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EXPOSED TWO-WIRE TRANSMISSION LINE ELECTROMAGNETICALLY COUPLED TO A ROCKET

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ABSTRACT

The electromagnetic compatibility of rockets is a research field of wide scope. One of its segments is concerned with the response of impedance-loaded conductors electromagnetically coupled to a rocket when this is illuminated by a plane wave. The currents in the load impedances are induced by the action on the line of both the incident field and the scattered field from the rocket. In this paper a coupled circuit consisting of a two-wire transmission line near a rocket is considered. The objective is to derive formulas for the currents in the impedances terminating the line when the rocket, line, and incident field are oriented in space to maximize the response in the line.

INTRODUCTION

The present paper is one in a series devoted to a study of the receiving characteristics of external unshielded conductors coupled to a rocket. The specific purpose is to derive formulas for the maximum currents in the impedances Z_0 and Z_s terminating a two-wire transmission line coupled to a rocket as shown in Fig. 1. The length of the line and its position along the rocket are arbitrary. The axes of the transmission-line wires are parallel to the axis of the rocket; all three conductors lie in the same plane. A plane monochromatic electromagnetic wave is incident on the conductors with its electric vector parallel to the common axis. It is this orientation of conductors and incident field that maximizes the load currents. Fig. 2, on which the theoretical discussion is based, is essentially the same as Fig. 1 except for minor differences in the arrangement of the loads. It shows a two-wire transmission line electromagnetically coupled to a rocket. The rocket is conductor 1, the transmission line conductors are numbered 2 and 3. The origin of cylindrical and Cartesian coordinates is at the centroid of conductor 1 which is a cylinder of radius a_1 that extends over the interval $-h_1 \leq z \leq h_1$. The distance between the centers of the wires forming the transmission line is b. The radii of the wires are equal, $a_2 = a_3$. The line extends from $z_2 = z_3 = -h_2'$ to $z_2 = z_3 = h_2$. At $z_2 = h_2$ the line is terminated in impedance Z_s; at $z_2 = -h_2'$ the terminating impedance is Z_0. The distance from the center of the rocket to the center of the line is d. The incident electric vector is parallel to and in the same plane as the three conductors; the wave propagates in the positive x direction with the velocity of light.

THE EXCITATION FUNCTIONS FOR THE THREE-CONDUCTOR CONFIGURATION

An incident plane wave traveling in the positive x direction with electric field parallel to the z axis is represented analytically by

$$E_z^{inc}(x) = E_z^{inc}(0)\, e^{-j\beta x} \tag{1}$$

where the reference for phase is at $x = 0$, $\beta = 2\pi/\lambda$ is the radian wave number, and λ is the free-space wavelength.

It is convenient to define an excitation function U for each of the conductors. For conductor 1 this function is

$$E_z^{inc}(0) = \beta U_1 \tag{2}$$

For conductors 2 and 3,

$$E_z^{inc}(-d + b/2) = E_z^{inc}(0)\ e^{j\beta(d - b/2)} = \beta U_2 \tag{3}$$

$$E_z^{inc}(-d - b/2) = E_z^{inc}(0)\ e^{j\beta(d + b/2)} = \beta U_3 \tag{4}$$

For later use it is necessary to express U_2 and U_3 in terms of their symmetrical and antisymmetrical components. This is accomplished by writing

$$U_2 = \frac{1}{2}(U_2 + U_3) + \frac{1}{2}(U_2 - U_3) \tag{5}$$

$$U_3 = \frac{1}{2}(U_2 + U_3) - \frac{1}{2}(U_2 - U_3) \tag{6}$$

and then setting

$$U_2 = U^s + U^a \quad ; \quad U_3 = U^s - U^a \tag{7}$$

where

$$U^s = (U_2 + U_3)/2 = \beta^{-1}\ E_z^{inc}(0)\ e^{j\beta d} \cos(\beta b/2) \tag{8}$$

$$U^a = (U_2 - U_3)/2 = -j\beta^{-1}\ E_z^{inc}(0)\ e^{j\beta d} \sin(\beta b/2) \tag{9}$$

VECTOR POTENTIALS ON THE SURFACES OF THE THREE CONDUCTORS

The resultant vector potentials on the surfaces of conductors 1, 2, and 3 due to the currents in all of them are, respectively,

$$A_{1z} = \frac{\mu_0}{4\pi}\left\{ \int_{-h_1}^{h_1} I_1(z_1') \frac{e^{-j\beta R_{11}}}{R_{11}} dz_1' + \int_{-h_2'}^{h_2} I_2(z_2') \frac{e^{-j\beta R_{12}}}{R_{12}} dz_2' + \int_{-h_2'}^{h_2} I_3(z_3') \frac{e^{-j\beta R_{13}}}{R_{13}} dz_3' \right\} \quad (10)$$

$$A_{2z} = \frac{\mu_0}{4\pi}\left\{ \int_{-h_1}^{h_1} I_1(z_1') \frac{e^{-j\beta R_{21}}}{R_{21}} dz_1' + \int_{-h_2'}^{h_2} I_2(z_2') \frac{e^{-j\beta R_{22}}}{R_{22}} dz_2' + \int_{-h_2'}^{h_2} I_3(z_3') \frac{e^{-j\beta R_{23}}}{R_{23}} dz_3' \right\} \quad (11)$$

$$A_{3z} = \frac{\mu_0}{4\pi}\left\{ \int_{-h_1}^{h_1} I_1(z_1') \frac{e^{-j\beta R_{31}}}{R_{31}} dz_1' + \int_{-h_2'}^{h_2} I_2(z_2') \frac{e^{-j\beta R_{32}}}{R_{32}} dz_2' + \int_{-h_2'}^{h_2} I_3(z_3') \frac{e^{-j\beta R_{33}}}{R_{33}} dz_3' \right\} \quad (12)$$

where

$$R_{11} = \sqrt{(z_1 - z_1')^2 + a_1^2} \quad ; \quad R_{12} = \sqrt{(z_1 - z_2')^2 + (d - b/2)^2}$$

$$R_{13} = \sqrt{(z_1 - z_3')^2 + (d + b/2)^2} \quad ; \quad R_{21} = \sqrt{(z_2 - z_1')^2 + (d - b/2)^2}$$

$$R_{22} = \sqrt{(z_2 - z_2')^2 + a_2^2} \quad ; \quad R_{23} = \sqrt{(z_2 - z_3')^2 + b^2}$$

$$R_{31} = \sqrt{(z_3 - z_1')^2 + (d + b/2)^2} \quad ; \quad R_{32} = \sqrt{(z_3 - z_2')^2 + b^2}$$

$$R_{33} = \sqrt{(z_3 - z_3')^2 + a_3^2} \tag{13}$$

and $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of free space. The several distances R_{ij} in (13) (of which several are shown in Fig. 2) are measured from the point z_i on the surface of conductor i where the vector potential is calculated to the point z_j' that locates the current element on the axis of conductor j that gives rise to this potential.

The currents and vector potentials on the surfaces of the wires nos. 2 and 3 comprising the transmission line can be combined into symmetrical and antisymmetrical parts as was done for the U functions in (5)-(9). This combination is useful because the antisymmetrical or balanced transmission-line currents $I^a(z_2) = \frac{1}{2}[I_{2z}(z_2) - I_{3z}(z_2)]$ (which are equal and opposite in conductors 2 and 3) maintain voltage drops across the impedances Z_0 and Z_s. On the other hand, the symmetrical currents $I^s(z_2) = \frac{1}{2}[I_{2z}(z_2) + I_{3z}(z_2)]$ in conductors 2 and 3 are equal and codirectional. They constitute radiating antenna currents which generate no voltage drops across Z_0 and Z_s.

With equations (11) and (12) the following expressions are obtained for the symmetrical and antisymmetrical vector potentials on conductor 2:

$$A_z^s(z_2) = \frac{1}{2}[A_{2z}(z_2) + A_{3z}(z_2)] = \frac{\mu_0}{8\pi} \int_{-h_1}^{h_1} I_1(z_1') \left[\frac{e^{-j\beta R_{21}}}{R_{21}} + \frac{e^{-j\beta R_{31}}}{R_{31}}\right] dz_1'$$

$$+ \frac{\mu_0}{4\pi} \int_{-h_2'}^{h_2} I^s(z_2') \left[\frac{e^{-j\beta R_{22}}}{R_{22}} + \frac{e^{-j\beta R_{23}}}{R_{23}}\right] dz_2' \tag{14}$$

$$A_z^a(z_2) = \frac{1}{2}[A_{2z}(z_2) - A_{3z}(z_2)] = \frac{\mu_0}{8\pi}\int_{-h_1}^{h_1} I_1(z_1')\left[\frac{e^{-j\beta R_{21}}}{R_{21}} - \frac{e^{-j\beta R_{31}}}{R_{31}}\right]dz_1'$$

$$+ \frac{\mu_0}{4\pi}\int_{-h_2'}^{h_2} I^a(z_2')\left[\frac{e^{-j\beta R_{22}}}{R_{22}} - \frac{e^{-j\beta R_{23}}}{R_{23}}\right]dz_2' \qquad (15)$$

Note that $A_z^s(z_3) = A_z^s(z_2)$, $A_z^a(z_3) = -A_z^a(z_2)$.

SIMULTANEOUS INTEGRAL EQUATIONS FOR THE CURRENTS ON THE THREE CONDUCTORS

The simultaneous integral equations for the currents on the rocket and the transmission-line conductors are obtained from the boundary conditions that require the axial electric fields to vanish on the surfaces of the three conductors. Since the part of the field due to the currents in the conductors has the component $E_z = (j\omega/\beta^2)(\partial^2/\partial z^2 + \beta^2)A_z$ and the incident fields are given by (2)-(4), simple differential equations in A_z are obtained which have the solutions:

$$4\pi\mu_0^{-1}A_{1z} = -j\,\frac{4\pi}{\zeta_0}\,[C_1 \cos \beta z_1 + D_1 \sin \beta z_1 + U_1] \qquad (16)$$

$$4\pi\mu_0^{-1}A_{2z} = -j\,\frac{4\pi}{\zeta_0}\,[C_2 \cos \beta z_2 + D_2 \sin \beta z_2 + U_2] \qquad (17)$$

$$4\pi\mu_0^{-1}A_{3z} = -j\,\frac{4\pi}{\zeta_0}\,[C_3 \cos \beta z_3 + D_3 \sin \beta z_3 + U_3] \qquad (18)$$

where $\zeta_0 = 120\pi$ ohms is the characteristic impedance of free space, and the constants of integration are C and D. If the integrals in (10)-(12) are substituted for A_{1z}, A_{2z}, and A_{3z} in (16)-(18), these become simultaneous integral equations for the three currents $I_1(z_1')$, $I_2(z_2')$, and $I_3(z_3')$.

The integral equations for the symmetrical and antisymmetrical currents on the transmission line may be written down immediately from (14) and (15) with (17) and (18). They are

$$4\pi\mu_0^{-1}A_z^s = \frac{1}{2}\int_{-h_1}^{h_1} I_1(z_1')\left[\frac{e^{-j\beta R_{21}}}{R_{21}} + \frac{e^{-j\beta R_{31}}}{R_{31}}\right]dz_1'$$

$$+ \int_{-h_2'}^{h_2} I^s(z_2')\left[\frac{e^{-j\beta R_{22}}}{R_{22}} + \frac{e^{-j\beta R_{23}}}{R_{23}}\right]dz_2'$$

$$= -j\frac{4\pi}{\zeta_0}[C^s \cos\beta z_2 + D^s \sin\beta z_2 + U^s] \tag{19}$$

$$4\pi\mu_0^{-1}A_z^a = \frac{1}{2}\int_{-h_1}^{h_1} I_1(z_1')\left[\frac{e^{-j\beta R_{21}}}{R_{21}} - \frac{e^{-j\beta R_{31}}}{R_{31}}\right]dz_1'$$

$$+ \int_{-h_2'}^{h_2} I^a(z_2')\left[\frac{e^{-j\beta R_{22}}}{R_{22}} - \frac{e^{-j\beta R_{23}}}{R_{23}}\right]dz_2'$$

$$= -j\frac{4\pi}{\zeta_0}[C^a \cos\beta z_2 + D^a \sin\beta z_2 + U^a] \tag{20}$$

where

$$C^s = \frac{1}{2}(C_2 + C_3) \quad , \quad C^a = \frac{1}{2}(C_2 - C_3)$$

$$D^s = \frac{1}{2}(D_2 + D_3) \quad , \quad D^a = \frac{1}{2}(D_2 - D_3) \tag{21}$$

Let

$$W^s = -j\frac{\zeta_0}{8\pi}\int_{-h_1}^{h_1} I_1(z_1')\left[\frac{e^{-j\beta R_{21}}}{R_{21}} + \frac{e^{-j\beta R_{31}}}{R_{31}}\right]dz_1' \tag{22}$$

and

$$W^a = -j\frac{\zeta_0}{8\pi}\int_{-h_1}^{h_1} I_1(z_1')\left[\frac{e^{-j\beta R_{21}}}{R_{21}} - \frac{e^{-j\beta R_{31}}}{R_{31}}\right]dz_1' \tag{23}$$

Then (19) and (20) become:

$$\int_{-h_2'}^{h_2} I^s(z_2')\left[\frac{e^{-j\beta R_{22}}}{R_{22}} + \frac{e^{-j\beta R_{23}}}{R_{23}}\right]dz_2'$$

$$= -j\frac{4\pi}{\zeta_0}[C^s \cos\beta z_2 + D^s \sin\beta z_2 + U^s + W^s] \tag{24}$$

$$\int_{-h_2'}^{h_2} I^a(z_2')\left[\frac{e^{-j\beta R_{22}}}{R_{22}} - \frac{e^{-j\beta R_{23}}}{R_{23}}\right]dz_2'$$

$$= -j\frac{4\pi}{\zeta_0}[C^a \cos\beta z_2 + D^a \sin\beta z_2 + U^a + W^a] \tag{25}$$

As mentioned earlier, the current $I^s(z_2)$ contributes nothing to the voltage drop across Z_0 and Z_s if these are lumped terminations centrally located. Hence, (24) need not be solved. However, it should be emphasized that the formulation is general to this point subject only to the usual restrictions: $\beta a_1 \ll 1$, $a_1 \ll h_1$; $\beta a_2 \ll 1$, $2a_2 \ll (h_2 + h_2')$; $\beta b \ll 1$, and $b \ll (h_2 + h_2')$. Attention is now directed toward the solution of (25) for the antisymmetrical current in the transmission line.

THE ANTISYMMETRICAL CURRENT IN THE TRANSMISSION LINE

Because the antisymmetric currents on the two conductors at z_2' and z_3' are equal and opposite and close together, their effects cancel at all distances $|z - z'|$ that are large compared with the distance b between the two conductors. It follows that the left side of (25) is well approximated by [1]:

$$\int_{-h_2'}^{h_2} I^a(z_2')\left[\frac{e^{-j\beta R_{22}}}{R_{22}} - \frac{e^{-j\beta R_{23}}}{R_{23}}\right] dz_2' \doteq 2I^a(z_2)\ \ln(b/a_2) \qquad (26)$$

Similarly, if $\beta d << 1$, $h_1 > h_2$, and $h_1 > h_2'$, W^a as given by (23) may be written as follows:

$$W^a = -j\frac{\zeta_0}{4\pi} I_1(z_1)\ \ln[(d + b/2)/(d - b/2)] \qquad (27)$$

The characteristic impedance of the two-wire line is

$$Z_c = \frac{\zeta_0}{\pi} \ln(b/a_2) = 1/Y_c = \sqrt{\ell^e/c} \qquad (28)$$

Then

$$I^a(z) = -j2Y_c[C^a \cos \beta z + D^a \sin \beta z + U^a + W^a(z)] \qquad (29)$$

Here U^a is the contribution to $I^a(z)$ from the incident field and $W^a(z)$ the contribution from the coupled rocket. For simplicity in writing (29) the subscripts on z have been omitted.

THE CURRENT IN THE ROCKET

A closely-spaced transmission line that extends a few meters in length near the skin of a rocket has only a relatively small localized effect on the current in that rocket. Accordingly, it is a satisfactory approximation to neglect the contributions to the vector potential on the surface of conductor 1 by the currents in conductors 2 and 3 in the process of determining the current in the rocket. Thus, (16) with (10) becomes:

$$\int_{-h_1}^{h_1} I_1(z')\frac{e^{-j\beta R_{11}}}{R_{11}} dz' \doteq -j\frac{4\pi}{\zeta_0}[C_1 \cos \beta z + U_1] \qquad (30)$$

(The coefficient D_1 vanishes because the current satisfies the symmetry condition $I_1(z) = I_1(-z)$ and has no discontinuity in slope at $z = 0$.) The approximate solution of the integral equation in (30) is [2]:

$$I_1(z) \doteq j \frac{4\pi}{\zeta_0} \frac{U_1(\cos \beta z - \cos \beta h_1)}{[\Psi_{dU} \cos \beta h_1 - \Psi_U(h_1)]}$$

$$= j \frac{4\pi}{\zeta_0} U_1 K_1 (\cos \beta z - \cos \beta h_1) \tag{31}$$

The functions Ψ_{dU} and $\Psi_U(h_1)$ are defined in the Appendix.

THE CURRENTS IN THE LOAD IMPEDANCES

For the antisymmetric current $I^a(z)$ ordinary transmission line equations apply so that the potential difference across the wires at any position z can be obtained directly from (29). It is given by

$$V^a(z) = - \frac{1}{j\omega c} \frac{\partial I^a(z)}{\partial z} = 2[-C^a \sin \beta z + D^a \cos \beta z + \frac{1}{\beta} \frac{\partial W^a(z)}{\partial z}] \tag{32}$$

where use has been made of the relation $\beta = \omega\sqrt{\ell^e c}$ for a dissipationless line.

The term $\partial W^a(z)/\partial z$ is easily evaluated from (27) and (31). It is

$$\frac{\partial W^a(z)}{\partial z} = -\beta U_1 K_1 \, \ell n[(d + b/2)/(d - b/2)] \sin \beta z \tag{33}$$

Since U^a is given and W^a and $\partial W^a/\partial z$ have been determined, $I^a(z)$ in (29) and $V^a(z)$ in (32) are known except for the constants C^a and D^a. These are easily evaluated in terms of the impedances Z_0 and Z_s at $z = -h_2'$ and $z = h_2$, respectively. Thus, with (29) and (32),

$$V^a(h_2) = 2[-C^a \sin \beta h_2 + D^a \cos \beta h_2 + \frac{1}{\beta} \left. \frac{\partial W^a(z)}{\partial z} \right|_{z=h_2}]$$

$$= Z_s I^a(h_2) = -j2Z_sY_c[C^a \cos \beta h_2 + D^a \sin \beta h_2 + U^a + W^a(h_2)] \quad (34)$$

Also,

$$V^a(-h_2') = 2[C^a \sin \beta h_2' + D^a \cos \beta h_2' + \frac{1}{\beta} \left. \frac{\partial W^a(z)}{\partial z} \right|_{z=-h_2'}]$$

$$= -Z_0 I^a(-h_2') = j2Z_0Y_c[C^a \cos \beta h_2' - D^a \sin \beta h_2' + U^a + W^a(-h_2')] \quad (35)$$

These equations may be rearranged as follows:

$$C^a[-\sin \beta h_2 + jZ_sY_c \cos \beta h_2] + D^a[\cos \beta h_2 + jZ_sY_c \sin \beta h_2] = G(h_2) \quad (36)$$

$$C^a[\sin \beta h_2' - jZ_0Y_c \cos \beta h_2'] + D^a[\cos \beta h_2' + jZ_0Y_c \sin \beta h_2'] = H(-h_2') \quad (37)$$

where

$$G(h_2) = -jZ_sY_c[U^a + W^a(h_2) - \frac{1}{\beta} \left. \frac{\partial W^a(z)}{\partial z} \right|_{z=h_2}] \quad (38)$$

$$H(-h_2') = jZ_0Y_c[U^a + W^a(-h_2') - \frac{1}{\beta} \left. \frac{\partial W^a(z)}{\partial z} \right|_{z=-h_2'}] \quad (39)$$

The simultaneous solution of (36) and (37) yields:

$$C^a = \frac{H(-h_2')[\cos \beta h_2 + jZ_sY_c \sin \beta h_2] - G(h_2)[\cos \beta h_2' + jZ_0Y_c \sin \beta h_2']}{D} \quad (40)$$

$$D^a = \frac{G(h_2)[\sin \beta h_2' - jZ_0Y_c \cos \beta h_2'] - H(-h_2')[-\sin \beta h_2 + jZ_sY_c \cos \beta h_2]}{D} \quad (41)$$

where

$$D = j[Y_c(Z_0 + Z_s) \cos \beta s + j(1 + Y_c^2 Z_0 Z_s) \sin \beta s] \tag{42}$$

The total length of the line is $s = h_2 + h_2'$. In (40) and (42),

$$G(h_2) = -jZ_sY_c\{U^a + U_1K_1 \ln[(2d + b)/(2d - b)](\cos \beta h_2 - \cos \beta h_1)\}$$
$$+ U_1K_1 \ln[(2d + b)/(2d - b)] \sin \beta h_2 \tag{43}$$

$$H(-h_2') = jZ_0Y_c\{U^a + U_1K_1 \ln[(2d + b)/(2d - b)](\cos \beta h_2' - \cos \beta h_1)\}$$
$$- U_1K_1 \ln[(2d + b)/(2d - b)] \sin \beta h_2' \tag{44}$$

Thus,

$$I^a(-h_2') = -j2Y_c[C^a \cos \beta h_2' - D^a \sin \beta h_2' + U^a + W^a(-h_2')]$$
$$= -j2Y_c\{C^a \cos \beta h_2' - D^a \sin \beta h_2' + U^a + U_1K_1 \ln[(2d + b)/(2d - b)]$$
$$\times (\cos \beta h_2' - \cos \beta h_1)\} \tag{45}$$

$$I^a(h_2) = -j2Y_c[C^a \cos \beta h_2 + D^a \sin \beta h_2 + U^a + W^a(h_2)]$$
$$= -j2Y_c\{C^a \cos \beta h_2 + D^a \sin \beta h_2 + U^a + U_1K_1 \ln[(2d + b)/(2d - b)]$$
$$\times (\cos \beta h_2 - \cos \beta h_1)\} \tag{46}$$

These are the final expressions for the load currents $I^a(-h_2')$ in Z_0 and $I^a(h_2)$ in Z_s. Y_c is defined by (28); C^a and D^a are given by (40) and (41); U^a and U_1 by (9) and (2), respectively, and K_1 by (31), i.e., $K_1 = 1/[\Psi_{dU} \cos \beta h_1 - \Psi_U(h_1)]$.

Note that in (45) and (46) the currents in the load impedances depend upon the length h_1 of the coupled rocket and on the relative axial location of the transmission line. The center of the rocket is at $z = 0$; the ends of the line are at $z = h_2$ and $z = -h_2'$. If the lower end of the line is above the center of the cylinder, h_2' must be replaced by $-h_2'$ in the formulas. On the other hand, if the line lies entirely in the interval $-h_1 \leq z \leq 0$, then $-h_2$ must be written for h_2 in the formulas.

Resonance in the cylinder with $\beta h_1 \doteq \pi/2$ increases $I_1(z)$, as given by (31), and hence $W^a(z)$ and $\partial W^a(z)/\partial z$. This significantly affects the currents in the transmission line and its terminating load impedances.

CONCLUSIONS

A unified treatment has been developed for the determination of the currents in the terminations of an open-wire transmission line that is electromagnetically coupled to a rocket. The relevant integral equations for all currents have been formulated and those related to the currents in the loads solved. In this manner full account is taken of the incident field and of the field scattered from the rocket as they affect the currents in the transmission line and its terminating loads. Since the formulation is based on the vector potentials A_z due to all axially directed currents and the associated charges, it necessarily includes the contributions from all components of the electromagnetic field that are derived from A_z. These are the radial and axial components of the electric field and the associated magnetic field maintained by the currents and charges in the rocket and the transmission line, and by the primary incident wave. Note that these distributions are not arbitrarily assumed but are derived from the boundary conditions; they include the effects of mutual coupling. The analysis as presented does not take account of the lateral displacement of the current in the rocket as the transmission line is brought nearer. This proximity effect is equivalent to a shift in the mean position of the current in the rocket from the central axis. This does not change the order of magnitude of the currents in the load impedances of the transmission line even when this is quite close to the surface of the rocket. A more detailed discussion of the significance of this effect and for its numerical evaluation is the subject of a separate note.

APPENDIX

The functions Ψ_{dU} and $\Psi_U(h_1)$ which appear in (31) are defined as follows:

$$\Psi_{dU} = (1 - \cos \beta h_1)^{-1} \int_{-h_1}^{h_1} (\cos \beta z' - \cos \beta h_1)[K(0,z') - K(h_1,z')]\, dz' \tag{47}$$

$$\Psi_U(h_1) = \int_{-h_1}^{h_1} (\cos \beta z' - \cos \beta h_1)\, K(h_1,z')\, dz' \tag{48}$$

with

$$K(z,z') = \frac{e^{-j\beta R}}{R} \tag{49}$$

$$R = [(z - z')^2 + a_1^2]^{1/2} \tag{50}$$

They are readily evaluated by computer.

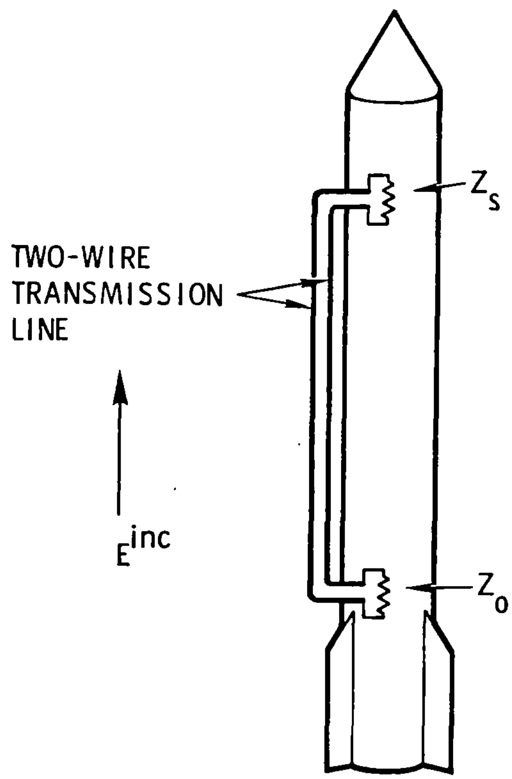

Fig. 1. Typical configuration of a transmission line electromagnetically coupled to a rocket

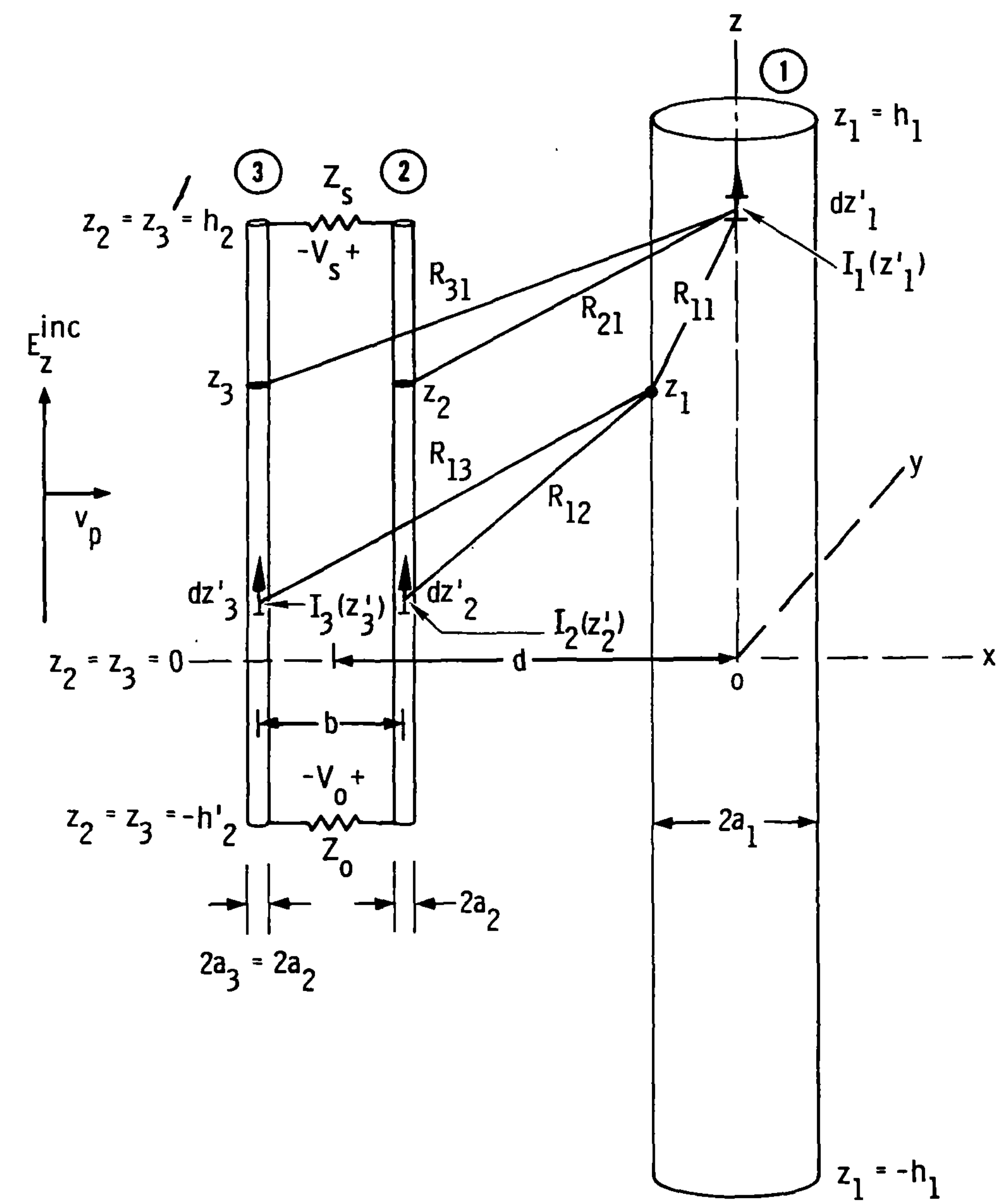

Fig. 2. Geometry of a coupled circuit approximately equivalent to Fig. 1

REFERENCES

[1] This is explained in detail in R. W. P. King, Transmission-Line Theory, pp. 60-61, Dover Publications, 1965.

[2] See, for example, R. W. P. King and C. W. Harrison, Jr., Antennas and Waves: A Modern Approach, Ch. 8, Sec. 3, M.I.T. Press, 1969.