdot \\ \cdot \\ \cdot \\ V_N^g \end{bmatrix} \tag{12}$$

in series with the above-ground terminals of $\underline{Y}^i$.

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*See Ref. 12, Chapter 2, Section 11.

**A source, with N terminals at N (generally) different, open-circuit potentials, plus a ground, or reference terminal.

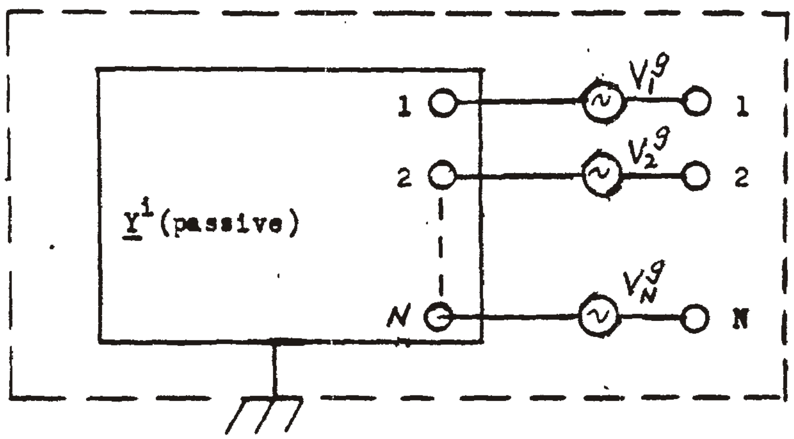


Fig. 5. Generalized Thévenin Generator.

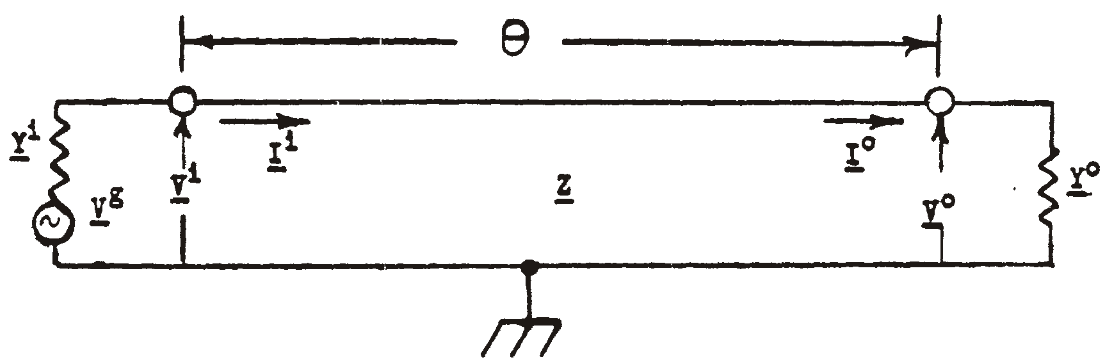


Fig. 6. Parameters and Terminations of an N-line.

Evidently the Thévenin load of Section 2.2.1 is the special case of a Thévenin source with $\underline{V}^g = \underline{0}$.

Schematically, when a Thévenin N-source and a Thévenin N-load are connected to an N-line, the result is shown in Fig. 6, again a simple generalization of the conventional 1-line diagram. In this case we are using the symbol $\underline{Y}^o$ to designate the terminating load matrix.

This completes the preliminary disucssion of basic model concepts. The next step consists of relating the problems of the contract work statement to the basic model.

2.3 Mathematical Models of Problem Type I - IV

Problem Type I, cable with a circumferential break in the shield, is exemplified in Fig. 7, which shows a shielded 4-line between two shielded terminations, and with a break in the shield at some intermediate point. A voltage is assumed impressed across the break.

This problem is treated as two lines in series, one on each side of the break, with the combination connected to a source representing the impressed voltage. Consult Fig. 8. The line to the left of the break has parameters with the "minus" subscript, - ; the parameters of the line to the right have the "plus" subscript, +. The termination matrices, $\underline{Y}^o_-$ and $\underline{Y}^o_+$, are

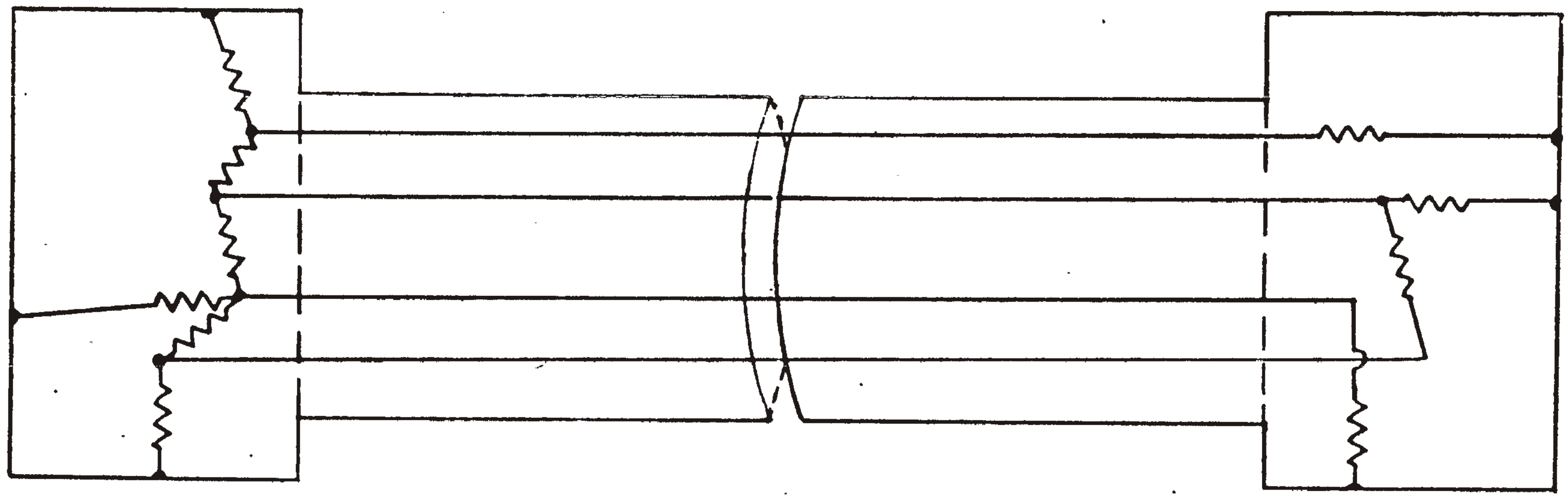


Fig. 7. Cable With a Broken Shield

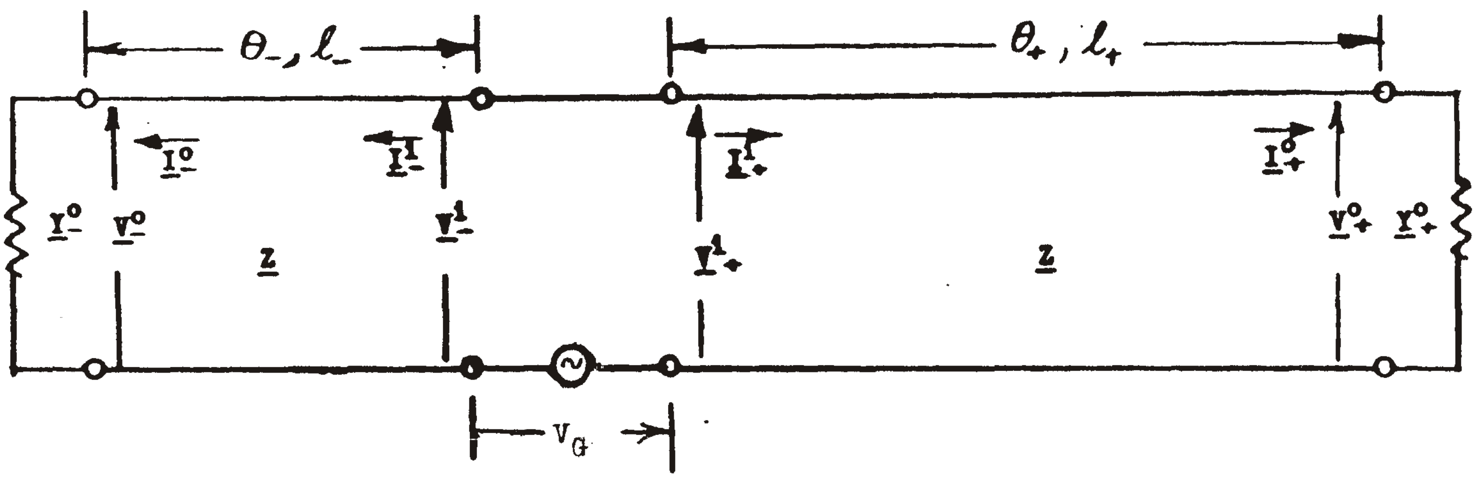


Fig.8. Schematic diagram, cable with broken shield.

Thévenin loads. The lines have the common line impedance matrix, $\underline{Z}$*

Problem Type II, excitation of an unshielded cable by a local source at some location along the cable, is shown schematically in Fig. 9. The components of the impressed voltage vector $\underline{V}_o$, are, in general, unequal. The diagram is similar to that of the preceding case except for the location of the source and the relative magnitudes of its elements.

Problem Types III and IV, cable excited from one end, is merely the conventional transmission line problem extended to an N-line. In fact, Fig. 6 is an appropriate representation.

2.4 The General Model

Consider the schematic of Fig. 10, which shows an N-line excited by a local source which includes both an N-vector in the N conductors above ground and a scalar in the reference conductor.

The whole source can be written as the single vector

$$\underline{V}^e = \underline{V}_o - V_G \underline{\mathcal{L}}_c \tag{13}$$

*A small difference in notation from Ref. 1 should be noted. Since that report was prepared it has appeared desirable to change V_g of the cited reference to $-V_G$ of Ref. 3 and the present report.

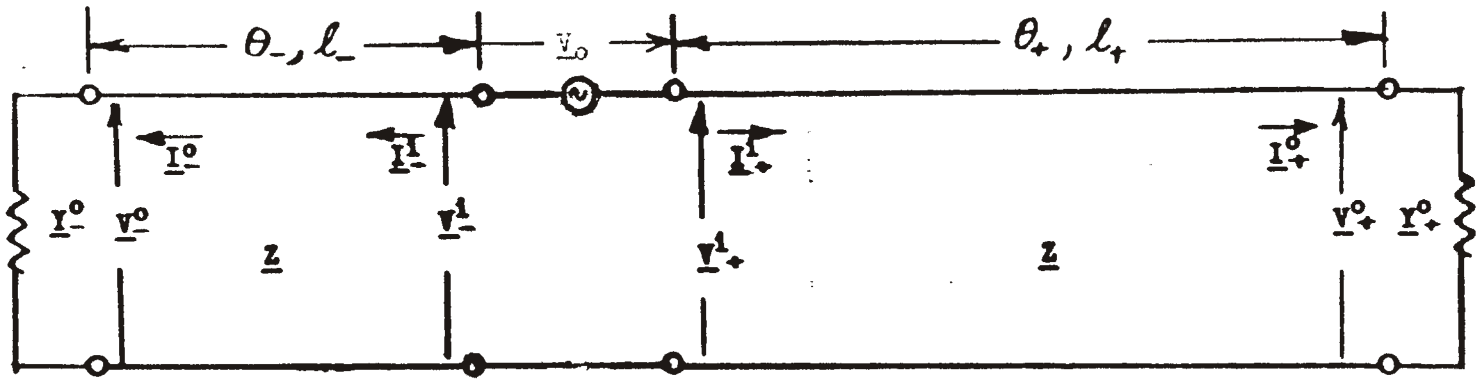


Fig.9. Schematic diagram, unshielded line with local excitation.

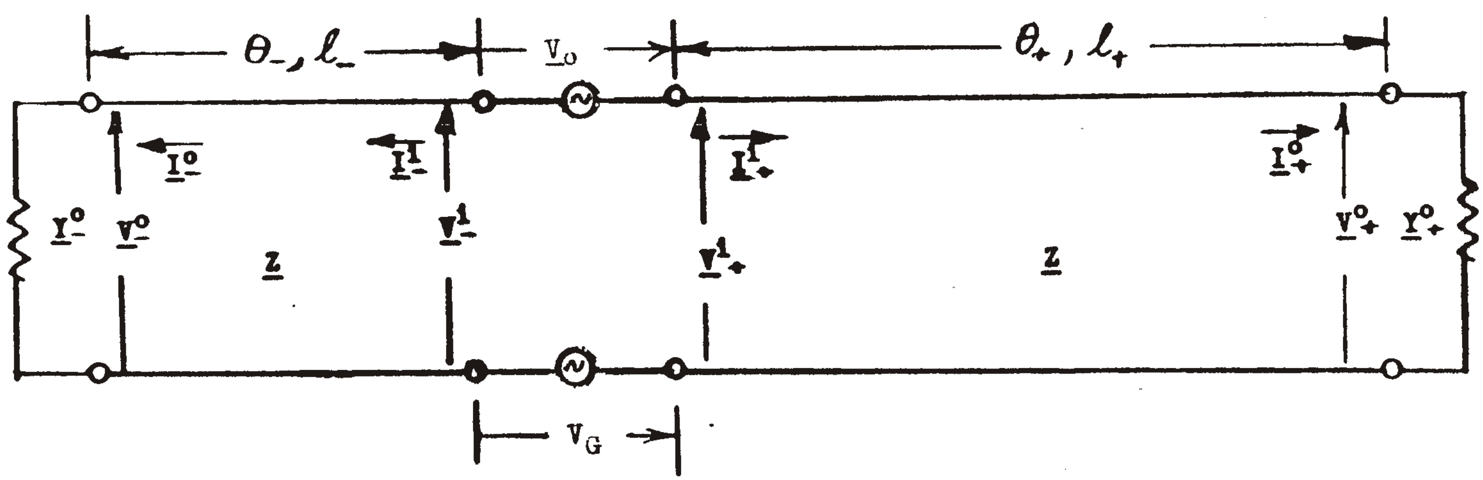


Fig.10. Schematic diagram, general model.

where $\mathcal{l}_c$ is the unit-element N-vector

$$\mathcal{l}_c = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}_N \qquad (14)$$

Before presenting the solution for this model we indicate the parameter values yielding the special cases, Problems Types I - IV:*

Type I: $\underline{V}_o = 0.$

Type II: $V_G = 0.$

Types III and IV: $V_G = 0;\ \theta_- = 0$.

The solution to the general case is[3]

*At the time of preparation of this final report Sandia had written a program only for problem Type I. It is now apparent that, in order to cover all cases, that program needs only to be modified by replacing $V_g\,\mathcal{l}_c$ with $\underline{V}^e$ as given by Equation 13. However, see footnote, page 34. See also, Ref. 3 for solution of Type IV problem as a line responding to a continuous distribution of voltage and current sources along the line.

$$
\left.\begin{aligned}
\underline{V}^{o}_{-} &= b_{-}\,(\underline{\Lambda}\,\underline{N}_{-})^{-1}\,\underline{V}^{e} && \text{(a)}\\
\underline{I}^{o}_{-} &= b_{-}\,\underline{Y}^{o}_{-}\,(\underline{\Lambda}\,\underline{N}_{-})^{-1}\,\underline{V}^{e} && \text{(b)}\\
\underline{V}^{i}_{-} &= -\underline{M}_{-}\,(\underline{\Lambda}\,\underline{N}_{-})^{-1}\,\underline{V}^{e} && \text{(c)}\\
\underline{I}^{i}_{-} &= -(\underline{\Lambda}\,\underline{Z})^{-1}\,\underline{V}^{e} && \text{(d)}\\
\underline{V}^{i}_{+} &= \underline{M}_{+}\,(\underline{\Lambda}\,\underline{N}_{+})^{-1}\,\underline{V}^{e} && \text{(e)}\\
\underline{I}^{i}_{+} &= -\underline{I}^{i}_{-} = (\underline{\Lambda}\,\underline{Z})^{-1}\,\underline{V}^{e} && \text{(f)}\\
\underline{V}^{o}_{+} &= -b_{+}\,(\underline{\Lambda}\,\underline{N}_{+})^{-1}\,\underline{V}^{e} && \text{(g)}\\
\underline{I}^{o}_{+} &= -b_{+}\,\underline{Y}^{o}_{+}\,(\underline{\Lambda}\,\underline{N}_{+})^{-1}\,\underline{V}^{e} && \text{(h)}
\end{aligned}\right\}\qquad (15)
$$

The meanings of the quantities

$$\underline{V}^{i}_{+},\ \underline{V}^{i}_{-},\ \underline{V}^{o}_{+},\ \underline{V}^{o}_{-}\quad ;$$

$$\underline{I}^{i}_{+},\ \underline{I}^{i}_{-},\ \underline{I}^{o}_{+},\ \underline{I}^{o}_{-}\quad ;$$

and

$$\underline{Z},\ \underline{Y}^{o}_{+},\ \underline{Y}^{o}_{-}\quad ,$$

are clear from Fig. 10, and from the foregoing discussion. In addition, with

$$\left.\begin{aligned} a_{\pm} &= -j \cot \theta_{\pm} \\ b_{\pm} &= j \csc \theta_{\pm} \\ j &= \sqrt{-1} \end{aligned}\right\} \quad (16)$$

we define

$$\left.\begin{aligned} \underline{P}_{\pm} &= \underline{Z}\ \underline{Y}^{o}_{\pm} \\ \underline{M}_{\pm} &= a_{\pm}\ \underline{\mathcal{I}} + \underline{P}_{\pm} \\ \underline{N}_{\pm} &= \underline{\mathcal{I}} + a_{\pm}\ \underline{P}_{\pm} \end{aligned}\right\} \quad (17)$$

$$\underline{\mathcal{I}} = \text{unit } N \times N \text{ matrix}$$

$$= \begin{bmatrix} 1 & & & \\ & 1 & & 0 \\ & & \ddots & \\ 0 & & & 1 \end{bmatrix} \quad (18)$$

Finally we define

$$\underline{\Lambda} = \underline{M}_{+}\ \underline{N}_{+}^{-1} + \underline{M}_{-}\ \underline{N}_{-}^{-1} \quad (19)$$

2.5 Discussion of Results

Certain degenerate forms taken by Equations (15) for special values of θ_{+} and θ_{-} are discussed in Appendix A. In the sections immediately following we discuss cases in which the matrices of the line and termination parameters have values that simplify interpolation of the results and yield some insight into the meaning of the more general solution.

2.5.1 Termination Matrices Proportional to Line Admittance Matrix

Write

$$\underline{Y}^{o}_{\pm} = k_{\pm} \underline{Y} \tag{20}$$

where $k_{\pm}$ are scalar constants. Then

$$\left.\begin{aligned} \underline{P}_{\pm} &= k_{\pm} \underline{\mathcal{Y}} \\ \underline{M}_{\pm} &= (a_{\pm} + k_{\pm}) \underline{\mathcal{Y}} \\ \underline{N}_{\pm} &= (1 + a_{\pm} k_{\pm}) \underline{\mathcal{Y}} \end{aligned}\right\} \tag{21}$$

Write

$$\left.\begin{aligned} L_{\pm} &= \frac{a_{\pm} + k_{\pm}}{1 + a_{\pm} k_{\pm}} \\ L &= L_{+} + L_{-} \end{aligned}\right\} \tag{22}$$

Thus

$$
\left.
\begin{aligned}
\underline{M}_{\pm}\ \underline{N}_{\pm}^{-1} &= L_{\pm}\ \mathscr{I} \\
\underline{\Lambda} &= L\ \mathscr{I} \\
\underline{\Lambda}\ \underline{Z} &= L\ \underline{Z} \\
\underline{\Lambda}\ \underline{N}_{\pm} &= L(1 + a_{\pm}\ k_{\pm})\ \mathscr{I}
\end{aligned}
\right\} \qquad (23)
$$

Equations (15) become

$$
\left.
\begin{aligned}
\underline{V}_{\pm}^{i} &= \pm \frac{L_{\pm}}{L_{+} + L_{-}}\ \underline{V}^{e} \\
\underline{I}_{\pm}^{i} &= \pm \frac{1}{L}\ \underline{Y}\ \underline{V}^{e} \\
\underline{V}_{\pm}^{o} &= \pm \frac{1}{L}\ \frac{-b_{\pm}}{1 + a_{\pm}\ k_{\pm}}\ \underline{V}^{e} \\
\underline{I}_{\pm}^{o} &= \pm \frac{k_{\pm}}{L}\ \frac{-b_{\pm}}{1 + a_{\pm}\ k_{\pm}}\ \underline{Y}\ \underline{V}^{e}
\end{aligned}
\right\} \qquad (24)
$$

One fact that is immediately apparent is that the elements of corresponding input and output vectors are in the same proportion, regardless of the nature of the generator vector. For instance, the elements of $\underline{V}_{+}^{i}$ have the same respective ratios as the corresponding elements of $\underline{V}_{+}^{o}$; the two vectors have the same "direction" but differ in amplitude and phase. Similar statements hold for $\underline{I}_{+}^{i}$ and $\underline{I}_{+}^{o}$, etc.

The next concepts are derived by reducing the N-line to a 1-line. This is done by applying a common value of emf to all conductors, retaining the condition of proportional terminations.

2.5.1.1 Common EMF on All Conductors, Proportional Terminations. Common Mode Characteristic Admittance. Load Impedance Mapping Function

Let

$$\underline{V}^e = V^g \underline{\mathcal{J}_c}$$

in Equations (24). Then these become

$$\left.\begin{aligned}
\underline{V}^i_\pm &= \pm \frac{L_\pm}{L_+ + L_-} V^g \underline{\mathcal{J}_c} && \text{(a)}\\
\underline{I}^i_\pm &= \pm \frac{V^g}{L} \underline{Y} \underline{\mathcal{J}_c} && \text{(b)}\\
\underline{V}^o_\pm &= \pm \frac{V^g}{L} \cdot \frac{-b_\pm}{1 + a_\pm k_\pm} \underline{\mathcal{J}_c} && \text{(c)}\\
\underline{I}^o_\pm &= \pm \frac{k_\pm V_g}{L} \cdot \frac{-b_\pm}{1 + a_\pm k_\pm} \underline{Y} \underline{\mathcal{J}_c} && \text{(d)}
\end{aligned}\right\} \qquad (25)$$

If we write

Y^c_m = common mode characteristic admittance of the m^{th} conductor with respect to ground

$$= \sum_{k=1}^{N} Y_{mk} \qquad (26)$$

then the quantity $\underline{Y} \underline{\mathcal{J}_c}$ appearing in Equations (25, b, d) may be written

$$\underline{Y}\,\underline{\mathcal{J}_c} = \begin{bmatrix} Y_1^c \\ Y_2^c \\ \cdot \\ \cdot \\ \cdot \\ Y_N^c \end{bmatrix} = \underline{Y}^c \qquad (27)$$

$\underline{Y}^c$ may be termed the common-mode admittance vector. A line with proportional terminations, excited by an equi-element emf source vector, operates in the common mode.

If $k_\pm = 1$, the line becomes match-terminated at both ends. For this condition we get

$$L_\pm = 1, \ L = 2$$

$$-\frac{b_\pm}{1 + a_\pm\, k_\pm} = \frac{-j \csc \theta_\pm}{1 - j \cot \theta_\pm} = e^{-j\theta_\pm}$$

Then with the help of Equation 27, Equations (25) become

$$\left.\begin{aligned} \underline{V}_\pm^i &= \pm\, V^g\, \underline{\mathcal{J}_c} && \text{(a)} \\ \underline{I}_\pm^i &= \pm\, \tfrac{1}{2}\, V^g\, \underline{Y}_c && \text{(b)} \\ \underline{V}_\pm^o &= \pm\, \tfrac{1}{2}\, V^g\, e^{-j\theta\pm}\, \underline{\mathcal{J}_c} && \text{(c)} \\ \underline{I}_\pm^o &= \pm\, \tfrac{1}{2}\, V^g\, e^{-j\theta\pm}\, \underline{Y}_c && \text{(d)} \end{aligned}\right\} \qquad (28)$$

Implications of the results in Equations (28 a - d) are:

1. When both ends of the cable are match-terminated, the input voltages to all conductors have the same magnitude, $|\frac{1}{2} V_g|$, but voltages on the opposite sides of the source have opposite signs.

2. The input currents to the lines on either side of the source satisfy the equations for waves traveling in one direction only, - away from the source.

3. The voltages at the terminations are all equal in magnitude, and have the values of the corresponding input voltages with phase delays equal to the electrical distance from the source to the termination.

These results accord with our usual notions for matched terminations which, in essence, require that no wave be reflected at the termination. Thus the condition

$$\underline{Y}_{\pm}^{o} = \underline{Y} \tag{29}$$

which follows from Equation (20) and the condition

$$k_{\pm} = 1$$

are consistent with the usual requirement for matched terminations on a 1-line.

For match conditions, the terminating admittance from each terminal to ground is the common-mode characteristic admittance for the conductor.[1] The terminating admittance required between any pair of terminals is apparently the negative of the mutual admittance coefficient for that pair of conductors. However, since by Equation (28 c), all terminal voltages at one end of the cable are equal, no current flows in the terminating admittances joining these terminals. Consequently these admittances may have any values, including zero. In fact, it is clear from Equations (25) that the last statement is true for a line operating in any common-mode condition.

Operation in the common mode implies that all conductors (except the shield) operate in parallel, so that the system is, in effect, a 1-line with characteristic admittance

$$Y_o^c = \sum_{k=1}^{N} Y_k^c = \sum_{k=1}^{N} \sum_{j=1}^{N} Y_{kj} \tag{30}$$

by Equation (26). Then $k_\pm$ are the VSWR's (or their reciprocals) corresponding to the total load admittances at either end of the cable.

From the first of Equations (22)

$$L_\pm = \frac{-j \cot \theta_\pm + \dfrac{Y_\pm^{oT}}{Y_o^c}}{1 - j\left(\dfrac{Y_\pm^{oT}}{Y_o^c}\right)\cot \theta_\pm} \tag{31}$$

where $Y_{\pm}^{OT}$ are the total output admittances in parallel at either end. Equation (31) is recognized as the usual normalized mapping of the load impedance of a 1-line to its input.* The quantity, L, (Equations 22)) is therefore the sum of these normalized impedances in series. The admittance seen by the voltage source is then L^{-1} multiplied by Y_o^c (cf. Equation (25 b)). Equation (25 a) then states that the driving voltages on opposite sides of the source divide in proportion to these input-impedances.

2.5.2 Interpretation of $\underline{M}_{\pm}\ \underline{N}_{\pm}^{-1}$ and $\underline{\Lambda}$

The interpretations above suggest that in the general case, (Equation (20) not satisfied), the quantities $\underline{M}_{\pm}\ \underline{N}_{\pm}^{-1}$ are to be interpreted as normalized mappings of the load impedance matrices to the input (voltage source) terminals, $\underline{\Lambda}$ is then the sum of these normalized impedances, while $\underline{\Lambda}\ \underline{Z}$ is the total actual input impedance matrix, since the line impedance matrix, $\underline{Z}$, is just the factor required to cancel the normalization. The quantities

$$R_{\pm} = \underline{M}_{\pm}\ (\underline{\Lambda}\ \underline{N}_{\pm})^{-1}$$

which appear in Equations (15c,e) are easily transformed to

$$R_{\pm} = (\underline{M}_{\pm}\ \underline{N}_{\pm})(\underline{M}_{+}\ \underline{N}_{+}^{-1} + \underline{M}_{-}\ \underline{N}_{-}^{-1})^{-1}$$

*See Ref. 13, page 22-4.

which express the input voltage division factors in proportion to the input matrix impedances, and so on.

2.5.3 Matched Line, Arbitrary Local Voltage Vector

With $k_{\pm} = 1$, Equations (24) become

$$\left.\begin{aligned} \underline{V}^{i}_{\pm} &= \pm \frac{1}{2} \underline{V}^{e} && \text{(a)} \\ \underline{I}^{i}_{\pm} &= \pm \frac{1}{2} \underline{Y}\, \underline{V}^{e} && \text{(b)} \\ \underline{V}^{o}_{\pm} &= \pm \frac{1}{2} e^{-j\theta_{\pm}} \underline{V}^{e} && \text{(c)} \\ \underline{I}^{o}_{\pm} &= \pm \frac{1}{2} e^{-j\theta_{\pm}} \underline{Y}\, \underline{V}^{g} && \text{(d)} \end{aligned}\right\} \qquad (32)$$

The important added information imparted by these equations is that the match condition results in reflectionless transmission independent of the relative values of the impressed voltage vector elements.

2.5.4 $\theta_{-} = 0$ (Problem Types III and IV). Proportional Terminations, Matched, Constant-Voltage, and Constant-Current Source Admittances

With $\theta_{-} \to 0$, we get, from Equations (22)

$$L_{-} \to \frac{1}{k_{-}}, \quad k_{-} \neq 0 \qquad (33a)$$

which express the input voltage division factors in proportion to the input matrix impedances, and so on.

2.5.3 Matched Line, Arbitrary Local Voltage Vector

With $k_{\pm} = 1$, Equations (24) become

$$\left.\begin{aligned} \underline{V}^i_{\pm} &= \pm \frac{1}{2} \underline{V}^e && \text{(a)} \\ \underline{I}^i_{\pm} &= \pm \frac{1}{2} \underline{Y}\, \underline{V}^e && \text{(b)} \\ \underline{V}^o_{\pm} &= \pm \frac{1}{2} e^{-j\theta\pm}\, \underline{V}^e && \text{(c)} \\ \underline{I}^o_{\pm} &= \pm \frac{1}{2} e^{-j\theta\pm}\, \underline{Y}\, \underline{V}^g && \text{(d)} \end{aligned}\right\} \qquad (32)$$

The important added information imparted by these equations is that the match condition results in reflectionless transmission independent of the relative values of the impressed voltage vector elements.

2.5.4 $\theta_- = 0$ (Problem Types III and IV). Proportional Terminations, Matched, Constant-Voltage, and Constant-Current Source Admittances

With $\theta_- \to 0$, we get, from Equations (22)

$$L_- \to \frac{1}{k_-}\ , \quad k_- \neq 0 \qquad (33a)$$

and

$$L \to \frac{\gamma}{1 + a_+ k_+} \qquad (33b)$$

where

$$\gamma(k_-, k_+; a_+) = \frac{1 + a_+(k_+ + k_-) + k_+ k_-}{k_-} \qquad (34)$$

Then the Equations (24) of interest become

$$\left.\begin{aligned} \underline{V}_+^i &= \frac{a_+ + k_+}{\gamma} \underline{V}^e & \text{(a)} \\ \underline{I}_+^i &= \frac{1 + a_+ k_+}{\gamma} \underline{Y} \, \underline{V}^e & \text{(b)} \\ \underline{V}_+^o &= -\frac{b_+}{\gamma} \underline{V}^e & \text{(c)} \\ \underline{I}_+^o &= -\frac{k_+ b_+}{\gamma} \underline{Y} \, \underline{V}^e & \text{(d)} \end{aligned}\right\} \qquad (35)$$

provided $k_- \neq 0$.

Consider the following special cases:

1. <u>$k_- = \infty$: Constant-Voltage Source</u>

From Equation (34),

$$\gamma(\infty, k_+; a_+) \to a_+ + k_+$$

and Equations (35) become

$$\left.\begin{aligned} \underline{V}_+^i &= \underline{V}^e && \text{(a)} \\ \underline{I}_+^i &= \frac{1 + a_+ k_+}{a_+ + k_+} \underline{Y}\, \underline{V}^e && \text{(b)} \\ \underline{V}_+^o &= - \frac{b_+}{a_+ + k_+} \underline{V}^e && \text{(c)} \\ \underline{I}_+^o &= - \frac{k_+ b_+}{a_+ + k_+} \underline{Y}\, \underline{V}^e && \text{(d)} \end{aligned}\right\} \quad (36)$$

2. <u>k = 1: Matched Source</u>

For $k_- = 1$ we have from Equation (34),

$$\gamma(1, k_+; a_+) = (1 + a_+)(1 + k_+)$$

Equations (35) become

$$\left.\begin{aligned} \underline{V}_+^i &= \frac{a_+ + k_+}{(1 + a_+)(1 + k_+)} \underline{V}^e && \text{(a)} \\ \underline{I}_+^i &= \frac{1 + a_+ k_+}{(1 + a_+)(1 + k_+)} \underline{Y}\, \underline{V}^e && \text{(b)} \\ \underline{V}_+^o &= \frac{e^{-j\theta_+}}{(1 + k_+)} V^e && \text{(c)} \\ \underline{I}_+^o &= \frac{k_+ e^{-j\theta_+}}{1 + k_+} \underline{Y}\, \underline{V}^e && \text{(d)} \end{aligned}\right\} \quad (37)$$

3. $k_- \to 0$: Constant-Current Source

For $k_- \to 0$ we have, from Equation (34),

$$\gamma(k_- \to 0,\ k_+;\ a_+) \to \frac{1 + a_+k_+}{k_+} \quad .$$

Equations (35) become

$$\left.\begin{aligned}
&\underline{V}_+^i \to \frac{a_+ + k_+}{1 + a_+k_+}\, k_-\ \underline{V}^e && \text{(a)}\\
&\underline{I}_+^i \to k_-\ \underline{Y}\ \underline{V}^e && \text{(b)}\\
&\underline{V}_+^o \to - \frac{b_+}{1 + a_+k_+}\, k_-\ \underline{V}^e && \text{(c)}\\
&\underline{I}_+^o \to - \frac{b_+k_+}{1 + a_+k_+}\, k_-\ \underline{Y}\ \underline{V}^e && \text{(d)}
\end{aligned}\right\} \qquad (38)$$

Now as $k_- \to 0$, assume that $\underline{V}^e \to \underline{\infty}$ (i.e., at least one element of $\underline{V}^e \to \infty$) in such a way that $k_-\ \underline{V}^e$ remains, in the limit, finite and different from $\underline{0}$. Then by Equation (38 b), this finite value is given by

$$k_-\ \underline{V}^e = \underline{Z}\ \underline{I}_+^i$$

Substituting this value in the remaining Equations (38) we get

$$
\left.
\begin{aligned}
\underline{V}_+^i &= \frac{a_+ + k_+}{1 + a_+k_+}\,\underline{Z}\,\underline{I}_+^i \\
\underline{V}_+^o &= -\,\frac{b_+}{1 + a_+k_+}\,\underline{Z}\,\underline{I}_+^i \\
\underline{I}_+^o &= -\,\frac{b_+k_+}{1 + a_+k_+}\,\underline{I}_+^i
\end{aligned}
\right\}
\qquad (39)
$$

where $\underline{I}_+^i$ is the strength of the constant-current source.

2.5.4.1 Source Matrix Alone Proportional to Line Matrix

In this section, although we permit the load matrix to be completely general, we write its value in such a way that it becomes relatively easy to use it for simple deviations from the cases discussed in the previous section. Thus, as before, take

$$\underline{P}_- = k_-\,\underline{\mathscr{I}}$$

but write, without loss of generality

$$\underline{Y}_+^o = k_+\,\underline{Y} + \underline{\Delta Y}^o \qquad (40)$$

and

$$
\begin{aligned}
\underline{P}_+ &= \underline{Z}\,\underline{Y}_+^o = \underline{Z}\,(k_+\,\underline{Y} + \underline{\Delta Y}^o) \\
&= k_+\,\underline{\mathscr{I}} + \underline{Z}\,\underline{\Delta Y}^o
\end{aligned}
$$

Then, with $\theta_- \to 0$, substitution in the appropriate Equations (15) yields[2]

$$\left.\begin{aligned}
\underline{V}_+^i &= \frac{a_+ + k_+}{\gamma}\left[\underline{\mathscr{I}} + (a_+ + k_+)^{-1}\,\underline{Z}\,\underline{\Delta Y}^o\right]\left[\underline{\mathscr{I}} + \frac{a_+ + k_-}{\gamma\,k_-}\,\underline{Z}\,\underline{\Delta Y}^o\right]^{-1}\underline{V}^e \\
\underline{I}_+^i &= \frac{1 + a_+k_+}{\gamma}\,\underline{Y}\left[\underline{\mathscr{I}} + \frac{a_+ + k_-}{\gamma\,k_-}\,\underline{Z}\,\underline{\Delta Y}^o\right]^{-1}\left[\underline{\mathscr{I}} + \frac{a_+}{1 + a_+k_+}\,\underline{Z}\,\underline{\Delta Y}\right]\underline{V}^e \\
\underline{V}_+^o &= -\frac{b_+}{\gamma}\left[\underline{\mathscr{I}} + \frac{a_+ + k_-}{\gamma\,k_-}\,\underline{Z}\,\underline{\Delta Y}^o\right]^{-1}\underline{V}^e \\
\underline{I}_+^o &= -\frac{b_+k_+}{\gamma}\,\underline{Y}\left[\underline{\mathscr{I}} + k_+^{-1}\,\underline{Z}\,\underline{\Delta Y}^o\right]\left[\underline{\mathscr{I}} + \frac{a_+ + k_-}{\gamma\,k_-}\,\underline{Z}\,\underline{\Delta Y}^o\right]^{-1}\underline{V}^e
\end{aligned}\right\} \quad (41)$$

We are particularly interested in further manipulation of the third and fourth of Equations (41).

Let

$$\beta = \frac{a_+ + k_-}{\gamma\,k_-} = \frac{a_+ + k_-}{1 + a_+(k_+ + k_-) + k_+\,k_-} \quad (42)$$

and

$$\left.\begin{aligned}
\underline{A} &= \underline{\mathscr{I}} + \beta\,\underline{Z}\,\underline{\Delta Y}^o \\
\underline{B} &= \underline{\mathscr{I}} + k_+^{-1}\,\underline{Z}\,\underline{\Delta Y}^o
\end{aligned}\right\} \quad (43)$$

Substituting in the third and fourth of Equations (41)

$$\left.\begin{aligned} \underline{V}^o_+ &= -\frac{b_+}{\gamma}\underline{A}^{-1}\underline{V}^e \\ \underline{I}^o_+ &= -\frac{b_+k_+}{\gamma}\underline{Y}\,\underline{B}\,\underline{A}^{-1}\underline{V}^e \end{aligned}\right\} \qquad (44)$$

2.5.5 Effect of Varying a Single Load Admittance, Starting With Proportional Terminations

The simplest deviation from proportional terminations involves making a single element of $\underline{\Delta Y}^o$ different from zero. Let

$$\underline{\Delta Y}^o = \begin{bmatrix} 0 & & & \\ & \ddots & & 0 \\ & & Y^L_{kk} & \\ 0 & & & \ddots \\ & & & & 0 \end{bmatrix} \qquad (45)$$

where Y^L_{kk} is an increment in the load admittance element connected between the k^{th} line terminal and ground.* To simplify the problem still further assume all applied emfs are equal:

$$\underline{V}^e = V^g \underline{\ell_c} \qquad (46)$$

*Note, however, that in general, $Y^L_{kk} \neq Y^o_{+(kk)}$, where $Y^o_{+(kk)}$ is the terminating admittance seen by the line between the k^{th} terminal and ground, while Y^L_{kk} is the actual load admittance element connected between the k^{th} terminal and ground. See Ref. 1.

Then the output current at the j^{th} conductor is[2]

$$I^{o}_{+j} = -\frac{b_+ k_+ V^g}{\gamma\, a_{kk}} \left\{ a_{kk}\, Y^c_j + (k_+^{-1} - \beta)\, \delta^j_k\, \Delta Y^L_{kk} \right\} \quad (47)$$

where

$$a_{kk} = 1 + \beta\, Z_{kk}\, \Delta Y^L_{kk} \quad (48)$$

and

$$\left. \begin{aligned} \delta^j_k &= \text{Kronecker's delta} \\ &= 0,\ k \neq j \\ &= 1,\ k = j \end{aligned} \right\} \quad (48a)$$

Y^c_j, the common-mode characteristic admittance of the j^{th} conductor, is defined by Equation (26).

The bulk output current is

$$I^o_T = \sum_{j=1}^{N} I^o_{+j}$$

$$= -\frac{b_+ k_+ V^g}{\gamma\, a_{kk}} \left\{ a_{kk} \sum_{j=1}^{N} Y^c_j + (k_+^{-1} - \beta)\, \Delta Y^L_{kk} \right\}$$

$$= -\frac{b_+ k_+ V^g}{\gamma\, a_{kk}} \left\{ Y^c_o + \left[\beta\, Y^c_o\, Z_{kk} + (k_+^{-1} - \beta) \right] \Delta Y^L_{kk} \right\} \quad (49)$$

where Y_o^c, the common-mode characteristic admittance of the line, is defined by Equation (30).

The output voltage on the i^{th} conductor is*

$$V_{+i}^o = -\frac{b_+ V^g}{\gamma a_{kk}} \left[a_{kk} - a_{ik} \right] \tag{50}$$

where

$$a_{ik} = \beta Z_{ik} \Delta Y_{kk}^L , \quad i \neq k \tag{51}$$

A quantity of particular interest in this study is the ratio of output voltage on any conductor to the bulk output current. From Equations (49) and (50) we get

$$\sigma_i = \frac{V_{+i}^o}{I_T^o} = \frac{1 + \beta(Z_{kk} - Z_{ik}) \Delta Y_{kk}^i}{k_+ Y_o^c + \left[\beta k_+(Y_o^c Z_{kk} - 1) + 1\right] \Delta Y_{kk}^L} ,$$

$$i = 1, \ldots , N \tag{52}$$

Analysis shows** that no set of parameters exists such that σ_i is independent of ΔY_{kk}^L.***

---

*See Ref. 2, Equation (55)

**See Ref. 2, Section 3.2.1.1.

***The discussion in Section 3.2.2 of Ref. 2 should be considered with care; it is probably misleading, in that it ignores the fact that as $k_+ \to \infty$, then must also $|\Delta Y_{kk}^2| \to \infty$ to make any significant change in Y_{kk}^L, which goes to infinity with k_+.

2.5.6 Special Discussion Involving a 7-line with Certain Symmetries

This section is included because of its relation to certain experiments conducted at Sandia in the course of the study. Consult Fig. 11 showing the cross-section of a seven-conductor cable, in which all conductors within the shield are circular, of radius a, in which one conductor is concentric with the shield and the remaining six are equi-angularly disposed with their centers on the perimeter of a circle concentric with with the shield. The conductors are numbered from 1 to 7 for reference, and for identifying the immittance parameters Z_{ij}, Y_{ij}, $(i, j = 1, \ldots, 7)$.

The coefficients of such a line have the following characteristics:

(a) There are $\frac{1}{2}(8)(7) = 28$ different coefficients.*

(b) Because of the symmetries, there are only six different coefficient values, grouped as follows:

(1) $Z_{11} = Z_{22} = Z_{33} = Z_{44} = Z_{55} = Z_{66}$ 6

(2) $Z_{12} = Z_{23} = Z_{34} = Z_{45} = Z_{56} = Z_{6}$ 6

(3) $Z_{13} = Z_{24} = Z_{35} = Z_{46} = Z_{51} = Z_{62}$ 6

*The impedance matrix (for instance) contains (7 x 7) = 49 coefficients; but since $Z_{ij} = Z_{ji}$ for every i, j, we have $\frac{1}{2}(7)(6) = 21$ dependent elements, leaving the number of independent coefficients at 28.

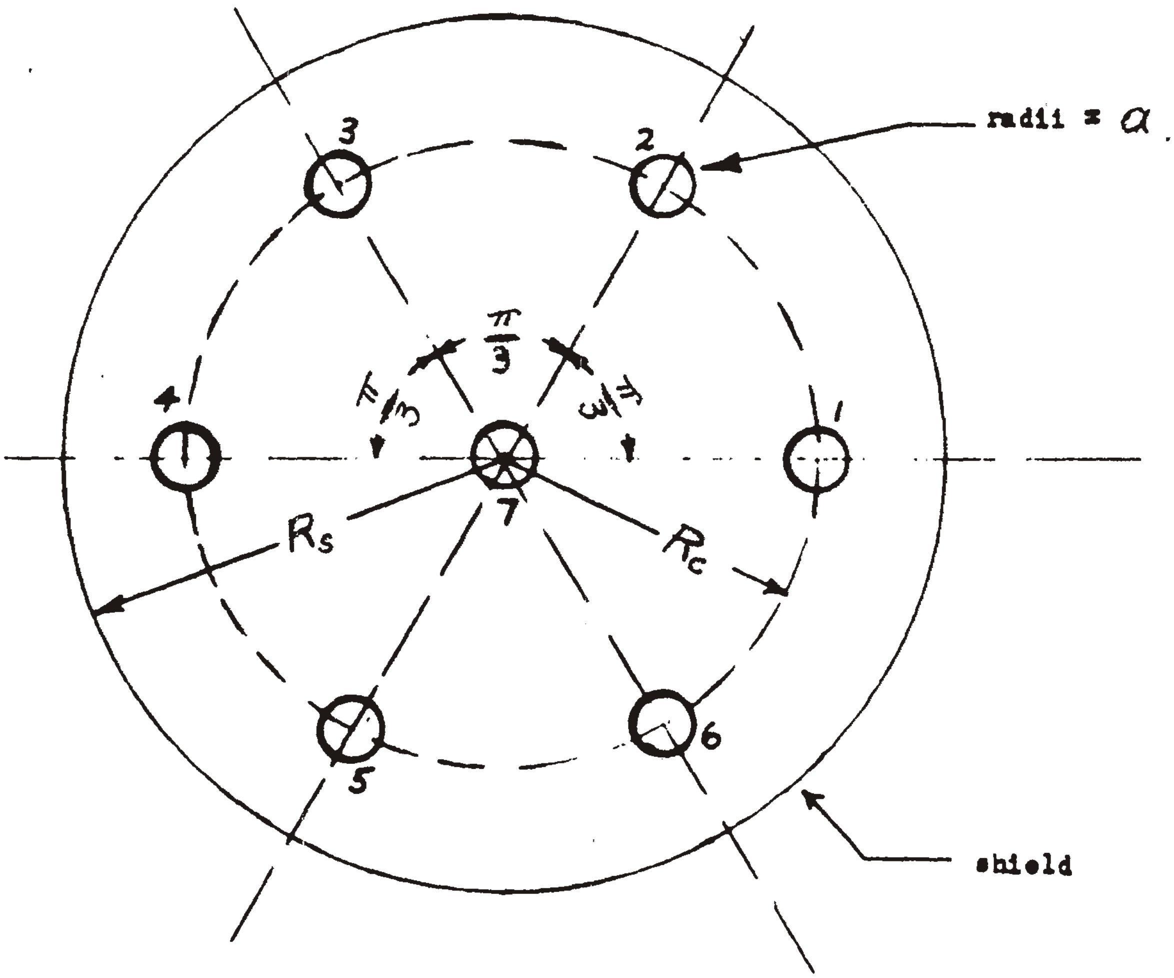


Fig. 11. Cross-section of seven-conductor shielded cable. Numbers adjacent to conductors correspond to subscripts of impedance coefficients, Z_{ij}.

(4) $Z_{14} = Z_{25} = Z_{36}$ 3

(5) $Z_{17} = Z_{27} = Z_{37} = Z_{47} = Z_{57} = Z_{67}$ 6

(6) Z_{77} . . . . $\frac{1}{28}$

with corresponding relations among the admittance coefficients.

All conductors are assumed driven in parallel from a common source. The six conductors on the circle of radius, R_c, are each terminated in a grounded admittance element having a common value. The central conductor is connected to a grounded admittance element of arbitrary value. All termination admittance elements between conductors (other than ground) are zero.

Because of the symmetrical situation of the outer six conductors, we can recognize that they carry equal potentials and currents at every line cross-section, and therefore operate, in effect, in parallel as a single conductor. The central conductor acts separately as a second conductor. Thus, the situation reduces to the much simpler case of a 2-line. Referring to quantities associated with the parallel group by the subscript, a, and those associated with the central conductor by b, we have

$$\left.\begin{aligned} I_a &= 6I_1 \\ I_b &= I_7 \\ V_a &= V_j, \; j = 1, \ldots, 6 \\ V_b &= V_7 \end{aligned}\right\}$$

Canonical equations for the infinite effective 2-line are[14]

$$\left.\begin{aligned} V_a &= Z_{aa} I_a + Z_{ab} I_b \\ V_b &= Z_{ab} I_a + Z_{bb} I_b \end{aligned}\right\} \quad (53)$$

where

$$\left.\begin{aligned} Z_{aa} &= \frac{1}{6} \sum_{i=1}^{6} Z_{i1} \\ Z_{ab} &= Z_{17} \\ Z_{bb} &= Z_{77} \end{aligned}\right\} \quad (54)$$

The 2-line admittance coefficients are[7]

$$\left.\begin{aligned} Y_{aa} &= Z_{bb}/\Delta_z \\ Y_{ab} &= -Z_{ab}/\Delta_z \\ Y_{bb} &= Z_{aa}/\Delta_z \end{aligned}\right\} \quad (55)$$

where

$$\Delta_z = Z_{aa} Z_{bb} - Z_{ab}^2 \quad (56)$$

These results may be used to solve any of the problem types when the terminations satisfy the special conditions of this section.

2.6 Line Parameters

Complete solution of the multiconductor cable problem includes evaluation of the cable impedance (or admittance) matrix. As stated in Section 2.1, it is only necessary, for this purpose, that the Maxwell coefficients of capacitance and the wave velocity in the cable be known (Equation (8)). The wave velocity is known from the permeability and the permittivity of the medium (Equations (5)). The capacitances must be determined as solutions of two-dimensional electrostatic field problems. Such problems involve solving Laplace's equation in two dimensions, subject to specification, for each conductor, either of its relative potential or of its total charge.* Explicit solutions by standard classical methods are available for a relatively small number of cases. Otherwise the necessary quantities may be determined by (a) direct measurement of a sample section of line, or of a suitably-scaled model of such a section, (b) by analog methods, including the electrolytic tank, resistance card or conductive sheet, and impedance network,[16] (c) by numerical solution of Laplace's equation with boundary conditions[16,21] or a Green's theorem integral

*See Ref. 15, Part II, Section 4.

equation formulation.[22,23] The problem to be solved is typified by the seven-conductor shielded cable shown in cross-section in Fig. 11. Three quantities suffice to specify this circularly symmetrical arrangement: a, the common radius of the cable wires; R_c, the locus of the center of the outer six wires; and R_s, the inner radius of the shield. Actually, a small redundancy appears here, since the line coefficients are completely determined by dimensional ratios, rather than absolute dimensions themselves. The ratios for the 7-line are taken as

$$\left.\begin{aligned} x &= \frac{R_s}{R_c} \\ p &= \frac{R_c}{a} \end{aligned}\right\} \qquad (57)$$

Explicit analytical solutions for the coefficients of this configuration are available for the following asymptotic conditions:

$$\left.\begin{aligned} &\text{A.} \quad x, p \to \infty \\ &\text{B.} \quad p \to 2; x \to 1 + \frac{1}{p} \to 3/2 \end{aligned}\right\} \qquad (58)$$

The first condition, A, corresponds to a cross-section geometry for which the wire radius is much less than the distance between wires, and much less than the distance from any wire to the shield. For this, the impedance coefficients are:[1]

$$Z_{11} = Z_{22} = Z_{33} = Z_{44} = Z_{55} = Z_{66} = \zeta \, \ell n \left[p \, \frac{x^2 - 1}{x} \right]$$

$$Z_{12} = Z_{23} = Z_{34} = Z_{45} = Z_{56} = Z_{61} = \frac{1}{2} \zeta \, \ell n \left[\frac{x^4 - x^2 + 1}{x^2} \right]$$

$$Z_{13} = Z_{24} = Z_{35} = Z_{46} = Z_{51} = Z_{62} = \frac{1}{2} \zeta \, \ell n \left[\frac{x^4 + x^2 + 1}{3x^2} \right]$$

$$Z_{14} = Z_{25} = Z_{36} = \zeta \, \ell n \left[\frac{x^2 + 1}{2x} \right]$$

$$Z_{17} = Z_{27} = Z_{37} = Z_{47} = Z_{57} = Z_{67} = \zeta \, \ell n \, x$$

$$Z_{77} = \zeta \, \ell n \, (px)$$

$$\zeta = 60/\sqrt{\varepsilon_r}$$

ε_r = relative dielectric constant (59)

The second condition, B, corresponds to a cross-section geometry for which the wires are so large and the shield diameter so small that all conductors, including shield, nearly touch their nearest neighbors. For this case it is easier to write expressions for the admittance coefficients. If

$$Y_a = \frac{1}{2\zeta(p - 2)^{1/2}}$$

$$Y_b = \frac{1}{\zeta} \left[\frac{px}{(px - 1)^2 - p^2} \right]^{1/2} \tag{60}$$

where ζ is defined in Equations (59), then

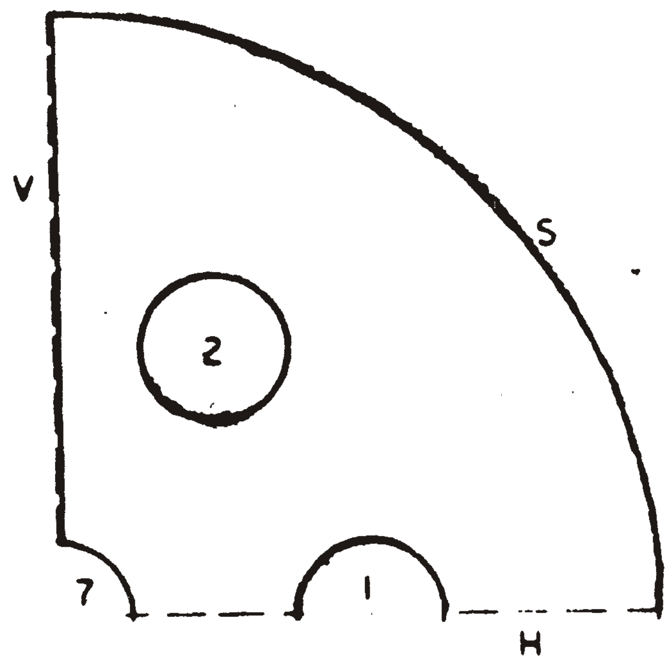
$$\left.\begin{aligned}
&Y_{11} = Y_{22} = Y_{33} = Y_{44} = Y_{55} = Y_{66} = 3Y_a + Y_b \\
&Y_{12} = Y_{23} = Y_{34} = Y_{45} = Y_{56} = Y_{61} = -Y_a \\
&Y_{13} = Y_{24} = Y_{35} = Y_{46} = Y_{51} = Y_{62} = 0 \\
&Y_{17} = Y_{27} = Y_{37} = Y_{47} = Y_{57} = Y_{67} = -Y_a \\
&Y_{77} = 6Y_a
\end{aligned}\right\} \qquad (61)$$

When, as is generally true, none of the simpler asymptotic solutions is suitable, one of the other methods previously mentioned must be used. The most common method, adapted for large computer use, is the solution of Laplace's equation replaced by a set of linear finite difference equations.[16 - 21]

For such an analysis the problem of the seven-conductor cable can be reduced to three problems requiring less computer capacity for a given accuracy.* The problem is summarized in Fig. 12.

*Or, rather, to a single simpler problem with three different sets of boundary conditions. See Ref. 24.

Fig. 1. . Reduced seven-conductor cable capacitance problem.



for all modes, $V_S = 0$ and boundary H is magnetic

Mode Designations: A, B, C

Charge Designations: Q_j^A, Q_j^B, Q_j^C, $j = 1,2,7$

MODE A: Boundary V is electric; $V_1 = 1$, $V_2 = V_7 = 0$

MODE B: Boundary V is magnetic; $V_1 = 1$, $V_2 = V_7 = 0$

MODE C: Boundary V is magnetic; $V_1 = V_2 = 0$, $V_7 = 1$

THEN:

$$C_{11} = Q_1^A + Q_1^B = C_{22} = \cdots = C_{66}$$

$$C_{12} = \tfrac{1}{2}(Q_2^A + Q_2^B) = C_{23} = \cdots = C_{61}$$

$$C_{13} = \tfrac{1}{2}(Q_2^B - Q_2^A) = C_{24} = \cdots = C_{62}$$

$$C_{14} = Q_1^B - Q_1^A = C_{25} = C_{36}$$

$$C_{17} = 2Q_1^C = C_{27} = \cdots = C_{67}$$

$$C_{77} = 4Q_7^C$$

2.7 Miscellaneous Briefly Considered Topics

Many questions arise in the course of a study of this nature which, because of schedule limitations, cannot be given the full consideration they deserve. Two topics arising from such questions are considered briefly in this section. They deal with (1) the subject of losses and (2) the problem of gaining additional intuitive insights into the physical phenomena taking place on a multiconductor cable, insights useful for making preliminary design decisions in order greatly to reduce the number of alternate choices requiring detailed consideration.

2.7.1 Losses

Losses occur as a result of finite conductivity in the conductors or finite resistivity in the dielectric. They affect the cable model in two ways: (1) The propagation constant becomes a complex quantity, $\gamma\ell$, instead of the pure imaginary quantity

$$j\theta = j\beta\ell$$

where

$$\left.\begin{aligned} \gamma = \alpha + j\beta,\ \alpha,\beta \text{ real} \\ \alpha \geq 0 \end{aligned}\right\}$$

and ℓ is the length of the cable. (2) The immittance coefficients become complex quantities rather than pure reals.

Following common practice in rf analysis of l-lines, we can ignore the second of these effects. As for the first, the problem becomes very complicated if conductor losses constitute a significant fraction of the total. However, in view of the fact that the cables under study here normally carry very low frequency signals, whereas the components of EMP signals are much higher, it is safe to assume that the cable design is such that dielectric losses control its behavior at EMP frequencies. In that case, transmission is still in the TEM mode at a single speed in each direction, and we can replace $a_{\pm}$ and $b_{\pm}$ of Equations (16) by

$$\left.\begin{aligned} a_{\pm} &= \coth \gamma\ell_{\pm} \\ b_{\pm} &= -\operatorname{csch} \gamma\ell_{\pm} \end{aligned}\right\} \qquad (62)$$

All other equations describing the model (viz., Equations (15), (17), (18)) remain the same as before.

2.7.2 Additional Insights

The information in this subsection is taken from the last interim report submitted in this study.[25] The object of that report was to attempt a simple "explanation" of the behavior of incompletely grounded shielded-twin cables in terms of certain well-accepted ideas, namely:

1. The whole body of information relating to conventional 2-conductor (1-line) transmission-line theory.

2. Elementary concepts of the behavior of TEM waves guided by multiple parallel conductor systems.[26]

3. The compensation theorem for investigating the behavior of networks.

Two configurations were considered: (1) A single twin cable, shield grounded at one end, load unbalanced (Fig. 13); (2) Two shielded twin cables, shields grounded at opposite ends, load unbalanced (Fig. 14).

The load imbalance in each case was predicated on the assumption that the metallic mass of the load was unbalanced, as is normal with loads having numerous conductors connected to a common terminal.

The purpose of the study was to gain approximate insights to the physical behavior of these configurations, rather than to obtain exact results, for which the canonical Equations (15) are available.

In each case the procedure consists in first finding the common voltage of twin leads and associated shield when no unbalance is present, and then finding the effect of a

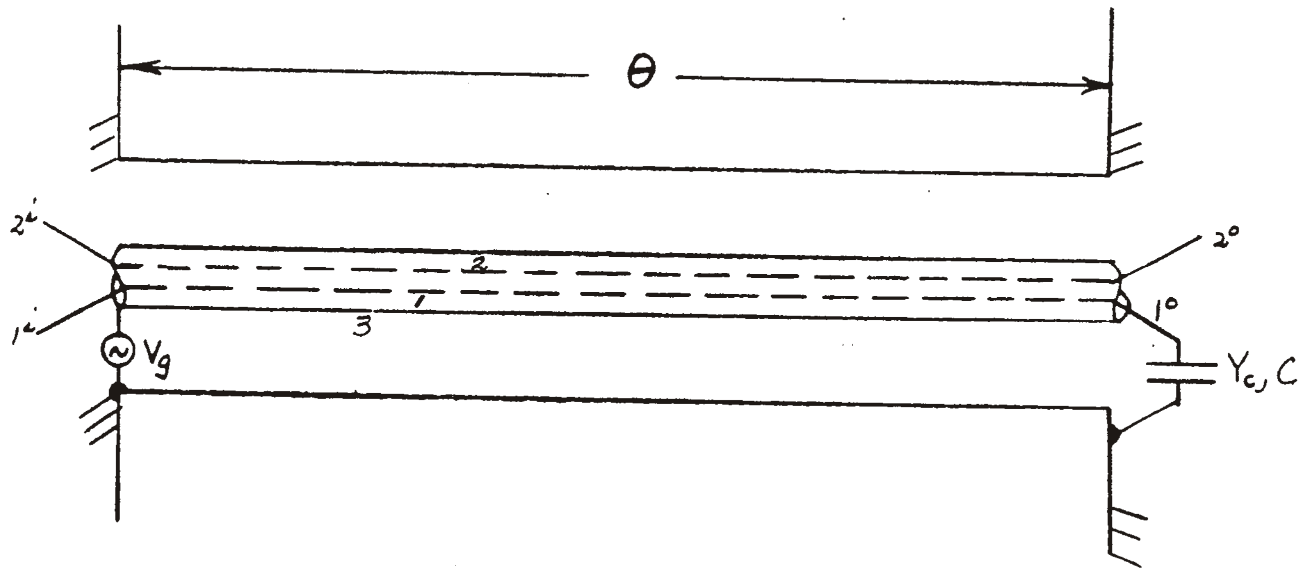


Fig. 13. Shielded twin pair with unbalanced load.

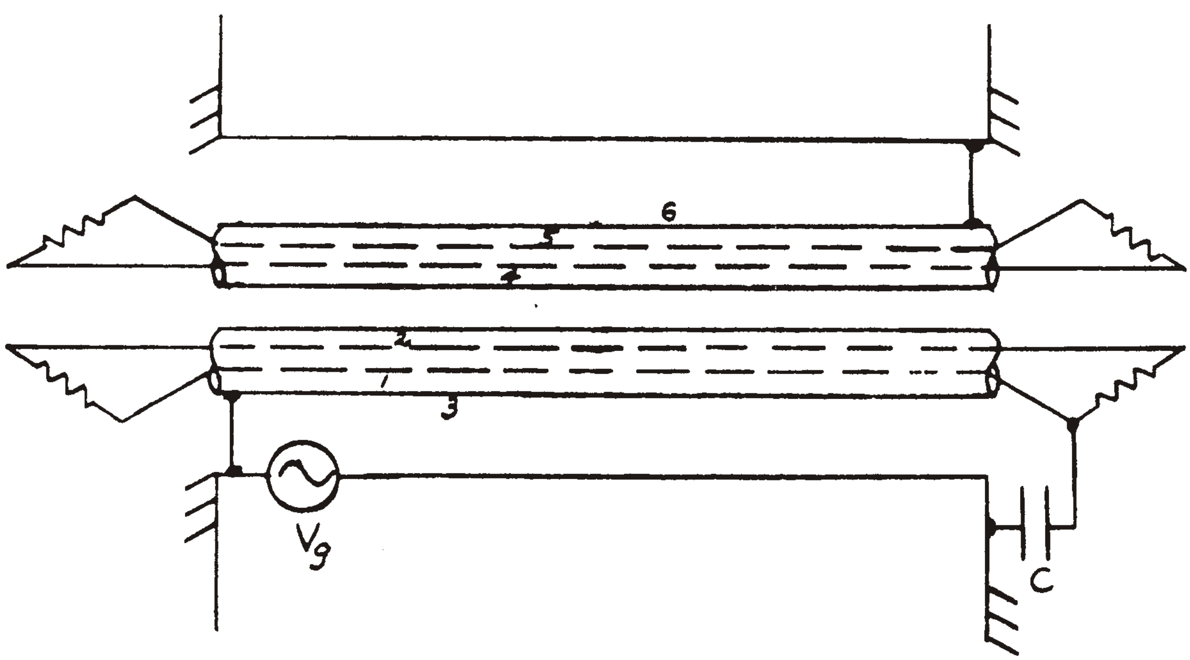


Fig. 14. Schematic, two shielded twin cables grounded at opposite ends.

compensating current source due to the unbalancing capacitance in producing a voltage difference across the twin leads.

Omitting the details, which can be found in the cited reference, the approximate analysis shows that the compensating current source strength is proportional to the voltage at the terminal in the absence of the unbalancing capacitance, and to the unbalancing capacitance. A fraction of this current flows into the line, in proportion to the line admittance compared to the unbalancing admittance. Finally, the voltage unbalance across the twin leads is proportional to this line current, and to the difference between the lead self-impedance and the mutual impedance between the two leads:

$$Z_{00} = Z_{11} - Z_{12} \tag{63}$$

Z_{00} is frequently referred to as the odd-mode, or balanced-line characteristic impedance of each of the twin leads with respect to ground.[27]

The two problems differ essentially only in the nature of the terminal voltages in the absence of terminal unbalance. In the single-pair case this is simply the open-circuit voltage of a 1-line stub-excited at the short. In the two-pair case the response is, as might be expected, entirely similar to that of over-coupled,* synchronously-tuned resonant circuits.

*Partly because losses were reflected.

3. SUMMARY: DISCUSSION OF RESULTS

In this study we undertook to define, quantitatively, the response of a multiconductor cable and its terminations to an externally impressed electromagnetic field. We assumed the cable has the following properties:

(1) It is lossless.

(2) It is, effectively, of invariant cross-section and with uniform dielectric properties throughout the cross-section.

(3) It operates in the TEM mode.

In addition, single-frequency, steady-state operation was assumed. However, the last assumption is essentially non-restrictive, since Sandia has programmed the Fourier-transform procedure for finding transient responses when the steady-state transfer function is known.

Given, (1) parameters consisting of the self- and mutual-impedance coefficients of the cable, (2) the most general terminations as seen from the cable at each end, and (3) the operating frequency, the following results were obtained:

1. For a shielded cable with a circumferential break at any arbitrary point, explicit (but complicated) expressions for the voltage and currents at all terminals.[1]

2. For an unshielded cable, the response at all terminals to an arbitrary applied field along the cable (again, explicit but complicated).[3]

3. As a special case, the response at all terminals at one end of a cable to an arbitrary set of voltages applied at the other end.[2]

As part of the overall problem it is necessary to determine the matrix of the impedance (or admittance) coefficients of the cable. Under the assumptions in this study this reduces to the problem of determining Maxwell's coefficients of capacitance for the particular cable configuration, in addition to the dielectric permittivity and magnetic permeability of the medium. Except in certain limiting cases it is impossible to determine these coefficients analytically in a practical general form. For a cable with a shield of circular cross-section it is possible to obtain solutions for the limiting cases where (a) the conductors are very small in diameter compared to their spacings and distances to the shield[1] (b) the conductors are so large they nearly touch each other and/or the shield.[4] These limiting cases were computed for a seven-conductor cable.

The results describing cable behavior, as reported in references 1 - 3, are too complicated for hand manipulation and computation, so that it was necessary to program the results for machine computation. This was done by Sandia for use with the CDC-6600 computer.*

However, for preliminary general understanding of the physical phenomena taking place, it is desirable, where possible, to develop simplified, if less accurate, models which help to explain cable behavior in terms of better-understood phenomena and, therefore, to make useful preliminary design decisions. This was done in a couple of cases to show that twin shielded leads, with the shield grounded at one end only, are subject to EMP interference when the load on the twin leads is unbalanced.[25]

In our opinion, the results so summarized represent a firm groundwork on which should be based a thorough study of the response and improvement of models representative of actual cable systems. Some of the factors to be considered may be listed as follows:

1. Cables terminate not only in sources and loads, but also in other cables. Computer programming should be generalized to include this contingency.

2. Cable dielectrics are not always uniform and are rarely lossless. Modifications required in the model to incorporate these facts, when significant, should be investigated.

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*See footnote, page 17.

APPENDIX

Line Equations for Special Values of θ_- and θ_+

1. $\theta_- = m\pi$, m integer. The case for m = 0 corresponds to Problem Types III and IV. We have

$$\left.\begin{aligned} a_- &= -j \cot \theta_- \rightarrow -j\infty \\ b_- &= -j \csc \theta_- \rightarrow j\infty \end{aligned}\right\}$$

Therefore,

$$\underline{M}_- = a_-(\underline{\ell} + a_-^{-1} \underline{P}_-)$$

$$\underline{N}_- = a_-(a_-^{-1} \underline{\ell} + \underline{P}_-)$$

$$\underline{M}_- \underline{N}_-^{-1} = (\underline{\ell} + a_-^{-1} \underline{P}_-)(a_-^{-1} \underline{\ell} + \underline{P}_-)^{-1}$$

$$\rightarrow \underline{P}_-^{-1} \, , \; \underline{P}_- \neq \underline{0}$$

$$\underline{\Lambda} \rightarrow \underline{M}_+ \underline{N}_+^{-1} + \underline{P}_-^{-1}$$

$$\underline{\Lambda} \, \underline{N}_- \rightarrow (\underline{M}_+ \underline{N}_+^{-1} + \underline{P}_-^{-1}) a_- \underline{P}_-$$

$$= a_-(\underline{\ell} + \underline{M}_+ N_+^{-1} \underline{P}_-)$$

In Equations (15),

$$\underline{V}^{o}_{-} \to \frac{b_-}{a_-}(\mathcal{I} + \underline{M}_+ \underline{N}_+^{-1} \underline{P}_-)^{-1} \underline{V}^e = -(\mathcal{I} + \underline{M}_+ \underline{N}_+^{-1} \underline{P}_-)^{-1} \underline{V}^e$$

$$\underline{V}^{i}_{-} = -a_- (\mathcal{I} + a_-^{-1} \underline{P}_-) \left[(\underline{M}_+ \underline{N}_+^{-1} + \underline{P}_-^{-1}) a_- \underline{P}_-\right]^{-1} \underline{V}^e$$

$$\to -\left[(\underline{M}_+ \underline{N}_+^{-1} + \underline{P}_-^{-1}) \underline{P}_-\right]^{-1} \underline{V}^e = -\underline{V}^{o}_{-}$$

Substitutions in the remaining Equations (15) are straightforward.

2. $\underline{\theta_+ = m\pi}$.

$$\left.\begin{aligned} a_+ &\to -j\infty \\ b_+ &\to j\infty \\ \underline{M}_+ \underline{N}_+^{-1} &\to \underline{P}_+^{-1} \\ \underline{\Lambda} &\to P_+^{-1} + \underline{M}_- \underline{N}_-^{-1} \\ \underline{\Lambda}\, N_+ &\to a_+(\mathcal{I} + \underline{M}_- \underline{N}_-^{-1} \underline{P}_+) \end{aligned}\right\}$$

Substitutions in $\underline{V}^{o}_{+}$, $\underline{V}^{i}_{+}$, Equations (15), are similar to substitutions in $\underline{V}^{o}_{-}$, $\underline{V}^{i}_{-}$ in the preceding case.

3. $\underline{\theta_- = (2m + 1)\frac{\pi}{2} \text{ or } \theta_+ = (2m + 1)\frac{\pi}{2}}$

$$
\left.
\begin{aligned}
&\text{For} \quad \theta_- = (2m + 1)\frac{\pi}{2} \\
&a_- = 0 \\
&b_- = (-1)^m\, j \\
&\underline{M}_- = \underline{P}_- \\
&\underline{N}_- = \underline{\mathcal{I}}, \\
&\text{provided } \underline{Y}^o_- \neq \underline{\infty} \\
&\underline{\Lambda} = \underline{M}_+ \underline{N}_+^{-1} + \underline{P}_-
\end{aligned}
\right\}
$$

Substitution in Equations (15) is straightforward.

Procedure is similar for $\theta_+ = (2m + 1)\frac{\pi}{2}$.

4. $\underline{\theta_- + \theta_+ = m\,\frac{\pi}{2}}$

a. m odd:

$$
\left.
\begin{aligned}
a_- &= -j \cot \theta_- = -j \tan \theta_+ = -\frac{1}{a_+} \\
b_- &= (-1)^{\frac{m-1}{2}} \sec \theta_+ = (-1)^{\frac{m-1}{2}} \left(\frac{b_+}{a_+}\right)
\end{aligned}
\right\}
$$

b. m even:

$$
\left.
\begin{aligned}
a_- &= j \cot \theta_+ = -\, a_+ \\
b_- &= -\, (-1)^{\frac{m}{2}}\, j \csc \theta_+ = -\, (-1)^{\frac{m}{2}}\, b_+
\end{aligned}
\right\}
$$

Subsequent substitutions and modifications of Equations (15) are straightforward.

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